

On the Wave Damping in Weakly Collisional Plasma

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Abstract The so-called Landau damping of electron waves in collisionless plasma does not exist due to the fact that when tending to zero the decrement $\text{Im } k$ in $\exp(-x \text{Im } k)$ characterizing the spatial wave damping, real part of divergent integral in the corresponding dispersion equation remains finite, making unnecessary addition to the complex dispersion equation the complex residue of the pole of the integral with logarithmical divergence in singularity point $v_x \sim \omega/\text{Re } k$ at $\text{Im } k = 0$, where k is complex wave number, ω is frequency, hence the collisionless Landau damping is resulting in violation of the law of energy conservation, causes problems with the rule of bypass around the pole in the calculation of the imaginary residue in the singularity point, leads to exponentially divergent solutions at $x \rightarrow \infty$ and other logical contradictions. The divergence of the imaginary part of dispersion relation at $\text{Im } k \rightarrow 0$ with the existence of solutions in the case $\text{Im } k = 0$ is already the reason for discarding solutions with $\text{Im } k \neq 0$. A relation is established between the averaged over oscillation period wave damping decrement and the collision term of kinetic equation. Collisionless Vlasov-Landau damping is explained finally by inadequate using mathematically correct results of transformations the nonlinear complex expressions.

Keywords Weakly collisional plasma waves, Landau damping, Collision damping, Collisionless damping

1. Introduction

Collisionless Landau damping is described in all textbooks on plasma physics as theoretically long ago predicted physical phenomenon of exponentially damping / growing electron waves in collisionless plasma [1], [2]. Theoretical deriving is based on the substitution of solutions in the form of complex running waves into the field equation and the kinetic equation with finding complex roots of the resulting complex dispersion equation, for example, for a given real frequency ω , where $k(\omega) \equiv k_1 + ik_2$ and k_1 is wave number, k_2 is decrement/increment of the electron wave. In this case the wave damping/growing is not consistent with the law of energy conservation and leads to a number of other contradictions [3].

Substitution into the original equations the real waves of the form

$$\exp(-k_2 x) [\cos(\omega t - k_1 x)] \text{ and } \exp(-k_2 x) [\sin(\omega t - k_1 x)]$$

leads to real dispersion equation containing two parameters k_1 and k_2 to be determined and (at given ω) the need to add the second equation which naturally is the equation of energy conservation with addition the energy-exchange collision term to the kinetic equation. It is shown that in the absence of such term the collisionless damping, which corresponds to the solution $k_2 \neq 0$, does not exist.

2. Analysis in Complex Variables

The well known solution for the electron velocity distribution function when considering electron waves in collisionless plasma is sought in the form of addition to the Maxwell electron distribution $f_0(v)$ the complex distribution function (with real ω)

$$f_1(v_x, x, t) \sim f_1(v_x) \exp[i(\omega t - kx)] \quad (1)$$

After its substitution to the collisionless kinetic equation

$$\frac{\partial f_1(v_x, x, t)}{\partial t} + v_x \frac{\partial f_1(v_x, x, t)}{\partial x} + \frac{e}{m_e} E(x, t) \frac{\partial f_0(v)}{\partial v_x} = 0 \quad (2)$$

we obtain

$$f_1(v_x, x, t) = -\frac{e}{m_e} \cdot \frac{(\partial f_0 / \partial v_x) E(x, t)}{i(\omega - kx)} \quad (3)$$

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where the complex electric field in Maxwell equation

$$\frac{\partial E(x, t)}{\partial x} = \frac{e}{m_e} \int f_1(v_x, x, t) dv_x \quad (4)$$

is assumed to be equal

$$E(x, t) = E_0 \cdot i \exp[i(\omega t - kx)], \quad (5)$$

and $k = \text{Re } k - i \text{Im } k \equiv k_1 - ik_2$. The complex dispersion equation $k = f(\omega)$ corresponding to these values contains logarithmically divergent integral of the form at $\text{Im } k = 0$

$$k \equiv \text{Re } k - i \text{Im } k = -\omega_0^2 \int \frac{(\partial f_0(v)/\partial v_x)}{\omega - kv_x} dv_x. \quad (6)$$

This leads to the relation between the actual observed real parameters of the real part of the dispersion equation ω , $\text{Re } k$, $\text{Im } k$, where ω_0 is Langmuir frequency $\omega_0^2 = 4\pi e^2/m_e$ and $\text{Im } k > 0$ characterizes the wave damping $\sim \exp(-x \text{Im } k)$

$$\text{Re } k = -\omega_0^2 \int \frac{(\partial f_0(v)/\partial v_x)(\omega - v_x \text{Re } k)}{(\omega - v_x \text{Re } k)^2 + (v_x \text{Im } k)^2} dv_x \quad (7)$$

with the real part of the electric field (5) in the plane $x = 0$

$$\text{Re } E(x, t) = -E_0 e^{-x \text{Im } k} \sin(\omega t - x \text{Re } k). \quad (8)$$

Virtual imaginary part of the dispersion equation

$$\text{Im } k = \omega_0^2 \int \frac{(\partial f_0/\partial v_x) v_x \text{Im } k}{(\omega - v_x \text{Re } k)^2 + (v_x \text{Im } k)^2} dv_x \quad (9)$$

leads when $\text{Im } k \rightarrow 0$ either to identity $0 = 0$ or to the divergent integral

$$\int \frac{v_x (\partial f_0/\partial v_x)}{(\omega - v_x \text{Re } k)^2} dv_x. \quad (10)$$

The latter result with the strange damping/growing solution $\pm k_2 \neq 0$ (which follows from the complex form of the Vlasov equations, “Vlasov damping” (9) at $k_2 \neq 0$, see also [3]) violating the energy conservation law, and the impossibility of the smooth transition $\text{Im } k \rightarrow 0$ indicates the inadequacy of using nonlinear complex expressions unrelated to the energy conservation law to describe the relationships between real physical quantities with a side effect of working with complex nonlinear equations.

But at the same time to each given value ω there corresponds the root of the complex dispersion equation with $\text{Re } k$, $\text{Im } k$, that is collisionless damping with

$\text{Im } k \equiv k_2 \neq 0$, while without having need to add a residuum/halfresiduum of any absent pole to the finite at $k_2 \neq 0$ integral (9).

3. Analysis in Real Variables

With direct substitution of initially real values $\text{Re } E(x, t)$ and $\text{Re } f_1(v_x, \text{Re } k, \text{Im } k, x, t)$ (in which $\text{Im } k \equiv k_2$ determines the wave damping $\sim \exp(-k_2 x)$) in the kinetic and Maxwell equations we obtain real dispersion equation in different form

$$1 = -\omega_0^2 \frac{ab}{k_1 a + k_2 b} \int \frac{(\partial f_0/\partial v_x)}{(\omega - k_1 v_x)b + k_2 v_x a} dv_x, \quad (11)$$

where we use the notation

$$\begin{aligned} \text{Re } k &\equiv k_1, \quad \text{Im } k \equiv k_2 > 0, \quad \cos(\omega t - k_1 x) \equiv a, \\ \sin(\omega t - k_1 x) &\equiv b. \end{aligned} \quad (12)$$

Since the dispersion equation contains two unknown variables k_1 , k_2 , to determine them one needs to add an independent energy conservation equation of collisional energy balance. However, the exact damping solution with independent of x , t values $k_1(x, t)$, $k_2(x, t)$ even neglecting derivatives $\partial k_1/\partial x$, $\partial k_1/\partial t$, $\partial k_2/\partial x$, $\partial k_2/\partial t$ of the next order of smallness (e.g., even by using the some average values $k_1(x, t)$ and $k_2(x, t)$) does not exist in this somewhat artificial initial formulation of the problem for any $k_2 \neq 0$ due to integrand terms with a and b .

This dispersion equation is analogous to equation (7) only if $k_2 \equiv \text{Im } k = 0$. Its distinction from the equation (7) can be understood by comparing with the elementary illustrative example of nonlinear inequality for arbitrary real values c , d , g , h :

$$cd \neq \text{Re}[(c + ig)(d + ih)]. \quad (13)$$

The right hand side of (13) contains mixed term gh and mixed imaginary terms whose appearance has nothing to do with physical nature of all values and their mutual relations (for example, in the case of the law of energy conservation). However, Eq. (11) is similar in form to the above expression (7). That is, even at cutting off the distribution function $f_1(v_x)$ in accordance with the condition of positivity of the total distribution function

$$|f_1(v_x, x, t)| < f_0(v) \quad (14)$$

and at $k_2 v_x a \rightarrow 0$, we obtain dispersion integral in the principal value sense with non-damping wave.

As a first step of the process of finding damping solution one can use some non zero value k_2 , but the collision term in kinetic equation of the type

$$-A f_1(v_x) \exp(-k_2 x) \cos(\omega t - k_1 x), \quad (15)$$

should be added then to the right hand side of kinetic equation with which the dispersion equation becomes

$$1 = \frac{-ab}{ak_1 + bk_2} \cdot \omega_0^2 \int \frac{\partial f_0 / \partial v_x}{(\omega - k_1 v_x)b + ak_2 v_x - A} dv_x. \quad (16)$$

In the process of tending $A \rightarrow ak_2 v_x$ the dispersion integral is finite, which corresponds to its calculation in the principal value sense in analogy with the expression (7).

Apparent internal contradiction is that in a hypothetical Landau damping with $k_2 \neq 0$, even without cutting off $f_1(v_x)$, the dispersion integral is finite, and in the absence of poles of the integrand (11) any need to add imaginary residua / halfresidua to dispersion integral that leads to Landau damping is completely absent.

In this simplified initial formulation of the problem (11) the dependences of k_1 and k_2 on x, t can not be calculated (the problem is insoluble), but apparently possibility exists of their calculation in the approach of averages values. The main result of the made analysis is proof that the so-called Landau damping does not exist as a real natural phenomenon.

It is possible, however, to obtain the averaged over oscillations value k_2 , at least approximately for $|k_2| \lambda \lesssim 1$, where λ is wave length $\lambda = 2\pi/k_1$, by setting $A = ak_2 v_x$ with

$$1 = \frac{-a}{ak_1 + bk_2} \cdot \omega_0^2 \int \frac{\partial f_0 / \partial v_x}{(\omega - k_1 v_x)} dv_x;$$

$$f_1(v_x) = \frac{e}{m_e} \frac{\partial f_0 / \partial v_x}{(\omega - k_1 v_x)} E_0, \quad (17)$$

where expression $a/(ak_1 + bk_2)$ is replaced by the value averaged over the oscillation period

$$\left(\frac{a}{ak_1 + bk_2} \right)_{av} = \frac{1}{2\pi} \int_0^{2\pi} \frac{k_1 - k_2 (\tan y)}{k_1^2 + (k_2 \tan y)^2} dy = \frac{1}{k_1 + k_2}. \quad (18)$$

Average value k_2 should be determined independently by the integral law of energy conservation, respectively, it could be large enough in (7), (9) to satisfy condition (14), but it can be small, and then one should implement cutting off

function $f_1(v_x)$ according to condition (14) which is necessary, unlike (7), in any case, according to the expression (17).

Parameter k_2 is crucial factor in the proof of non-existence of collisionless damping (contrary to the collisionless Landau damping) at the apparent absence of collisional terms in kinetic equation and determines very small collisional damping of the wave and of the electric field in plasma at $k_2 \rightarrow 0$ when taking into account the real-life small collision additives to the kinetic equation in the form determined by the balance energy parameter $k_2 \neq 0$.

Integral (17) is calculated in the principal value sense that corresponds to the form of the integral (7) obtained using complex variables.

It is inconceivable that there would not be arbitrarily small wave damping in the presence of arbitrarily small but non-zero terms of collisional kinetic equation.

Thus, it is clear that we obtain by (16) or (17) at $\text{Im } k \rightarrow 0$ not-damping collisionless Vlasov solution [1] with the integral in the principal value sense, hence proposed by Landau [2] eight years later adding to the complex Vlasov dispersion equation the imaginary residue at the pole $v_x = \omega/\text{Re } k$ leading to the so-called Landau damping is erroneous. The real problem is to find the roots of a complex dispersion equation without any artificial additives.

Due to the smallness of the range $|\Delta v_x|$ near the singularity point $v_x \sim \omega/\text{Re } k$ and smallness $\partial f_0 / \partial v_x$ at large v_x , corrections due to the missing collision term in the kinetic equation influence usually very weakly on the solution of the dispersion equations (16), (17), however obtaining solution for k_2 requires to attract additional new equation (since the dispersion equation contains at least two parameters k_1 and k_2 to be determined) which takes into account the energy loss in collisions of electrons with each other and with atoms and molecules present in plasma.

In computing, for example, the modulated flux of electrons entrained by non-damping Van Kampen waves at $\text{Im } k = 0$ it is necessary to carry out a smooth cutoff of the distribution function $|f_1(v_x, x, t)|$ near the critical point $v_x \sim \omega/\text{Re } k$ in accordance with the condition (14).

Integral energy balance can be written, for example, as relation of the type

$$\int f_1(v_x) k_2 v_x \Delta \varepsilon_1(v_x) dv_x =$$

$$\int f_1(v_x) [f_0 n_e \sigma_1(v_x) v_x \Delta \varepsilon_2(v_x) + n_m \sigma_2(v_x) v_x \Delta \varepsilon_3(v_x)] dv_x, \quad (19)$$

where $\Delta \varepsilon_1(v_x)$ is energy loss per electron associated with

a decrease in the number of electrons with an additional wave propagation velocity v_x ; $\Delta\mathcal{E}_2(v_x)$, $\Delta\mathcal{E}_3(v_x)$ are portions of transmitted energy; $\sigma_n(v_x)$ are collision cross sections; n_e is electron density; n_m is density of atoms and molecules. In this

$$\Delta\mathcal{E}_1 \sim \frac{m_e}{2} \left[(v_{xM} + v_x)^2 - v_{xM}^2 \right], \quad (20)$$

where v_{xM} is proper velocity of electron along the axis x in Maxwell distribution $f_0(v)$ independent on the wave speed. In expression (19) one can add also any other energy-exchange processes.

It is assumed that at extreme complexity of calculations using original kinetic equation, the proposed approximations with averaging k_1 , k_2 and calculation averaged $k_2 = \text{const}$ based on the energy balance represent the main features of the damping plasma waves.

A more detailed exposition is presented in [3], where are marked some obvious logical contradictions that arise when using Landau rules: violation of the energy conservation law in the damping waves; the appearance of exponentially divergent solution at $|x| \rightarrow \infty$; uncertainties in calculating the residue or the half residue near/on the real axis of the wave velocity v_x ; jump transition between the non-zero values of the damping parameter $\pm \text{Im } k \rightarrow \mp \text{Im } k$ when choosing the bypass direction around the pole (cf. inspiring causality principle in [4]) or at infinitesimal displacement of the pole across the real axis v_x ; contradictions of the calculating residue or half residue at the pole.

4. Conclusions

There is considered general ideology of solving the problem of wave propagation in a collisionless plasma with the phenomenological account of collisional damping parameter (decrement) $\text{Im } k \neq 0$ determined by collision term in the kinetic equation, with the substantiation of impossibility of collisionless Landau damping, which leads to a number of obvious logical contradictions [3]. Thereby:

1. Using technology of complex variables to solve the equations for waves $\sim \exp[i(\omega t - kx)]$ with complex $k \equiv k_1 - ik_2$, $k_2 \neq 0$ leads to results that are not equivalent to the solution of equations with real physical variables and waves

$$\sim \exp(-k_2 x) \times [\cos(\omega t - k_1 x) \text{ or } \sin(\omega t - k_1 x)]. \quad (21)$$

Nonlinear complex expressions lead to errors in the transition to the real relations of physically observable real

quantities with violation energy conservation law.

2. Smoothed transition to the singularity point $v_x \rightarrow \omega/k_1$ in which the real integrand of the real dispersion integral remains finite for any values v_x arbitrarily close to ω/k_1 leads to the dispersion integral in the principal value sense without the need to supplement it with imaginary residue at the singularity point.

3. Moreover, the requirement of positivity of the total distribution function (14) leads to the necessity of its cutting off near the singularity point with resulting the finite integrand and the finite dispersion integral.

4. When adding collision terms into the kinetic equation, surprisingly simple and natural averaging of the real dispersion equation over the period of oscillations leads to the dispersion equation for the two unknown parameters $(k_1, k_2) = \text{const}$.

5. The second equation (19) to determine the values (k_1, k_2) is averaged over the period $2\pi/\omega$ energy balance equation, and the exponential damping of the wave at arbitrarily infinitesimal collision term can lead to arbitrarily infinitesimal value k_2 .

6. In the case of the collisionless plasma such averaging in (11) is not possible, and the solution of the dispersion equation (11) with $k_2 \neq 0$ and exponentially damping wave with $(k_1, k_2 \neq 0)$ does not exist.

Thus, the main results are: (1) nonphysical negative electron distribution functions must be cut off, what leads to elimination of singular points in the dispersion equation; (2) the use of complex expressions for the wave functions with solving nonlinear complex dispersion equation leads to erroneous relations between physically observable real wave parameters and the emergence of collisionless damping; (3) solving the real wave dispersion equation with constant parameters k_1 and $k_2 \neq 0$ (at constant real ω) is possible only when averaged over the period oscillation values k_1, k_2 . At the same time their determination requires the addition of the second equation with k_1, k_2 , which may be an energy balance equation when adding collisional terms in kinetic equation.

Carried out in this paper relatively very simple analysis leads to the conclusion that an analytical description of the damping/growing electron waves in plasma is impossible without adding appropriate collision (energy-exchange) terms in the collisionless kinetic equation.

The principal result of this paper is demonstration of the serious consequences of commonly used in physics of wave processes in weak collisional plasma the techniques in the form of mathematically correct transformations of nonlinear complex expressions that leads to completely erroneous conclusions about the true relations of real physical observables. In particular, values k_1, k_2 in Eq. (11) depend

on x, t unlike equations (7), (9).

These results are correction of the author's comment pp. 515 - 516 to the Russian translation of [5].

Trivial explanation of many-years ambiguities of Vlasov-Landau wave damping in weakly collisional plasma by uncritical using results of transformations the complex nonlinear equations is shocking.

There is the open question: whether now Landau damping is considered and can be considered as an outstanding discovery of plasma physics?

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