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Bianchi Type-IX Bulk Viscous String Cosmological Model in *f*(*R*,*T*) Gravity with Special Form of Deceleration Parameter

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Abstract A Bianchi type-IX bulk viscous string cosmological model has been investigated in the frame work of $f(R,T)$ gravity proposed by Harko *et al.* (Phys. Rev. D 84:024020, 2011). The source for energy momentum tensor is bulk viscous fluid containing one dimensional cosmic strings. The field equations have been solved by applying a special form of deceleration parameter proposed by Singha and Debnath (IJTP 48:351, 2009). A barotropic equation of state for the pressure and energy density is assumed to get a determinate solution of the field equations. Also the bulk viscous pressure is assumed to be proportional to energy density. The physical and geometrical properties of the models are also discussed.

Keywords $f(R,T)$ Gravity, Cosmic Strings, Bulk Viscosity, Bianchi type-IX, Special form of Deceleration Parameter

1. Introduction

It is well known that Einstein's general theory of relativity has been successful in developing various cosmological models to study a structure formation and early stages of evolution of the universe. Einstein pointed out that general relativity does not account satisfactorily for inertial properties of matter i.e. Mach's principle is not substantiated by general relativity. Several attempts have been made to generalize the general theory of gravitation by incorporating Mach's principle and other desired features which were lacking in the original theory. Alternative theories of gravitation have been proposed to Einstein's theory to incorporate certain desirable features in the general theory. In the last decades, as an alternative to general relativity, scalar tensor theories and modified theories of gravitation have been proposed. Brans and Dicke [1], Saez and Ballester [2] have much importance amongst the scalar-tensor theories of gravitation. Recently $f(R)$ gravity and $f(R,T)$ gravity [3, 4] theories have much importance amongst the modified theories of gravity because these theories are supposed to provide natural gravitation alternatives to dark energy. From the cosmological observations, it is known that the energy composition of the universe has 76% dark energy, 20% dark matter and 4% Baryon matter. This is confirmed by the high red shift supernovae experiments [5, 6] and cosmic microwave background radiation observations [7, 8]. Coperland *et al.* [9] have given a comprehensive review of $f(R)$ gravity and Harko *et al.* [4] proposed $f(R,T)$ theory of gravity as an alternative amongst the modified theory of gravity of Einstein's theory of gravitation. Several authors have studied $f(R,T)$ theory of gravity in different contexts [10-15]. Recently Chirde and Kadam [16] have studied A new class of Bianchi type bulk viscous and string cosmological model in $f(R,T)$ theory of gravitation.

In $f(R,T)$ theory of gravity, the field equations are obtained from the Hilbert-Einstein type variation principle. The action for this modified theory of gravity is given by

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi G}f(R,T)+L_m\right)d^4x\,, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the stress-energy tensor of the matter $T_{\mu\nu}$. L_m is the matter Lagrangian density.

The stress-energy tensor of matter is defined as [17]

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{\mu\nu}}\,, \tag{2}$$

and its trace by $T=g^{\mu\nu}T_{\mu\nu}$ respectively. By assuming that the Lagrangian density L_m of matter depends only on the metric tensor components $g_{\mu\nu}$ and not on its derivatives, we obtain

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$$T_{\mu\nu} = g_{\mu\nu}L_m - 2\frac{\partial L_m}{\partial g^{\mu\nu}}.$$

In the present paper, we use the natural system of units with $G = c = 1$ so that the Einstein gravitational constant is defined as $k^2 = 8\pi$.

The corresponding field equations of the $f(R,T)$ gravity are found by varying the action with respect to the metric $g_{\mu\nu}$ [4]:

$$\{f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T)\}$$
$$= 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}, \tag{3}$$

where $f_R \equiv \frac{\delta f(R,T)}{\delta R}$, $f_T \equiv \frac{\delta f(R,T)}{\delta T}$, $\Box \equiv \nabla^\mu\nabla_\mu$,

∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

Contracting (3) gives the relation between the Ricci scalar R and the trace T of the stress-energy tensor,

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T)$$
$$= 8\pi T - f_T(R,T)T - f_T(R,T)\Theta, \tag{4}$$

where $\Theta = \Theta_\mu{}^\mu$.

Generally the field equations also depend [through the tensor $\Theta_{\mu\nu}$] on the physical nature of the matter field. Hence several theoretical models corresponding to different matter sources in $f(R,T)$ gravity can be obtained.

By considering some particular class of $f(R,T)$ modified gravity model obtained by explicitly specifying the functional form of f. Assuming that the function $f(R,T)$ given by

$$f(R,T) = R + 2T, \tag{5}$$

where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor of matter.

From equation (3), the gravitational field equations are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij}$$
$$= 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\Theta_{ij} - f(T)g_{ij}, \tag{6}$$

where the prime denotes a derivative with respect to the argument.

If the matter source is a perfect fluid,

$$\Theta = -2T_{\mu\nu} - pg_{\mu\nu}$$

then the above equation (6) reduces to

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij}$$
$$= 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}, \tag{7}$$

where the prime denotes derivative with respect to the argument.

The Deceleration parameters (q) are useful in studying expansion of the universe. We know that the universe has

(i) decelerating expansion if $q > 0$,

(ii) an expansion with constant rate if $q = 0$,

(iii) accelerating power law expansion if $-1 < q < 0$,

(iv) exponential expansion (de-Sitter expansion) if $q = -1$

(v) super-exponential expansion if $q < -1$.

Many relativists have studied DE cosmological models with different form of deceleration parameters. Singha and Debnath [18], Adhav *et al.* [19, 20] have obtained DE models with special form of deceleration parameter.

The study of Bianchi type cosmological models are important in achieving better understanding of anisotropy in the universe. Moreover the anisotropic universes have greater generality than FRW isotropic models. The simplicity of the field equations made Bianchi type space-times useful. Bianchi type I-IX cosmological models are homogeneous and anisotropic. Bianchi type-IX universe are studied by the number of cosmologists because of familiar solutions like Robertson–Walker Universe, the de-sitter universe, the Taub-Nut solutions etc. Chakraborty [21], Bali and Dave [22], Bali and Yadav [23], Pradhan *et al.* [24], Tyagi *et al.* [25], Ghate and Sontakke [26, 27] have obtained Bianchi type-IX cosmological models in different contexts.

Bulk viscosity plays a very important role in getting accelerated expansion of the universe. The matter behaves like viscous fluid in the early stages of the universe when neutrinos decoupling occurs. The effect of viscosity on the evolution of the universe had been studied by Misner [28, 29]. Several authors have studied bulk viscous cosmological models in general relativity [30-37] Johri and Sudharsan [38], Pimental [39], Banerjee and Beesham [40], Singh *et al.* [41], Rao *et al.* [42], Naidu *et al.* [43], Reddy *et al.* [44] have studied bulk viscous string cosmological models in scalar-tensor theories of gravitation. Recently Naidu *et al.* [45], Reddy *et al.* [46] have studied Bianchi type-V and Kaluza-Klein cosmological models with bulk viscosity and cosmic strings in $f(R,T)$ theory of gravitation. Motivated by the above discussion and investigations in $f(R,T)$ modified theory of gravity, in this paper we have investigated Bianchi type-IX bulk viscous string cosmological model in $f(R,T)$ theory of gravity with special form of deceleration parameter.

Bianchi type-IX space-time has considered when universe is filled with cosmic strings and bulk viscosity in $f(R,T)$ theory of gravity with special form of deceleration parameter. This work is organized as follows: In Section 2, the model and field equations have been presented. The field equations have been solved in Section 3 by applying special form of deceleration parameter. The physical and geometrical behavior of the model have been discussed in Section 4. In the last Section 5, concluding remarks have been expressed.

2. Metric and Field Equations

Bianchi type-IX metric is considered in the form,

$$ds^2 = \left\{\begin{array}{l} -dt^2 + a^2dx^2 + b^2dy^2 + \left(b^2\sin^2 y + a^2\cos^2 y\right)dz^2 \\ -2a^2\cos y dx dz \end{array}\right\}, \quad (8)$$

where a, b are scale factors and are functions of cosmic time t.

We consider the energy momentum tensor for a bulk viscous fluid containing one dimensional string as

$$T_{ij} = (\rho + p)u_i u_j - \overline{p}\, g_{ij} - \lambda_s x_i x_j, \quad (9)$$

and

$$\overline{p} = p - 3\xi H, \quad (10)$$

where ρ is the rest energy density of the system, $\xi(t)$ is the coefficient of bulk viscosity, $3\xi H$ is bulk viscous pressure, H is Hubble's parameter, x^i is the direction of the string and λ_s is the string tension density. Also $u^i = \delta_4^i$ is a four-velocity vector which satisfies $g_{ij}u^i u_j = -1,\ x^i x_j = 1,\ u^i x_i = 0$. Here we consider ρ, $\overline{p}$ and λ_s as function of time t only.

We assume the string to be lying along the x-axis. One-dimensional strings are assumed to be loaded with particles and the particle energy density is $\rho_p = \rho - \lambda_s$.

In co-moving coordinate system, we get

$$T_0^0 = \rho,\ T_1^1 = -\overline{p} - \lambda_s,$$
$$T_2^2 = T_3^3 = -\overline{p},\ T = \rho - 3\overline{p} - \lambda_s. \quad (11)$$

Now we choose the function $f(T)$ of the trace of the stress-energy tensor of the matter so that

$$f(T) = \mu T, \quad (12)$$

where μ is constant.

In the co-moving coordinate system, the field equations (7) for the metric (8) and with the help of (9)-(12) can be written as

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{a^2}{4b^4} = \rho(8\pi + 3\mu) - \mu\overline{p} - \mu\lambda_s, \quad (13)$$

$$2\frac{\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{3a^2}{4b^4} = \mu\rho - (8\pi + 3\mu)(\overline{p} + \lambda_s), \quad (14)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{4b^4} = \mu\rho - (8\pi + 3\mu)\overline{p} - \mu\lambda_s, \quad (15)$$

where the overdot $(\dot{\ })$ denotes the differentiation with respect to t.

3. Solutions of Field Equations

The field equations (13)–(15) are a system three independent equations in six unknowns $a, b, \overline{p}, \rho, \xi, \lambda_s$. Three additional conditions relating these unknowns may be used to obtain explicit solutions of the systems.

(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$a = b^m,\ (m \neq 1), \quad (16)$$

where m is proportionality constant.

The motive behind assuming condition is explained with reference to Thorne [47], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent [48, 49]. Collin *et al.* [50] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

(ii) Secondly for a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\overline{p} = p - 3\xi H = \varepsilon\rho, \quad (17)$$

where

$$\varepsilon = \varepsilon_0 - \gamma\ (0 \le \varepsilon_0 \le 1),\ p = \varepsilon_0\rho \text{ and } \rho = \eta\lambda_s.$$

Here $\varepsilon_0, \gamma, \eta$ are constants.

(iii) Lastly following Singha and Debnath [17], we use a special form of deceleration parameter as:

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 + \frac{k}{1 + R^k}, \quad (18)$$

where R is the average scale factor, $k > 0$ is constant.

Solving equation (18), the average scale factor R is given by

$$R = \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{1}{k}}, \quad (19)$$

where α_1 and α_2 are constants of integration.

For the metric (8), the scale factor R is given by

$$R = (ab^2)^{\frac{1}{3}}. \quad (20)$$

Solving field equations (13)-(15), with the help of equations (19) and (20), we obtain

$$a = \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{3m}{k(m+2)}}, \quad (21)$$

$$b = \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{3}{k(m+2)}}. \quad (22)$$

Using equations (21) and (22), the metric (8) takes the form

$$ds^2 = \left\{ \begin{array}{l} -dt^2 + \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6m}{k(m+2)}} dx^2 \\ + \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6}{k(m+2)}} dy^2 \\ + \left(\begin{array}{l} \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6}{k(m+2)}} \sin^2 y \\ + \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6m}{k(m+2)}} \cos^2 y \end{array} \right) dz^2 \\ -2\left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6m}{k(m+2)}} \cos y\, dx\, dz \end{array} \right\}. \quad (23)$$

Equation (23) represents Bianchi type-IX bulk viscous strings cosmological model in $f(R,T)$ gravity with special form of deceleration parameter.

4. Some Physical Properties of the Model

For the cosmological model (23), the physical quantities Spatial volume V, Hubble parameter H, Expansion scalar θ, Mean Anisotropy parameter A_m, Shear scalar σ^2, Energy density ρ, String tension density λ_s, Bulk viscosity pressure $\overline{p}$, Coefficient of bulk viscosity ξ and Particle energy density ρ_p are obtained as follows :

Spatial volume,

$$V = \left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{3}{k}}. \quad (24)$$

Hubble parameter,

$$H = \alpha_1 \alpha_2 \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-1}. \quad (25)$$

Expansion scalar,

$$\theta = 3H = 3\alpha_1 \alpha_2 \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-1}. \quad (26)$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant} \ (\neq 0 \ \text{for} \ m \neq 1). \quad (27)$$

Shear scalar,

$$\sigma^2 = \frac{3\alpha_1^2 \alpha_2^2 (m-1)^2}{(m+2)^2} \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-2}, \quad (28)$$

$$\text{where } \frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} \neq 0. \quad (29)$$

Energy density,

$$\rho = \left(\frac{1}{\left(8\pi + \mu\left(3 - \varepsilon - \frac{1}{\eta}\right)\right)} \right) \left[\frac{9(2m+1)\alpha_1^2 \alpha_2^2}{(m+2)^2} \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-2} + \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{\frac{-6}{k(m+2)}} - \frac{1}{4}\left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6(m-2)}{k(m+2)}} \right]. \quad (30)$$

String tension density,

$$\lambda_s = \frac{1}{\eta\left(8\pi + \mu\left(3 - \varepsilon - \frac{1}{\eta}\right)\right)} \left[\frac{9(2m+1)\alpha_1^2 \alpha_2^2}{(m+2)^2} \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-2} + \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{\frac{-6}{k(m+2)}} - \frac{1}{4}\left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6(m-2)}{k(m+2)}} \right]. \quad (31)$$

Particle energy density,

$$\rho_p = \frac{(\eta - 1)}{\eta\left(8\pi + \mu\left(3 - \varepsilon - \frac{1}{\eta}\right)\right)} \left[\frac{9(2m+1)\alpha_1^2 \alpha_2^2}{(m+2)^2} \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-2} + \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{\frac{-6}{k(m+2)}} - \frac{1}{4}\left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6(m-2)}{k(m+2)}} \right]. \quad (32)$$

Bulk viscous pressure,

$$\overline{p} = \frac{\varepsilon}{\left(8\pi + \mu\left(3 - \varepsilon - \frac{1}{\eta}\right)\right)}\left[\frac{9(2m+1)\alpha_1^2\alpha_2^2}{(m+2)^2}\left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-2} + \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{\frac{-6}{k(m+2)}} - \frac{1}{4}\left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6(m-2)}{k(m+2)}}\right]. \tag{33}$$

Coefficient of bulk viscosity,

$$\xi = \frac{(\varepsilon - \varepsilon_0)\left(\alpha_1 - e^{-\alpha_2 kt}\right)}{3\alpha_1\alpha_2\left(8\pi + \mu\left(3 - \varepsilon - \frac{1}{\eta}\right)\right)}\left[\begin{array}{l}\frac{9(2m+1)\alpha_1^2\alpha_2^2}{(m+2)^2}\left(\alpha_1 - e^{-\alpha_2 kt}\right)^{-2} \\ + \left(\alpha_1 - e^{-\alpha_2 kt}\right)^{\frac{-6}{k(m+2)}} - \frac{1}{4}\left(\alpha_1 e^{\alpha_2 kt} - 1\right)^{\frac{6(m-2)}{k(m+2)}}\end{array}\right]. \tag{34}$$

4.1. Physical Behavior of the Model

For the cosmological model (23), we observed that, the spatial scale factors has finite value at the initial epoch $t = 0$ and increases for large values of t. In particular for $\alpha_1 = 1$ and $t = 0$, the model has a big bang type singularity. It can also be observed that, at $t = 0$, the universe starts evolving with finite volume and expands infinitely with the increase in cosmic time t (Figure 1). The values of expansion θ and shear σ tend to infinity for large values of t $(t \to \infty)$ showing that the universe is expanding with increase of time (Figure 3, 4).

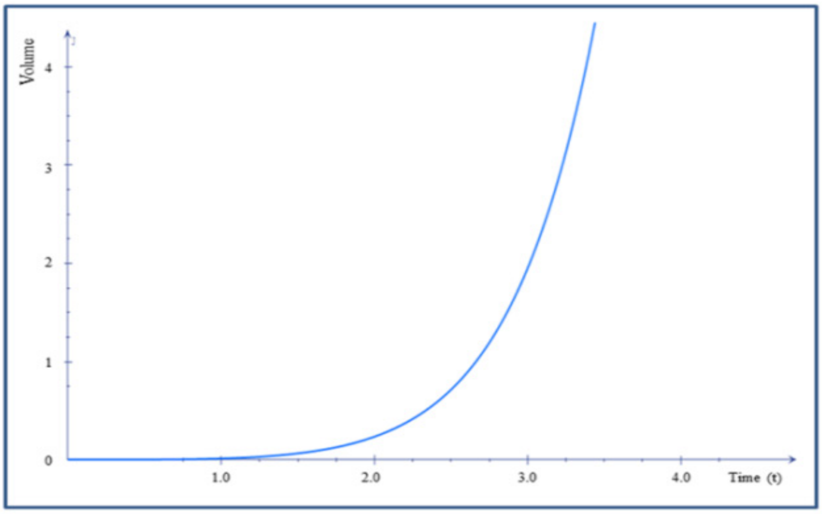

Figure 1. The Plot of Volume verses time

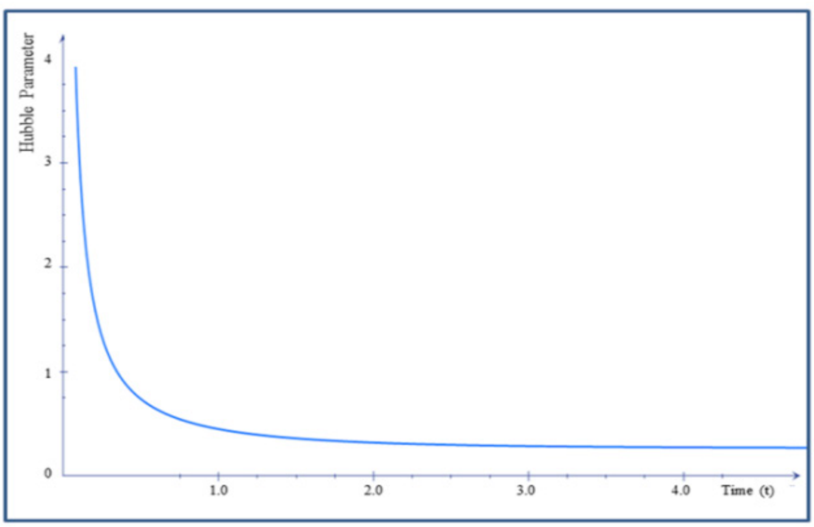

Figure 2. The Plot of Hubble parameter verses time

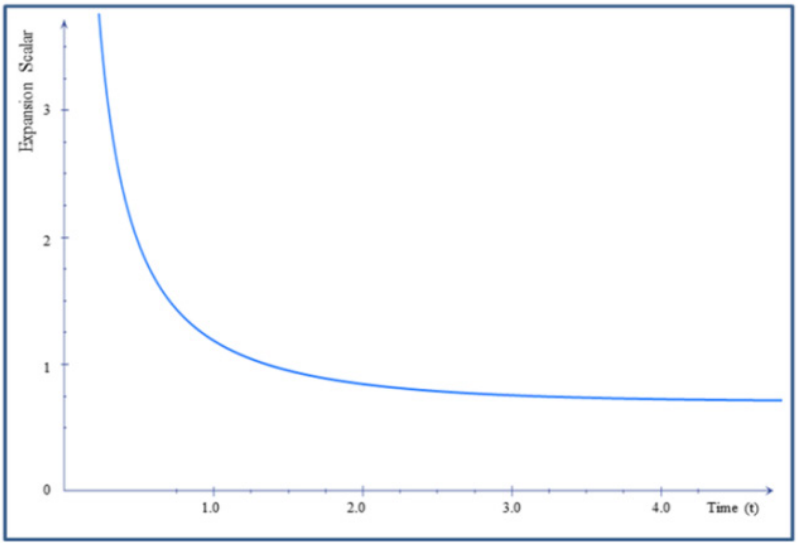

Figure 3. The Plot of Expansion scalar verses time

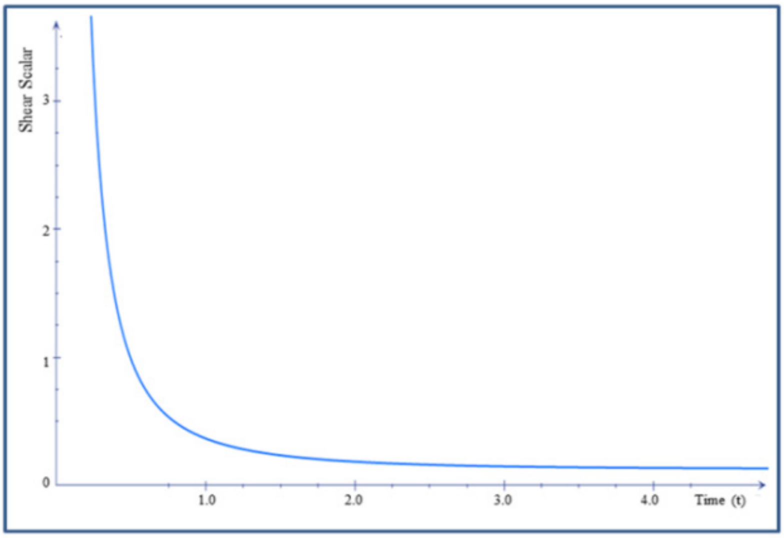

Figure 4. The Plot of Shear scalar verses time

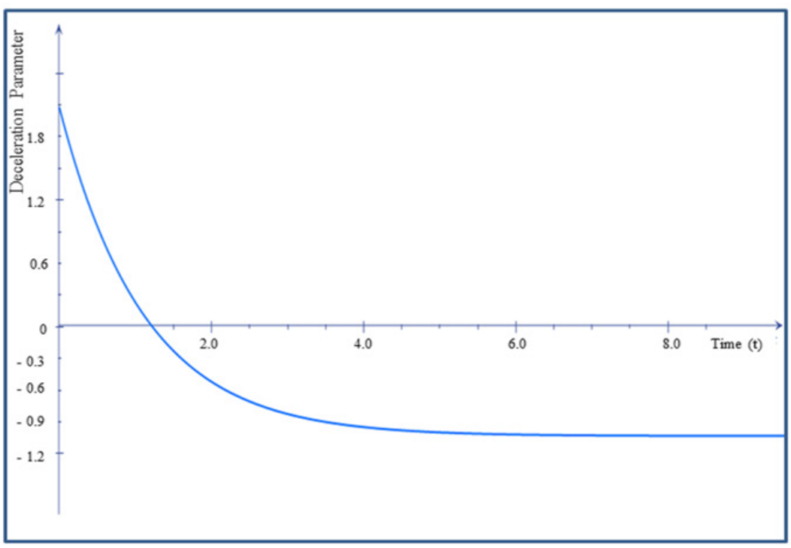

Figure 5. The Plot of deceleration parameter verses time

From equations (27) and (29), the mean anisotropy parameter A_m = constant ($\neq 0$ for $m \neq 1$) and $\frac{\sigma^2}{\theta^2}$ = constant ($\neq 0$ for $m \neq 1$) indicates that the model is anisotropic throughout the evolution of the universe except at $m = 1$ (*i.e.* the model does not approach isotropy).

In figure 5, the plot of deceleration parameter verses time is given from which we conclude that the model is decelerating at an initial phase and changes the expansion from decelerating to accelerating. Hence the model is consistent with the recent cosmological observations [5-6, 51-55].

Bulk viscosity in the model decreases with increase in time which is in accordance with the well-known fact that bulk viscosity decreases with time and leads to inflationary model [56].

For illustrative purposes, evolutionary behaviors of some cosmological parameters are shown graphically (Figure 1-5).

5. Conclusions

Bianchi type-IX bulk viscous strings cosmological model has been discussed in the frame work of $f(R,T)$ gravity proposed by Harko *et al.* (Phys. Rev. D 84:024020, 2011) when the source for energy momentum tensor is bulk viscous fluid containing one dimensional cosmic strings. The solution of the field equations has obtained by choosing the special form of deceleration parameter $q = -1 + \frac{k}{1+R^k}$. It is observed that in early phase of universe, the value of deceleration parameter is positive while as $t \to \infty$, the value of $q = -1$. Hence the universe had a decelerated expansion in the past and has accelerated expansion at present which is in good agreement with the recent observations of SN Ia. It is worth to mention that, the model obtained is point type singular, expanding, shearing, non-rotating and do not approach isotropy for large t. Further the models are anisotropic throughout the evolution. This shows that the present model not only represent accelerating universe but also confirms the well-known fact that the bulk viscosity will play a vital role in getting the accelerated expansion. We hope that our model will be useful in the discussion of structure formation in the early universe and an accelerating expansion of the universe at present.

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