

Exchange Rate Forecasting Using Non-linear Threshold Models

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Abstract Foreign exchange is one of the most important financial instruments very volatile and chaotic in nature. Nowadays, the role of the foreign exchange market is becoming more and more important in the financial markets around the world; it remains the only instruments worldwide to measure the standard of living, Economic performance and country standing among the committee of nations. The foreign exchange market which is an over-the-counter market is used for the trading of currencies. It makes the foreign exchange market the largest and most liquid market among the financial markets. Necessary mathematical frame work was put in place, this was illustrated with data from record of Central Bank of Nigeria through their official website. Through this the performance of nonlinear threshold models in forecasting the exchange rate of Nigeria in relation to United States of American dollar as bench mark. The software used for the analysis was Econometrics View (E-view). Stationarity tests were carried out before the analysis, (the original data was not stationary, but at first difference it was stationary, thereafter comprehensive data analysis was performed). The forecasting results indicate that SETAR models did not outperform Random Walk in any period. TAR models offered promising results in the period. This study supports the general belief that the exchange rates are chaotic, volatile and very difficult to forecast and this applies to Exchange rate system of Nigeria as it is the case in the developed world.

Keywords Forecasting, Stationarity test, Exchange rate, Nonlinearity, SETAR, TAR

1. Introduction

Foreign exchange is one of the most important financial instruments. Nowadays, the role of the foreign exchange market is becoming more and more important in the financial markets around the world. The foreign exchange market which is an over-the-counter market is used for the trading of currencies. The trading is happening 24 hours a day around the world and a great number of currencies is being transacted every hour. It makes the foreign exchange market the largest and most liquid market among the financial markets.

There are so many studies on exchange rate forecasting carried out worldwide. Until now, there are a lot of forecasting models, each having its point of strengths and weaknesses used in exchange rate forecasting, such study

include ARIMA model (Tseng, 2001), [19], Least Squared model [10] or Purchasing Power Parity model and [3]. [13] used ARIMA model for forecasting inflation in Irish, [14] used Arima model for forecasting stock price. ARIMA is also used for predicting stock price in the research of [12], [11]. It is also used for forecasting the price of gold [8].

According to [1] Triennial Survey, turnover in global foreign exchange markets averaged \$5.1 trillion per day. This is down from \$5.4 trillion in April 2013, a month which had seen heightened activity in Japanese yen against the background of monetary policy developments at that time. In addition, exchange rate movements are used to compare influence of the previous surveys on the current year. In particular, the appreciation of the US dollar between 2013 compared with 2016. When valued at constant (April 2016) exchange rates, turnover increased slightly, by about 4% between April 2016 and April 2013. Nevertheless, the latest developments contrast with the strong growth in turnover observed between Triennial Surveys since 2001.

The foreign exchange rate has two main uses. Firstly, it allows the businesses to exchange its currency to a target currency in the determined foreign exchange rate. Thus, it benefits the global trade and investment. Secondly, it also

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provides the speculation and expedites the carry trade, in which there are substantial profits available. However, there

also exists high risk in the speculation.

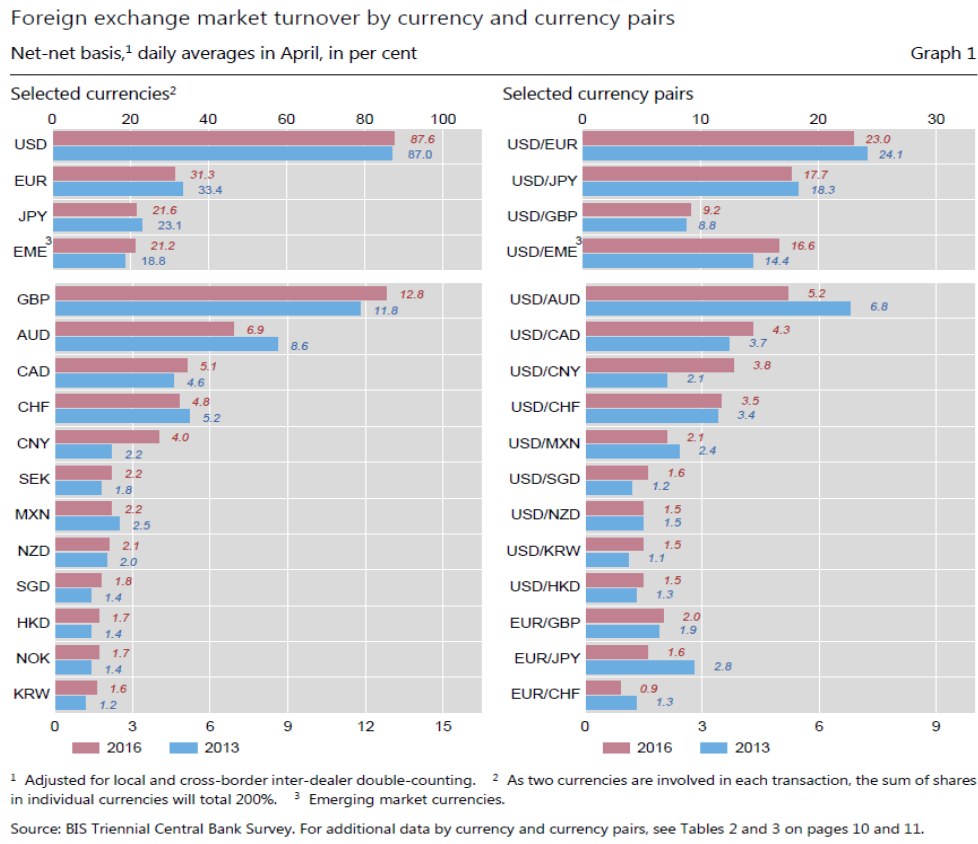


Figure 1. Foreign Exchange Market Turnover by Currency Pairs

The figure 1 above summarizes the exchange rate relative to some selected currencies in the world market.

2. Review of Relevant Literature

So many Empirical econometric modeling works in Agricultural Economics, finance and economic data assume that relationships are linear. Economic theory plays a passive role on this issue, and thus most applied research finds it convenient to assume linearity. In the 2000's, some researchers try to challenge the empirical theory —the random walk model predicts the exchange rate best ||. They used different methods and chose different models. For examples, data from Central and Eastern European countries was used to compare the forecasting models in transition economies [2]. Intraday foreign exchange rates were used as observations [9]. They found that some sophisticated time series models such as the Markov regime-switching model have better performance than the random walk models under the condition of intensive time period. Thus, the empirical theory —the random walk model predicts the exchange rate best || does not work well sometimes. The thought expressed began to change based on irregularities observed in economic and financial data, that non-linear specifications

is gradually becoming a more realistic representation of data generation processes. In finance, for instance, stock returns tend to be more correlated when there is low volatility than when volatility is high. A similar behavior has been observed in exchange rate mechanisms where the exchange rate may be constrained to lie within a pre-defined target zone [4]. To accommodate this kind of dynamic behavior using time series data, regime-switching models (RSM) have been introduced ([15] & [16]; [7]). Threshold autoregressive (TAR) model begins to be regularly appears in the agricultural economics literature as a model that is popularly used [18], and extensively discussed in Tong [17].

3. Mathematical Preliminaries of TAR and SETAR Models

3.1. SETAR Models

The SETAR model is a convenient way to specify a TAR model because q_t is defined simply as the dependent variable y_t . In this case, the process can be formally written as

$$y_t = \begin{cases} \phi_{0,1} + \phi_{1,1}y_{t-1} + \phi_{2,1}y_{t-2} + \dots + \phi_{p,1}y_{t-p} + \varepsilon_t & \text{if } y_{t-1} \leq c \\ \phi_{0,2} + \phi_{1,2}y_{t-1} + \phi_{2,2}y_{t-2} + \dots + \phi_{p,2}y_{t-p} + \varepsilon_t & \text{if } y_{t-1} > c \end{cases}$$

It is interesting to highlight that the estimation of SETAR models requires the application of least squares procedures only, more specifically, sequential conditional least squares. For the two-regime SETAR model, the steps can be outlined as follows:

Step 1: Set $P_1 = P_2 = P$ for simplicity and estimate the AR coefficients conditional on the value of the threshold (c).

Step 2: Calculate conditional residuals $\hat{\varepsilon}_t$ and estimated variances $\hat{\sigma}_t^2$ from the coefficients in step 1.

Step 3: Obtain least squares estimates of c by minimizing the residual variance $\hat{\sigma}(c)$ over all possible values of the threshold coefficient c , that is,

$$\hat{C} = \arg \min \hat{\sigma}^2(c)$$

This minimization requires a direct search over the ordered values of y_t [6] provides an approach free of nuisance parameters that are involved in estimation. Also note that in more complex models, the direct search may be over values of c and d , and thus, the estimate of the variance in Step 3 would be conditional on values of c and d (e.g., [6]).

Step 4: Obtain final estimates of $\hat{\phi}$ and $\hat{\sigma}^2$ from results in Step 3.

The estimation of the threshold values in Step 3 requires that each regime contains enough observations for reliable estimation of the AR coefficients. About fifteen percent (15%) of the observations on each regime seems to work well.

3.2. TAR Models

$$\Delta_{ut} = \begin{cases} \rho_1 u_{t-1} + \varepsilon_t & \text{if } \Delta_{t-1} > 0 \\ \rho_2 u_{t-1} + \varepsilon_t & \text{if } \Delta_{t-1} < 0 \end{cases}$$

If the above sequence is stationary, the least squares estimates of P_1 and P_2 have an asymptotic multivariate normal distribution. The process is formally specified as:

$$\Delta_{ut} = I_t \rho_1 u_{t-1} + (I - I_t) \rho_2 u_{t-1} + \varepsilon_t$$

$$I_t = \begin{cases} 1 & \text{if } u_t \geq 0 \\ 0 & \text{if } u_t < 0 \end{cases}$$

4. Description and Data Analysis

The exchange rate data used in the study was extracted from Central Bank of Nigeria Bulletin of 2017. The data was obtained for 1996 to 2018.

ORIGINAL DATA

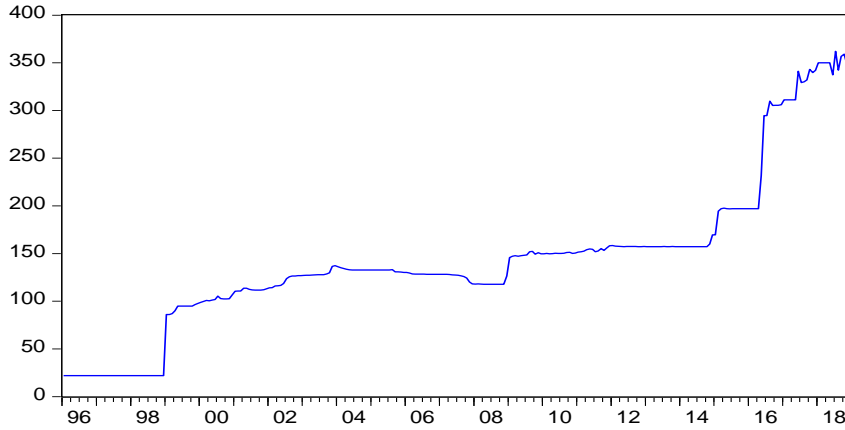


Figure 2. Line Graph of the Leveled Exchange Rate of Naira/ Dollar

first diff

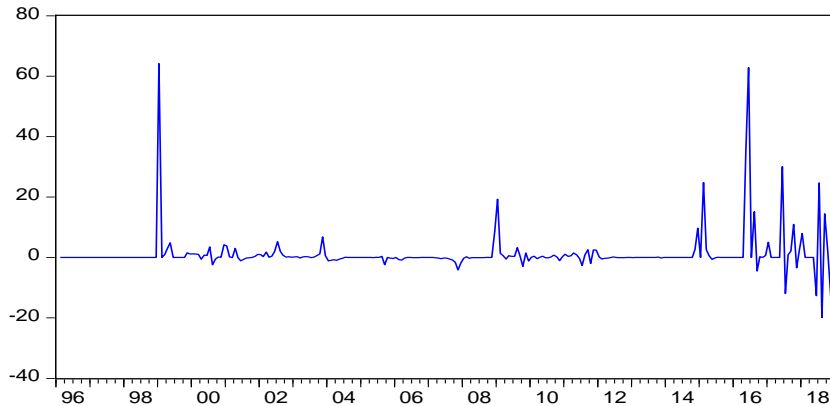


Figure 3. Line Graph of the First Difference Exchange Rate of Naira/Dollar

4.1. Identification of a Stationary Condition of the Series

The line graph of the series Figure 2 indicates the non-stationarity of the series. There is evidence of volatility as the values do not fluctuate around a constant mean. The first differences of the series were taken (figure 3) and the graphs seem to fluctuate around a constant mean of zero value.

The results on the correlogram of the leveled for the series shows stronger evidence of non-stationarity since its autocorrelation coefficient function (ACF) of the residuals does not quickly decay to zero. On the other hand, the correlogram of the first difference shows that it is consistent with mean stationarity because most of the values promptly decay to zero. Tables 1 and 2 below show both the correlograms for level and first difference.

Table 1. Correlogram for Level

Date: 11/10/19 Time: 17:20

Sample: 1996M01 2018M12

Included observations: 276

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. *****	. *****	1	0.983	0.983	269.50	0.000
. *****	* .	2	0.962	-0.110	528.75	0.000
. *****	. .	3	0.940	-0.061	776.84	0.000
. *****	. .	4	0.917	0.001	1014.0	0.000
. *****	. .	5	0.895	0.019	1241.0	0.000
. *****	* .	6	0.872	-0.069	1457.1	0.000
. *****	. .	7	0.851	0.059	1663.6	0.000
. *****	* .	8	0.828	-0.067	1859.9	0.000
. *****	. .	9	0.805	-0.011	2046.3	0.000
. *****	. .	10	0.782	-0.014	2222.9	0.000
. *****	. .	11	0.759	-0.011	2389.8	0.000
. *****	. .	12	0.736	-0.023	2547.4	0.000
. *****	. .	13	0.713	0.000	2695.7	0.000
. *****	. .	14	0.690	-0.036	2835.0	0.000
. *****	. .	15	0.666	0.001	2965.6	0.000
. *****	. .	16	0.644	-0.006	3087.8	0.000
. *****	. .	17	0.619	-0.054	3201.4	0.000
. *****	. .	18	0.595	-0.019	3306.6	0.000
. *****	. .	19	0.569	-0.053	3403.3	0.000
. *****	. .	20	0.546	0.072	3492.5	0.000
. *****	. .	21	0.522	-0.037	3574.5	0.000
. *****	. .	22	0.498	-0.019	3649.5	0.000
. *****	. .	23	0.475	-0.015	3717.9	0.000
. *****	. .	24	0.451	-0.015	3779.9	0.000
. *****	. .	25	0.428	-0.002	3836.0	0.000
. *****	. .	26	0.405	-0.024	3886.3	0.000
. *****	. .	27	0.381	-0.022	3931.1	0.000
. *****	. .	28	0.358	-0.022	3970.7	0.000
. *****	. .	29	0.333	-0.045	4005.1	0.000
. *****	. .	30	0.310	0.048	4035.2	0.000
. *****	. .	31	0.289	0.008	4061.4	0.000
. *****	. *	32	0.274	0.176	4085.0	0.000
. *****	. .	33	0.263	0.045	4106.8	0.000
. *****	. .	34	0.251	-0.037	4126.8	0.000
. *****	. .	35	0.240	-0.023	4145.1	0.000
. *****	. .	36	0.228	0.001	4161.7	0.000

Table 2. Correlogram for First Difference

Date: 11/10/19 Time: 17:15

Sample: 1996M01 2018M12

Included observations: 275

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
. .	. .	1	0.069	0.069	1.3351	0.248
. *	. *	2	0.091	0.086	3.6266	0.163
. .	. .	3	-0.007	-0.019	3.6400	0.303
. .	. .	4	0.032	0.026	3.9241	0.416
. .	. .	5	-0.062	-0.064	5.0043	0.415
. .	. .	6	0.012	0.016	5.0478	0.538
. .	. .	7	0.017	0.028	5.1342	0.644
. .	. .	8	-0.019	-0.028	5.2423	0.731
. .	. .	9	-0.008	-0.004	5.2599	0.811
. .	. .	10	-0.004	-0.003	5.2637	0.873
. .	. .	11	-0.029	-0.028	5.5060	0.904
. .	. *	12	0.066	0.076	6.7682	0.873
. .	. .	13	0.064	0.057	7.9414	0.847
* .	* .	14	-0.105	-0.130	11.145	0.675
. *	. *	15	0.075	0.089	12.785	0.619
. *	. *	16	0.158	0.169	20.126	0.215
. .	. .	17	0.021	-0.018	20.251	0.262
. .	. .	18	0.020	0.005	20.372	0.312
. .	. .	19	0.008	-0.015	20.390	0.371
. .	. .	20	0.012	0.013	20.433	0.431
. .	. .	21	-0.016	0.016	20.506	0.489
. .	* .	22	-0.050	-0.071	21.252	0.505
. .	. .	23	0.027	0.035	21.467	0.553
* .	* .	24	-0.086	-0.078	23.688	0.480
. *	. .	25	0.075	0.071	25.398	0.440
. .	. .	26	-0.042	-0.012	25.945	0.466
. .	. .	27	0.009	-0.003	25.970	0.520
. .	. .	28	0.045	0.018	26.589	0.541
* .	* .	29	-0.093	-0.124	29.262	0.451
* .	* .	30	-0.152	-0.110	36.444	0.194
* .	* .	31	-0.080	-0.068	38.457	0.168
. .	. .	32	0.006	0.006	38.468	0.200
. .	. .	33	-0.021	-0.011	38.608	0.231
. .	. .	34	0.002	0.001	38.609	0.269
. .	. .	35	0.008	-0.003	38.631	0.309
. .	. .	36	-0.003	-0.011	38.634	0.351

The stationary conditions of the series were formally verified by using Unit Root test (URT) for the leveled and first differences of the series. We tested for a unit root using the augmented Dickey-Fuller (ADF) statistic. At level, (table 3) all the series are not stationary but at first difference (table 4) all series appeared stationary as shown in the tables below:-

Table 3. Original Data

Null Hypothesis: VALUES has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=15)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	0.117358	0.9666
Test critical values: 1% level	-3.453997	
5% level	-2.871845	
10% level	-2.572334	

*MacKinnon (1996) one-sided p-values.

Table 4. First Difference

Null Hypothesis: D(VALUEs) has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic - based on SIC, maxlag=15)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-14.89460	0.0000
Test critical values: 1% level	-3.454085	
5% level	-2.871883	
10% level	-2.572354	

*MacKinnon (1996) one-sided p-values.

4.2. Results of ARIMA Model Selection

To identify the appropriate model parameter of ARIMA (p, d, q), AIC is adopted. From the unit root test, we obtain d to be 1. As to parameters p and q, we run the regression through the combinations of from $p = 1$ to $p = 10$ and from $q = 1$ to $q = 10$. For sake of saving space, we just list the top 10 with higher AIC values here. As shown in Table 5 below the model with the smallest value (9.14334) of AIC is the optimal ARIMA (6, 1, 9) chosen.

Table 5. Arima Model Results

RANK	MODELS	AIC
1	ARIMA (6,1, 9)	9.14340
2	ARIMA (9,1,7)	9.14403
3	ARIMA (9,1,9)	9.14545
4	ARIMA (9,1,6)	9.14505
5	ARIMA (8,1,9)	9.14871
6	ARIMA (10,1,9)	9.14879
7	ARIMA (6,1,6)	9.14978
8	ARIMA (9,1,8)	9.15125
9	ARIMA (6,1,5)	9.15178
10	ARIMA (6,1,10)	9.15368

4.3. Results of SETAR Model

Table 6. Panel A Regime 1 Observations Included 764

RANK	MODELS	AIC
1	AR(7)	9.8432
2	AR(9)	9.8120
3	AR(1)	9.8743
4	AR(6)	9.8321
5	AR(3)	9.8845
6	AR(4)	9.8764
7	AR(2)	9.8221
8	AR(10)	9.8765
9	AR(8)	9.9872
10	AR(5)	9.9721

In our self-exciting threshold autoregressive (SETAR) model, we assume that a variable Naira is a linear autoregression within a regime. As there are two regimes in the study, the model could be written as SETAR (2, p, p).

Panel A (table 6) is regime 1 of the two-regime SETAR and panel B (table 7) is regime 2. We select the most appropriate model by minimizing value of AIC. We build up a two-regime SETAR model, SETAR (2, 7, 4).

Table 7. Panel B Regime 2 Observations Included 308

RANK	MODELS	AIC
1	AR(4)	9.0982
2	AR(10)	9.0021
3	AR(3)	9.0882
4	AR(9)	9.0071
5	AR(7)	9.0765
6	AR(4)	9.0974
7	AR(5)	9.0053
8	AR(1)	9.0642
9	AR(2)	9.0523
10	AR(6)	9.7432

4.4. Results Out-of-sample Forecasting Performance

In this section, we check out-of-sample forecast performance of the two models. By checking multi-criteria (MSE, MAE, AMPE, and MAPE) on ARIMA and SETAR, we can compare the residual of these two models. For the mean squared error (MSE), SETAR is smaller than ARIMA with 4479.28 and 5744.65. This means SETAR has a fewer errors in the standard of MSE. For the mean absolute error (MAE), the adjusted mean absolute property error (AMAPE) and mean absolute property error (MAPE), SETAR is also smaller than ARIMA with fewer errors. According to our result shown in the table 8 below, the SETAR model is better than the ARIMA model over the sample period (second regime).

Comparison of Forecasting Power.

Table 8. Comparison of Forecasting Power by Models

	MSE	MAE	AMAPE(%)	MAPE (%)
SETAR	4479.28	45.51	90.68	89.02
ARIMA	5744.65	58.32	304.20	190.91

5. Conclusions

At first, we believed there is no way for a linear regression to suit a series forever where stock prices follow a non-linear trend. Due to the economic environment changing, the stock market will be affected and change over time. Therefore, non-linear regression should be better than linear regression in the exchange rate market. First step of handling time series data is to check stationary state in the mean. We found out there was a unit root existed so we analyzed the first-difference. Next, we constructed an ARIMA model by using AIC selection criteria. And we build up a SETAR model by AIC as well. Afterward, we checked the forecasting power by four criteria (MSE, MAE, AMAPE, and MAPE) and all of those standards showed that SETAR

has a stronger predicting power than ARIMA. The results also support previous assumptions of this thesis. Non-linear SETAR model is better than linear ARIMA model in Nigeria exchange rate market.

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