International Journal of Theoretical and Mathematical Physics 2014, 4(2): 37-52
DOI: 10.5923/j.ijtmp.20140402.02

On Achievements, Shortcomings and Errors of Einstein

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Abstract The electromagnetic energy is not equivalent to mass. Thus, $E = mc^2$ is not generally valid. Experimentally, a piece of heated-up metal has reduced weight instead of increased as predicted by $E = mc^2$. Moreover, for an electromagnetic wave to have a gravitational solution, the photonic energy-momentum tensor with an anti-gravity coupling must be added to the Einstein field equation. To have a dynamic solution for massive sources, the Einstein equation must be modified to the Lorentz-Levi-Einstein equation, which additionally has the gravitational energy-stress tensor with the anti-gravity coupling because the Einstein equation has no bounded dynamic solution as Gullstrand suspected. Moreover, the linearized equation is actually a linearization of the Lorentz-Levi-Einstein equation. Thus, the space-time singularity theorems of Penrose & Hawking are irrelevant to physics because its implicit assumption of unique sign for all couplings is invalid. The positive mass theorem of Yau & Schoen (and Witten) is misleading in physics because of the same invalid assumption. Thus, the Fields Medal has been wrong at least twice. Also, the 2011 Shaw Prize to Christodoulou actually awards his errors against the honorable Gullstrand. Due to the absence of the radiation reaction force, general relativity is incomplete. The static charge-mass interaction necessarily implies Einstein's unification between gravitation and electromagnetism. This repulsive interaction would explain the weight reduction of charged capacitors and the Space-Probe Pioneer Anomaly. The repulsive force breaks the basis for black holes, that gravity is always attractive. However, such force was overlooked for 80 years due to believing in $E = mc^2$ as unconditionally true. In conclusion, Einstein's general relativity is incomplete and needs rectifications. Additional difficulties were due to the confusion created by the Wheeler School and the misleading results of Yau & Schoen and Witten. However, the main source of errors is that Einstein's claim on the existence of dynamic solutions for his equation is invalid.

Keywords Dynamic solution, Gravitational radiation, Anti-gravity coupling, Principle of causality

"Unthinking respect for authority is the greatest enemy of truth." – A. Einstein

1. Introduction

Einstein is commonly recognized as a genius because he created new theories and led us with accurate predictions to new areas of physics, namely, special relativity, quantum mechanics and general relativity. In special relativity, he revolutionized the notion of space-time and established a formula between energy and mass $E = mc^2$.[1)] The notion of photon plays a central role in quantum mechanics. In general relativity, he amazed us by his three accurate predictions. Thus, a faith in Einstein was developed. In particular, many believed that the Einstein equation is correct.

However, as time has gone by the shortcomings of his theories are gradually shown and even include mistakes. We admire Einstein not because he was perfect as we previously believed, but rather for his ingenuity in starting new chapters of physics. In what follows, we shall point out the current problems. This leads to the necessary rectification of his theories, in particular, the Einstein equation.[2)] Then, it also becomes clear that his conjecture of unification between gravitation and electromagnetism is established.

Nevertheless because of the blind faith to Einstein and inadequacy in mathematics, many theorists erroneously claimed to have explicit dynamic solutions.[3)] Apparently, such claims were mistaken by some mathematicians as the truth because they do not understanding the related physics. Thus, the subsequent works done by those mathematicians are misleading in physics.[4)] These have led to the difficulty to prove Einstein's conjecture of unification. Thus, such errors must be pointed out to remove the obstacles to progress in general relativity.

Finally, Einstein's conjecture of unification turns out to be correct at the end. It is up to us to further explore what he has started and perform further investigations.

2. Special Relativity and the Formula $E = mc^2$

While special relativity has the least problems, the unconditional validity of the formula $E = mc^2$ is only a speculation that has never been verified [1]. In fact, Einstein

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Published online at http://journal.sapub.org/ijtmp

had tried very hard for years (1905-1909) to prove this formula to be generally valid, but failed [2].

The formula can be traced back to special relativity, which suggested a rest inertial mass m_0 has the rest energy of m_0c^2. This is supported by the nuclear fissions with $\Delta E = \Delta mc^2$, where Δm is the mass difference after the fission and ΔE the total energy created and is usually a combination of different types of energy.[1)] However, for an arbitrary energy E, the relation $m = E/c^2$ is not only an unverified speculation, but also an obstacle for progress in physics [1].

According to general relativity, such a claim is incorrect for electromagnetic energy. The Einstein field equation is,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R = - K\ T_{\mu\nu}, \qquad (1)$$

where the energy stress tensor $T_{\mu\nu}$ is the sum of any type of energy-stress tensor. The electromagnetic energy-stress tensor, being traceless, cannot affect R in eq. (1). Therefore, the electromagnetic energy is not equivalent to mass.[5)] Moreover, the Reissner-Nordstrom metric illustrated that electromagnetic energy and mass are clearly different in terms of gravity [1]. Also, unconditional validity of $E = mc^2$ is the primary cause of overlooking the charge-mass interaction [1]. This non-equivalence is due to that the trace of an electromagnetic energy-stress tensor is zero. Experimentally, in contrast to Einstein [3], it has been shown that a piece of heated up metal has reduced weight [4].

Thus, mass and energy are not equivalent. It follows that the coupling signs of all the energy-stress tensors in general relativity need not be the same as Penrose and Hawking implicitly assumed in their space-time singularity theorems [5]. Thus, it is clear that their singularity theorems are irrelevant to physics.

3. The Question of Photons

When Einstein proposed the notion of photon, it consists of a quantum of electromagnetic energy, and it acts like a particle in the photo–electric effect. It has been observed that the particle π^0 meson decays into two photons (i.e., $\pi^0 \to \gamma + \gamma$). This was mistakenly considered as evidence that the electromagnetic energy is equivalent to mass. However, there would be a conflict if a photon includes only electromagnetic energy since the electromagnetic energy-stress tensor is traceless. Therefore, the photons must consist of non-electromagnetic energy.

Since a charged particle has mass, it is natural that the non-electromagnetic energy is the gravitational energy. When Einstein proposed the notion of photon, he had not conceived general relativity yet. Moreover, since Hawking and Penrose claimed that general relativity was not applicable in microscopic scale, the possibility of including gravitational energy in photons was ignored. It will be shown that Hawking and Penrose are incorrect (see sections 4 & 5). Moreover, it has been shown that the energy of photons is, indeed, the sum of the energies of the electromagnetic wave component and that of the gravitational wave component [6]. In other words, the energy-stress of photons $T(L)_{ab}$ is

$$T(L)_{ab}=T(E)_{ab}+T(N)_{ab} \text{ or } T(N)_{ab}=T(L)_{ab}-T(E)_{ab} \qquad (2)$$

where $T(E)_{ab}$ and $T(N)_{ab}$ are respectively the electromagnetic energy-stress tensor and the non-electromagnetic energy-stress tensor. Besides, $T(E)_{cb}$ being intrinsically traceless ($T(E)^{mn}g_{mn} \equiv 0$), would not be compatible with Einstein's formula $\Delta E = \Delta mc^2$. Based on the fact that the electromagnetic energy is dominating experimentally, it is natural to assume as shown later that $T(N)_{ab}$ is in fact the gravitational energy-stress tensor $T(g)_{ab}$.

Physics requires that the energy-stress tensor for photons $T(L)_{ab}$ is: 1) traceless, 2) $T(L)_{ab}$ equal to $T(E)_{ab}$ approximately but $[T(L)_{tt} - T(E)_{tt}] \geq 0$ on the average, and 3) satisfying an equation as follows,

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab}R = KT(g)_{ab} = -K[T(E)_{ab} - T(L)_{ab}]. \qquad (3)$$

where R_{ab} is Ricci curvature tensor. Eq. (3) is different from Einstein equation with an additional term $T(L)_{ab}$ with a coupling of different sign. From eq. (3), we have $\nabla_c T(L)^{cb} = 0$ since $\nabla_c T(E)^{cb} = 0$.

Let us consider the energy-stress tensor $T(L)_{ab}$ for photons (of a plane wave) defined by formula (2). If a geodesic equation must be produced, for a monochromatic wave with frequency ω, the form of a photonic energy tensor should be similar to that of massive matter. Observationally, there is very little interaction, if any, among photons of the same ray. Since photons travel at the velocity of light, there should not be any interaction (other than collision) among them.

Thus, the photons can be treated as a bundle of massless particles just as Einstein [7] did. Therefore, the photonic energy tensor of a wave of frequency ω should be dust-like, whose trace is zero, as follows:

$$T^{ab}(L) = \rho\ P^a\ P^b, \qquad (4)$$

where ρ is a scalar and is a function of u (= ct – z). In the units $c = \hbar = 1$, $P_t = \omega$ is the energy. If the photons are a bundle of massless particles, the photonic energy tensor has been obtained [8] as follows:

$$\rho\ (u) = -\ A_m g^{mn} A_n \geq 0 \qquad (5)$$

is a scalar and is a function of u (= ct - z). Since light intensity is proportional to the square of the wave amplitude, ρ which is Lorentz gauge invariant, and the density function of photons. Then

$$T_{ab} = -T(g)_{ab} = T(E)_{ab} - T(L)_{ab} = T(E)_{ab} + A_m g^{mn} A_n P_a P_b\ . \qquad (6)$$

Note that ρ(u) is a positive non-zero scalar consisting of A_k and/or fields such that, on the average, $T(L)_{ab}$ is approximately $T(E)_{ab}$, and eq. (3) would have physically valid solutions. Note that $T(g)_{ab}$ has an antigravity coupling.

For instance, consider an electromagnetic plane-wave of circular polarization, propagating to the z-direction

$$`A_x = \frac{1}{\sqrt{2}} A_0 \cos \omega u\ , \text{ and } A_y = \frac{1}{\sqrt{2}} A_0 \sin \omega u\ , \qquad (7)$$

where A_0 is a constant. The rotational invariants with respect to the z-axis are constants. These invariants are: G_{tt}, R_{tt}, $T(E)_{tt}$, G, $(g_{xx} + g_{yy})$, g_{tz}, g_{tt}, g, and etc. It follows that [6]

$$g_{xx}=-1-C+B_\alpha \cos(\omega_1 u+\alpha),\quad g_{yy}=-1-C-B_\alpha \cos(\omega_1 u+\alpha), \quad (8)$$

and

$$g_{xy} = \pm B_\alpha \sin(\omega_1 u + \alpha),$$

where C and B_α are small constants, and $\omega_1 = 2\omega$. Thus, metric (8) is a circularly polarized wave with the same direction of polarization as the electromagnetic wave (7). On the other hand, one also has

$$G_{tt} = 2\omega^2 B_\alpha^2/G \geq 0 \text{, where } G = (1 + C)^2 - B_\alpha^2 > 0 , \quad (9)$$

but

$$T(E)tt = \frac{1}{2G}\omega^2 A_0^2(1 + C - B\alpha \cos\alpha) > 0.$$

Then eq. (4), eq. (5) and eq. (6) give

$$T_{tt} = T(E)_{tt} - T(L)_{tt} = -\frac{1}{G}\omega^2 A_0^2 B_\alpha \cos\alpha < 0, \text{ since}$$

$$B_\alpha = \frac{K}{2} A_0^2 \cos\alpha . \quad (10)$$

It follows that eq. (3) is satisfied. This shows also eq. (5) is valid. To confirm the general validity of eq. (2) further, consider an electromagnetic plane-wave linearly polarized in the x-direction,

$$A_X = A_0 \cos\omega(ct - z). \quad (11)$$

For this electromagnetic wave, a gravitational wave was also obtained [6]. Thus, a photonic energy-stress tensor has been obtained to satisfy the demanding physical requirements [6].

The energy and momentum of a photon is proportional to its frequency, as expected from a classical theory. Just as expected from special relativity, indeed, the gravity of an electromagnetic wave is an accompanying gravitational wave propagating with the same speed. Concurrently, for this case, the need of modifying the Einstein equation is accomplished. Then, clearly the gravity due to the light is negligible in calculating the light bending [7]. However, if $T(E)_{ab}$ were alone in eq. (3), the gravitational effect would be very large and that cannot be neglected. Note that, because of the anti-gravity coupling, the energy conditions in the singularity theorems of Hawking and Penrose [5] failed.

Thus, the fact that gravity of the light is neglectible in the calculation of the bending of light together with the symmetry due to polarization of the light ray lead to the natural conclusion that the Einstein equation must be modified with the antigravity coupling of the photonic energy-stress tensor [8]. Then, it should not be a surprise that an anti-gravity coupling of gravitational energy-stress tensor must be added to obtain a dynamic solution for massive sources. Thus, Gullstrand is right on his suspicion of the invalidity of Einstein's calculation on the perihelion of Mercury.

Based on special relativity, it is known that the energy of an electromagnetic wave is distinct from the energy of a rest mass. Interestingly, *it is precisely because of this non-equivalence of mass and energy that the photonic energy-stress tensor (6) is valid, and the formula* $E = mc^2$ *can be proven*.

One might argue that experiment shows the notion of massless photons is valid, and thus believed the equivalence of mass and electromagnetic energy. However, *while the addition of two massless particles may end up with a rest mass, the energy-stress tensor of electromagnetism cannot represent a rest mass since such a tensor is traceless* ($T(E)^{mn} g_{mn} = 0$). Thus, Einstein's formula $\Delta E = \Delta mc^2$ necessarily implies that $T(L)_{ab}$ must include non-electromagnetic energy. This makes it clear that the photonic energy tensor is intrinsically different from the electromagnetic energy tensor.

Clearly, gravity is also a microscopic phenomena of crucial importance to the formula $E = mc^2$. Both quantum theory and relativity are based on the phenomena of light. The gravity of photons finally shows that there is a link between them. It is gravity that makes the notion of photons compatible with electromagnetic waves [6]. Einstein probably would smile heartily since his formula confirms the link that relates gravity to quantum theory.

4. Problems and Errors in General Relativity

In general relativity, the most important statement of Einstein is the existence of a bounded dynamic solution for the Einstein equation. On this issue, Einstein and Gullstrand [9], the chairman (1922-1929) of the Nobel Committee for Physics are in opposition to each other. To clarify the issue, one must be clear on some obscure errors.

It was generally believed that the linearized Einstein equation would give a first order approximation of the solution for the Einstein equation [10]. For space-time metric $g_{\mu\nu}$, the Einstein equation of 1915 is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -KT(m)_{\mu\nu} \quad (12)$$

where $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and K is the coupling constant.

The linearized Einstein equation [10] with the linearized harmonic gauge $\partial^\mu \bar{\gamma}_{\mu\nu} = 0$ is

$$\frac{1}{2}\partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} = \kappa T(m)_{\mu\nu} \text{ where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma$$

$$\text{and } \gamma = \eta^{\alpha\beta}\gamma_{\alpha\beta} . \quad (13)$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \text{ and}$$

$$G_{\mu\nu}^{(1)} = \frac{1}{2}\partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} - \partial^\alpha \partial_\mu \bar{\gamma}_{\nu\alpha} - \partial^\alpha \partial_\nu \bar{\gamma}_{\mu\alpha} + \frac{1}{2}\eta_{\mu\nu}\partial^\alpha \partial^\beta \bar{\gamma}_{\alpha\beta} \quad (14)$$

where $G_{\mu\nu}^{(2)}$ represents the second order terms. This linearized equation is obtained by neglecting the second order terms in the non-linear Einstein equation with a harmonic gauge. Thus, it is expected that its solution is the first approximation of a solution of the Einstein equation. This result has been verified for the static case [10].

Many believed the linearized equation would give the first order approximation for other circumstances. However, this has been proven to be not true for the dynamic case when gravitational waves are involved [11-13]. In terms of mathematics, for a linear equation such as the Maxwell equation, a weak solution always exists if the sources are weak. However, for a non-linear equation, there is no compelling reason that a bounded solution exists for a weak source.

Since a linearized equation such as eq. (2) always produces a bounded solution, if the non-linear Einstein equation has only unbounded solutions, the Einstein equation and its linearized equation would have no compatible solutions. Thus, if a bounded solution does not exist, then the procedure of "linearization" is not valid. Then, the non-linear Einstein equation and its "linearization" are essentially independent equations.

There are explicit examples [14, 15] that show the non-linear Einstein equation and the linearized equation are independent equations that have no compatible solutions. We shall illustrate such characteristics such that the readers can readily accept the fact that Einstein was wrong and there is no dynamic solution for the Einstein equation.

4.1. Examples of no Bounded Dynamic Solutions

Many believed that it would be very difficult to show that there is no dynamic solution for the Einstein equation. However, if one would look carefully, it is possible to have very remarkable evidence such as the two examples.

a) The metric obtained by Bondi, Pirani, & Robinson [14]

The metric is as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2 \begin{bmatrix} \cosh 2\beta\left(d\eta^2 + d\varsigma^2\right) \\ +\sinh 2\beta \cos 2\theta\left(d\eta^2 - d\varsigma^2\right) \\ -2\sinh 2\beta \sin 2\theta \, d\eta \, d\varsigma \end{bmatrix} \quad (15a)$$

where ϕ, β and θ are functions of u ($= \tau - \xi$). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right), \quad (15b)$$

a special case of $G_{\mu\nu} = 0$. They claimed this is a wave from a distant source. Note that *ϕ cannot be a periodic function.* The metric is irreducibly unbounded because of the factor u^2. Thus, for this case, there is no bounded dynamic solution.

Moreover, when gravity is absent, it needs to have $2\phi = \sinh 2\beta = \sin 2\theta = 0$. These reduce (15a) to

$$ds^2 = \left(d\tau^2 - d\xi^2\right) - u^2\left(d\eta^2 - d\zeta^2\right) \quad (15c)$$

Thus this metric is not equivalent to the flat metric. Hence, metric (15c) violates the principle of causality again. Also, in metric (15a) there is no parameter to be adjusted such that metric (15a) becomes equivalent to the flat metric.

Clearly, metric (15) is not a bounded dynamic solution, and thus this illustrates that the non-linear Einstein equation and the linearized equation are independent equations. Moreover, linearization of (15b) does not make sense since variable u is not bounded. Thus, many theorists claim Einstein's notion of weak gravity invalid.

However, they overlooked that for this case, there is no bounded dynamic solution. This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid. These conflicting views are supported respectively by the editors of the "Royal Society Proceedings A" and the "Physical Review D". The Royal Society correctly pointed out [16-18] that Einstein's notion of weak gravity is inconsistent with his covariance principle. Moreover, Einstein's weak gravity is supported by the principle of causality [11].

b) The Metric of Misner, Thorne, & Wheeler

The general Einstein equation is complicated. However, Misner, Thorne, and Wheeler [15] inadvertently give a simple example that illustrates the non-existence of a dynamic solution.

The "wave" form considered by Misner, Thorne, & Wheeler [15] is as follows:

$$ds^2 = c^2dt^2 - dz^2 - L^2\left(e^{2\beta}dx^2 + e^{-2\beta}dy^2\right) \quad (16)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - z$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \quad (17)$$

Misner et al. [15] claimed that Eq. (17) has a bounded solution, compatible with a linearization of metric (16).

Such a claim is clearly in conflict with the non-existence of dynamic solutions. Apparently, they incorrectly believe this is a case different from the metric of Bondi et. al [14]. It will be shown that such a claim is due to a blind faith on Einstein's claim on the existence of the dynamic solution together with a careless calculation at the undergraduate level.

They further claimed [15], "The linearized version of L'' = 0 since $(\beta')^2$ is a second–order quantity. Therefore the solution corresponding to linearized theory is

$$L = 1, \ \beta(u) \text{ arbitrary but small.} \quad (18)$$

The corresponding metric is

$$ds^2 = (1+2\beta)dx^2 + (1-2\beta)dy^2 + dz^2 - dt^2, \ \beta = \beta(t-z)." \quad (19)$$

However, careful calculation shows that these claims are also incorrect. In other words, Misner et al. [15] are incorrect and Eq. (17) does not have a physical solution that satisfies Einstein's requirement on weak gravity.

In fact, *L(u)* is unbounded even for a very small $\beta(u)$. Linearization of (17) yields $L'' = 0$, and this leads to $\beta'(u) = 0$. Thus, this leads to a solution $L = C_1 u + C_2$ where C_1 & C_2 are constants. Therefore, the requirement $L \approx 1$ implies $C_1 = 0$. *However,* $\beta'(u) = 0$ *implies* $\beta(u)$*=constant. Thus, metric (19) cannot be derived from the Einstein equation.*

To prove Eq. (17) having no wave solution, it is sufficient to consider the case of weak gravity since a reduction of source strength would lead to weak gravity. According to Einstein, for weak gravity of metric (16), one would have

$$L^2 e^{2\beta} \cong 1 \text{ and } L^2 e^{-2\beta} \cong 1 \tag{20a}$$

It follows that

$$L^4 \cong 1, \quad e^{\pm 2\beta} \cong 1 \text{ and } L(u) >> |\beta(u)|. \tag{20b}$$

Since *L(u)* is bounded, *L'(u)* cannot be a monotonic function of u, unless L'–> 0. Thus, there is an interval of *u* such that the average,

$$\langle L'' \rangle = 0 \tag{21}$$

However, the average of the second term of equation (17) is larger than zero unless $\beta'(u) = 0$ in the whole interval of u.

Also, from eq. (20), one would obtain $L\ (\cong 1) > 0$, and one has $0 > L''(u)$ *if* $\beta'(u) \neq 0$. Thus, $-L'(u)$ is a monotonic increasing function in any finite interval of u since $\beta'(u) = 0$ means $L'' = 0$, i.e., no wave. In turn, *since* $\beta'(u)$ *is a "wave factor",* this implies that *L(u)* is an unbounded function of *u*. Therefore, this would contradict the requirement that L is bounded. In other words, eq. (17) does not have a bounded wave solution. Moreover, the second order term *L''* would give a very large term to *L*, after integration. Also, linearizing eq. (17) to $L'' = 0$ leads to no wave.

Now, let us investigate the errors of Misner et al. [15; p. 958]. They assumed that the signal $\beta(u)$ has duration of *2T*. For simplicity, it is assumed that definitely $|\beta'(u)| = \delta$ in the period *2T*. Before the arrival of the signal at $u = x$, one has

$$L(u) = 1 \text{, and } \beta(u) = 0 \tag{22}$$

If weak gravity is compatible with Eq. (16), one would have $L(u) \cong 1$. It thus follows from Eq. (17), one has

$$L'(u) = 0 - \int_x^u \beta'^2 dy \approx -\int_x^u \delta^2 dy = \delta^2 (u - x) \quad \text{for } x + 2T > u > x,$$

$$\text{or } \approx -\delta^2 2T \quad \text{for } u > x + 2T \tag{23}$$

Hence

$$\left. \begin{aligned} L(u) &= 1 + \int_x^u L' dy \\ &\approx 1 - \int_x^u \delta^2 (y - x) dy = 1 - \frac{\delta^2 (u - x)^2}{2} \end{aligned} \right)$$

for $x + 2T > u > x$

$$\text{or} \quad \left. \begin{aligned} &\approx 1 - \int_x^{x+2T} \delta^2 (y - x) dy - \delta^2 2T \int_{x+2T}^u dy \\ &= 1 - \delta^2 2T (u - T - x) \end{aligned} \right) \text{ for}$$

$$u > x + 2T \tag{24}$$

Thus, independent of the smallness of $2\delta^2 T$ (or details of $|\beta'(u)|^2$), *L* could be approximately zero and violates the condition for weak gravity. In other words, eq. (17) has no weak wave solution. Moreover, |*L(u)*| is not bounded since it would become very large as u increases. Thus, restriction of $2\delta^2 T$ being small [15] does not help.

Thus, one can get a no wave solution through linearization of Eq. (17), which has no bounded solution. The assumption of metric form (16) is bounded [15], and has a weak form (19), is not valid. Thus, there is no bounded wave solution for the non-linear Einstein equation, which violates the principle of causality.

The root of their errors was that they incorrectly assumed that a linearization of the Einstein non-linear equation would produce a valid approximation. Thus, they implicitly and incorrectly assume the existence of a bounded wave solution without the necessary verification, and thus obtain incorrect conclusions.

On the other hand, from the linearization of the Einstein equation (the Maxwell-Newton approximation) in vacuum, Einstein [19] independently obtained a solution as follows:

$$ds^2 = c^2 dt^2 - dz^2 - (1 + 2\phi) dx^2 - (1 - 2\phi) dy^2 \tag{25}$$

where ϕ is a bounded function of $u\ (= ct - z)$. Note that metric (25) is the linearization of metric (16) if $\phi = \beta(u)$, but it cannot be obtained through the non-linear Einstein equation.

Thus, the problem of waves illustrates that the linearization is not valid for the dynamic case when gravitational waves are involved since eq. (17) does not have a bounded wave solution. In other words, the Einstein equation and its linearization are essentially independent equations.

4.2. Other Errors in the Non-existence of a Dyanmic Solution

Since the error of the non-existence of dynamic solution was made by Einstein, other theorists also made errors by having unverified faith on Einstein. Wald [5] is the better known among them.

According to Einstein [10], in general relativity weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } 1 >> |\gamma_{\mu\nu}| \tag{26}$$

and $\eta_{\mu\nu}$ is the flat metric when there is no source. However, this is true only if the equation is valid in physics. Many failed to see this because they failed to see the difference between physics and mathematics [20]. When the Einstein equation has a weak solution, an approximate weak solution can be derived through the approach of the field equation being linearized. However, the non-linear equation may not have a bounded solution. The linearized Einstein equation with the linearized harmonic gauge $\partial^{\mu}\bar{\gamma}_{\mu\nu} = 0$ is eq. (13). Then, the linearized vacuum Einstein equation means

$$G^{(1)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0 \qquad (27)$$

Thus, as pointed out by Wald [5], in order to maintain a solution of the vacuum Einstein equation to second order we must correct $\gamma^{(1)}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}_{\mu\nu}$, where $\gamma^{(2)}_{\mu\nu}$ satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}{}_{\alpha\beta}] + G^{(2)}_{\mu\nu}\left[\gamma_{\alpha\beta}\right] = 0, \text{ where } \gamma_{\mu\nu}=\gamma^{(1)}_{\mu\nu}+\gamma^{(2)}_{\mu\nu} \qquad (28)$$

which is the correct form of eq. (4.4.52) in Wald's book. In Wald's book, he did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}_{\mu\nu}$.

This equation does have a solution for the static case. However, detailed calculation shows that this equation does not have a solution for the dynamic case [11-13]. The fact that, for a dynamic case, there is no bounded solution for eq. (28) means also that the Einstein equation does not have a dynamic solution. The examples are the metric of Bondi, Pirani, & Robinson [14] and the metric of Misner et al. [15].

Due to confusion between mathematics and physics, Wald [5] made errors at the undergraduate level. The principle of causality requires the existence of a dynamic solution, but Wald did not see that the Einstein equation can fail this requirement [20]. Thus, his theory does not include the dynamic solutions.

There are others who also make errors on dynamic solutions. Christodoulou & Klainerman [21], and 't Hooft [22, 23] claimed to have explicit examples of bounded dynamic solutions. However, these are also due to errors in calculation and/or misconceptions as the case of Misner et. al [15]. Christodoulou and Klainerman [21] claimed to have constructed dynamic solutions, but their construction is actually incomplete [24]. Moreover, J. A. Taylor of Princeton was unable to justify their calculation on the binary pulsar experiments [25] when Morrison of MIT questioned him [26].

In defense of the errors of the Nobel Committee for Physics of 1993, 't Hooft [22, 23] comes up with a bounded time-dependent cylindrical symmetric solution as follows:

$$\Psi(r,t) = A\int_0^{2\pi} d\varphi e^{-\alpha(t-r\cos\varphi)^2}, \qquad (29)$$

where A and α (> 0) are free parameters. $|\Psi|$ is everywhere bounded. Then, 't Hooft [22, 23] claimed that his solution, Ψ is obtained by superimposing plane wave packets of the form $exp[-\alpha(x-t)^2]$ rotating them along the z axis over angle φ, so as to obtain a cylindrical solution. Note that since the integrand $exp[-\alpha(t - r\cos\varphi)^2] = exp[-\alpha(t - x)^2]$, there is no rotation along the z axis. The function $exp[-\alpha(t - x)^2]$ is propagating from $x = -\infty$ to $x = \infty$ as time t increases.

Note, however, that in a superimposition the integration is over a parameter of frequency ω unrelated to the x-axis; whereas the solution (29) is integrated over $\varphi(x, y)$. Since, (29) is a combination that involves the coordinate $\varphi(x, y)$, it is not a superimposition of plane waves propagating along the x-axis. Furthermore, the integration over all angles φ is a problem that would violate the requirement of the idealization because it requires that the plane wave is valid over the whole x-y plane. Thus, function (29) is not valid as an idealization in physics.

Therefore, in solution (29), two errors have been made, namely: 1) the plane wave has been implicitly extended beyond its physical validity, and 2) the integration over $d\varphi$ is a process without a valid physical justification. Moreover, it has been shown that there are no valid sources that can be related to solution (29) [23]. Thus, since the principle of causality is also violated, his solution is not valid in physics.

5. The Dynamic Case for the Einstein Equation with Massive Sources

Einstein started his faith on his theory of general relativity with the remarkable calculation of the remaining perihelion of Mercury. However, Gullstrand suspected that his calculation is questionable since he failed to show such a calculation is derivable from a many-body problem approach. This is a very insightful criticism. Thus, Einstein was awarded a Nobel Prize for his photo-electric effects, and a controversy accompanies the theory since then.

In 1995, it is proven that for the dynamic case, i.e., when the gravitational waves are involved, there is no bounded dynamic solution for the Einstein equation. In other words, Einstein has been proven wrong, but Gullstrand is right. However, because the proof is rather long, many theorists just do not have the time to go over it. Besides, well-known theorists such Misner et al and Wald insisted as Einstein claimed the existence of the dynamic solutions.

In this paper, we show through the theories of Misner et al. [15] and Wald [5] and examples in the literature that they are mistaken. The main error is that they believe that the Einstein equation has bounded dynamic solutions and thus is valid for the dynamic case. However, it has been shown that there is no dynamic solution for the Einstein equation. In short, the Einstein equation is valid only for the static and stable cases, but invalid for the dynamic case.

5.1. The Question of a Correct Field Equation for Massive Sources

An obvious question is what is the correct field equation for the dynamic case? Moreover, how such an equation would be related to the "linearized" equation, which is supported by observation. A meaningful answer would be that the "linearized" equation is obtained though a valid mathematical linearization of the correct field equation. Moreover, since the "linearized" equation with massive sources is supported by observation, there must be a way to justify its validity independently. This leads to the Maxwell-Newton Approximation [11-13].

$$\frac{1}{2}\partial_c\partial^c\bar{\gamma}_{ab} = -\kappa T(m)_{ab}\,, \quad (13a)$$

where $\bar{\gamma}_{ab} = \gamma_{ab} - (1/2)\eta_{ab}\gamma$, $\gamma_{ab} = g_{ab} - \eta_{ab}$, $\gamma = \eta^{cd}\gamma_{cd}$, and η_{ab} is the flat metric. A solution of eq. (13a) is

$$\bar{\gamma}_{ab}(x_i,t) = -\frac{\kappa}{2\pi}\int\frac{1}{R}T_{ab}[y^i, (t-R)]\,d^3y, \text{ where}$$

$$R^2 = \sum_{i=1}^{3}\left(x^i - y^i\right)^2 \quad (13b)$$

Solution (13b) would represent a wave if T_{ab} has a dynamical dependency on time t' (= t – R).

Thus, the theoretical existence of gravitational waves seems to be assured as a certainty as believed.

Eq, (13) is currently considered as the linearization of the Einstein equation of 1915 [7], which is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T(m)_{\mu\nu}\,. \quad (12)$$

Einstein believed that his equation satisfied this requirement since its linearized "approximation" gives a wave. However, as has been shown that, for the dynamic case, equation (12) and the "linearized" equation (13) are actually independent equations Therefore, eq. (13) cannot be a mathematical linearization of equation (12).

Moreover, for non-linear equations, the physical second order terms can be crucial for the mathematical existence of bounded solutions. For the Einstein equation, the Cauchy initial condition is restricted by four constraints since there is no second order time derivatives in G_{at} (a = x, y, z, t) [27]. This suggests that the Einstein equation (12) and Eq. (13) may not be compatible for a dynamic problem. Einstein discovered that his equation does not admit a propagating wave solution [28, 29]. It has also been known that the linearization procedure is not generally valid. Thus, it is necessary to justify waver solution (13b) independently since it is the basis of Einstein's radiation formula.

The question of dynamic solutions was raised by Gullstrand [9]. He challenged Einstein and also D. Hilbert who approved Einstein's calculations [30]. However, Hilbert did not participate in the subsequent defense and he would probably have seen the deficiency. Nevertheless, theorists such as Christodoulou & Klainerman [21], Misner et al. [15] and Wald [5] etc. failed to see this, and tried very hard to prove otherwise.

The failure of producing a dynamic solution would cast a strong doubt to the validity of the linearized equation that produces many effects including the gravitational waves. In fact, for the case that the source is an electromagnetic plane wave, the linearized equation actually does not have a bounded solution [31].

Nevertheless, when the sources are massive, some of such results from the linearized equation have been verified by observation. Thus, there must be a way to justify the linearized equation, independently. To this end, Einstein's equivalence principle is needed [12], The non-existence of a bounded dynamic solution for massive sources is due to a violation of the principle of causality [32].

5.2. The Weak Gravity of Massive Matter and Einstein Equation of the 1995 Update

It will be shown that eq. (13a) can be derived from Einstein's equivalence principle. Based on this, the equation of motion for a neutral particle is the geodesic equation. In comparison with Newton's second law, one obtains that the Newtonian potential of gravity is approximately $c^2g_{tt}/2$. Then, in accord with the Poisson equation and special relativity, the most general equation for the first order approximation of g_{ab} is,

$$\frac{1}{2}\partial_c\partial^c\gamma_{ab} = -\frac{\kappa}{2}[\alpha T(m)_{ab} + \beta\hat{T}(m)\eta_{ab}], \quad (30a)$$

where

$$\hat{T}(m) = \eta^{cd}T(m)_{cd}\,,\ \kappa = 8\pi Kc^{-2}\,, \text{ and } \alpha + \beta = 1, \quad (30b)$$

where α and β are constants since Newton's theory is not gauge invariant.

Then, according to Riemannian geometry [27], the exact equation would be

$$R_{ab} + X^{(2)}_{ab} = -\frac{\kappa}{2}[\alpha T(m)_{ab} + \beta\, T(m)g_{ab}], \text{ where}$$

$$T(m) = g^{cd}T(m)_{cd} \quad (31a)$$

and $X^{(2)}_{ab}$ is an unknown tensor of second order in K, if R_{ab} consists of no net sum of first order other than the term (1/2) $\partial_c\partial^c\gamma_{ab}$. This requires that the sum

$$-\frac{1}{2}\partial^c[\partial_b\gamma_{ac} + \partial_a\gamma_{bc}] + \frac{1}{2}\partial_a\partial_b\gamma\,, \quad (31b)$$

must be of second order. To this end, let us consider eq. (30a), and obtain

$$\frac{1}{2}\partial_c\partial^c(\partial^a\gamma_{ab}) = -\frac{\kappa}{2}[\alpha\partial^a T(m)_{ab} + \beta\partial_b\hat{T}(m)] \quad (32a)$$

From $\nabla^c T(m)_{cb} = 0$, it is clear that K $\partial^c T(m)_{cb}$ is of second order but K$\partial_b\hat{T}(m)$ is not. However, one may obtain a second order term by a suitable linear combination of $\nabla^c\gamma_{cb}$ and $\partial_b\gamma$. From (32a), one has

$$\frac{1}{2}\partial_c\partial^c(\partial^a\gamma_{ab} + C\,\partial_b\gamma)$$

$$= -\frac{\kappa}{2}[\alpha\partial^a T(m)_{ab} + (\beta + 4C\beta + C\alpha)\partial_b\hat{T}(m)]\,. \quad (32b)$$

Thus, the harmonic coordinates (i.e., $\partial^a\gamma_{ab} - \partial_b\gamma/2 \approx 0$), can lead to inconsistency. It follows eqs. (31b) and (32b) that, for the other terms to be of second order, one must have C = -1/2, $\alpha = 2$, and $\beta = -1$.

Hence, eq. (30a) becomes,

$$\frac{1}{2}\partial_c\partial^c\gamma_{ab} = -\kappa[T(m)_{ab} - \frac{1}{2}\hat{T}(m)\,\eta_{ab}]. \quad (33)$$

which is equivalent to eq. (13a). This derivation is independent of the exact form of equation (32a). The implicit gauge condition is that the flat metric η_{ab} is the asymptotic limit. Eq. (33) is compatible with the equivalence principle as demonstrated by Einstein in his calculation of the bending of light. Thus, the derivation is self-consistent.

Einstein obtained the same values for α and β by considering eq. (31a) with the implicit assumption $X^{(2)}{}_{ab} = 0$. An advantage of the approach of considering eq. (30) and eq. (31b) is that the over simplification $X^{(2)}{}_{ab} = 0$ is not needed.

Then, it is possible to obtain from eq. (31a) an equation different from (12),

$$G_{ab} \equiv R_{ab} - \frac{1}{2}g_{ab}R = -\kappa\,[T(m)_{ab} - Y^{(1)}{}_{ab}], \quad (34)$$

where

$$-\kappa Y^{(1)}{}_{ab} = X^{(2)}{}_{ab} - \frac{1}{2}g_{ab}\{X^{(2)}{}_{cd}\,g^{cd}\}.$$

The conservation law $\nabla^c T(m)_{cb} = 0$ and $\nabla^c G_{cb} \equiv 0$ implies also $\nabla^a Y^{(1)}{}_{ab} = 0$. If $Y^{(1)}{}_{ab}$ is identified as the gravitational energy tensor of $t(g)_{ab}$, Einstein equation of the 1995 update [11] is reaffirmed. Note that eq. (13a) is the first order approximation of eq. (34). Note, also that $t(g)_{ab}$ must be a tensor [11] because the radiation energy cannot be zero.

A crucial discovery is that for the existence of dynamic solutions there must be different signs for the couplings [11]. (Thus, as in the case of eq. (3), the energy conditions in the space-time singularity theorems of Hawking and Penrose [5] also failed for the above case.) For the dynamic case, a modified Einstein equation [11-13] is,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa[T(m)_{\mu\nu} - t(g)_{\mu\nu}], \quad (35)$$

i.e. the Lorentz-Levi-Einstein equation,[6)] where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity. Note that $t(g)_{\mu\nu}$ with the anti-gravity coupling has appeared in eq. (3). Such a calculation is necessary to justify that in the bending of light, the gravitational effect of an electromagnetic wave is negligible.

5.3. Implications of the Modified Einstein Equation

The equation (35) explains why the nonlinear Einstein equation always results in violating the principle of causality because $t(g)_{\mu\nu}$ is neglected. Moreover, since the popular notion of black holes is a speculation based on the Einstein equation, which is not valid for the dynamic case, this notion must be thoroughly reviewed.

From (35), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\, t(g)_{\mu\nu}. \quad (36)$$

Note that $Kt(g)_{\mu\nu}$ is equivalent to $G^{(2)}{}_{\mu\nu}$ (and Einstein's gravitational pseudotensor) in terms of his radiation formula [11]. Thus, $Kt(g)_{\mu\nu}$ makes that eq. (30) satisfies the principle of causality and has a bounded dynamic solution [11-13].

The gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero for gravitational waves. Thus, a radiation does carry energy-momentum as required. This explains also that the absence of an anti-gravity coupling, which is determined by Einstein's radiation formula, is the physical reason that the 1915 Einstein equation (1) is incompatible with radiation. Moreover, it is clear now the linearized equation is a mathematical linearization of the modified Einstein equation.

Now, the non-existence of dynamic solutions has been established. It necessarily follows that their singularity theorems are actually irrelevant to physics. Moreover, general relativity is applicable to micro-phenomena. In fact, it has been shown that the photons must include gravitational energy [6]. Both quantum theory and relativity are based on the phenomena of light. The gravity of photons finally shows that there is a link between them. It is gravity that makes the notion of photons compatible with electromagnetic waves [6].

In the above, errors are due to inadequacy in mathematics. Thus, one might expect that the mathematicians would remedy all these problems. Unfortunately, things are not that simple, and the mathematicians such as M. Atiyah,[7)] D. Hilbert, E. Witten [8)] and S. T. Yau, have a common problem, they do not understand the related physics.

6. Problems in Mathematics due to a Failure in Understanding the Physics

In mathematics, a theorem need not be simply right or wrong, but misleading because some invalid implicit assumptions. Such misleading theorems could be very damaging because of its superficial validity in mathematics. In fact, such an error can be made by top mathematicians such as M. Atiyah. Thus, such misleading results were cited as a main reason to award the 1982 and the 1990 Fields Medal to Yau and Witten, and to award the 2011 Shaw Prize in mathematics to Christodoulou. [9)]

Briefly, the positive mass conjecture [33, 34] says that if a three-dimensional manifold has positive scalar curvature and is asymptotically flat, then the mass in the asymptotic expansion of the metric is positive (Wikipedia). The unique coupling signs are also implicitly used in the positive energy theorem of Schoen and Yau [33, 35]. A crucial assumption in the theorem is that the solution is asymptotically flat. Yau [33] requires the metric,

$$g_{ij} = \delta_{ij} + O(r^{-1}). \quad (37)$$

The motivation of (37) is clearly the linearized equation (13). Assumption (37) can be considered as common since it is satisfied in stable solutions such as the Schwarzschild solution, the harmonic solution, the Kerr solution, etc. [10)] Thus, this theorem was almost universally accepted for a long time as a standard argument or the truth in general relativity.

However, if one understands the physics in general relativity as well as Gullstrand [9] does, the above statement is clearly incorrect. Note that the condition of asymptotically flat does not necessarily imply the inclusion of a dynamic solution. Apparently, Schoen and Yau assumed it did [11)] because they failed to see that, for a dynamic case the linearized equation and the non-linear Einstein equation is not compatible [36].

In fact, it has been proven that the Einstein equation has no dynamic solution, which is bounded [11-13]. Thus, the assumption of asymptotically flat implies the exclusion of the dynamic solutions. However, Schoen and Yau failed to see this because it is difficult to see whether there is a dynamic solution in their approach. In other words, the conclusions drawn from the positive theorem are grossly misleading.[12)]

The problem rises from the Einstein equation that does not have a bounded dynamic solution as Gullstrand suspected [6]. Thus, Yau and Schoen used an invalid implicit assumption, the existence of bounded dynamic solutions, but was not stated in their theorem. Atiyah, being a pure mathematician, was not aware of the problem of non-existence of bounded dynamic solutions. Moreover, this made Witten and associated string theorists failed to understand general relativity, and thus would not be able to do meaningful work on unification.

Theorists such as Yau [33], Christodoulou [21], Wald [5], and Penrose & Hawking [5] make essentially the same error of defining a set of solutions that actually includes no dynamic solutions [11-13, 24]. This is why many theorists agree with each other in making the same errors. The fatal error is that they neglected to find explicit examples to support their claims. Had they tried, they should have discovered their errors. Moreover, the same error [37] was cited in awarding to Christodoulou the 2011 Shaw Prize.[9)] Subsequently, Christodoulou was elected to the Member of U.S. National Academy of Sciences (2012).[13)] It would be interesting to see how this special case would end up since the contributions of Christodoulou to general relativity are just errors.

Many theorists use the linearized equation and obtained satisfactory results. However, they did not stop to think what does a linearization means when the non-linear Einstein equation has no bounded solutions. From the explicit calculations, clearly for the dynamic case, the linearized equation is independent of the non-linear Einstein equation. The calculation for the gravitational wave illustrates the following:

1) The linearized equation and the non-linear Einstein equation have no compatible solutions.

2) The non-linear Einstein equation for waves has no bounded solution.

3) Since, for the dynamic case, the Einstein equation and its linearized equation are independent equations, the linearization procedure is not valid for this case.

Although we have formally obtained a modified Einstein equation, this rectification is still incomplete because the exact form of the gravitational energy-stress tensor $t(g)_{\mu\nu}$ is still not known.

Moreover, in a dynamic situation, the geodesic as the equation of motion is also inadequate because there is no radiation reaction force in general relativity. Although an accelerated massive particle would create radiation [5, 7], the metric elements in the geodesic equation are created by particles other than the test particle [7]. Thus, Einstein's general relativity is intrinsically incomplete even for the case of massive sources.

7. Unification of Gravitation and Electromagnetism

Now, let us consider the case that electromagnetism is involved. Since the photons include gravitational energy, the unification of gravitation and electromagnetism is necessary. In general relativity, it is clear that a charge can create a field that couples with a mass. This is shown [1] by the Reissner-Nordstrom metric [15] (with c = 1) created by a particle with charge q and mass M is as follows (see also remark 11) in Appendix A):

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \quad (37)$$

However, it is not clear that a mass can create a field that couples with a charge.

Consider the static force that acts on a test particle P with mass m. For the first order approximation this force is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \quad (38)$$

since $g^{rr} \cong -1$. Note that it is clear that the second term is a repulsive force due to the static charge-mass interaction [1]. Since the reaction force is equal to but in the opposite direction of the acting force, the test particle P must create a field m/r^3 that couples to q^2. This would mean that unification between electromagnetism and gravitation is necessary. However, the new force that should have been discovered in 1916 was over-looked until 1997 [38].

This was so because of two misconceptions: 1) Gravity is always attractive; 2) $E = mc^2$ is incorrectly considered as unconditional. The non-existence of dynamic solution for the Einstein equation leads to the discovery that there must be different coupling signs for the dynamic case [11]. This non-uniqueness of couplings leads to the investigation of whether $E = mc^2$ is conditional. Thus, the charge-mass interaction is discovered [39]. The experimental confirmation of the charge-mass interaction would confirm the unification of electromagnetism and gravitation. Einstein over-looked the coupling of charge square in the five-dimensional theory [40] because he believed that, unlike Maxwell, that a new interaction should not be created. In other words, Einstein has over confidence in general relativity. Since formula (38) is generated by general relativity and thus is also a test for general relativity.

However, the charge square coupling is clearly beyond Einstein's general relativity. Moreover, in a four-dimensional theory the electromagnetic field is subjected to electromagnetic screening. On the other hand, the q^2 factor is independent of the sign of the charge and therefore the force should not be subjected to electromagnetic screening. To find out this, the best way would be to test whether such a force would act on a charged capacitor. Experiments show that such a force does act on a charged capacitor and its force is proportional to square of potential difference as expected [39]. This is possible in a five-dimensional theory because the repulsive force can be generated by the mass m through the new metric element g_{55} [41]. If $dx^5/d\tau = q/Mc^2K$, one would have

$$\frac{mq^2}{\rho^3} \approx -\Gamma_{\rho,55}\frac{1}{K^2}\frac{q^2}{Mc^2}\, g^{\rho\rho} \quad \text{where}$$

$$\Gamma_{k,55} \equiv \frac{\partial g_{i5}}{\partial x^5} - \frac{1}{2}\frac{\partial g_{55}}{\partial x^k} = -\frac{1}{2}\frac{\partial g_{55}}{\partial x^k} \quad . \qquad (39)$$

Thus, a five-dimensional theory would be a potential candidate for unification as Einstein envisioned [40]. Moreover, the repulsive force can be verified with experiments (see Appendix B).

In short, it is clear that unification of electromagnetism and gravitation is necessary although the exact theory of unification is not yet very clear. For this, one must understand the fifth variable in a five-dimensional theory.

8. Conclusions and Discussions

Now, since the Einstein equation has no dynamic solution, rectification of the theory is necessary. The important conclusions are: 1) $E = mc^2$ is not generally valid. In particular, the electromagnetic energy alone is not equivalent to mass. 2) The photons include energy from its gravitational components. 3) Einstein's general relativity is valid only for the static and stable cases. However, for the dynamic case, it remains to be rectified [36] in at least two aspects: a) The exact form of the gravitational energy-stress tensor is not known; and b) The radiation reaction force is also not known. Due to the radiation reaction force, considering general relativity as a theory of geometry is questionable. Since the full field equation is unclear, the notion of black holes should also be reviewed. Moreover, since the photons include gravitational energy, the unification of gravitation and electromagnetism is necessary. A five-dimensional theory [39] would be a candidate because it includes the charge-mass interaction with charge-square coupling. This analysis reveals also that some physicists are inadequate in understanding the mathematics and the principle of causality. [14)]

Also, there are positive results learned from the issue of dynamic solutions. The existence of a dynamic solution requires [11], an additional gravitational energy-momentum tensor with an antigravity coupling, i.e., the Lorentz-Levi-Einstein equation.[6)] Thus, the space-time singularity theorems, which require the same sign for couplings, are actually irrelevant to physics. Recall the need of modifying the Einstein equation for the gravity of the electromagnetic wave. In such a modification, the anti-gravity coupling of the photonic energy-stress tensor must also be added [8]. Otherwise there is no solution for the gravity of an electromagnetic wave. Thus, the anti-gravity coupling is important. However, string theorists, such as Witten [34] failed in understanding the necessity of such anti-gravity coupling.

Using the assumption of unique coupling sign implicitly, the positive energy theorem of Schoen and Yau is misleading [35].[12)] For instance, without a proof, they act as if that their theorem includes the case of dynamic solutions. The tragic is that Yau [33] and Witten [34] produced a misleading result, but failed to see this as a problem. Such error should be responsible for the lack of progress in String theory. Moreover, $E = mc^2$ is not generally valid [20], and such recognition is crucial to identify, after 80 years, the charge-mass interaction [1]. This repulsive force explains the weight reduction of charged capacitors and potentially the Space-Probe Pioneer anomaly [42]. This force is not subjected to electromagnetic screening because it is proportional to the charge square. Thus, it would be useful to detect things since the strength of such detection can be adjusted with the potential of a charged capacitor [39].

The verifications of the charge-mass interaction imply that Einstein's unification between electromagnetism and gravitation is proven valid [39, 42]. Einstein failed to see this himself because of his two shortcomings: 1) He failed to see, as Maxwell showed, that unification is necessary to create new interactions. 2) $E = mc^2$ was mistaken as unconditional. Although this newly discovered repulsive force can be detected from its action to a charged capacitor, such a force cannot be generated in current four-dimensional theories. Perhaps, this is why Einstein's unification was not recognized for a long time. Hence, Einstein turns out to be the biggest winner at the end.

Many theorists started their errors because of their faith to Einstein's claim on the existence of dynamic solutions for the Einstein equation. A theoretical obstacle was the invalid speculation that $E = mc^2$ being unconditionally true, and consequently Harvard University and the Princeton University were the sources of major errors [43,44]. It should be noted that the charge-mass interaction implies that the assumption of Galileo that the acceleration of gravity is universal for all neutral massive matter may not always be valid [45], and thus a new revolution in physics has begun.

ACKNOWLEDGMENTS

This paper is dedicated to Prof. P. Morrison of MIT for his unfailing guidance over 15 years. The author is grateful to Prof. A. J. Coleman for pointing out some errors in general relativity, and to Prof. I. Halperin for the mathematical foundation of Einstein's theory. The author wishes to express his appreciation to S. Holcombe for valuable suggestions. This publication is supported by Innotec Design, Inc., U.S.A. and the Chan Foundation, Hong Kong.

Appendix A: Summary of Misrepresentations and Errors in General Relativity

For the convenience of the readers, the errors and misinterpretations in general relativity are summarized in this Appendix. The first error, suspected by Gullstrand [9], is the non-existence of dynamic solutions. However, in 2011 half of a Shaw Prize for mathematics was awarded to Christodoulou [37] for his errors against Gullstrand. This error has been well-established because it can be illustrated with examples understandable at the undergraduate level.

The fundamental issues that historically relate to errors are:

1) Einstein's 1911 assumption of equivalence between acceleration and Newtonian gravity [46]: It was used to derive the correct gravitational redshifts, but the so-obtained light bending deflection disagrees with observation.

2) Einstein's equivalence principle [7]: The effects of an accelerated frame are equivalent to a uniform gravity (generated by a metric). In physics, the local metric of a particle under the influence of gravity is a local Minkowski metric [7]. This principle can be illustrated with explicit examples and is supported by experiments. Since the local metric of the earth is only a locally constant metric at one point, Einstein pointed out that the gravity cannot be transformed away by using an accelerated frame. Thus, gravity and acceleration are not generally equivalent.

a) Pauli's misinterpretation [47]: Pauli claimed that the gravity of an infinitesimal region can be transformed away; but the local metric of a particle need not be locally Minkowski.

b) The misinterpretation of Misner, Thorne & Wheeler [15]: They agree with Pauli and claimed that gravity is equivalent to acceleration in a small region of the local metric. What they referred to is the Newtonian gravity (since they agree with Fock [48] and reject the principle). Moreover, they claimed that in such a small region the local metric is necessarily Minkowski (the so-called Lorentz invariance). However, their notion of Lorentz invariance is incorrect in mathematics and is not favored by the 2009 experiment of Chu et al. [49].

c) Fock [48] misinterpreted that Einstein's equivalence principle as the 1911 assumption. He shows that it is impossible to have a metric for the Newtonian gravity in general relativity; and invalidly rejected the principle.

3) Einstein's covariance principle: Einstein extended his principle of general relativity to unrestricted mathematical covariance and called it as the "principle of covariance". Since different gauges would lead to different physical interpretations of the coordinates [50, 51], this is in conflict with his equivalence principle which implies the local time dilation and space contractions are unique. The experiment supports Einstein's equivalence principle.

4) Einstein's measurement of the distance [7]: Einstein adapted the notion of distance in a Riemannian space. Such an adaptation has been pointed out by Whitehead [52] as invalid in physics. Also, it is found that his justifications for his adaptation are due to invalid applications of special relativity [53]. It turns out that the correct theory of measurement [54] is just what Einstein practiced in his calculation of light bending. Then, the measurement of distance is consistent with the observed bending of a light ray [55]. Thus, it becomes clear that to regard the Hubble redshifts as due to the Doppler effects is invalid [56], as Hubble himself also disagrees.

5) The question of a physical gauge: The invalidity of the covariance principle exposed an urgent issue, i.e., to find a valid physical gauge for a given problem. Fortunately, the Maxwell-Newton approximation has been proven to be an independently valid first order approximation for gravity due to massive sources [12], so that the binary pulsar radiation experiments can be explained satisfactorily [11, 13]. Thus, Einstein's notion of weak gravity (including gravitomagnetism and gravitational radiation [57] [15)]) is valid [12, 17, 18].

6) The principle of causality is implicitly used in any scientific research. In general relativity, this principle is implicitly used by Einstein in symmetry considerations [7]. However, theorists such as Penrose [58] and 't Hooft [22, 23] do not understand this principle adequately. The Physical Review also failed to understand the principle of causality adequately and thus mistakenly believed that the non-linear Einstein equation has wave solutions [22].

7) Invalidity of linearization [20]: Currently, to obtain an approximation through linearizing the Einstein equation is incorrectly believed as generally valid because linearization has been successful for the static case of massive source. However, this process of linearization for the dynamic cases is invalid since the Einstein equation actually has no bounded dynamics solutions [11, 13] because the principle of causality is violated.

8) Bounded dynamic solutions: The Einstein equation has no bounded dynamic solution. Thus the perihelion of Mercury is beyond the reach of Einstein as Gullstrand [9] suspected; and the calculation for the gravitational radiation of binary pulsars is actually invalid. A conclusion from this result is that all the coupling constants cannot have the same sign, and thus the physical assumption of the space-time singularity theorems [5] is invalid.

9) The sign of coupling constants being unique was

accepted since $E = mc^2$ was considered as unconditional. However, the electromagnetic energy cannot be equivalent to mass since the trace of an electromagnetic energy-stress tensor is zero. In fact, for several years, Einstein had tried and failed to prove this formula for other type of energy [2].

10) The photons must have non-electromagnetic energy because the meson π_0 decays into two photons. The immature assumption that the photons have only electromagnetic energy was proposed before general relativity. Since a charged particle is massive, it is not surprising that the photons should include also gravitational energy.

11) The static Einstein equation with the source of a charged particle implies the existence of a static repulsive force between a charge and a massive particle. Moreover, such a repulsive effect has been inadvertently observed by Tsipenyuk & Andreev [59]. Verification of such a force is necessary for general relativity. Thus, unification of gravitation and electromagnetism is actually necessary. Some argued that the effective mass of such a charged particle should be $(M - q^2/2r)$ or M should include the electromagnetic energy outside the particle. However they have been proven as theoretically invalid and against observation.

Note that all the errors are directly or indirectly related to distortions of Einstein's equivalence principle. The invalid speculation of unconditional validity of $E = mc^2$ is the source of many errors in general relativity, and thus Einstein's general relativity is not yet complete. Its completion would be crucial to explain the Hubble redshifts and the pioneer anomaly discovered by NASA [60-63], and may even be needed to explain problem of renormalization.

Appendix B: Experimental Verification of the Mass-Charge Repulsive Force

The repulsive force in metric (37) can be detected with a neutral mass. To see the repulsive effect, one must have

$$\frac{1}{2}\frac{\partial}{\partial r}\left(1-\frac{2M}{r}+\frac{q^2}{r^2}\right)=\frac{M}{r^2}-\frac{q^2}{r^3}< 0 \qquad \text{(A1)}$$

Thus, repulsive gravity would be observed at $q^2/M > r$. For the electron the repulsive gravity would exist only inside the classical electron radius r_0 $(= 2.817\times 10^{-13}cm)$. Thus, it would be very difficult to test a single charged particle.

However, for a charged metal ball with mass M and charge Q, the formula is similarly $0 > M/R^2 - Q^2/R^3$, where R is the distance from the center of the ball [64]. Consequently, the attractive effect in gravity is proportional to mass related to the number of electrons, but the repulsive effect in gravity is proportional to square of charge related to the square of the number of electrons. Thus, when the electrons are numerous enough accumulated in a metal ball, the effect of repulsive gravity will be shown in a macroscopic distance.

Consider Q and M is consist of N electrons, i.e., $Q = Ne$, $M = Nm_e + M_0$, where M_0 is the mass of the metal ball, m_e and e are the mass and charge of an electron. To have sufficient electrons, the necessary condition is

$$N > \frac{R}{r_0}, \text{ where } r_0 = \frac{e^2}{m_e c^2} = 2.817\times 10^{-13} \text{ cm.} \qquad \text{(A2)}$$

For example, if $R = 10\ cm$, then it requires $N > 3.550\times 10^{13}$. Thus $Q = 5.683\times 10^{-7}$ *Coulomb*. Then, one would see the attractive and repulsive additional forces change hands. For this case, the repulsive force is

$\frac{Q^2 m_p}{R^3}$ where m_p is the mass of the testing particle P.(A3)

And the total force is

$$\left(\frac{M_0 + Nm_e}{R^2} - \frac{N^2e^2}{R^3}\right)m_p \qquad \text{(A4)}$$

When condition (A2) is satisfied for a certain R, the repulsive effect will be observed as the charge increases. However, since the repulsive force is very small, the interference of electricity would be comparatively large.

Thus, it would be desirable to screen the electromagnetic effects out. The modern capacitor is such a piece of simple equipment. When a capacitor is charged, it separates the electron from the atomic nucleus, but there is no change of mass due to increase of charged particles. Before such separation the effect of the charge-mass interaction is cancelled out by the current-mass interaction (see Appendix C). Thus, after charged, the capacitor would have less weight due to the charge-mass repulsive force, a nonlinear force towards charges. This simple experiment would confirm the mass-charge repulsive force, and thus the unification in term of a five-dimensional theory.

One may ask whether the lighter weight of a capacitor after charged could be due to a decrease of mass. Such a speculation is ruled out. Inside a capacitor the increased energy due to being charged would not be pure electromagnetic energy such that, for the total internal energy, Einstein's formula is valid.

In the case of charged capacitor, the repulsive force would be proportional to the potential square, V^2 where V is the electric potential difference of the capacitor. This has been verified by the experiments of Musha [65]. However, the weigh reduction phenomenon is currently mixed up with the B-B effect which is directional to the electric field applied. However, the weigh reduction effect is not directional and it stays if the potential does not change. This is verified by Liu [66], who measured the effect of weight reduction with the roll-up capacitors.

Appendix C: The Current-mass Interaction

If the electric energy leads to a repulsive force toward a mass, according to general relativity, the magnetic energy would lead to an attractive force from a current toward a mass [67, 68]. The existence of such a current-mass attractive force has been verified by Martin Tajmar and Clovis de Matos [69] from the European Space Agency.

They found that a spinning ring of superconducting material increases its weight much more than expected. Thus, they believed that general relativity had been proven wrong. However, according to quantum theory, spinning super-conductors should produce a weak magnetic field. Thus, they are also measuring the interaction between an electric current and the earth, i.e. an effect of the current-mass interaction!

The existence of the current-mass attractive force would solve a puzzle, i.e., why a charged capacitor exhibits the charge-mass repulsive force since a charged capacitor has no additional electric charges? In a normal situation, the charge-mass repulsive force would be cancelled by other forms of the current-mass force as Galileo, Newton and Einstein implicitly assumed. This general force is related to the static charge-mass repulsive force in a way similar to the Lorentz force is related to the Coulomb force.

One may ask what is the formula for the current-mass force? However, unlike the static charge-mass repulsive force, which can be derived from general relativity; this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generates electromagnetic radiation. Moreover, when the radiation is involved, the radiation reaction force and the variable of the fifth dimension must be considered [41]. Thus, we are not ready to derive the current-mass interaction yet.

Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass [4]. The irradiated ball has the extra electrons compared to a normal ball. A spinning ring of superconducting material has the electric currents that are attractive to the earth. This also explains a predicted phenomenon, which is also reported by Liu [66] that it takes time for a capacitor to recover its weight after being discharged [4]. This was observed by Liu because his rolled-up capacitors keep heat better. A discharged capacitor needs time to dissipate the heat generated by discharging, and the motion of its charges would accordingly recover to normal.

Endnotes

1) For a thorough discussion on the relation between the mass and the total energy of a particle, one can read the 1989 paper of L. B. Okun [70]. However, Okun did not understand that the electromagnetic energy is not equivalent to mass [1] as shown in his 2008 paper [71].

2) Currently a major problem in general relativity is not only that Einstein's errors are over-looked, but also that some theorists claimed as "experts" [5, 14, 15, 21, 25, 33, 34] who additionally make their own errors.

3) Theorists such a Misner et al. [15], Wald [5], Christodoulou & Klainerman [21], 't Hooft [23], and etc. claimed to have explicit dynamic solutions. It turns out that all these are due to errors in mathematics [35, 43, 44].

4) A well-known misleading result is the positive mass theorem of Schoen & Yau [33] (and the positive energy theorem of Witten [34]).

5) Some considered that even the photon has mass in terms of $m = E/c^2$, the so-called electromagnetic mass. This is due to confusing the difference between mathematics and physics. To define a mass in terms of mathematics would be misleading if it is not supported with experiments. They must show also that such an electromagnetic mass would produce the same gravitational effect as the inertial mass.

6) This equation was first proposed by Lorentz [72] and later Levi-Civita [73] as a possibility in the following form,

$$\kappa t(g)_{ab} = G_{ab} + \kappa T_{ab} \qquad \text{(LL)}$$

where $t(g)_{ab}$ is the gravitational energy-stress tensor, G_{ab} is the Einstein tensor, and T_{ab} is the sum of other massive energy-stress tensors. Then, the gravitational energy-stress tensor takes a covariant form, although they have not proved its necessity with calculations. Thus, the anti-gravity coupling was first proposed by Lorentz [72] and Levi-Civita [73]. However, Einstein [74] objected to this form on the grounds that his field equation implies $t(g)_{ab} = 0$. Now, Einstein is clearly wrong since his equation is proven invalid for the dynamic case [11-13]. Thus, eq. (35) should be called the Lorentz-Levi-Einstein equation.

7) Michael Francis Atiyah has been leader of the Royal Society (1990-1995), master of Trinity College, Cambridge (1990-1997), chancellor of the University of Leicester (1995-2005), and President of the Royal Society of Edinburgh (2005-2008). Since 1997, he has been an honorary professor at the University of Edinburgh (Wikipedia). Apparently, Atiyah does not understand the physics and the non-existence of a dynamic solution.

8) Ludwig D. Faddeev, the Chairman of the Fields Medal Committee, wrote ("On the work of Edward Witten"):

"Now I turn to another beautiful result of Witten – proof of positivity of energy in Einstein's theory of gravitation. Hamiltonian approach to this theory proposed by Dirac in the beginning of the fifties and developed further by many people has led to the natural definition of energy. In this approach a metric γ and external curvature h on a space-like initial surface $S^{(3)}$ embedded in space-time $M^{(4)}$ are used as parameters in the corresponding phase space. These data are not independent. They satisfy Gauss-Codazzi constraints – highly non-linear PDE, The energy H in the asymptotically flat case is given as an integral of indefinite quadratic form of $\nabla \gamma$ and h. Thus, it is not manifestly positive. The important statement that it is nevertheless positive may be proved only by taking into the account the constraints – a formidable problem solved by Yau and Schoen in the late seventy as Atiyah mentions, 'leading in part to Yau's Fields Medal at the Warsaw Congress'.

Witten proposed an alternative expression for energy in terms of solutions of a linear PDE with the coefficients expressed through γ and h ….."

9) The 2011 Shaw Prize also made a mistake by awarding a half prize to Christodoulou for his work, based on obscure errors, against the honorable Gullstrand [9] Chairman (1922-1929) of the Nobel Committee for Physics. Professor Christodoulou has misled many including the 1993 Nobel Committee [75] because of their inadequacy in mathematics. However, his errors are now well-established since they have been illustrated with mathematics at the undergraduate level [44]. Starting from J. A. Wheeler, it seems, members of the Wheeler School have a problem of competency in mathematics. Christodoulou claimed in his autobiography that his work is essentially based on two sources: 1) The claims of Christodoulou and Klainerman on general relativity as shown in their book ***The Global Nonlinear Stability of the Minkowski Space*** [21]; 2) Roger Penrose had introduced, in 1965, the concept of a trapped surface and had proved that a space-time containing such a surface cannot be complete [5]. However, this work of Penrose, which uses an implicit assumption of unique sign for all coupling constants, actually depends on the errors of Christodoulou and Klainerman [21, 24]. However, such a relation was not clear until 1995 [11].

10) These solutions have no gravitational radiation.

11) n 1993, I met Prof. Yau in The Chinese University of Hong Kong and discussed the non-existence of dynamic solutions for the Einstein equation. Then, Yau avoids contacting me since he failed to defend such existence.

12) I have written to Horng-Tzer Yau, Editor in Chief of Communications in Mathematical Physics, on the misleading nature of the papers of Yau & Schoen [33] and Witten [34]. However, I have no response from him on this so far.

13) The Ph. D. degree advisor of D. Christodoulou is J. A. Wheeler, whose mathematics is also unreliable. The honors awarded to Christodoulou reflected the blind faith toward Einstein and accumulated errors in general relativity [35]. For instance, Prof. Yum-Tong Siu of Harvard University, who approved awarding him the 2011 Shaw Prize against the honorable Gullstrand [44], is also a mathematician with little background in general relativity or physics. I have asked about the reason of giving Christodoulou the award, but he could not give a clear explanation.

14) These problems explain why, for a long time, there is little progress in general relativity. It is hoped that the situation would change after the rectifications.

15) However, Edmund Bertschinger does not understand that, for the dynamic case, the non-linear Einstein equation and the linearized equation are not compatible [36].

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