

Phase Velocities of Three-Dimensional and Axis-Symmetrical Elastic Waves in Isotropic Cylindrical Shell

S. L. Ilmenkov*, A. A. Kleshchev

Saint-Petersburg State Navy Technical University, Saint-Petersburg, Lotsmanskaya str., 3, 190008, Russia

Abstract Based on use of Debye's potentials one can find the direct solution of the problem of definition of the characteristic equations for wave numbers of three-dimensional and axis-symmetrical (flexural and longitudinal) elastic waves in steel and aluminium cylindrical shells of various thickness.

Keywords Phase Velocity, Debye's Potential, Flexural Wave, Longitudinal Wave, Three-Dimensional Problem, Axis-Symmetrical Problem

1. Introduction

The Debye's potentials are used for the first time for a study of the three-dimensional flexural waves. The paper shows the existing of the three-dimensional and axis-symmetrical flexural waves in cylindrical shell (contrary to the cylindrical bar). The paper demonstrates the calculated values of the phase velocities of axis-symmetrical and three-dimensional flexural waves, while the phase velocities of the non-zero forms of three-symmetrical flexural waves are calculated for the first time.

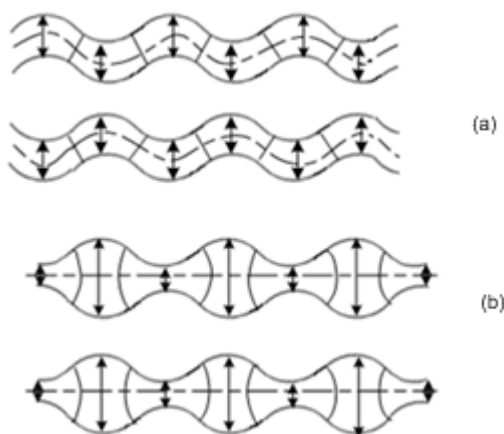


Figure 1. The deformation of cylindrical shell for flexural (a) and longitudinal (b) axis-symmetrical waves

2. The Three-Dimensional Flexural Waves in Cylindrical Shell

The first part of the article substantiates the effectiveness of usage of Debye's potentials[1 – 3] for studying of three-dimensional flexural waves in cylindrical shell.

In contrast to the bar the flexural wave in cylindrical shell can be three-dimensional and two-dimensional (axis-symmetrical). The deformation of cylindrical shell in the propagation of axis-symmetrical flexural (a) and longitudinal (b) waves in it is schematically shown in fig. 1.

Let's start with examination of three-dimensional flexural wave in isotropic cylindrical shell. In this case the same mathematical apparatus is used as in the study of flexural waves in bar[4], but the number of unknown quantities and the number of boundary conditions increase with the account of the second (internal) boundary surface. Now the expansions of the potentials Φ , V , U [5 – 15] take the form:

$$\left. \begin{aligned} \Phi &= e^{ikz} \sum_{m=0}^{\infty} \cos m\phi \left[A_m J_m(h'r) + B_m N_m(h'r) \right]; \\ V &= e^{ikz} \sum_{m=0}^{\infty} \cos m\phi \left[C_m J_m(\kappa'r) + D_m N_m(\kappa'r) \right]; \\ U &= e^{ikz} \sum_{m=1}^{\infty} \sin m\phi \left[E_m J_m(\kappa'r) + F_m N_m(\kappa'r) \right], \end{aligned} \right\} \quad (1)$$

where $h' = (k_1^2 - k^2)^{1/2}$; $\kappa' = (k_2^2 - k^2)^{1/2}$;

$N_m(h'r)$ - are the Naman's cylindrical function; A_m , B_m , C_m , D_m , E_m , F_m - are the unknown coefficients of expanding, which are calculated basing on the following physical

* Corresponding author:

ilms@rambler.ru (S. L. Ilmenkov)

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boundary conditions on the external ($r = a$) and inside ($r = b$) surfaces of the shell: there are no normal and tangent tensions on the both boundaries of the shell.

We desire the analytical form of boundary conditions as following:

$$(\lambda + 2\mu) \left(\frac{\partial U_r}{\partial r} \right) + \lambda \left[\frac{1}{r} \left(\frac{\partial U_\varphi}{\partial \varphi} \right) + \frac{1}{r} U_r + \left(\frac{\partial U_z}{\partial z} \right) \right] \Big|_{r=a; r=b} = 0 \quad (2)$$

$$\left(\frac{\partial U_\varphi}{\partial r} \right) + \frac{1}{r} U_\varphi + \frac{1}{r} \left(\frac{\partial U_r}{\partial \varphi} \right) \Big|_{r=a; r=b} = 0 \quad (3)$$

$$\left(\frac{\partial U_r}{\partial r} \right) + \left(\frac{\partial U_z}{\partial z} \right) \Big|_{r=a; r=b} = 0 \quad (4)$$

If we substitute (1) in the boundary conditions (2) - (4), we'll get the determinant of six order [5 - 15]:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{vmatrix}, \quad (5)$$

where

$$\begin{aligned} a_{11} &= -(\lambda + 2\mu) J_m''(h'a) + \\ &+ \lambda [a^{-2} m^2 J_m(h'a) - a^{-1} J_m'(h'a) + k^2 J_m(h'a)]; \\ a_{12} &= -(\lambda + 2\mu) N_m''(h'a) + \\ &+ \lambda [a^{-2} m^2 N_m(h'a) - a^{-1} N_m'(h'a) + k^2 N_m(h'a)]; \\ a_{13} &= (\lambda + 2\mu) \{ -i k_2 m^2 [J_m(\kappa'a) - J_m'(\kappa'a)] + \\ &+ i k_2 J_m''(\kappa'a) + i k_2 k^2 [J_m(\kappa'a) + a J_m'(\kappa'a)] \} + \\ &+ \lambda \{ i k_2 a^{-2} J_m(\kappa'a) (k^2 + m^2) - i k_2 a^{-1} J_m'(\kappa'a) (m^2 + k^2) - \\ &- 2 i k_2 k^2 J_m(\kappa'a) - i k_2 J_m''(\kappa'a) \}; \\ a_{14} &= (\lambda + 2\mu) \{ -i k_2 m^2 [N_m(\kappa'a) - N_m'(\kappa'a)] + \\ &+ i k_2 N_m''(\kappa'a) + i k_2 k^2 [N_m(\kappa'a) + a N_m'(\kappa'a)] \} + \\ &+ \lambda \{ i k_2 a^{-2} N_m(\kappa'a) (k^2 + m^2) - i k_2 a^{-1} N_m'(\kappa'a) \times \\ &\times (m^2 + k^2) - 2 i k_2 k^2 N_m(\kappa'a) - i k_2 N_m''(\kappa'a) \}; \\ a_{15} &= 2(\lambda + 2\mu) (a^{-2} - 1) i k m J_m(\kappa'a); \\ a_{16} &= 2(\lambda + 2\mu) (a^{-2} - 1) i k m N_m(\kappa'a); \\ a_{21} &= -(\lambda + 2\mu) J_m''(h'b) + \\ &+ \lambda [b^{-2} m^2 J_m(h'b) - b^{-1} J_m'(h'b) + k^2 J_m(h'b)]; \\ a_{22} &= -(\lambda + 2\mu) N_m''(h'b) + \\ &+ \lambda [b^{-2} m^2 N_m(h'b) - b^{-1} N_m'(h'b) + k^2 N_m(h'b)]; \end{aligned}$$

$$\begin{aligned}
 a_{23} &= (\lambda + 2\mu) \{-ik_2 m^2 [J_m(\kappa'b) - J'_m(\kappa'b)] + \\
 &\quad + ik_2 J''_m(\kappa'b) + ik_2 k^2 [J_m(\kappa'b) + b J'_m(\kappa'b)]\} + \\
 &\quad + \lambda \{ik_2 b^{-2} J_m(\kappa'b)(k^2 + m^2) - ik_2 b^{-1} J'_m(\kappa'b)(m^2 + k^2) - \\
 &\quad - 2ik_2 k^2 J_m(\kappa'b) - ik_2 J''_m(\kappa'b)\}; \\
 a_{24} &= (\lambda + 2\mu) \{-ik_2 m^2 [N_m(\kappa'b) - N'_m(\kappa'b)] + \\
 &\quad + ik_2 N''_m(\kappa'b) + ik_2 k^2 [N_m(\kappa'b) + b N'_m(\kappa'b)]\} + \\
 &\quad + \lambda \{ik_2 b^{-2} N_m(\kappa'b)(k^2 + m^2) - ik_2 b^{-1} N'_m(\kappa'b) \times \\
 &\quad \times (m^2 + k^2) - 2ik_2 k^2 N_m(\kappa'b) - ik_2 N''_m(\kappa'b)\}; \\
 a_{25} &= 2(\lambda + 2\mu)(b^{-2} - 1)ikm J_m(\kappa'b); \\
 a_{26} &= 2(\lambda + 2\mu)(b^{-2} - 1)ikm N_m(\kappa'b); \\
 a_{31} &= 2ma^{-1}[J'_m(h'a) - a^{-1}J_m(h'a)]; \\
 a_{32} &= 2ma^{-1}[N'_m(h'a) - a^{-1}N_m(h'a)]; \\
 a_{33} &= ik_2 m \{J_m(\kappa'a)[a^{-2}(2 - m^2) - k^2] - J''_m(\kappa'a) - a^{-1}J'_m(\kappa'a)\}; \\
 a_{34} &= ik_2 m \{N_m(\kappa'a)[a^{-2}(2 - m^2) - k^2] - N''_m(\kappa'a) - a^{-1}N'_m(\kappa'a)\}; \\
 a_{35} &= ik J'_m(\kappa'a)ak_2^2; \\
 a_{36} &= ik N'_m(\kappa'a)ak_2^2; \\
 a_{41} &= 2mb^{-1}[J'_m(h'b) - b^{-1}J_m(h'b)]; \\
 a_{42} &= 2mb^{-1}[N'_m(h'b) - b^{-1}N_m(h'b)]; \\
 a_{43} &= ik_2 m \{J_m(\kappa'b)[b^{-2}(2 - m^2) - k^2] - J''_m(\kappa'b) - b^{-1}J'_m(\kappa'b)\}; \\
 a_{44} &= ik_2 m \{N_m(\kappa'b)[b^{-2}(2 - m^2) - k^2] - N''_m(\kappa'b) - b^{-1}N'_m(\kappa'b)\}; \\
 a_{45} &= ik J'_m(\kappa'b)bk_2^2; \\
 a_{46} &= ik N'_m(\kappa'b)bk_2^2; \\
 a_{51} &= -2ik J'_m(h'a); \\
 a_{52} &= -2ik N'_m(h'a); \\
 a_{53} &= -kk_2 \{J_m(\kappa'a)(m^2 + a^{-1}k^2) + 5J'_m(\kappa'a) + aJ''_m(\kappa'a)\}; \\
 a_{54} &= -kk_2 \{N_m(\kappa'a)(m^2 + a^{-1}k^2) + 5N'_m(\kappa'a) + aN''_m(\kappa'a)\}; \\
 a_{55} &= k_2^2 m [a^{-1}J_m(\kappa'a) - J'_m(\kappa'a)]; \\
 a_{56} &= k_2^2 m [a^{-1}N_m(\kappa'a) - N'_m(\kappa'a)]; \\
 a_{61} &= -2ik J'_m(h'b); \\
 a_{62} &= -2ik N'_m(h'b); \\
 a_{63} &= -kk_2 \{J_m(\kappa'b)(m^2 + b^{-1}k^2) + 5J'_m(\kappa'b) + bJ''_m(\kappa'b)\}; \\
 a_{64} &= -kk_2 \{N_m(\kappa'b)(m^2 + b^{-1}k^2) + 5N'_m(\kappa'b) + bN''_m(\kappa'b)\}; \\
 a_{65} &= k_2^2 m [b^{-1}J_m(\kappa'b) - J'_m(\kappa'b)]; \\
 a_{66} &= k_2^2 m [b^{-1}N_m(\kappa'b) - N'_m(\kappa'b)];
 \end{aligned}$$

Equating the determinant (5) zero and opening him, we receive the characteristic equation for wave numbers of form m of three-dimensional flexural waves in a isotropic cylindrical shell of any (but constant) thickness.

The decision of the characteristic equation for steel and aluminium shells of various thickness is submitted in a fig. 2

and 3, thus external radius was accepted equal 1,0, and radius b accepted two meanings: $b = 0,99$ (continuous curve) and $b = 0,8$ (dotted curve).

The meanings of velocities longitudinal (C_1), lateral (C_2) and Rayleigh's wave (C_R) are shown on the diagrams. The meaning $m = 1$ corresponds to zero form of flexural wave,

which velocity aspires asymptotically to velocity of Rayleigh's wave C_R , Λ - length of a longitudinal wave in the shell's material; $\Lambda = c_l / f$, where f - frequency of a wave in Hz. In a fig. 2 the phase velocities of three-dimensional flexural waves in steel shells, in a fig. 3 - in aluminium shells are shown.

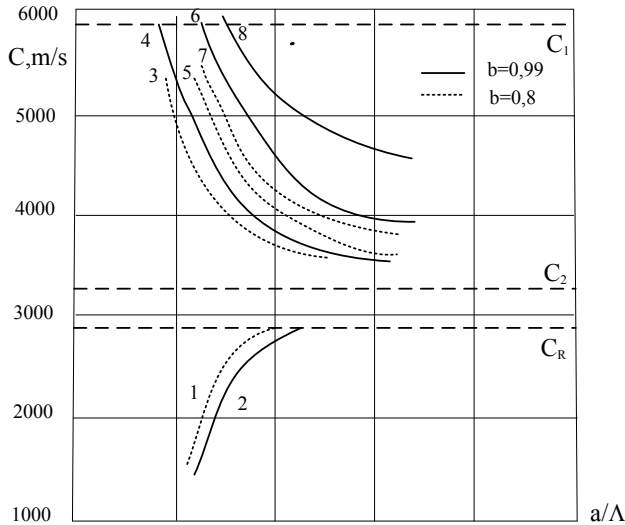


Figure 2. Phase velocities of three-dimensional flexural waves in steel shells

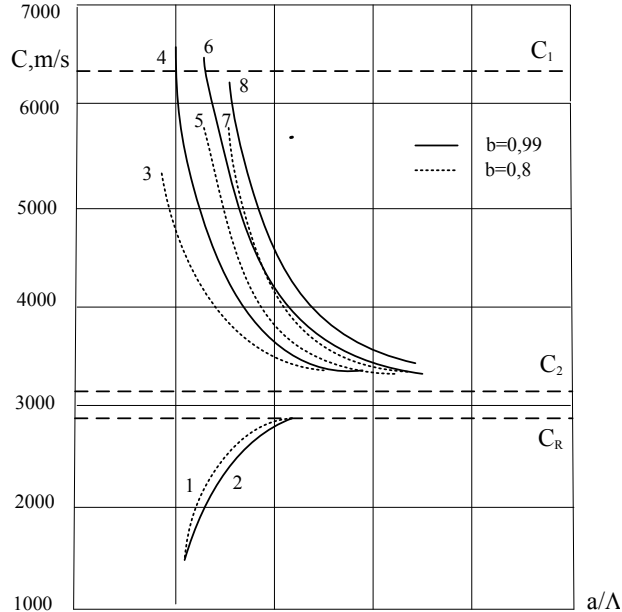


Figure 3. Phase velocities of three-dimensional flexural waves in aluminum shells

Let's turn to axis-symmetrical longitudinal and flexural waves. In accordance with [5 – 15] in axis-symmetrical case the boundary conditions (2) – (4) become simpler: the condition (3) disappears and the condition (2) takes the following form (in this case the index $m=0$ or l):

$$(\lambda + 2\mu) \left(\frac{\partial U_r}{\partial r} \right) + \lambda \left[\frac{1}{r} U_r + \left(\frac{\partial U_z}{\partial z} \right) \right] \Big|_{r=a, r=b} = 0 \quad (6)$$

The determinant of the fourth order derived from boundary conditions takes the form [5]*:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}; \quad (7)$$

where

$$\begin{aligned} a_{11} &= -(\lambda + 2\mu) J_0''(h'a) + \lambda [k^2 J_0(h'a) - a^{-1} J_0'(h'a)]; \\ a_{12} &= -(\lambda + 2\mu) N_0''(h'a) + \lambda [k^2 N_0(h'a) - a^{-1} N_0'(h'a)]; \\ a_{13} &= -2i\mu k J_1'(\kappa'a); \\ a_{14} &= -2i\mu k N_1'(\kappa'a); \\ a_{21} &= -(\lambda + 2\mu) J_0''(h'b) + \lambda [k^2 J_0(h'b) - b^{-1} J_0'(h'b)]; \\ a_{22} &= -(\lambda + 2\mu) N_0''(h'b) + \lambda [k^2 N_0(h'b) - b^{-1} N_0'(h'b)]; \\ a_{23} &= -2i\mu k J_1'(\kappa'b); \\ a_{24} &= -2i\mu k N_1'(\kappa'b); \\ a_{32} &= -2ik N_0'(h'a); \end{aligned}$$

* In the present work the derivatives of cylindrical functions at radial coordinate r are marked the following way: $J_0'(h'r) = \partial J_0(h'r) / \partial r$; $N_0'(h'r) = \partial N_0(h'r) / \partial r$; in the work [5] differently: $J_0'(h'r) = \partial J_0(h'r) / \partial(h'r)$; $N_0'(h'r) = \partial N_0(h'r) / \partial(h'r)$. Both approaches are competent.

$$\begin{aligned} a_{33} &= k^2 J_1(\kappa'a) - J_1(\kappa'a) + J_1'(\kappa'a) + J_1''(\kappa'a); \\ a_{34} &= k^2 N_1(\kappa'a) - N_1(\kappa'a) + N_1'(\kappa'a) + N_1''(\kappa'a); \\ a_{41} &= -2ik J_0'(h'b); \\ a_{42} &= -2ik N_0'(h'b); \end{aligned}$$

$$\begin{aligned} a_{43} &= k^2 J_1(\kappa'b) - J_1(\kappa'b) + J_1'(\kappa'b) + J_1''(\kappa'b); \\ a_{44} &= k^2 N_1(\kappa'b) - N_1(\kappa'b) + N_1'(\kappa'b) + N_1''(\kappa'b). \end{aligned}$$

Equating the determinant (7) zero and opening it, we get the characteristic equation for wave numbers of flexural and longitudinal axis-symmetrical waves.

The determinant for axis-symmetrical torsion waves are represented in [5]:

$$\Delta = \begin{vmatrix} a^{-1} J_1'(\kappa_1 a) - J_1(\kappa_1 a) a^{-2} & a^{-1} N_1'(\kappa_1 a) - N_1(\kappa_1 a) a^{-2} \\ b^{-1} J_1'(\kappa_1 b) - b^{-2} J_1(\kappa_1 b) & b^{-1} N_1'(\kappa_1 b) - b^{-2} N_1(\kappa_1 b) \end{vmatrix}, \quad (8)$$

where $\kappa_1 = (k_2^2 - k^2)^{1/2}$, k - is the desired wave number of axis-symmetrical torsion wave in the shell.

Equating the determinant (8) zero and opening it, we get the characteristic equation for wave numbers of axis-symmetrical torsion waves.

3. The Results of Numerical Experiment for Determination of Phase Velocities of Elastic Waves

The second part of the article investigates the results of numerical experiment for determination of phase velocities of elastic waves (axis-symmetrical and three-dimensional) in cylindrical shell and analyses the calculated dependences.

The results of calculations of phase velocities are represented in the fig.4.

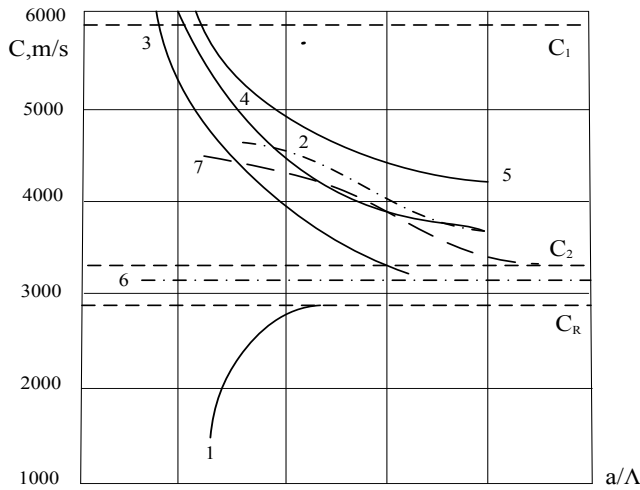


Figure 4. Phase velocities of axis-symmetrical flexural, longitudinal and torsion waves in steel cylindrical shell

One can find the following designations in the picture: the curve 1 - is the phase velocity of zero mode of flexural wave; the curve 2 - is the phase velocity of zero mode of longitudinal wave; the curves 3, 4, 5 - are the phase velocities of nonzero modes of longitudinal or of flexural waves; the straight line 6 - is the phase velocity of zero mode of torsion wave; the curve 7 - is the phase velocity of the first mode of torsion wave: $\Lambda = c_l / f$.

In order to define the influence of external and internal liquid mediums on dispersion curves of phase velocities of shell to the determinant of the sixth order (5) are added two columns, and two lines and it converts into the determinant of the eighth order, and determinant of the fourth order - into the determinant of the sixth order. The potentials of sound waves (in external environment) and in the filling of the shell expand by the cylindrical functions the following way:

$$\Phi_1 = \sum_{m=0}^{\infty} G_m H_m^{(1)}(\gamma_1 r) \cos m\varphi e^{ikz}; \quad (9)$$

$$\Phi_2 = \sum_{m=0}^{\infty} K_m J_m(\gamma_2 r) \cos m\varphi e^{ikz}, \quad (10)$$

where:

$\gamma_1 = \sqrt{(\omega/c_3)^2 - k^2}$; $\gamma_2 = \sqrt{(\omega/c_4)^2 - k^2}$; c_3 and c_4 - sound velocities in external and internal environments correspondingly.

The component of wave vector lengthwise of axis $z \Rightarrow k$, as in the shell in the force of Snellius law.

In this case the boundary conditions (2) for normal tensions on the both surfaces of the shell transforms:

$$(\lambda + 2\mu)(\partial U_r / \partial r) + \lambda \left[\frac{1}{r} \left(\frac{\partial U_\varphi}{\partial \varphi} \right) + \frac{1}{r} U_r + (\partial U_z / \partial z) \right] + i\omega \rho_l \Phi_1 = 0, \quad |_{r=a} \quad (11)$$

where: ρ_l - the solidity of the external environment.

$$(\lambda + 2\mu)(\partial U_r / \partial r) + \lambda \left[\frac{1}{r} \left(\frac{\partial U_\varphi}{\partial \varphi} \right) + \frac{1}{r} U_r + (\partial U_z / \partial z) \right] + i\omega \rho_2 \Phi_2 = 0, \quad |_{r=b} \quad (12)$$

where: ρ_2 - the solidity of the internal environment.

Two more boundary conditions are added: the normal components of displacement vector are continuous in the both borders of the shell:

$$\frac{\partial \Phi}{\partial r} + \frac{1}{r} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} - \frac{\partial \Phi_l}{\partial r} \Big|_{r=a} = 0 \quad (13)$$

$$\left. \frac{\partial \Phi}{\partial r} + \frac{1}{r} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} - \frac{\partial \Phi_2}{\partial r} \right|_{r=b} = 0 \quad (14)$$

In axis-symmetrical case the condition (6) transforms:

$$(\lambda + 2\mu) \left(\frac{\partial U_r}{\partial r} \right) + \lambda \left[\frac{1}{r} U_r + \left(\frac{\partial U_z}{\partial z} \right) \right] + i\omega \rho_1 \Phi_1 \Big|_{r=a} = 0 \quad (15)$$

$$(\lambda + 2\mu) \left(\frac{\partial U_r}{\partial r} \right) + \lambda \left[\frac{1}{r} U_r + \left(\frac{\partial U_z}{\partial z} \right) \right] + i\omega \rho_2 \Phi_2 \Big|_{r=b} = 0 \quad (16)$$

The added boundary conditions in axis-symmetrical case:

$$\left. \frac{\partial \Phi}{\partial r} - \frac{\partial A}{\partial z} - \frac{\partial \Phi_1}{\partial r} \right|_{r=a} = 0 \quad (17)$$

$$\left. \frac{\partial \Phi}{\partial r} - \frac{\partial A}{\partial z} - \frac{\partial \Phi_2}{\partial r} \right|_{r=b} = 0 \quad (18)$$

In axis-symmetrical case:

$$\Phi_1 = G_0 H_0^{(1)}(\gamma_1 r) e^{ikz}; \quad (19)$$

$$\Phi_2 = K_0 J_0(\gamma_2 r) e^{ikz}. \quad (20)$$

The boundary conditions (11), (12) and (17), (18) will add two lines to determinant, and the multipliers of unknown coefficients G_m, K_m or G_0, K_0 (axis-symmetrical problem) will add two columns. In this case only two lines in each of these two columns will be different from zero.

If the shell borders the liquid only from one side (and from the other is still the vacuum) the determinant for determination of wave numbers will have the seventh order in three-dimensional problem and the fifth order (in axis-symmetrical case), that is to determinants (5) и (7) one line and one column are added correspondingly.

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4. Conclusions

Based on use of Debye's potentials one can find the direct solution of the problem of definition of the characteristic equation for phase velocities of three – dimensional flexural waves. The numerical solution of this equation destines to obtain the dispersive curves of phase velocities. The dispersive dependencies of phase velocities of axis – symmetrical flexural, longitudinal and torsion waves are represented.

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