

# Longitudinal Traction of a Fiber-Reinforced Composite Beam with Debonded Fibers

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**Abstract** The present paper is devoted to the study of longitudinal traction problem of a composite beam with debonded fibers but in contact with the matrix. We show, using the behavior results of "debonded" composites obtained in [Berrehili, Y. and J.-J. Marigo, 2013, The homogenized behavior of unidirectional fiber-reinforced composite materials in the case of debonded fibers, Mathematics and Mechanics of Complex Systems sous presse.] and some intuitive remarks on the symmetry of the problem, that when the period of the microstructure is small enough, the displacement field solution is approximated, in the first order, by a explicit displacement field depending on the fineness of the microstructure. Moreover, unlike that found in the literature, this field depends not only on classical homogenized tensor but also new homogenized tensors characterizing the "debonded" composite.

**Keywords** Homogenization, Composite beam, Debonded fibers, Modeling, Behavior, Traction problem

## 1. Introduction

The study and the direct calculation, by finite element, of the behavior of composite structures, are generally difficult mainly due to the large number of heterogeneity which requires a very fine discretization of the different constituents (fibers and matrix). To overcome these difficulties, multi-scale methods (the homogenization in particular) were introduced to approach the real problem by a limit problem when the parameter characterizing the fineness of the microstructure tends to zero [1]. If the results are well established on the effective behavior of composite materials with perfectly bonded fibers [2-12], few studies have been made in the case where the constituents are debonded [13-19]. The objective of this work is to illustrate the effective behavior of a composite beam with debonded fibers (but in contact with the matrix) on a classical problem of traction. Although simple, it is of paramount importance for practical applications. For solving this problem, we will try, with intuitive remarks, to guess the analytical form of a part of solution which will serve as a starting assumption. We then calculate the full solution using the equilibrium equations and the boundary conditions, i.e. to determine all constants uniquely. The uniqueness of these constants ensures, thanks to the Lax-Milgram theorem, the uniqueness of the sought solution of the problem considered.

Specifically, the paper is organized as follows. The next section is devoted to the setting of the problem which consists to study the longitudinal traction of a fiber-reinforced composite beam with debonded fibers but in contact with the matrix. The third section is devoted to a brief general recall of homogenization results obtained in [20] and adaptation of these results to our problem. One formulates thus the homogenized problem associated with our problem of traction which consists to solve an equilibrium equations system, coupled and not classic. One solves therefore this system in the fourth section. And one ends with a conclusion in the fifth section.

## 2. Problem Statement

We consider a cylindrical fibrous composite beam with circular cross section which occupies, in its natural reference configuration, the solid cylinder  $\Omega = \mathbf{D}_R ]0, L[$  of  $\mathbb{R}^3$  where  $\mathbf{D}_R$  is the disk with center  $\mathbf{O}$  and radius  $R$ . We denote by  $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$  the canonical basis of  $\mathbb{R}^3$  and  $(x_1, x_2, x_3)$  the coordinates of a point  $\mathbf{x}$  of  $\Omega$ . The beam is assumed of axis  $\mathbf{Ox}_3$  and with two circular straight sections  $\Sigma_0$  and  $\Sigma_L$  located respectively in the planes  $x_3=0$  and  $x_3=L$ . The two sections  $\Sigma_L$  and  $\Sigma_0$  are submitted respectively to a surface force densities  $\mathbf{F} = \mathbf{F}_e$  and  $-\mathbf{F}$ ,  $\mathbf{F}$  being a constant vector directed along  $\mathbf{Ox}_3$  (see Figure 1.). The lateral surface force applied on  $\Sigma_{\text{Lat}}$  is supposed null and the density of the volume forces applied on  $\Omega$  is assumed negligible.

The fibers of the beam are supposed all debonded from the matrix. We assume further that, during the deformation, the fibers remain in contact with the matrix and can sliding

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without friction. This express that the normal displacement field is continuous and the shear is null on the interfaces fibers-matrix  $\mathbf{I}^\varepsilon$  ( $\varepsilon$  is the period of the microstructure destined to tend to zero [22, 23]).

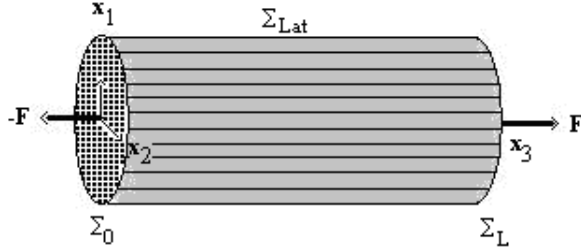


Figure 1. The cylindrical composite beam

Working in the framework of linear elasticity, the problem consists in seeking for the displacement and the associated stress fields, denoted respectively by  $\mathbf{u}^\varepsilon$  and  $\boldsymbol{\sigma}^\varepsilon$  so that the dependence in  $\varepsilon$  be explicit, solutions to the following traction problem:

$$\text{Div } \boldsymbol{\sigma}^\varepsilon = \mathbf{0} \text{ in } \Omega \mathbf{I}^\varepsilon \quad (1)$$

$$\boldsymbol{\sigma}^\varepsilon = \mathbf{A}^\varepsilon \boldsymbol{\varepsilon}(\mathbf{u}^\varepsilon) \text{ in } \Omega \mathbf{I}^\varepsilon \quad (2)$$

$$\boldsymbol{\varepsilon}(\mathbf{u}^\varepsilon) = \frac{1}{2} (\nabla \mathbf{u}^\varepsilon + \nabla^T \mathbf{u}^\varepsilon) \text{ in } \Omega \mathbf{I}^\varepsilon \quad (3)$$

$$\boldsymbol{\sigma}^\varepsilon \mathbf{n} = -\mathbf{F} \text{ on } \Sigma_0, \quad \boldsymbol{\sigma}^\varepsilon \mathbf{n} = \mathbf{F} \text{ on } \Sigma_L, \quad \boldsymbol{\sigma}^\varepsilon \mathbf{n} = \mathbf{0} \text{ on } \Sigma_{\text{Lat}} \quad (4)$$

$$[\mathbf{u}^\varepsilon] \cdot \mathbf{n} = 0, \quad [\boldsymbol{\sigma}^\varepsilon] \mathbf{n} = \mathbf{0}, \quad \boldsymbol{\sigma}^\varepsilon \mathbf{n} \wedge \mathbf{n} = \mathbf{0} \text{ on } \mathbf{I}^\varepsilon \quad (5)$$

where  $\mathbf{A}^\varepsilon$  denote the elasticity tensor of the composite beam,  $\boldsymbol{\varepsilon}(\mathbf{u}^\varepsilon)$  the strain field associated with the displacement field  $\mathbf{u}^\varepsilon$ ,  $\mathbf{n}$  the outer normal of  $\Sigma_0$  ( $\mathbf{n} = -\mathbf{e}_3$ ),  $\Sigma_L$  ( $\mathbf{n} = \mathbf{e}_3$ ) or  $\Sigma_{\text{Lat}}$  ( $\mathbf{n} = n_1 \mathbf{e}_1 + n_2 \mathbf{e}_2$  with  $n_1^2 + n_2^2 = 1$ ) and  $[\mathbf{u}^\varepsilon]$  the jump of the displacement field  $\mathbf{u}^\varepsilon$  across the interface  $\mathbf{I}^\varepsilon$ . The three relations of the last line reflect the continuity of normal displacement field, the continuity of the stress vector and the nullity of the shear on the debonded interfaces  $\mathbf{I}^\varepsilon$  (see [10, 13, 17] for other interface conditions).

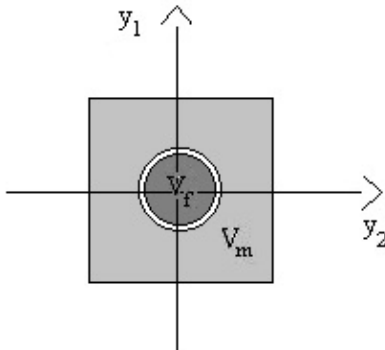


Figure 2. The unit periodic cell  $\mathbf{V} = \mathbf{V}_f \cup \mathbf{V}_m$

It is assumed that the two materials, fibers and matrix, constituting the composite structure, are elastics homogeneous and isotropic. Their Lamé coefficients are respectively denoted by  $(\lambda_f, \mu_f)$  and  $(\lambda_m, \mu_m)$ . The coefficients of the elasticity tensor of the composite structure,  $\mathbf{A}^\varepsilon_{ijkl}$ ,  $1 \leq i, j, k, l \leq 3$ , are given therefore by [2]:

$$\mathbf{A}^\varepsilon(\mathbf{x})_{ijkl} = \lambda \left( \frac{\mathbf{x}}{\varepsilon} \right) \delta_{ij} \delta_{kl} + \mu \left( \frac{\mathbf{x}}{\varepsilon} \right) (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad (6)$$

where we conventionally denote by  $\delta_{ij}$  the Kronecker symbol equal to 1 if  $i=j$  and 0 if  $i \neq j$ .  $\lambda$  and  $\mu$  are the Lamé coefficients, defined in the periodic cell  $\mathbf{V} = \mathbf{V}_f \cup \mathbf{V}_m$  (reunion of the fiber part  $\mathbf{V}_f$  and the matrix part  $\mathbf{V}_m$ ) by:

$$(\lambda(\mathbf{y}), \mu(\mathbf{y})) = \begin{cases} (\lambda_f, \mu_f) & \text{if } \mathbf{y} \in \mathbf{V}_f \\ (\lambda_m, \mu_m) & \text{if } \mathbf{y} \in \mathbf{V}_m \end{cases} \quad (7)$$

### 3. The Macroscopic Problem

Following the classical two-scale procedure in homogenization theory of periodic medium [1, 20, 21, 24], we assume that  $\mathbf{u}^\varepsilon$  can be expanded as follows:

$$\mathbf{u}^\varepsilon(\mathbf{x}) = \mathbf{u}^0(\mathbf{x}, y_1, y_2) + \varepsilon \mathbf{u}^1(\mathbf{x}, y_1, y_2) + \varepsilon^2 \mathbf{u}^2(\mathbf{x}, y_1, y_2) \dots \quad (8)$$

where  $\mathbf{x}$  is the macroscopic variable ( $\mathbf{x} \in \Omega$ ) and,  $y_1 = x_1/\varepsilon$  and  $y_2 = x_2/\varepsilon$  are the coordinates of the microscopic variable  $\mathbf{y}$  describing the unit cell  $\mathbf{V}\Gamma$ , with  $\mathbf{V} = ]-1/2, +1/2[ \times ]-1/2, +1/2[$  [and  $\Gamma = \{(y_1, y_2) \in \mathbf{V} / y_1^2 + y_2^2 = \alpha^2\}$  where  $0 < \alpha < 1/2$  ( $\mathbf{V}_f = \pi \alpha^2$  denotes the volume fraction of fibers into the matrix). And the  $\mathbf{u}^i = (u^i_1, u^i_2, u^i_3)$ ,  $i \geq 0$ , are the  $\mathbf{V}\Gamma$ -periodic fields with respect to the variable microscopic  $\mathbf{y}$ . We show in [20] that the result obtained differs in general from the usual property of the homogenization theory. Indeed, because of debonding of the fibers from the matrix, the leading term  $\mathbf{u}^0$  of the asymptotic displacement field expansion depends on the microscopic coordinates  $\mathbf{y}$ . Moreover a new macroscopic field enters in the effective kinematics of the composite structure. Specifically, we obtain a classical vector field representing the macroscopic displacement of the matrix  $\mathbf{u}$  and additional macroscopic scalar fields  $\delta$  and  $\omega$  interpreted respectively as the sliding and the rotation of the fibers with respect to the matrix. And the homogenized problem obtained is an equilibrium equations system, coupled and not classical. It contains in addition of homogenized stiffness tensor  $\mathbf{A}^{\text{hom}}$  (of the debonded composite structure) new homogenized tensors  $\mathbf{K}$ ,  $\mathbf{T}$  and  $\boldsymbol{\Sigma}$ .  $\mathbf{K}$  and  $\mathbf{T}$  are interpreted respectively as the effective rigidity tensors to the extension and to the torsion (of the debonded fibers) and  $\boldsymbol{\Sigma}$  as an effective stress tensor. These three tensors are obtained by solving new cell problems ignored in the existent literature [20]. It appears in the equilibrium equations of the effective problem in coupling the three macroscopic fields  $\mathbf{u}$ ,  $\delta$  and  $\omega$ .

The leading term  $\mathbf{u}^0(\mathbf{x}, \mathbf{y})$  of the asymptotic displacement field expansion  $\mathbf{u}^\varepsilon(\mathbf{x})$  can be written as in [20]:

$$\mathbf{u}^0(\mathbf{x}, \mathbf{y}) = \mathbf{u}(\mathbf{x}) + \chi_f(\mathbf{y})(\delta(\mathbf{x})\mathbf{e}_3 + \omega(\mathbf{x})\mathbf{e}_3 \wedge \mathbf{y}) \quad (9)$$

where  $\chi_f(\mathbf{y})$ , with  $\mathbf{y} \in \mathbf{V}$ , is the characteristic function of  $\mathbf{V}_f$  (equal to 1 on  $\mathbf{V}_f$  and 0 on  $\mathbf{V}_m$ ).

But the forces applied on the beam do not work in rotation (since it is a problem of pure traction), we will obtain therefore simply  $\omega = 0$  and  $\mathbf{T} = \mathbf{0}$  [21, 24].  $\mathbf{u}^0$  is then written:

$$\mathbf{u}^0(\mathbf{x}, \mathbf{y}) = \mathbf{u}(\mathbf{x}) + \chi_f(\mathbf{y})\delta(\mathbf{x})\mathbf{e}_3, \quad (10)$$

where the couple of macroscopic fields  $(\mathbf{u}, \delta)$  is a solution of the following homogenized problem (coupled not classic) posed on the same domain  $\Omega$  but which we have replaced (see Figure 3.) the fibred composite material by an equivalent homogeneous material characterized by the tensors  $\mathbf{A}^{\text{hom}}$ ,  $\mathbf{K}$  and  $\Sigma$  [20, 25]:

$$\text{Div} \left( \boldsymbol{\sigma} + \frac{\partial \delta}{\partial \mathbf{x}_3} \Sigma \right) = \mathbf{0} \quad \text{in } \Omega \quad (11)$$

$$\frac{\partial}{\partial \mathbf{x}_3} (\Sigma \cdot \boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K} \frac{\partial \delta}{\partial \mathbf{x}_3}) = 0 \quad \text{in } \Omega \quad (12)$$

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{A}^{\text{hom}} \boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x}) \quad \text{in } \Omega \quad (13)$$

$$\boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x}) = \frac{1}{2} (\nabla \mathbf{u} + \nabla^T \mathbf{u})(\mathbf{x}) \quad \text{in } \Omega \quad (14)$$

$$\left( \boldsymbol{\sigma} + \frac{\partial \delta}{\partial \mathbf{x}_3} \Sigma \right) \mathbf{n}(\mathbf{x}) = \mathbf{F} \quad \text{on } \Sigma_0 \quad (15)$$

$$\left( \boldsymbol{\sigma} + \frac{\partial \delta}{\partial \mathbf{x}_3} \Sigma \right) \mathbf{n}(\mathbf{x}) = \mathbf{F} \quad \text{on } \Sigma_L \quad (16)$$

$$\left( \boldsymbol{\sigma} + \frac{\partial \delta}{\partial \mathbf{x}_3} \Sigma \right) \mathbf{n}(\mathbf{x}) = 0 \quad \text{on } \Sigma_{\text{Lat}} \quad (17)$$

$$(\Sigma \cdot \boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K} \frac{\partial \delta}{\partial \mathbf{x}_3})(\mathbf{x}) \mathbf{n}_3(\mathbf{x}) = V_f F \quad \text{on } \Sigma_0 \quad (18)$$

$$(\Sigma \cdot \boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K} \frac{\partial \delta}{\partial \mathbf{x}_3})(\mathbf{x}) \mathbf{n}_3(\mathbf{x}) = V_f F \quad \text{on } \Sigma_L \quad (19)$$

$$(\Sigma \cdot \boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K} \frac{\partial \delta}{\partial \mathbf{x}_3})(\mathbf{x}) \mathbf{n}_3(\mathbf{x}) = 0 \quad \text{on } \Sigma_{\text{Lat}} \quad (20)$$

The first equation is a three-dimensional equilibrium. The second one is a family of scalar equations, identified by the indices  $(x_1, x_2)$ . It is the equations of beam type ( $\mathbf{K} \frac{\partial \delta}{\partial \mathbf{x}_3}$  represents the normal effort). We see that, in the first equation, the term  $\frac{\partial \delta}{\partial \mathbf{x}_3} \Sigma$  plays the role of a pre-stressed field whereas in the second one, the term  $\Sigma \boldsymbol{\varepsilon}(\mathbf{u})$  plays the role of a pre-tensioning field of the beam composite. This system of equations is completed by the boundary conditions (15)-(20).

## 4. Resolution

It should be noted that the problem considered is symmetrical: One has geometrical and loading symmetries with respect to the axis  $\mathbf{Ox}_3$  and with respect to the plan  $x_3 = L/2$ . The couple-solution  $(\mathbf{u}, \delta)$  can be then searched under the following form:

$$\mathbf{u}(\mathbf{x}) = a_1(x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2) + a_2 x_3 \mathbf{e}_3 \quad \text{and} \quad \delta(\mathbf{x}) = a_3 x_3 \quad (21)$$

where  $a_1$ ,  $a_2$  and  $a_3$  are real constants to be sought for.

The proposed forms of displacements  $\mathbf{u}$  and  $\delta$  are for instance starting assumptions. The strain field associated with the displacement field  $\mathbf{u}$  is then given by:

$$\boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x}) = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix}. \quad (22)$$

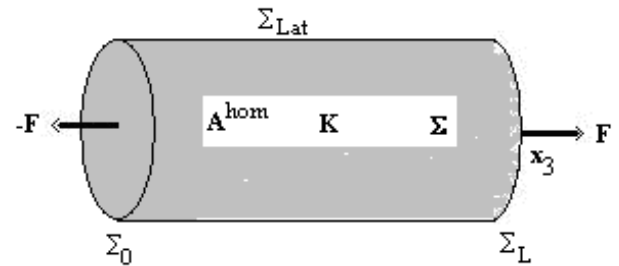


Figure 3. The homogenized composite beam

Recall now the form of the stress tensor  $\Sigma$ , obtained in [20], resulting of the internal stresses generated in the cell by an extension of the debonded fiber (but in contact with the matrix):

$$\Sigma = \begin{pmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{11} & 0 \\ 0 & 0 & \Sigma_{33} \end{pmatrix}. \quad (23)$$

In the case of a disposition of fibers at the vertices of a square lattice (as in our case), the homogenized stiffness tensor  $\mathbf{A}^{\text{hom}}$  is given, in the axis system  $(\mathbf{O}; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ , by the following matrix [8]:

$$\mathbf{A}^{\text{hom}} = \begin{bmatrix} A_{1111}^{\text{hom}} & A_{1122}^{\text{hom}} & A_{1133}^{\text{hom}} & 0 & 0 & 0 & 0 \\ & A_{1111}^{\text{hom}} & A_{1133}^{\text{hom}} & 0 & 0 & 0 & 0 \\ & & & A_{3333}^{\text{hom}} & 0 & 0 & 0 \\ & & & & 2A_{3131}^{\text{hom}} & 0 & 0 \\ & & & & & 2A_{3131}^{\text{hom}} & 0 \\ & & & & & & 2A_{1212}^{\text{hom}} \end{bmatrix}. \quad (24)$$

Let us check now or write the relations that must satisfy the equilibrium equations and the boundary conditions (11)-(20), we obtain:

- Firstly we check without difficulty that the equilibrium equations (11) and (12) are satisfied by the proposed displacement fields  $\mathbf{u}$  and  $\delta$  given by the expressions (21).

- Then we remark that the conditions (15) and (16) are equivalent. By making explicit these two conditions, we see that the unknown coefficient  $a_3$  satisfies the following equation:

$$2 A_{1212}^{\text{hom}} \boldsymbol{\varepsilon}(\mathbf{u})_{12} + a_3 \Sigma_{33} = F. \quad (25)$$

Taking into account (22), i.e.  $\boldsymbol{\varepsilon}(\mathbf{u})_{12} = 0$ , we obtain:

$$a_3 = \frac{F}{\Sigma_{33}}. \quad (26)$$

- Let us write now explicitly the boundary condition (17), taking into account (22) and (23), we obtain:

$$. (A_{1111}^{\text{hom}} A_{1122}^{\text{hom}}) a_1, A_{1133}^{\text{hom}} a_2 \frac{F \Sigma_{11}}{\Sigma_{33}}. \quad (27)$$

• The equations (18) and (19) are equivalent. By writing explicitly these two equations we obtain, as above, the following relation:

$$2\Sigma_{11}a_1 + a_2\Sigma_{33} = F(V_f - \frac{K}{\Sigma_{33}}). \quad (28)$$

$V_f$  is the volume fraction of fibers into the matrix i.e.  $V_f = \pi\alpha^2$ .

• And finally, the equation (20) is checked.

In summary, one has then to solve the following system of two equations with two unknowns, giving the constants  $a_1$  and  $a_2$ :

$$\begin{cases} 2a_1\Sigma_{11} + a_2\Sigma_{33} = F(\pi\alpha^2 - \frac{K}{\Sigma_{33}}) \\ (A_{1111}^{\text{hom}} + A_{1122}^{\text{hom}})a_1 + A_{1133}^{\text{hom}}a_2 = \frac{F\Sigma_{11}}{\Sigma_{33}} \end{cases} \quad (29)$$

which has for solution :

$$a_1 = \frac{F(\pi\alpha^2 + \Sigma_{33} - \frac{KA_{1133}^{\text{hom}}}{\Sigma_{33}})}{2\Sigma_{11}A_{1133}^{\text{hom}} - \Sigma_{33}(A_{1111}^{\text{hom}} + A_{1122}^{\text{hom}})} \quad (30)$$

$$\begin{aligned} a_2 = & \frac{F(K - \pi\alpha^2\Sigma_{33}A_{1133}^{\text{hom}})(A_{1111}^{\text{hom}} + A_{1122}^{\text{hom}})}{\Sigma_{33}A_{1133}^{\text{hom}}(2\Sigma_{11}A_{1133}^{\text{hom}} - \Sigma_{33}(A_{1111}^{\text{hom}} + A_{1122}^{\text{hom}}))} \\ & - \frac{2F(\Sigma_{11})^2A_{1133}^{\text{hom}}}{\Sigma_{33}A_{1133}^{\text{hom}}(2\Sigma_{11}A_{1133}^{\text{hom}} - \Sigma_{33}(A_{1111}^{\text{hom}} + A_{1122}^{\text{hom}}))} \end{aligned} \quad (31)$$

So the displacement field defined explicitly by

$$\begin{aligned} \mathbf{u}^0(\mathbf{x}, \mathbf{y}) &= \mathbf{u}(\mathbf{x}) + \chi_f(\mathbf{y})\delta(\mathbf{x})\mathbf{e}_3 \\ &= a_1x_1\mathbf{e}_1 + a_1x_2\mathbf{e}_2 + (a_2 + \chi_f(\mathbf{y})a_3)x_3\mathbf{e}_3, \end{aligned} \quad (32)$$

where  $a_1$ ,  $a_2$  and  $a_3$  are given respectively by (30), (31) and (26), is then an approached solution, in first order, of the longitudinal traction problem (1)-(5). Any other solution is obtained by adding to the expression (32) a rigid displacement [1], without consequences for the mechanical problem considered.

**Remark:** The obtained solution is a general solution. Indeed, when it comes to fibers perfectly bonded to the matrix, the term in  $\delta$  disappears and therefore obtained a transverse displacement equal to  $a_1x_1\mathbf{e}_1 + a_1x_2\mathbf{e}_2$  and a longitudinal displacement equal to  $a_2x_3\mathbf{e}_3$ . By cons when it comes to debonded fibers (but in contact with the matrix) we obtain  $a_1x_1\mathbf{e}_1 + a_1x_2\mathbf{e}_2$  as transverse displacement but  $(a_2 + \chi_f(\mathbf{y})a_3)x_3\mathbf{e}_3 = (a_2 + \chi_f(\mathbf{x}/\varepsilon)a_3)x_3\mathbf{e}_3$  as longitudinal displacement (with  $\varepsilon$  here is a fixed parameter (assumed small enough) characterizing the structure composite of beam. We see that for a given cross-section ( $x_3$  fixed) of the composite beam, the longitudinal displacement, for a perfectly bonded structure, is the same at every point of this section. By cons when it comes to a debonded structure, the longitudinal displacement depends, this time, on the position in the section: we obtain a longitudinal displacement of the matrix part and another different one of the fiber part. Which explains then the appearance of a new macroscopic field,

noted  $\delta$ . Physically, this field represents the inner sliding relative of the fibers with respect to the matrix [25].

## 5. Conclusions

We have shown that the displacement field, solution of the longitudinal traction problem of a fiber-reinforced composite beam with debonded fibers (but in contact with the matrix) is approximated, in the first order, by a displacement field depending on the fineness of the microstructure (presence of the microscopic variable  $\mathbf{y}$  in its expression). Moreover this field is given explicitly by the expression (32) because the traction problem considered is symmetrical (symmetry materiel and loading). And unlike that found in the literature, this field depends not only on usual terms, i.e. applied forces  $\mathbf{F}$ , volume fraction of the fibers  $V_f = \pi\alpha^2$  and homogenized stiffness tensor  $\mathbf{A}^{\text{hom}}$ , but also new homogenized tensors  $\mathbf{K}$  and  $\Sigma$  (calculated once for all in [20]) characterizing the structure composite with debonded fibers.

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