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Lossless Transmission Lines Terminated by in-Series Connected *RL*-Loads Parallel to *C*-Load

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Abstract The paper deals with an analysis of transmission line terminated by in series connected *RL*-loads parallel to *C*-load. The mixed problem for Telegrapher equations to a periodic problem on the boundary is reduced. The obtained neutral system of equations in an operator form is presented. The fixed points of the operator in question are solutions of the periodic problem.

Keywords Lossless Transmission Lines, *RLC*-Loads, Periodic Solutions, Fixed Point Theorems

1. Introduction

The main purpose of the present paper is to consider a lossless transmission line loaded by in series connected nonlinear *RL*-loads parallel to *C*-load. Such a configuration arises not only in radio frequencies devices but in various geophysical studies as well (cf.[1]). Our goal is to demonstrate the advantages of our method[2] used in analogous problems. So we came up with an approach to solve this set of problems (cf.[3]-[7]).

The primary purpose of the present paper is twofold. First, to formulate the mixed problem for hyperbolic Telegrapher equations corresponding to the nonlinear circuits on Figure 1. The boundary conditions are nonlinear ones in view of the nonlinear characteristics of the loads. Important first step on the base of Kirchoff's law is to derive the boundary conditions in the form of differential equations on the boundary. This is done in **2.1**. Reducing the mixed problem to a neutral system on the boundary is made in **2.2** following the technique from[2],[5] and[6].

Second, to present a method for solving neutral equations with "bad" (non-Lipschitz) nonlinearities. In **2.3** the domains of the nonlinear characteristics are defined. They have to possess strictly positive lower bounds and namely Assumption **(C)** for capacitive functions and **(L)** for inductive functions ensure that. In **2.3** the choice of functional spaces and a family of pseudo-metrics is directly related to the operator representation of the periodic problem in **2.4**. Let us point out that the extension of Bielecki norm allows to overcome the difficulties caused by nonlinearity of the characteristic functions.

The key role plays Lemma 3. It guaranties that the fixed points of the operator defined in **2.4** are periodic solutions of the neutral system and conversely. The main result is Theorem 1 (cf. **2.5**) and its proof consists of two parts. The first one is to show that operator B maps the set $M_0 \times M_U \times M_1 \times M_I$ into itself. The second one is to show that B is contractive operator. The fixed point of B is the required periodic solution. The numerical example in **2.6** shows that for applications are required only inequalities obtained in the proof of the main theorem.

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2. Main Results

2.1. Formulation of the Problem

Let Λ be the length of the transmission line and $T = \Lambda / v = \Lambda /(1/\sqrt{LC}) = \Lambda\sqrt{LC}$, $Z_0 = \sqrt{L/C}$, where L is per unit length inductance and C – per unit-length capacitance. In accordance of Kirchoff's voltage-law (Figure 1) we have to add the voltages of the elements R_0 and L_0 after that to define the current of $R_0 L_0$ and finally to add it with the current of C_0. In the real cases parallel to $R_0 L_0$ and C_0 is connected an input voltage $g_m \breve{U}_{in}(t)$, where g_m is the amplification coefficient. Since we have to add the currents one can replaced $g_m \breve{U}_{in}(t)$ by an equivalent current source $\breve{I}_{in}(t)$. We assume that the second end is terminated by the same configuration.

Assume that R_0, L_0 and C_0 are nonlinear elements, that is, $R_0 = R_0(i), L_0 = L_0(i)$ and $C_0 = C_0(u)$ are nonlinear functions. So we have

$$u_{R_0} = R_0(i),$$

$$u_{\Psi_0} = \frac{d\Psi_0}{dt} = \frac{d\widetilde{L}_0(i)}{dt} \equiv \frac{d\left(L_0(i).i\right)}{dt} = \left[i\frac{dL_0(i)}{di} + L_0(i) \right] \frac{di}{dt}$$

and then

$$u_{R_0L_0} = R_0(i_{R_0L_0}) + \left[i_{R_0L_0} \frac{dL_0(i_{R_0L_0})}{di_{R_0L_0}} + L_0(i_{R_0L_0}) \right] \frac{di_{R_0L_0}}{dt}.$$

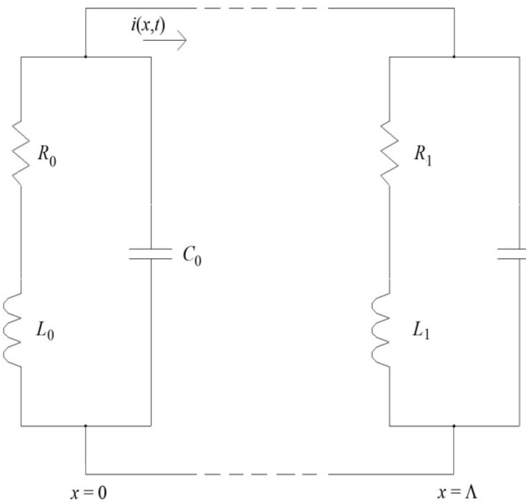

Figure 1. Lossless transmission line terminated at both ends by in series connected *RL*-loads parallel to *C*-load

Since current $I_{input}(t)$ is connected parallel to *RL*-elements then Kirchoff's current-law yields

$$-i(0,t) = I_{input}(t) + i_{R_0L_0} + i_{C_0}. \qquad (1)$$

Since $u_{R_0L_0}(t) = u(0,t),\ i_{C_0} = \dfrac{dq_{C_0}}{dt} = \dfrac{d(C_0(u).u)}{dt}$ then $i_{R_0L_0}$ can be found as a solution of differential equation

$$\left[i_{R_0L_0} \frac{dL_0(i_{R_0L_0})}{di_{R_0L_0}} + L_0(i_{R_0L_0}) \right] \frac{di_{R_0L_0}(t)}{dt} = u(0,t) - R_0(i_{R_0L_0}).$$

But $u(0,t)$ is unknown too and then from (1) we have

$$-i(0,t) = I_{input}(t) + i_{R_0L_0}(t) + \frac{d(C_0(u(0,t)).u(0,t))}{dt}$$

or one more differential equation:

$$-i(0,t) = I_{input}(t) + i_{R_0L_0}(t) + \\ + \frac{du(0,t)}{dt} \left[\frac{dC_0(u(0,t))}{du} u(0,t) + C_0(u(0,t)) \right].$$

Analogously for the right end we have

$$\left[i_{R_1L_1}(t) \frac{dL_1(i_{R_1L_1}(t))}{di_{R_1L_1}} + L_1(i_{R_1L_1}(t)) \right] \frac{di_{R_1L_1}(t)}{dt} = \\ = u(\Lambda,t) - R_1(i_{R_1L_1}(t)),$$

$$\frac{du(\Lambda,t)}{dt} \left[\frac{dC_1(u(\Lambda,t))}{du} u(\Lambda,t) + C_1(u(\Lambda,t)) \right] = \\ = -i(\Lambda,t) - i_{R_1L_1}(t).$$

Now we are able to formulate the initial-boundary value (mixed) problem for the transmission line equations: to find a solution $(u(x,t), i(x,t))$ of the first order partial differential system of hyperbolic type

$$\frac{\partial u(x,t)}{\partial t} + L\frac{\partial i(x,t)}{\partial x} = 0,\ \frac{\partial i(x,t)}{\partial t} + C\frac{\partial u(x,t)}{\partial x} = 0 \qquad (2)$$

for

$$(x,t) \in \Pi = \{(x,t) \in R^2 : 0 \le x \le \Lambda,\ t \ge 0\}$$

satisfying the initial conditions

$$u(x,0) = u_0(x),\ i(x,0) = i_0(x) \text{ for } x \in [0,\Lambda] \qquad (3)$$

and the boundary conditions for $x = 0$

$$\begin{aligned} &\left[i_{R_0L_0}(t) \frac{dL_0(i_{R_0L_0}(t))}{di_{R_0L_0}} + L_0(i_{R_0L_0}(t)) \right] \frac{di_{R_0L_0}(t)}{dt} = u(0,t) - R_0(i_{R_0L_0}(t)), \\ &\left[\frac{dC_0(u(0,t))}{du} u(0,t) + C_0(u(0,t)) \right] \frac{du(0,t)}{dt} = -i(0,t) - \breve{I}_{in}(t) - i_{R_0L_0}(t) \end{aligned} \qquad (4)$$

and for $x = \Lambda$

$$\begin{aligned} &\left[i_{R_1L_1}(t) \frac{dL_1(i_{R_1L_1}(t))}{di_{R_1L_1}} + L_1(i_{R_1L_1}(t)) \right] \frac{di_{R_1L_1}(t)}{dt} = u(\Lambda,t) - R_1(i_{R_1L_1}(t)), \\ &\left[\frac{dC_1(u(\Lambda,t))}{du} u(\Lambda,t) + C_1(u(\Lambda,t)) \right] \frac{du(\Lambda,t)}{dt} = -i(\Lambda,t) - i_{R_1L_1}(t). \end{aligned} \qquad (5)$$

2.2. Reducing the Mixed Problem to an Initial Value Problem on the Boundary

We proceed from the lossless transmission line equations

$$\frac{\partial u(x,t)}{\partial x} + L\frac{\partial i(x,t)}{\partial t} = 0,\ \frac{\partial i(x,t)}{\partial x} + C\frac{\partial u(x,t)}{dt} = 0. \qquad (6)$$

Rewrite system (6) in the form

$$\frac{\partial u(x,t)}{\partial t} + \frac{1}{C}\frac{\partial i(x,t)}{\partial x} = 0, \sqrt{\frac{L}{C}}\frac{\partial i(x,t)}{\partial t} + \sqrt{\frac{1}{LC}}\frac{\partial u(x,t)}{\partial x} = 0. \qquad (7)$$

Adding and subtracting (6) and (7) in view of $Z_0 = \sqrt{L/C}$ we get:

$$\begin{aligned}&\frac{\partial}{\partial t}(u+Z_0\,i)+v\frac{\partial}{\partial x}(u+Z_0\,i)=0,\\&\frac{\partial}{\partial t}(u-Z_0\,i)-v\frac{\partial}{\partial x}(u-Z_0\,i)=0.\end{aligned}\tag{8}$$

Let us put

$$U(x,t)=u(x,t)-Z_0\,i(x,t),\quad I(x,t)=u(x,t)+Z_0\,i(x,t)$$

and hence

$$\frac{\partial U}{\partial t}-v\frac{\partial U}{\partial x}=0,\quad \frac{\partial I}{\partial t}+v\frac{\partial I}{\partial x}=0.$$

It follows

$$u(x,t)=\frac{U(x,t)}{2}+\frac{I(x,t)}{2},\quad i(x,t)=-\frac{U(x,t)}{2Z_0}+\frac{I(x,t)}{2Z_0}.\tag{9}$$

For $x=0$ and $x=\Lambda$ we have

$$u(0,t)=\frac{U(0,t)}{2}+\frac{I(0,t)}{2},\quad i(0,t)=-\frac{U(0,t)}{2Z_0}+\frac{I(0,t)}{2Z_0},$$

$$u(\Lambda,t)=\frac{U(\Lambda,t)}{2}+\frac{I(\Lambda,t)}{2},\quad i(\Lambda,t)=-\frac{U(\Lambda,t)}{2Z_0}+\frac{I(\Lambda,t)}{2Z_0}.$$

Replace them in (4) and (5) we obtain

$$\left[i_{R_0L_0}(t)\left(dL_0(i_{R_0L_0}(t))/di_{R_0L_0}\right)+L_0(i_{R_0L_0}(t))\right]\frac{di_{R_0L_0}(t)}{dt}=$$

$$=\frac{1}{2}U(0,t)+\frac{1}{2}I(0,t)-R_0(i_{R_0L_0}(t)),$$

$$\left[u(0,t)\left(dC_0(u(0,t))/du\right)+C_0(u(0,t))\right]\times$$

$$\times\frac{1}{2}\left(\frac{dU(0,t)}{dt}+\frac{dI(0,t)}{dt}\right)=-\frac{1}{2Z_0}U(0,t)+$$

$$+\frac{1}{2Z_0}I(0,t)-\breve{I}_{in}(t)-i_{R_0L_0}(t)$$

and

$$\left[i_{R_1L_1}(t)\left(dL_1(i_{R_1L_1}(t))/di_{R_1L_1}\right)+L_1(i_{R_1L_1}(t))\right]\frac{di_{R_1L_1}(t)}{dt}=$$

$$=\frac{1}{2}U(\Lambda,t)+\frac{1}{2}I(\Lambda,t)-R_1(i_{R_1L_1}(t)),$$

$$\left[u(\Lambda,t)\left(dC_1(u(\Lambda,t))/du\right)+C_1(u(\Lambda,t))\right]\times$$

$$\times\frac{1}{2}\left(\frac{dU(\Lambda,t)}{dt}+\frac{dI(\Lambda,t)}{dt}\right)=-\frac{1}{2Z_0}U(\Lambda,t)+$$

$$+\frac{1}{2Z_0}I(\Lambda,t)-i_{R_1L_1}(t).$$

But

$$U(0,t)=U(\Lambda,t+T),\ I(0,t+T)=I(\Lambda,t).$$

We assume that the unknown functions are

$$U(0,t)\equiv U(t),\ I(\Lambda,t)\equiv I(t)$$

and then in view of

$$u(0,t)=\left(U(0,t)+I(0,t)\right)/2=\left(U(t)+I(t-T)\right)/2,$$

$$u(\Lambda,t)=\left(U(\Lambda,t)+I(\Lambda,t)\right)/2=\left(U(t-T)+I(t)\right)/2$$

solve with respect to the derivatives we reach the system

$$\begin{aligned}&\frac{di_{R_0L_0}(t)}{dt}=\frac{1}{2}\frac{U(t)+I(t-T)-2R_0(i_{R_0L_0}(t))}{i_{R_0L_0}(t)\left(dL_0(i_{R_0L_0}(t))/di_{R_0L_0}\right)+L_0(i_{R_0L_0}(t))},\\
&\frac{dU(t)}{dt}=-\frac{dI(t-T)}{dt}+\\
&+\frac{1}{Z_0}\frac{-U(t)+I(t-T)-2Z_0\breve{I}_{in}(t)-2Z_0i_{R_0L_0}(t)}{u(0,t)\left(dC_0(u(0,t))/du\right)+C_0(u(0,t))},\\
&\frac{di_{R_1L_1}(t)}{dt}=\frac{1}{2}\frac{U(t-T)+I(t)-2R_1(i_{R_1L_1}(t))}{i_{R_1L_1}(t)\left(dL_1(i_{R_1L_1}(t))/di_{R_1L_1}\right)+L_1(i_{R_1L_1}(t))},\\
&\frac{dI(t)}{dt}=-\frac{dU(t-T)}{dt}+\\
&+\frac{1}{Z_0}\frac{-U(t-T)+I(t)-2Z_0i_{R_1L_1}(t)}{u(\Lambda,t)\left(dC_1(u(\Lambda,t))/du\right)+C_1(u(\Lambda,t))}.\end{aligned}\tag{10}$$

So we have obtained a neutral system of differential equations with retarded arguments.

2.3. Estimates of the Arising Nonlinearities and Introducing Metrics

We consider C-V characteristics

$$C_p(u)=c_p/\sqrt[h]{\left(1-u/\Phi_p\right)},$$

$h\in[2,3]\,,c_p>0,\Phi_p>0\ (p=0,1)$

for

$|u|\le\phi_0<\min\{\Phi_0,\Phi_1\}$

where $C_p(u)$ have a strictly positive lower bounds. Put

$$\overline{C}_p(u)=C_p(u).u\ ,\ \widetilde{C}_p(u)=d\overline{C}_p(u)/du.$$

Then

$$\widetilde{C}_p(u)=c_p\sqrt[h]{\Phi_p}\,\frac{\Phi_p-\left((h-1)/h\right)u}{\left(\Phi_p-u\right)^{(h+1)/h}};$$

$$\frac{d\widetilde{C}_p(u)}{du}=\frac{2c_p\sqrt[h]{\Phi_p}}{\left(\Phi_p-u\right)^{(1+2h)/h}}\frac{\Phi_p-\left(2+(1/h)\right)u}{h};$$

$$\left|\frac{d\overline{C}_p(u)}{du}\right|\le\frac{2c_p\sqrt[h]{\Phi_p}}{\left(\Phi_p-\phi_0\right)^{(1+2h)/h}}\frac{\Phi_p+\left(2+(1/h)\right)\phi_0}{h};$$

$$\left|\frac{d\widetilde{C}_p(u)}{du}\right|\le\frac{2c_p\sqrt[h]{\Phi_p}\left(h\Phi_p+\phi_0\right)}{h^2\sqrt[h]{\left(\Phi_p-\phi_0\right)^{1+2h}}};$$

$$\left|\frac{d\overline{C}_p}{du}\right|\ge\min\left\{\widetilde{C}_p(u):u\in[-\phi_0,\phi_0]\right\}\ge$$

$$\ge\frac{2c_p\sqrt[h]{\Phi_p}}{\left(\Phi_p+\phi_0\right)^{(1+2h)/h}}\frac{\Phi_p-\left(2+(1/h)\right)\phi_0}{h}=\hat{C}_p>0.$$

For the *I-V* characteristics we assume

$$R_0(i)=\sum_{n=1}^{m}r_n^{(0)}i^n,\quad L_p(i)=\sum_{n=1}^{m}l_n^{(p)}i^n\ (p=0,1).$$

Then $\widetilde{L}_p(i)=i\,.L_p(i)=i.\sum_{n=1}^{m}l_n^{(p)}i^n$. For $\widetilde{L}_p(i)$ we get

$$\frac{d\widetilde{L}_p(i)}{di}=i\frac{dL_p(i)}{di}+L_p(i)=\sum_{n=1}^{m}(n+1)l_n^{(p)}i^n;$$

$$\frac{d^2\widetilde{L}_p(i)}{di^2}=\sum_{n=1}^{m}(n+1)nl_n^{(p)}i^{n-1}.$$

Assumptions (L): $|i(t)|\le i_0\Rightarrow$

$$\frac{d\tilde{L}_p(i(t))}{di}=\sum_{n=1}^{m}(n+1)l_n^{(p)}\left(i(t)\right)^n\geq\hat{L}_p>0\ (p=0,1).$$

Assumptions (C): $\left|u(0,t)\right|\leq e^{\mu T_0}\left(U_0+I_0e^{-\mu T}\right)/2\leq\phi_0$; $\left|u(\Lambda,t)\right|\leq e^{\mu T_0}\left(U_0e^{-\mu T}+I_0\right)/2\leq\phi_0$.

For the *V-I* characteristics we assume that they are of polynomial type:

$$R_0(i_{R_0L_0})=\sum_{n=1}^{m}r_n^{(0)}(i_{R_0L_0})^n,\ R_1(i_{R_1L_1})=\sum_{n=1}^{m}r_n^{(1)}(i_{R_1L_1})^n.$$

We introduce the sets for the unknown functions $i_{R_0L_0}(t),\ U(t),\ i_{R_1L_1}(t),\ I(t)$

$$M_0=\left\{i_{R_0L_0}(t)\in C^1_{T_0}[T,2T\}:\left|i_{R_0L_0}(t)\right|\leq I_{R_0}e^{\mu(t-T-kT_0)}\right\},$$

$$M_U=\left\{u\in C^1_{T_0}[T,2T]:\left|U(t)\right|\leq U_0e^{\mu(t-T-kT_0)}\right\},$$

$$M_1=\left\{i_{R_1L_1}(t)\in C^1_{T_0}[T,2T]:\left|i_{R_1L_1}(t)\right|\leq I_{R_1}e^{\mu(t-T-kT_0)}\right\},$$

$$M_I=\left\{u\in C^1_{T_0}[T,2T]:\left|I(t)\right|\leq I_0e^{\mu(t-T-kT_0)}\right\},$$

$t\in[T+kT_0,T+(k+1)T_0]$ $(k=0,1,2,...,m-1)$, where $C^1_{T_0}[T,2T]$ is the set of all continuously differentiable T_0 -periodic functions and $I_{R_0},U_0,I_{R_1},I_0,T_0,\ \mu$ are positive constants (chosen below) and $\mu T_0=\mu_0=\text{const}$.

Introduce the metrics

$$\rho^{(k)}(i_{R_0L_0},\bar{i}_{R_0L_0})=\max\left\{\left|i_{R_0L_0}(t)-\bar{i}_{R_0L_0}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\hat{\rho}(i_{R_0L_0},\bar{i}_{R_0L_0})=\max\left\{\left|i_{R_0L_0}(t)-\bar{i}_{R_0L_0}(t)\right|:t\in[T,2T]\right\},$$

$$\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0})=\max\left\{e^{-\mu(t-T-kT_0)}\left|\dot{i}_{R_0L_0}(t)-\dot{\bar{i}}_{R_0L_0}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\rho^{(k)}(U,\bar{U})=\max\left\{\left|U(t)-\bar{U}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\hat{\rho}(U,\bar{U})=\max\left\{\left|U(t)-\bar{U}(t)\right|:t\in[T,2T]\right\},$$

$$\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})=\max\left\{e^{-\mu(t-T-kT_0)}\left|\dot{U}(t)-\dot{\bar{U}}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\rho^{(k)}(i_{R_1L_1},\bar{i}_{R_1L_1})=\max\left\{\left|i_{R_1L_1}(t)-\bar{i}_{R_1L_1}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\hat{\rho}(i_{R_1L_1},\bar{i}_{R_1L_1})=\max\left\{\left|i_{R_1L_1}(t)-\bar{i}_{R_1L_1}(t)\right|:t\in[T,2T]\right\},$$

$$\rho_\mu^{(k)}(\dot{i}_{R_1L_1},\dot{\bar{i}}_{R_1L_1})=\max\left\{e^{-\mu(t-T-kT_0)}\left|\dot{i}_{R_1L_1}(t)-\dot{\bar{i}}_{R_1L_1}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\rho^{(k)}(I,\bar{I})=\max\left\{\left|I(t)-\bar{I}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\},$$

$$\hat{\rho}(I,\bar{I})=\max\left\{\left|I(t)-\bar{I}(t)\right|:t\in[T,2T]\right\},$$

$$\rho_\mu^{(k)}(\dot{I},\dot{\bar{I}})=\max\left\{e^{-\mu(t-T-kT_0)}\left|\dot{I}(t)-\dot{\bar{I}}(t)\right|:t\in[T+kT_0,T+(k+1)T_0]\right\}.$$

The set $M_0\times M_U\times M_1\times M_I$ turns out into a complete metric space with respect to the metric:

$$\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)=$$
$$=\max\left\{\hat{\rho}(i_{R_0L_0},\bar{i}_{R_0L_0}),\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0}),\hat{\rho}(U,\bar{U}),\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}}),\right.$$
$$\left.\hat{\rho}(i_{R_1L_1},\bar{i}_{R_1L_1}),\rho_\mu^{(k)}(\dot{i}_{R_1L_1},\dot{\bar{i}}_{R_1L_1}),\hat{\rho}(I,\bar{I}),\rho_\mu^{(k)}(\dot{I},\dot{\bar{I}}):k=0,1,2,...,m-1\right\}.$$

2.4. Operator Presentation of the Periodic Problem

Now we formulate the main problem: to find a T_0 -periodic solution $\left(i_{R_0L_0}(t),U(t),i_{R_1L_1}(t),I(t)\right)$ of the system (10) on the interval $[T,2T]$ coinciding with prescribed T_0 -periodic initial functions $i^{(0)}_{R_0L_0}(t),U_0(t),\ \ i^{(0)}_{R_1L_1}(t),I_0(t)$ on the interval respectively:

$$U(t)=U_0(t),\ dU(t)/dt=dU_0(t)/dt,t\in[0,T];$$
$$I(t)=I_0(t),\ dI(t)/dt=dI_0(t)/dt,t\in[0,T];$$
$$i_{R_1L_1}(T)=0,\ i_{R_0L_0}(T)=0,\ U_0(T)=0,\ I_0(T)=0.$$

Remark 1. As in[3] one can shift the initial function of the mixed problem from the interval $[0,\Lambda]$ along the characteristic to the interval $[0,T]$.

The main difficulty is to define a suitable operator whose fixed points are solutions sought. We define it in the following way: the four-tuple functions $B=(B_0(t),B_U(t),B_1(t),B_I(t))$ are defined on every interval $[T+kT_0,T+(k+1)T_0]$ (for every $k=0,1,2,\ \dots,\ m$-1) by the expressions

$$B_0^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t):=\int_{T+kT_0}^{t}I_{R_0}(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds-\frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0}I_{R_0}(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds,$$

$$B_U^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t):=\int_{T+kT_0}^{t}V(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds-\frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0}V(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds,$$

$$B_1^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t):=\int_{T+kT_0}^{t}I_{R_1}(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds-\frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0}I_{R_1}(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds,$$

$$B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t):=\int_{T+kT_0}^{t}J(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds-\frac{t-T-kT_0}{T_0}\int_{T+kT_0}^{T+(k+1)T_0}J(i_{R_0L_0},U,i_{R_1L_1},I)(s)ds,$$

where

$$I_{R_0}(i_{R_0L_0},U,I)(t)=\frac{1}{2}\frac{U(t)+\bar{I}_0(t)-2R_0(i_{R_0L_0}(t))}{i_{R_0L_0}(t)\left(dL_0(i_{R_0L_0}(t))/di_{R_0L_0}\right)+L_0(i_{R_0L_0}(t))},$$

$$V(i_{R_0L_0},U,I)(t)=$$
$$=-\frac{d\bar{I}_0(t)}{dt}+\frac{1}{Z_0}\frac{-U(t)+\bar{I}_0(t)-2Z_0\breve{I}_{in}(t)-2Z_0 i_{R_0L_0}(t)}{\left(dC_0(u(0,t))/du\right)u(0,t)+C_0(u(0,t))},$$
$$I_{R_1}(U,i_{R_1L_1},I)(t)=$$
$$=\frac{\bar{U}_0(t)+I(t)-2R_1(i_{R_1L_1}(t))}{2\left[i_{R_1L_1}(t)\left(dL_1(i_{R_1L_1}(t))/di_{R_1L_1}\right)+L_1(i_{R_1L_1}(t))\right]},$$
$$J(U,i_{R_1L_1},I)=$$
$$=-\frac{d\bar{U}_0(t)}{dt}+\frac{1}{Z_0}\frac{-\bar{U}_0(t)+I(t)-2Z_0 i_{R_1L_1}(t)}{\left(dC_1(u(\Lambda,t))/du\right)u(\Lambda,t)+C_1(u(\Lambda,t))},$$
$$u(0,t)=\frac{U(t)+I(t-T)}{2};\ u(\Lambda,t)=\frac{U(t-T)+I(t)}{2}$$

and $\bar{U}_0(t),\bar{I}_0(t)$ are translated to the right initial functions $U_0(t),I_0(t)$ over $[T,2T]$.

From now on the following assumptions will be fulfilled:

(E): $\breve{I}_{in}(.)\in C^1_{T_0}[0,\infty);\left|\breve{I}_{in}(t)\right|\le e^{\mu(t-T-kT_0)}I_{R_0}$;

(IN):

$U_0(.)\in C^1_{T_0}[0,T]$, $I_0(.)\in C^1_{T_0}[0,T], T=mT_0, m\in\{2,3,...\}$

$\left|U_0(t)\right|\le e^{-\mu T}U_0 e^{\mu(t-kT_0)},\left|I_0(t)\right|\le e^{-\mu T}U_0 e^{\mu(t-kT_0)}$

$(k=0,1,2,...,m-1)$.

It follows

$$\left|\bar{U}_0(t)\right|\le e^{-\mu T}U_0 e^{\mu(t-T-kT_0)},\left|\bar{I}_0(t)\right|\le e^{-\mu T}U_0 e^{\mu(t-T-kT_0)}.$$

Lemma 1. If (E) and (IN) are satisfied and

$$(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I$$

then

$$B_0(i_{R_0L_0},U,i_{R_1L_1},I)(t), B_U(i_{R_0L_0},U,i_{R_1L_1},I)(t),$$
$$B_1(i_{R_0L_0},U,i_{R_1L_1},I)(t), B_I(i_{R_0L_0},U,i_{R_1L_1},I)(t)$$

are T_0-periodic ones.

Lemma 2. If

$$(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I$$

then

$$\left(B_0(i_{R_0L_0},U,i_{R_1L_1},I)(t), B_U(i_{R_0L_0},U,i_{R_1L_1},I)(t),\right.$$
$$\left.B_1(i_{R_0L_0},U,i_{R_1L_1},I)(t), B_I(i_{R_0L_0},U,i_{R_1L_1},I)(t)\right)\in\left(C^1_{T_0}[T,2T]\right)^4.$$

The proofs can be accomplished as in[5],[6].

The following lemma guaranties that the fixed points of the above defined operator are periodic solutions of the neutral system (10).

Lemma 3. The periodic problem (10) has a solution

$$(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I$$

iff the operator B has a fixed point

$$(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I,$$

that is,

$$i_{R_0L_0}=B_0(i_{R_0L_0},U,i_{R_1L_1},I),\ U=B_U(i_{R_0L_0},U,i_{R_1L_1},I),$$
$$i_{R_1L_1}=B_1(i_{R_0L_0},U,i_{R_1L_1},I),\ I=B_I(i_{R_0L_0},U,i_{R_1L_1},I).$$

***Proof*:** Let

$$(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I$$

be a T_0-periodic solution of (10).

Then after integration of the first equation we have (recall that $i_{R_0L_0}(T)=0$):

$$i_{R_0L_0}(t)=\int_{T+kT_0}^{t}I_{R_0L_0}(t)dt\Rightarrow$$
$$i_{R_0L_0}(T+(k+1)T_0)=\int_{T+kT_0}^{T+(k+1)T_0}I_{R_0}(t)dt\Rightarrow\int_{T+kT_0}^{T+(k+1)T_0}I_{R_0}(t)dt=0.$$

Therefore

$$B_0^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)=\int_{T+kT_0}^{t}I_{R_0L_0}(s)ds.$$

Analogously we obtain

$$B_U^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)=\int_{T+kT_0}^{t}U(s)ds,$$
$$B_1^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)=\int_{T+kT_0}^{t}I_{R_1}(s)ds,$$
$$B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)=\int_{T+kT_0}^{t}J(s)ds$$

that is, $(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I$ is a fixed point of B.

Conversely, let B have a fixed point $(i_{R_0L_0},U,i_{R_1L_1},I)\in M_0\times M_U\times M_1\times M_I$. Then

$$\left|\int_{T+kT_0}^{T+(k+1)T_0}I_{R_0}(i_{R_0L_0},U,I)(s)ds\right|\le$$
$$\le\frac{1}{2}\int_{T+kT_0}^{T+(k+1)T_0}\left|\frac{U(t)+\bar{I}_0(t)-2R_0(i_{R_0L_0}(t))}{i_{R_0L_0}(t)\left(dL_0(i_{R_0L_0}(t))/di_{R_0L_0}\right)+L_0(i_{R_0L_0}(t))}\right|dt\le$$
$$\le\frac{1}{2\hat{L}_0}\int_{T+kT_0}^{T+(k+1)T_0}\left(U_0e^{\mu(t-T-kT_0)}+I_0e^{\mu(t-T-kT_0)}e^{-\mu T}+\right.$$
$$\left.+2\sum_{n=1}^{m}\left|r_n^{(0)}\right|\left|i_{R_0L_0}(t)\right|^n\right)dt\le\frac{U_0}{\hat{L}_0}\frac{e^{\mu T_0}-1}{\mu}+\frac{I_0e^{-\mu T}}{\hat{L}_0}\frac{e^{\mu T_0}-1}{\mu}+$$
$$+\frac{2}{\hat{L}_0}\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^n\int_{T+kT_0}^{T+(k+1)T_0}e^{n\mu(t-T-kT_0)}dt\le$$
$$\le\frac{e^{\mu_0}-1}{\mu\hat{L}_0}\left(U_0+I_0e^{-\mu T}+2\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^n e^{(n-1)\mu_0}\right)\equiv M_{I_{R_0}}(\mu);$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0}V(i_{R_0L_0},U,I)(s)ds\right|\le$$
$$\le\frac{1}{Z_0\hat{C}_0}\int_{T+kT_0}^{T+(k+1)T_0}\left(U_0e^{\mu(t-T-kT_0)}+I_0e^{\mu(t-T-kT_0)}e^{-\mu T}+\right.$$
$$\left.+2Z_0I_{R_0}e^{\mu(t-T-kT_0)}+2Z_0I_{R_0}e^{\mu(t-T-kT_0)}\right)dt\le$$
$$\le\frac{U_0+I_0e^{-\mu T}+4Z_0I_{R_0}}{Z_0\hat{C}_0}\frac{e^{\mu T_0}-1}{\mu}\equiv M_V(\mu);$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0} I_{R_1}(U, i_{R_1L_1}, I)(s)ds\right| \le$$

$$\le \frac{U_0 e^{-\mu T}}{2\hat{L}_1}\int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(t-T-kT_0)}dt +$$

$$+\frac{I_0}{2\hat{L}_1}\int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(t-T-kT_0)}dt + \frac{1}{\hat{L}_1}\int_{T+kT_0}^{T+(k+1)T_0}\left|R_1(i_{R_1L_1}(t))\right|dt \le$$

$$\le \frac{e^{\mu_0}-1}{\mu\hat{L}_1}\left(U_0 e^{-\mu T} + I_0 + \sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n e^{(n-1)\mu_0}\right) \equiv M_{I_{R_1}}(\mu);$$

$$\left|\int_{T+kT_0}^{T+(k+1)T_0} J(U, i_{R_1L_1}, I)(s)ds\right| \le$$

$$\le \frac{1}{Z_0\hat{C}_1}\int_{T+kT_0}^{T+(k+1)T_0}\left(\left|\bar{U}_0(t)\right| + \left|I(t)\right| + 2Z_0\left|i_{R_1L_1}(t)\right|\right)dt \le$$

$$\le \frac{U_0 e^{-\mu T} + I_0 + 2Z_0 I_{R_1}}{Z_0\hat{C}_1}\frac{e^{\mu_0}-1}{\mu}.$$

If we assume $\left|\int_{T+kT_0}^{T+(k+1)T_0} I_{R_0}(s)ds\right| \ne 0$ in view of $\lim_{\mu\to\infty} M_{I_{R_0}}(\mu) = 0$ one obtains a contradiction.

It follows

$$\int_{T+kT_0}^{T+(k+1)T_0} I_{R_0}(s)ds = 0.$$

Analogously

$$\int_{T+kT_0}^{T+(k+1)T_0} V(t)dt = 0, \quad \int_{T+kT_0}^{T+(k+1)T_0} I_{R_1}(s)ds = 0, \quad \int_{T+kT_0}^{T+(k+1)T_0} J(t)dt = 0.$$

Consequently

$$i_{R_0L_0}(t) = \int_{T+kT_0}^{t} I_{R_0}(i_{R_0L_0}, U, i_{R_1L_1}, I)(s)ds,$$

$$U(t) = \int_{T+kT_0}^{t} V(i_{R_0L_0}, U, i_{R_1L_1}, I)(s)ds,$$

$$i_{R_1L_1}(t) = \int_{T+kT_0}^{t} I_{R_1}(i_{R_0L_0}, U, i_{R_1L_1}, I)(s)ds,$$

$$I(t) = \int_{T+kT_0}^{t} J(i_{R_0L_0}, U, i_{R_1L_1}, I)(s)ds.$$

Differentiating the last equalities we conclude that (10) has T_0-periodic solution.

Lemma 3 is thus proved.

2.5. Existence-Uniqueness of Periodic Solution

The main result contains in the following

Theorem 1. Let assumptions **(L)**, **(C)**, **(E)** and **(IN)** be fulfilled. Then there exists a unique T_0-periodic solution of (10).

Proof: We show that B maps $M_0 \times M_U \times M_1 \times M_I$ into itself. Indeed

$$\left|B_0^{(k)}(i_{R_0L_0}, U, i_{R_1L_1}, I)(t)\right| \le$$

$$\le \int_{T+kT_0}^{t}\left|I_{R_0L_0}(s)\right|ds + \left|\int_{T+kT_0}^{T+(k+1)T_0} I_{R_0L_0}(s)ds\right| \equiv J_1 + J_2.$$

We have

$$J_1 \le \frac{U_0}{2\hat{L}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + \frac{I_0 e^{-\mu T}}{2\hat{L}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} +$$

$$+\frac{1}{\hat{L}_0}\sum_{n=1}^{m}\left|r_n^{(0)}\right| I_{R_0}^n \int_{T+kT_0}^{t} e^{n\mu(s-T-kT_0)}ds$$

$$\le \frac{U_0}{2\hat{L}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + \frac{I_0 e^{-\mu T}}{2\hat{L}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} +$$

$$+\frac{1}{\hat{L}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu}\sum_{n=1}^{m}\left|r_n^{(0)}\right| I_{R_0}^n e^{(n-1)\mu T_0} \le$$

$$\le e^{\mu(t-T-kT_0)}\frac{1}{\mu\hat{L}_0}\left(\frac{U_0 + I_0 e^{-\mu T}}{2} + \sum_{n=1}^{m}\left|r_n^{(0)}\right| I_{R_0}^n e^{(n-1)\mu T_0}\right);$$

$$J_2 \le \frac{U_0}{2\hat{L}_0}\frac{e^{\mu T_0}-1}{\mu} + \frac{I_0 e^{-\mu T}}{2\hat{L}_0}\frac{e^{\mu T_0}-1}{\mu} +$$

$$+\frac{1}{\hat{L}_0}\sum_{n=1}^{m}\left|r_n^{(0)}\right| I_{R_0}^n \int_{T+kT_0}^{T+(k+1)T_0} e^{n\mu(s-T-kT_0)}ds \le$$

$$\le e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}-1}{\mu\hat{L}_0}\left(\frac{U_0 + I_0 e^{-\mu T}}{2} + \sum_{n=1}^{m}\left|r_n^{(0)}\right| I_{R_0}^n e^{(n-1)\mu_0}\right);$$

$$\left|B_0^{(k)}(i_{R_0L_0}, U, i_{R_1L_1}, I)(t)\right|$$

$$\le e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}}{\mu\hat{L}_0}\left(\frac{U_0 + I_0 e^{-\mu T}}{2} + \sum_{n=1}^{m}\left|r_n^{(0)}\right| I_{R_0}^n e^{(n-1)\mu_0}\right)$$

$$\le e^{\mu(t-T-kT_0)} I_{R_0}.$$

Further on we have

$$\left|B_U^{(k)}(i_{R_0L_0}, U, i_{R_1L_1}, I)(t)\right| \le \left|\int_{T+kT_0}^{t} V(i_{R_0L_0}, U, i_{R_1L_1}, I)(s)ds\right| +$$

$$+\left|\int_{T+kT_0}^{T+(k+1)T_0} V(i_{R_0L_0}, U, i_{R_1L_1}, I)(s)ds\right| \equiv W_1 + W_2$$

and

$$W_1 \le \left|\bar I_0(t)\right| + \frac{1}{\hat C_0 Z_0}\int_{T+kT_0}^{t} U_0 e^{\mu(s-T-kT_0)}ds +$$
$$+\frac{1}{\hat C_0 Z_0}\int_{T+kT_0}^{t} I_0 e^{-\mu T} e^{\mu(s-T-kT_0)}\, ds +$$
$$+\frac{2Z_0}{\hat C_0 Z_0} I_{R_0}\int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)}ds + \frac{2Z_0}{\hat C_0 Z_0} I_{R_0}\int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)}ds \le$$
$$\le I_0 e^{-\mu T} e^{\mu(t-T-kT_0)} + \frac{U_0}{\hat C_0 Z_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} +$$
$$+\frac{2Z_0}{\hat C_0 Z_0} I_{R_0}\int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)}ds + \frac{2Z_0}{\hat C_0 Z_0} I_{R_0}\int_{T+kT_0}^{t} e^{\mu(s-T-kT_0)}ds \le$$
$$+\frac{I_0 e^{-\mu T}}{\hat C_0 Z_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + \frac{4Z_0}{\hat C_0 Z_0} I_{R_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} \le$$
$$\le e^{\mu(t-T-kT_0)}\left[I_0 e^{-\mu T} + \frac{1}{\mu \hat C_0 Z_0}\left(U_0 + I_0 e^{-\mu T} + 4Z_0 I_{R_0}\right)\right]$$

and

$$W_2 \le \left|\int_{T+kT_0}^{T+(k+1)T_0}\frac{d\bar I_0(s)}{ds}ds\right| +$$
$$+\frac{1}{\hat C_0 Z_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|U(s)\right|ds + \frac{1}{\hat C_0 Z_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|\bar I_0(s)\right| ds +$$
$$+\frac{2Z_0}{\hat C_0 Z_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|\bar I_{in}(s)\right|ds + \frac{2Z_0}{\hat C_0 Z_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|i_{R_0L_0}(s)\right|ds \le$$
$$\le \frac{1}{\hat C_0 Z_0}\left(\int_{T+kT_0}^{T+(k+1)T_0} U_0 e^{\mu(s-T-kT_0)}ds + \int_{T+kT_0}^{T+(k+1)T_0} I_0\, e^{-\mu T} e^{\mu(s-T-kT_0)}ds\right) +$$
$$+\frac{2Z_0}{\hat C_0 Z_0}\left(I_{R_0}\int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)}ds + I_{R_0}\int_{T+kT_0}^{T+(k+1)T_0} e^{\mu(s-T-kT_0)}ds\right) \le$$
$$\le e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}-1}{\mu \hat C_0 Z_0}\left(U_0 + I_0 e^{-\mu T} + 4Z_0 I_{R_0}\right).$$

So $\left|B_U^{(k)}(i_{R_0L_0},U,I)(t)\right| \le$

$$= e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}\left(U_0 + I_0 e^{-\mu T} + 4Z_0 I_{R_0}\right)}{\mu \hat C_0 Z_0} \le U_0 e^{\mu(t-T-kT_0)}.$$

On the other hand

$$\left|B_1^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)\right| \le$$
$$\le \int_{T+kT_0}^{t}\left|I_{R_1}(s)\right|ds + \left|\int_{T+kT_0}^{T+(k+1)T_0} I_{R_1}(s)ds\right| \equiv I_1 + I_2.$$

We have

$$I_1 \le \frac{1}{2\hat L_1}\left(U_0 e^{-\mu T}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + I_0\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + \right.$$
$$\left. +2\sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n \int_{T+kT_0}^{t} e^{n\mu(s-T-kT_0)}ds\right) \le$$
$$\le \frac{1}{2\hat L_1}\left(U_0 e^{-\mu T}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + I_0\frac{e^{\mu(t-T-kT_0)}-1}{\mu} + \right.$$
$$\left. +\frac{e^{\mu(t-T-kT_0)}-1}{\mu}2\sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n e^{(n-1)\mu_0}\right) \le$$
$$\le e^{\mu(t-T-kT_0)}\frac{1}{\mu \hat L_1}\left(\frac{U_0 e^{-\mu T}+I_0}{2} + \sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n e^{(n-1)\mu_0}\right)$$

and

$$I_2 \le \frac{1}{2\hat L_1}\int_{T+kT_0}^{T+(k+1)T_0}\left(\left|\bar U_0(s)\right| + \left|I(s)\right| + \left|R_1(i_{R_1L_1}(s))\right|\right)ds \le$$
$$\le \frac{1}{2\hat L_1}\left(U_0 e^{-\mu T}\frac{e^{\mu T_0}-1}{\mu} + I_0\frac{e^{\mu T_0}-1}{\mu} + \right.$$
$$\left. +2\sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n \int_{T+kT_0}^{T+(k+1)T_0} e^{n\mu(s-T-kT_0)}ds\right) \le$$
$$\le e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}-1}{\mu \hat L_1}\left(\frac{U_0 e^{-\mu T}+I_0}{2} + \sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n e^{(n-1)\mu_0}\right)$$

Then

$$\left|B_1^{(k)}(i_{R_0L_0},u,i_{R_1L_1})(t)\right| \le e^{\mu(t-T-kT_0)} \times$$
$$\times \frac{e^{\mu_0}}{\mu \hat L_1}\left(\frac{U_0 e^{-\mu T}+I_0}{2} + \sum_{n=1}^{m}\left|r_n^{(1)}\right| I_{R_1}^n e^{(n-1)\mu T_0}\right) \le$$
$$\le I_{R_1} e^{\mu(t-T-kT_0)}.$$

Finally

$$\left|B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)\right| \le \left|\int_{T+kT_0}^{t} J(s)ds\right| + \left|\int_{T+kT_0}^{T+(k+1)T_0} J(s)ds\right| \equiv V_1 + V_2.$$

We have

$$V_1 \le \left|\bar U_0(t)\right| + \frac{1}{Z_0 \hat C_1}\int_{T+kT_0}^{t}\left(\left|\bar U_0(s)\right| + \left|I(s)\right| + 2Z_0\left|i_{R_1L_1}(s)\right|\right)ds \le$$
$$\le e^{\mu(t-T-kT_0)}U_0 e^{-\mu T} + \frac{1}{Z_0 \hat C_1}\int_{T+kT_0}^{t}\left(U_0 e^{-\mu T} e^{\mu(s-T-kT_0)} + \right.$$
$$\left. + I_0 e^{\mu(s-T-kT_0)} + 2Z_0 I_{R_1} e^{\mu(s-T-kT_0)}\right)ds \le$$
$$\le e^{\mu(t-T-kT_0)}U_0 e^{-\mu T} +$$
$$+\frac{U_0 e^{-\mu T} + I_0 + 2Z_0 I_{R_1}}{Z_0 \hat C_1}\frac{e^{\mu(t-T-kT_0)}-1}{\mu} \le$$

$$\le e^{\mu(t-T-kT_0)}\left(U_0 e^{-\mu T}+\frac{U_0 e^{-\mu T}+I_0+2Z_0 I_{R_1}}{\mu Z_0 \hat{C}_1}\right)$$

and

$$V_2 \le \left|\int_{T+kT_0}^{T+(k+1)T_0}\frac{d\bar{U}_0(s)}{ds}ds\right|+$$

$$+\frac{1}{Z_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|\frac{-\bar{U}_0(s)+I(s)-2Z_0 i_{R_1L_1}(s)}{u(\Lambda,s)\left(dC_1(u(\Lambda,s))/du\right)+C_1(u(\Lambda,s))}\right|ds \le$$

$$\le \frac{1}{Z_0\hat{C}_1}\int_{T+kT_0}^{T+(k+1)T_0}\left(\left|\bar{U}_0(s)\right|+\left|I(s)\right|+2Z_0\left|i_{R_1L_1}(s)\right|\right)ds \le$$

$$\le \frac{1}{Z_0\hat{C}_1}\int_{T+kT_0}^{T+(k+1)T_0}\left(U_0 e^{-\mu T}e^{\mu(s-T-kT_0)}+I_0 e^{\mu(s-T-kT_0)}+\right.$$

$$\left.+2Z_0 I_{R_1}e^{\mu(s-T-kT_0)}\right)ds \le \frac{U_0 e^{-\mu T}+I_0+2Z_0 I_{R_1}}{Z_0\hat{C}_1}\frac{e^{\mu T_0}-1}{\mu}\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{\left(e^{\mu T_0}-1\right)\left(U_0 e^{-\mu T}+I_0+2Z_0 I_{R_1}\right)}{\mu Z_0\hat{C}_1};$$

$$\left|B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\left(U_0 e^{-\mu T}+e^{\mu 0}\frac{U_0 e^{-\mu T}+I_0+2Z_0 I_{R_1}}{\mu Z_0\hat{C}_1}\right)\le$$

$$\le e^{\mu(t-T-kT_0)}I_0.$$

It remains to obtain the Lipschitz estimates for the right-hand sides of the equations. Omitting calculation of the partial derivatives we obtain:

$$\left|I_{R_0}(i_{R_0L_0},U,I)-I_{R_0}(\bar{i}_{R_0L_0},\bar{U},\bar{I})\right|\le$$

$$\le\frac{1}{\hat{L}_0^2}\left(\sum_{n=1}^{m}n\left|r_n^{(0)}\right|\left|i_{R_0L_0}\right|^{n-1}.\sum_{n=1}^{m}(n+1)\left|l_n^{(p)}\right|\left|i_{R_0L_0}\right|^{n}+\right.$$

$$+\left(\left|U(t)\right|+\left|I(t-T)\right|+\sum_{n=1}^{m}\left|r_n^{(0)}\right|\left|i_{R_0L_0}\right|^{n}\right)\times$$

$$\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(p)}\right|\left|i_{R_0L_0}\right|^{n-1}\right)\left|i_{R_0L_0}-\bar{i}_{R_0L_0}\right|+$$

$$+\frac{1}{\hat{L}_0}\left|U(t)-\bar{U}(t)\right|+\frac{1}{\hat{L}_0}\left|I(t-T)-\bar{I}(t-T)\right|;$$

$$\left|V(i_{R_0L_0},U,I)-V(\bar{i}_{R_0L_0},\bar{U},\bar{I})\right|\le$$

$$\le\frac{2}{\hat{C}_0}\left|i_{R_0L_0}(t)-\bar{i}_{R_0L_0}(t)\right|+$$

$$+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+\frac{\left|U(t)\right|+\left|I(t-T)\right|+2Z_0\left|\breve{I}_{in}(t)\right|+2Z_0\left|i_{R_0L_0}(t)\right|}{2}\times$$

$$\left.\times\frac{2c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+|u|(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\left|U-\bar{U}\right|+$$

$$+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+\frac{\left|U(t)\right|+\left|I(t-T)\right|+2Z_0\left|\breve{I}_{in}(t)\right|+2Z_0\left|i_{R_0L_0}(t)\right|}{2}\times$$

$$\left.\times\frac{2c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+|u|(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\left|I(t-T)-\bar{I}(t-T)\right|+$$

$$+\left|\dot{I}(t-T)-\dot{\bar{I}}(t-T)\right|;$$

$$\left|I_{R_1}(U,i_{R_1L_1},I)-I_{R_1}(\bar{U},\bar{i}_{R_1L_1},\bar{I})\right|\le$$

$$\le\frac{1}{2\hat{L}_1}\left(\left|U(t-T)-\bar{U}(t-T)\right|+2\left|\frac{dR_1(i_{R_1L_1})}{di_{R_1L_1}}\right|\left|\frac{d\widetilde{L}_1(i_{R_1L_1})}{di_{R_1L_1}}\right|+\right.$$

$$\left.+\left(\left|U(t-T)\right|+\left|I(t)\right|+2\left|R_1(i_{R_1L_1})\right|\right)\left|d^2\widetilde{L}_1(i_{R_1L_1})/di_{R_1L_1}^2\right|\times\right.$$

$$\times\left|i_{R_1L_1}-\bar{i}_{R_1L_1}\right|+\frac{1}{2\hat{L}_1}\left|I(t)-\bar{I}(t)\right|;$$

$$\left|J(U,i_{R_1L_1},I)-J(\bar{U},\bar{i}_{R_1L_1},\bar{I})\right|\le\frac{2}{\hat{C}_1}\left|i_{R_1L_1}(t)-\bar{i}_{R_1L_1}(t)\right|+$$

$$+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+\frac{\left|U(t-T)\right|+\left|I(t)\right|+2Z_0\left|i_{R_1L_1}(t)\right|}{2}\times$$

$$\left.\times\frac{2c_1\sqrt[h]{\Phi_1}\left[h(\Phi_1-\phi_0)+|u|(1+h)\right]}{h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\left|I-\bar{I}\right|+$$

$$+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_1+(2+(1/h))\phi_0}{h}+\right.$$

$$+\frac{\left|U(t-T)\right|+\left|I(t)\right|+2Z_0\left|i_{R_1L_1}(t)\right|}{2}\times$$

$$\left.\times\frac{2c_1\sqrt[h]{\Phi_1}\left[h(\Phi_1-\phi_0)+|u|(1+h)\right]}{h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\left|U(t-T)-\bar{U}(t-T)\right|+$$

$$+\left|\dot{U}(t-T)-\dot{\bar{U}}(t-T)\right|.$$

Then

$$\left|B_0^{(k)}(i_{R_0L_0},U,I)(t)-B_0^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le\int_{T+kT_0}^{t}\left|I_{R_0}(i_{R_0L_0},U,I)(s)-I_{R_0}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(s)\right|ds+$$

$$+\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(I_{R_0}(i_{R_0L_0},U,I)(s)-I_{R_0}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(s)\right)ds\right|\equiv I_1+I_2;$$

$$I_1\le\frac{\rho_\mu^{(k)}(i_{R_0L_0},\bar{i}_{R_0L_0})}{\hat{L}_0^2}\frac{e^{\mu(t-T-kT_0)}-1}{\mu}\times$$

$$\times\left[\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})}{\mu\hat{L}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu}\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{1}{\mu^2}\left\{\frac{\rho_\mu^{(k)}(i_{R_0L_0},\dot{\bar{i}}_{R_0L_0})}{\hat{L}_0^2}\left[\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\times\right.\right.$$

$$\times\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})}{\hat{L}_0}\right\};$$

$$I_2\le\frac{1}{\hat{L}_0^2}\int_{T+kT_0}^{T+(k+1)T_0}\left[\sum_{n=1}^{m}n\left|r_n^{(0)}\right|\left|i_{R_0L_0}(s)\right|^{n-1}.\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|\left|i_{R_0L_0}(s)\right|^{n}+\right.$$

$$+\left(\left|U(s)\right|+\left|I(s-T)\right|+\sum_{n=1}^{m}\left|r_n^{(0)}\right|\left|i_{R_0L_0}(s)\right|^{n}\right)\times$$

$$\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|\left|i_{R_0L_0}(s)\right|^{n-1}\right]\times$$

$$\times\left|i_{R_0L_0}(s)-\bar{i}_{R_0L_0}(s)\right|ds+\frac{1}{\hat{L}_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|U(s)-\bar{U}(s)\right|ds\le$$

$$\le\frac{\rho_\mu^{(k)}(i_{R_0L_0},\bar{i}_{R_0L_0})}{\hat{L}_0^2}\frac{e^{\mu T_0}-1}{\mu}\left[\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}.\right.$$

$$\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})}{\mu\hat{L}_0}\frac{e^{\mu_0}-1}{\mu}\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}-1}{\mu^2}\left\{\frac{\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0})}{\hat{L}_0^2}\times\right.$$

$$\times\left[\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})}{\hat{L}_0}\right\};$$

$$\left|B_0^{(k)}(i_{R_0L_0},U,I)(t)-B_0^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{e^{\mu_0}}{\mu^2}\left\{\frac{1}{\hat{L}_0^2}\left[\sum_{n=1}^{m}n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}.\right.\right.$$

$$\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{\hat{L}_0}\right\}\times$$

$$\times\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}K_0\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\le$$

$$\le e^{\mu T_0}K_0\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

It follows

$$\rho^{(k)}(B_0^{(k)}(i_{R_0L_0},U,I),B_0^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I}))\le$$

$$\le e^{\mu_0}K_0\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)$$

or

$$\hat{\rho}(B_0(i_{R_0L_0},U,I),B_0(\bar{i}_{R_0L_0},\bar{U},\bar{I}))\le$$

$$\le e^{\mu_0}K_0\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

Further on we get

$$\left|B_U^{(k)}(i_{R_0L_0},U,I)(t)-B_U^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le\int_{T+kT_0}^{t}\left|V(i_{R_0L_0},U,I)(s)-V(\bar{i}_{R_0L_0},\bar{U},\bar{I})(s)\right|ds+$$

$$+\int_{T+kT_0}^{T+(k+1)T_0}\left|V(i_{R_0L_0},U,I)(s)-V(\bar{i}_{R_0L_0},\bar{U},\bar{I})(s)\right|ds\equiv W_1+W_2.$$

But

$$W_1 \le \frac{2}{\hat{C}_0}\int_{T+kT_0}^{t}\left|i_{R_0L_0}(s)-\bar{i}_{R_0L_0}(s)\right|ds+$$

$$+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+\int_{T+kT_0}^{t}\left|U(s)-\bar{U}(s)\right|ds+\frac{U_0e^{\mu 0}+I_0e^{\mu 0}e^{-\mu T}+4Z_0I_{R_0}e^{\mu 0}}{2}\times$$

$$\left.\times\frac{2c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]+$$

$$+\frac{U_0e^{\mu 0}+I_0e^{\mu 0}e^{-\mu T}+4Z_0I_{R_0}e^{\mu 0}}{2}\times$$

$$\left.\times\frac{2c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\times$$

$$\times\int_{T+kT_0}^{t}\left|I(s-T)-\bar{I}(s-T)\right|ds+\int_{T+kT_0}^{t}\left|\dot{I}(s-T)-\dot{\bar{I}}(s-T)\right|ds\le$$

$$\le\frac{2\rho_\mu^{(k)}(i_{R_0L_0},\bar{i}_{R_0L_0})}{\hat{C}_0}\frac{e^{\mu(t-T-kT_0)}-1}{\mu}+$$

$$+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+e^{\mu 0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\times$$

$$\times\rho_\mu^{(k)}(U,\bar{U})\frac{e^{\mu(t-T-kT_0)}-1}{\mu}\le$$

$$\le\frac{e^{\mu(t-T-kT_0)}-1}{\mu}\left\{\frac{2\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0})}{\mu\hat{C}_0}+\right.$$

$$+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+e^{\mu 0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\frac{\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})}{\mu}\right\}\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\frac{1}{\mu^2}\left\{\frac{2}{\hat{C}_0}+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\hat{C}_1^2+\right.\right.$$

$$+e^{\mu 0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}$$

and

$$W_2\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\frac{e^{\mu 0}-1}{\mu^2}\left\{\frac{2}{\hat{C}_0}+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+e^{\mu 0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}.$$

Then

$$\left|B_U^{(k)}(i_{R_0L_0},U,I)(t)-B_U^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\frac{e^{\mu 0}}{\mu^2}\left\{\frac{2}{\hat{C}_0}++\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+e^{\mu 0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}\le$$

$$\le e^{\mu T_0}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\frac{e^{\mu 0}}{\mu^2}\times$$

$$\times\left\{\frac{2}{\hat{C}_0}+\frac{1}{Z_0\hat{C}_0^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+e^{\mu 0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu 0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}\equiv$$

$$\equiv e^{\mu 0}K_U\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

It follows

$$\hat{\rho}(B_U(i_{R_0L_0},U,I),B_U(\bar{i}_{R_0L_0},\bar{U},\bar{I}))\le$$

$$\le e^{\mu 0}K_U\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

For the third component we obtain:

$$\left|B_1^{(k)}(U,i_{R_1L_1},I)(t)-B_1^{(k)}(\bar{U},\bar{i}_{R_1L_1},\bar{I})(t)\right|\le$$

$$\le\int_{T+kT_0}^{t}\left|I_{R_1}(U,i_{R_1L_1},I)(s)-I_{R_1}(\bar{U},\bar{i}_{R_1L_1},\bar{I})(s)\right|ds+$$

$$+\int_{T+kT_0}^{T+(k+1)T_0}\left|I_{R_1}(U,i_{R_1L_1},I)(s)-I_{R_1}(\bar{U},\bar{i}_{R_1L_1},\bar{I})(s)\right|ds\equiv J_1+J_2;$$

$$J_1 \le \frac{1}{2\hat{L}_1}\int_{T+kT_0}^{t}\left|U(s-T)-\overline{U}(s-T)\right|ds+$$

$$+\int_{T+kT_0}^{t}\frac{2\left|dR_1(i_{R_1L_1})/di_{R_1L_1}\right|\left|d\tilde{L}_1(i_{R_1L_1})/di_{R_1L_1}\right|+\left(\left|U(s-T)\right|+\right.}{2\hat{L}_1^{\,2}}$$

$$\frac{\left.+\left|I(s)\right|+2\left|R_1(i_{R_1L_1})\right|\right)\left|d^2\tilde{L}_1(i_{R_1L_1})/di_{R_1L_1}^2\right|}{2\hat{L}_1^{\,2}}\left|i_{R_1L_1}(s)-\overline{i}_{R_1L_1}(s)\right|ds+$$

$$+\frac{1}{2\hat{L}_1}\int_{T+kT_0}^{t}\left|I(s)-\overline{I}(s)\right|ds\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{\rho_\mu^{(k)}(i_{R_1L_1},\dot{\overline{i}}_{R_1L_1})}{\mu^2}\frac{1}{\hat{L}_1^2}\left[\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\times\right.$$

$$\times\sum_{n=1}^{m}(n+1)\left|l_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}\right)\times$$

$$\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+e^{\mu(t-T-kT_0)}\frac{1}{2\hat{L}_1}\frac{\rho_\mu^{(k)}(\dot{I},\dot{\overline{I}})}{\mu^2}\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})\right)\frac{1}{\mu^2\hat{L}_1^2}\times$$

$$\times\left\{\left[\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{2}\right\};$$

$$J_2\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})\right)\frac{e^{\mu_0}-1}{\mu^2\hat{L}_1^2}\times$$

$$\times\left\{\left[\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{2}\right\};$$

$$\left|B_1^{(k)}(U,i_{R_1L_1},I)(t)-B_1^{(k)}(\overline{U},\overline{i}_{R_1L_1},I)(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})\right)\frac{e^{\mu_0}-1}{\mu^2\hat{L}_1^2}\times$$

$$\times\left\{\left[\sum_{n=1}^{m}n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(1)}\right|I_{R_1}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{2}\right\};$$

$$\equiv e^{\mu(t-T-kT_0)}K_1\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})\right)$$

or

$$\hat{\rho}(B_1(i_{R_0L_0},U,i_{R_1L_1},i_{R_1L_1}),B_1(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I}))\le$$

$$\le e^{\mu_0}K_1\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})\right).$$

For the fourth component we have

$$\left|B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)-B_I^{(k)}(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})(t)\right|\le$$

$$\le\int_{T+kT_0}^{t}\left|J(i_{R_0L_0},U,i_{R_1L_1},I)(s)-J(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})(s)\right|ds+$$

$$+\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(J(i_{R_0L_0},U,i_{R_1L_1},I)(s)-J(\overline{i}_{R_0L_0},\overline{U},\overline{i}_{R_1L_1},\overline{I})(s)\right)ds\right|\equiv$$

$$\equiv K_1+K_2;$$

$$K_1\le\frac{2}{\hat{C}_1}\rho_\mu^{(k)}(i_{R_1L_1},\overline{i}_{R_1L_1})\frac{e^{\mu(t-T-kT_0)}-1}{\mu}+$$

$$+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_0+\left(2+(1/h)\right)\phi_0}{h}+\right.$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\left.\times\frac{c_1\sqrt[h]{\Phi_1}\left[2h(\Phi_1-\phi_0)+\left(U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\times$$

$$\times\rho_\mu^{(k)}(I,\overline{I})\frac{e^{\mu(t-T-kT_0)}-1}{\mu}\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{1}{\mu^2}\Bigg\{\frac{2}{\hat{C}_1}+\frac{1}{Z_0\hat{C}_1^2}\Bigg[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\times$$

$$\times\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\times\frac{c_1\sqrt[h]{\Phi_1}\big[2h(\Phi_1-\phi_0)+e^{\mu_0}(U_0e^{-\mu T}+I_0)(1+h)\big]}{2h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\Bigg]\frac{\rho_\mu^{(k)}(\dot{I},\dot{\bar{I}})}{\mu}\Bigg\}\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\big((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})\big)\times$$

$$\times\frac{1}{\mu^2}\Bigg\{\frac{2}{\hat{C}_1}+\frac{1}{Z_0\hat{C}_1^2}\Bigg[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\times\frac{2c_1\sqrt[h]{\Phi_1}\big[2h(\Phi_1-\phi_0)+e^{\mu_0}(U_0e^{-\mu T}+I_0)(1+h)\big]}{2h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\Bigg]\Bigg\}$$

and

$$K_2\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\big((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})\big)\times$$

$$\times\frac{e^{\mu_0}-1}{\mu^2}\Bigg\{\frac{2}{\hat{C}_1}+\frac{1}{Z_0\hat{C}_1^2}\Bigg[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\times$$

$$\times\frac{\Phi_0+(2+(1/h))\phi_0}{h}+e^{\mu_0}\frac{U_0e^{-\mu T}+I_0+2Z_0I_{R_1}}{2}\times$$

$$\times\frac{c_1\sqrt[h]{\Phi_1}\big[2h(\Phi_1-\phi_0)+e^{\mu_0}(U_0e^{-\mu T}+I_0)(1+h)\big]}{h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\Bigg]\Bigg\};$$

$$\left|B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)-B_I^{(k)}(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})(t)\right|\le$$

$$\le e^{\mu_0}\hat{\rho}_\mu\big((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})\big)\times$$

$$\times\frac{e^{\mu_0}}{\mu^2}\Bigg\{\frac{2}{\hat{C}_1}+\frac{1}{Z_0\hat{C}_1^2}\Bigg[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_1+(2+(1/h))\phi_0}{h}+$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\times\frac{c_1\sqrt[h]{\Phi_1}\big[2h(\Phi_1-\phi_0)+e^{\mu_0}(U_0e^{-\mu T}+I_0)(1+h)\big]}{h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\Bigg]\Bigg\}\equiv$$

$$\equiv e^{\mu_0}K_I\hat{\rho}_\mu\big((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})\big).$$

It follows

$$\hat{\rho}(B_I(i_{R_0L_0},U,i_{R_1L_1},i_{R_1L_1}),B_I(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I}))\le$$

$$\le e^{\mu_0}K_I\hat{\rho}_\mu\big((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})\big).$$

For the derivatives we obtain

$$\left|\dot{B}_0^{(k)}(i_{R_0L_0},U,I)(t)-\dot{B}_0^{(k)}(\bar{i}_{R_0L_0},\overline{U},\bar{I})(t)\right|\le$$

$$\le\left|I_{R_0}(i_{R_0L_0},U,I)(t)-I_{R_0}(\bar{i}_{R_0L_0},\overline{U},\bar{I})(t)\right|+$$

$$+\frac{1}{T_0}\left|\int_{T+kT_0}^{T+(k+1)T_0}\big(I_{R_0}(i_{R_0L_0},U,I)(s)-I_{R_0}(\bar{i}_{R_0L_0},\overline{U},\bar{I})(s)\big)ds\right|\equiv$$

$$\equiv\dot{A}_1+\dot{A}_2;$$

$$\dot{A}_1\le e^{\mu(t-T-kT_0)}\frac{1}{\hat{L}_0^2}\Bigg[\sum_{n=1}^m n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu(t-T-kT_0)}\times$$

$$\times\sum_{n=1}^m(n+1)\left|l_n^{(0)}\right|I_{R_0}^n e^{n\mu(t-T-kT_0)}+$$

$$+\begin{pmatrix}U_0e^{\mu(t-T-kT_0)}+I_0e^{\mu(t-T-kT_0)}e^{-\mu T}\\+\sum_{n=1}^m\left|r_n^{(0)}\right|I_{R_0}^n e^{n\mu(t-T-kT_0)}\end{pmatrix}\times$$

$$\times\sum_{n=1}^m(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu(t-T-kT_0)}\Bigg]\rho_\mu^{(k)}(i_{R_0L_0},\bar{i}_{R_0L_0})+$$

$$+e^{\mu(t-T-kT_0)}\frac{\rho_\mu^{(k)}(U,\overline{U})}{\hat{L}_0}\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0})}{\mu\hat{L}_0^2}\Bigg[\sum_{n=1}^m n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\times$$

$$\times\sum_{n=1}^m(n+1)\left|l_n^{(0)}\right|I_{R_0}^n e^{n\mu_0}+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^m\left|r_n^{(0)}\right|I_{R_0}^n e^{n\mu_0}\right)\times$$

$$\times\sum_{n=1}^m(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\Bigg]+e^{\mu(t-T-kT_0)}\frac{\rho_\mu^{(k)}(\dot{U},\dot{\overline{U}})}{\mu\hat{L}_0}\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{1}{\mu}\Bigg\{\frac{\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0})}{\hat{L}_0^2}\Bigg[\sum_{n=1}^m n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\times$$

$$\times\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^m\left|r_n^{(0)}\right|I_{R_0}^n e^{n\mu_0}\right)+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^m\left|r_n^{(0)}\right|I_{R_0}^n e^{n\mu_0}\right)\times$$

$$\times\sum_{n=1}^m(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\Bigg]+\frac{\rho_\mu^{(k)}(\dot{U},\dot{\overline{U}})}{\hat{L}_0}\Bigg\}\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\big((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\overline{U},\bar{i}_{R_1L_1},\bar{I})\big)\frac{1}{\mu}\times$$

$$\times\left\{\frac{1}{\hat{L}_0^2}\left[\sum_{n=1}^{m} n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{\hat{L}_0}\right\}$$

and

$$\dot{A}_2\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\frac{e^{\mu_0}-1}{\mu_0}\frac{1}{\mu}\left\{\frac{1}{\hat{L}_0^2}\left[\sum_{n=1}^{m} n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{\hat{L}_0}\right\}.$$

Consequently

$$\left|\dot{B}_0^{(k)}(i_{R_0L_0},U,I)(t)-\dot{B}_0^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\left(1+\frac{e^{\mu_0}-1}{\mu_0}\right)\frac{1}{\mu}\left\{\frac{1}{\hat{L}_0^2}\left[\sum_{n=1}^{m} n\left|r_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\times\right.\right.$$

$$\times\sum_{n=1}^{m}(n+1)\left|l_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}+\begin{pmatrix}U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}\\ +\sum_{n=1}^{m}\left|r_n^{(0)}\right|I_{R_0}^{n}e^{n\mu_0}\end{pmatrix}\times$$

$$\left.\left.\times\sum_{n=1}^{m}(n+1)n\left|l_n^{(0)}\right|I_{R_0}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{\hat{L}_0}\right\}\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}\dot{K}_0\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

It follows

$$\rho_\mu^{(k)}(\dot{B}_0^{(k)}(i_{R_0L_0},U,I),\dot{B}_0^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I}))\le$$

$$\le\dot{K}_0\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

Further on we have

$$\left|\dot{B}_U^{(k)}(i_{R_0L_0},U,I)(t)-\dot{B}_U^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le\left|V(i_{R_0L_0},U,I)(t)-V(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|+$$

$$+\frac{1}{T_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|V(i_{R_0L_0},U,I)(s)-V(\bar{i}_{R_0L_0},\bar{U},\bar{I})(s)\right|ds\equiv\dot{W}_1+\dot{W}_2;$$

$$\dot{W}_1\le e^{\mu(t-T-kT_0)}\frac{2\rho_\mu^{(k)}(i_{R_0L_0},\bar{i}_{R_0L_0})}{\hat{C}_0}+$$

$$+e^{\mu(t-T-kT_0)}\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+e^{\mu_0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu_0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\rho_\mu^{(k)}(U,\bar{U})\le$$

$$\le e^{\mu(t-T-kT_0)}\left\{\frac{2\rho_\mu^{(k)}(\dot{i}_{R_0L_0},\dot{\bar{i}}_{R_0L_0})}{\mu\hat{C}_0}+\right.$$

$$+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+e^{\mu_0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu_0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\frac{\rho_\mu^{(k)}(\dot{U},\dot{\bar{U}})}{\mu}\right\}\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\frac{1}{\mu}\times$$

$$\times\left\{\frac{2}{\hat{C}_0}+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+e^{\mu_0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu_0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}$$

and

$$\dot{W}_2\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\frac{e^{\mu T_0}-1}{\mu T_0}\times$$

$$\times\frac{1}{\mu}\left\{\frac{2}{\hat{C}_0}+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+e^{\mu_0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu_0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}.$$

Therefore

$$\left|\dot{B}_U^{(k)}(i_{R_0L_0},U,I)(t)-\dot{B}_U^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{I})(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\left(1+\frac{e^{\mu_0}-1}{\mu_0}\right)\times$$

$$\times\frac{1}{\mu}\left\{\frac{2}{\hat{C}_0}+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_0\sqrt[h]{\Phi_0}}{\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+e^{\mu_0}\left(U_0+I_0e^{-\mu T}+4Z_0I_{R_0}\right)\times$$

$$\left.\left.\times\frac{c_0\sqrt[h]{\Phi_0}\left[h(\Phi_0-\phi_0)+e^{\mu_0}\left(U_0+I_0e^{-\mu T}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_0-\phi_0)^{1+2h}}}\right]\right\}\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}\dot{K}_U\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

Consequently

$$\rho_\mu^{(k)}(\dot B_U^{(k)}(i_{R_0L_0},U,I),\dot B_U^{(k)}(\bar i_{R_0L_0},\bar U,\bar I))\le$$

$$\le \dot K_U\hat\rho_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)\right).$$

Further on we have

$$\left|\dot B_1^{(k)}(U,i_{R_1L_1},I)(t)-\dot B_1^{(k)}(\bar U,\bar i_{R_1L_1},\bar I)(t)\right|\le$$

$$\le\left|I_{R_1}(U,i_{R_1L_1},I)(t)-I_{R_1}(\bar U,\bar i_{R_1L_1},\bar I)(t)\right|+$$

$$+\frac{1}{T_0}\int_{T+kT_0}^{T+(k+1)T_0}\left|I_{R_1}(U,i_{R_1L_1},I)(s)-I_{R_1}(\bar U,\bar i_{R_1L_1},\bar I)(s)\right|ds\equiv$$

$$\equiv\dot J_1+\dot J_2;$$

$$\dot J_1\le e^{\mu(t-T-kT_0)}\hat\rho_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)\right)\frac{1}{\mu}\times$$

$$\times\left\{\frac{1}{\hat L_1^2}\left[\sum_{n=1}^m n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^m(n+1)\left|l_n^{(1)}\right|I_{R_1}^n e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^m\left|r_n^{(1)}\right|I_{R_1}^n e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^m(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{2\hat L_1}\right\}$$

and

$$\dot J_2\le e^{\mu(t-T-kT_0)}\hat\rho_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)\right)\times$$

$$\times\frac{e^{\mu_0}-1}{\mu_0}\frac{1}{\mu}\left\{\frac{1}{\hat L_1^2}\left[\sum_{n=1}^m n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}.\sum_{n=1}^m(n+1)\left|l_n^{(1)}\right|I_{R_1}^n e^{n\mu_0}+\right.\right.$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^m\left|r_n^{(1)}\right|I_{R_1}^n e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^m(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{2\hat L_1}\right\};$$

$$\left|\dot B_1^{(k)}(U,i_{R_1L_1},I)(t)-\dot B_1^{(k)}(\bar U,\bar i_{R_1L_1},\bar I)(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\left(1+\frac{e^{\mu_0}-1}{\mu_0}\right)\frac{1}{\mu}\left\{\frac{1}{\hat L_1^2}\left[\sum_{n=1}^m n\left|r_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\times\right.\right.$$

$$\times\sum_{n=1}^m(n+1)\left|l_n^{(1)}\right|I_{R_1}^n e^{n\mu_0}+$$

$$+\left(U_0e^{\mu_0}+I_0e^{\mu_0}e^{-\mu T}+\sum_{n=1}^m\left|r_n^{(1)}\right|I_{R_1}^n e^{n\mu_0}\right)\times$$

$$\left.\left.\times\sum_{n=1}^m(n+1)n\left|l_n^{(1)}\right|I_{R_1}^{n-1}e^{(n-1)\mu_0}\right]+\frac{1}{2\hat L_1}\right\}\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}\dot K_1\hat\rho_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)\right).$$

Then

$$\rho_\mu^{(k)}\left(\dot B_1^{(k)}(U,i_{R_1L_1},I),\dot B_1^{(k)}(\bar U,\bar i_{R_1L_1},\bar I)\right)\le$$

$$\le\dot K_1\hat\rho_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)\right)$$

$$(k=0,1,2,\ \dots,m-1).$$

For the last component of the derivative we obtain

$$\left|\dot B_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)-\dot B_I^{(k)}(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)(t)\right|\le$$

$$\le\left|J(i_{R_0L_0},U,i_{R_1L_1},I)(t)-J(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)(t)\right|+$$

$$+\frac{1}{T_0}\left|\int_{T+kT_0}^{T+(k+1)T_0}\left(J(i_{R_0L_0},U,i_{R_1L_1},I)(s)-J(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)(s)\right)ds\right|\equiv$$

$$\equiv\dot K_1+\dot K_2;$$

$$\dot K_1\le e^{\mu(t-T-kT_0)}\frac{2}{\hat C_1}\rho_\mu^{(k)}(i_{R_1L_1},\bar i_{R_1L_1})+$$

$$+e^{\mu(t-T-kT_0)}\frac{1}{Z_0\hat C_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\left.\times\frac{c_1\sqrt[h]{\Phi_1}\left[2h(\Phi_1-\phi_0)+\left(U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}\right)(1+h)\right]}{h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\rho_\mu^{(k)}(I,\bar I)\le$$

$$\le e^{\mu(t-T-kT_0)}\frac{1}{\mu}\left\{\frac{2}{\hat C_1}\rho_\mu^{(k)}(\dot i_{R_1L_1},\dot{\bar i}_{R_1L_1})+\right.$$

$$+\frac{1}{Z_0\hat C_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\left.\left.\times\frac{2c_1\sqrt[h]{\Phi_1}\left[2h(\Phi_1-\phi_0)+e^{\mu_0}\left(U_0e^{-\mu T}+I_0\right)(1+h)\right]}{2h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\rho_\mu^{(k)}(\dot I,\dot{\bar I})\right\}\le$$

$$\le e^{\mu(t-T-kT_0)}\hat\rho_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar i_{R_0L_0},\bar U,\bar i_{R_1L_1},\bar I)\right)\times$$

$$\times\frac{1}{\mu}\left\{\frac{2}{\hat C_1}+\frac{1}{Z_0\hat C_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\right.\right.$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\left.\left.\times\frac{2c_1\sqrt[h]{\Phi_1}\left[2h(\Phi_1-\phi_0)+e^{\mu_0}\left(U_0e^{-\mu T}+I_0\right)(1+h)\right]}{2h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\right\}$$

and

$$\dot{K}_2 \le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\frac{e^{\mu_0}-1}{\mu_0}\frac{1}{\mu}\left\{\frac{2}{\hat{C}_1}+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\varphi_0)^{1+2h}}}\frac{\Phi_0+(2+(1/h))\varphi_0}{h}+\right.\right.$$

$$+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\left.\left.\times\frac{2c_1\sqrt[h]{\Phi_1}\left[2h(\Phi_1-\varphi_0)+e^{\mu_0}\left(U_0e^{-\mu T}+I_0\right)(1+h)\right]}{2h^2\sqrt[h]{(\Phi_1-\varphi_0)^{1+2h}}}\right]\right\}.$$

On the other hand

$$\left|\dot{B}_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I)(t)-\dot{B}_I^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})(t)\right|\le$$

$$\le e^{\mu(t-T-kT_0)}\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\times$$

$$\times\left(1+\frac{e^{\mu_0}-1}{\mu_0}\right)\frac{1}{\mu}\left\{\frac{2}{\hat{C}_1}+\frac{1}{Z_0\hat{C}_1^2}\left[\frac{2c_1\sqrt[h]{\Phi_1}}{\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\times\right.\right.$$

$$\times\frac{\Phi_0+(2+(1/h))\phi_0}{h}+\frac{U_0e^{-\mu T}e^{\mu_0}+I_0e^{\mu_0}+2Z_0I_{R_1}e^{\mu_0}}{2}\times$$

$$\left.\left.\times\frac{2c_1\sqrt[h]{\Phi_1}\left[2h(\Phi_1-\phi_0)+e^{\mu_0}\left(U_0e^{-\mu T}+I_0\right)(1+h)\right]}{2h^2\sqrt[h]{(\Phi_1-\phi_0)^{1+2h}}}\right]\right\}\equiv$$

$$\equiv e^{\mu(t-T-kT_0)}\dot{K}_I\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

Consequently

$$\rho_\mu^{(k)}(\dot{B}_I^{(k)}(i_{R_0L_0},U,i_{R_1L_1},I),\dot{B}_I^{(k)}(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I}))\le$$
$$\le\dot{K}_I\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right).$$

Finally we have

$$\hat{\rho}_\mu\left(B_0(i_{R_0L_0},U,i_{R_1L_1},I),B_U(i_{R_0L_0},U,i_{R_1L_1},I),\right.$$
$$B_1(i_{R_0L_0},U,i_{R_1L_1},I),B_I(i_{R_0L_0},U,i_{R_1L_1},I),$$
$$B_0(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I}),B_U(\bar{i}_{R_0L_0},\bar{U},i_{R_1L_1},\bar{I}),$$
$$\left.B_1(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I}),B_I(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right)\le$$
$$\le K\hat{\rho}_\mu\left((i_{R_0L_0},U,i_{R_1L_1},I),(\bar{i}_{R_0L_0},\bar{U},\bar{i}_{R_1L_1},\bar{I})\right),$$

where

$$K=\max\left\{e^{\mu_0}K_0,e^{\mu_0}K_U,e^{\mu_0}K_1,e^{\mu_0}K_I,\dot{K}_0,\dot{K}_U,\dot{K}_1,\dot{K}_I\right\}<1.$$

Then B has a unique fixed point which is a periodic solution of (10).

Theorem 1 is thus proved.

2.6. Numerical Example

For a transmission line with length $\Lambda=1m$, $L=0{,}45\,\mu H/m$, $C=80\,pF/m$, $v=1/\sqrt{LC}=1/\left(6.10^{-9}\right)=1{,}66.10^8$; $Z_0=\sqrt{L/C}=75\Omega$. Then $T=\Lambda\sqrt{LC}=6.10^{-9}\,\mathrm{sec}$.

Let us check the propagation of waves with $\lambda_0=(1/6)10^{-3}m$. We have

$$f_0=1/\left(\lambda_0\sqrt{LC}\right)=10^{12}\,Hz\Rightarrow T_0=1/f_0=10^{-12}.$$

We choose $\mu=10^{12}$, then $\mu T_0=\mu_0=1$ and $T=12000.T_0\Rightarrow m=12000\Rightarrow e^{-\mu T}=e^{-12000}\approx 0$.

We choose resistive elements with the following *V-I* characteristics $R_0(i)=R_1(i)=0{,}028i-0{,}125i^3$ and inductive elements with $L_0(i)=L_1(i)=3i-(1/12)i^3$. Then $\bar{L}_0(i)=i(dL_0(i)/di)+L_0(i)=6i-(1/3)i^3$. For $i_0=1$ one obtains $\bar{L}_0(i)>6-(1/3)=17/3\Rightarrow 1/\hat{L}_0=3/17$.

Let us take $C_0(u)=C_1(u)=c_0/\sqrt{1-(u/\Phi_0)}=c_0\sqrt{\Phi_0}/\sqrt{\Phi_0-u}$ where $h=2$. Let us choose $\phi_0=0{,}2$; $c_0=c_1=50pF=5.10^{-11}F$ and $\Phi_0=\Phi_1=0{,}4\,V\Rightarrow U_0<\phi_0<0{,}4$. Then

$$C_0(u)=c_0/\sqrt{1-(u/\Phi_0)}=c_0\sqrt{\Phi_0}/\sqrt{\Phi_0-u}\ge$$
$$\ge C_0(-U_0)=c_0\sqrt{\Phi_0}/\sqrt{\Phi_0+U_0}=\hat{C}_0$$
$$\hat{C}_0=5.10^{-11}\sqrt{0{,}4}/\sqrt{0{,}4+0{,}2}\Rightarrow\hat{C}_0=\hat{C}_1=8{,}2.10^{-11}.$$

Then the above inequalities for $h=2$; $U_0=0{,}01$; $I_0=0{,}01$; $I_{R_0}=I_{R_1}=0{,}1$ become:

$$1{,}36.0{,}01\le 0{,}2;\quad 1{,}36.0{,}01\le 0{,}2;$$
$$0{,}18e.10^{-12}\left(7{,}8.10^{-3}+1{,}25.10^{-4}e^2\right)\le 10^{-1};\quad \frac{30{,}01e}{8{,}2.75}\le 0{,}1;$$
$$0{,}18e.10^{-12}\left(7{,}8.10^{-3}+1{,}25.10^{-4}e^2\right)\le 0{,}1;\quad \frac{30{,}01e}{8{,}2.75}\le 0{,}1.$$

We calculate just $\dot{K}_0,\dot{K}_U,\dot{K}_1,\dot{K}_I$ since K_0,K_U,K_1,K_I are of order $1/\mu^2$: $\dot{K}_0=0{,}544.10^{-12}<1$; $\dot{K}_U=9/82<1$; $\dot{K}_1=0{,}544.10^{-12}<1$; $\dot{K}_I=9/82<1\Rightarrow K\approx 0{,}111$.

3. Conclusions

We consider transmission lines neglecting the lossies. This makes it possible to find conditions for the existence and uniqueness of periodic regimes. This natural physical fact is confirmed by the mathematical method we apply.

• In order to prove an existence-uniqueness theorem we introduce an operator (unknown in the literature up to now) whose fixed points are periodic solutions of the problem stated. We apply contractive fixed point theorems in metric spaces. By extended Bielecki metrics we overcome the difficulties caused by polynomial and transcendental nonlinearities.

• The numerical example demonstrates a frame of applicability of the theory exposed (for instance to design of circuits) and shows that the method could be applied checking few simple inequalities between the basic specific parameter of the lines and loads.

• We show a unified approach for solving problems for analysis of transmission lines terminated various configurations of nonlinear loads. In contrast to various results devoted to numerical methods[8]-[14] we obtain an explicit approximated solution.

• We emphasize on the fact that first we prove not only an existence but and uniqueness of the solution as well. So our successive approximations tend to this solution. All other methods need such a uniqueness result. Unfortunately_in most papers the uniqueness is not ensured.

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