

# Biswas Theorem

Deapon Biswas

Transport Officer, Private Concern, Chittagong, Bangladesh

**Abstract** In this chapter we can find various numbers of EB members of the experiment “M sided N dice taken V at a time”. Beside this we can find various numbers of IB members of different IB experiment i.e., of the experiments M sided N dice taken V at a time in which first v dice are identified in the first way or second way or both way. Again various numbers of B events can be calculated from different CB experiments i.e., of the experiments M sided N dice taken V at a time in which first v dice are identified in the first way or second way or both way. Moreover we can find various numbers of GB members of different GB experiment i.e., of the experiments M sided N dice taken V at a time in which X is observed in the first way or Y is observed in the second way or X, Y are observed in the both way at a time. X and Y are variables. Beside this we can get combination theorems, permutation theorems, formation theorems and homogenation theorems from these theorems.

**Keywords** Biswas theorem, EB theorem, IB theorem, CB theorem, GB theorem

## 1. Introduction

First of all I discussed Biswas theorem and then elementary B theorems. They are divided into four parts; both way selected B (SB) theorem, both way arranged B (AB) theorem, first way selected and second way arranged B (SAB) theorem and first way arranged and second way selected B (ASB) theorem.

Then identified B theorems are divided into four parts i.e.,

- (i) both way selected B (SB) theorem
- (ii) both way arranged B (AB) theorem
- (iii) first way selected and second way arranged B (SAB) theorem
- (iv) first way arranged and second way selected B (ASB) theorem.

Similarly characterizing B theorems and general B theorems are divided into the same.

## 2. Findings

### 2.1. Biswas Theorem

**Theorem 2.1 Biswas theorem:** The number of GB members of the GB space  $GB\left\{\begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix}\right\}$  denoted by

$GB\left(\begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix}\right)$  is

$$GB\left(\begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M & U \\ V & X \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N & U \\ V & Y \end{smallmatrix}\right) \quad (2.1)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

And  $B^1$  is the B first number to be hold the number of homogenations or formations and  $B^2$  is the B second number to be hold the number of permutations or combinations.

**Proof:** We know the B number is to be product of B first number and B second number. As the theorem is from a GB experiment so we get the B first number from general homogenation theorem or general formations theorem and it is to be  $B^1\left(\begin{smallmatrix} M & U \\ V & X \end{smallmatrix}\right)$ . Again B second number get from general permutation theorem or general combination theorem and it is to be  $B^2\left(\begin{smallmatrix} N & U \\ V & Y \end{smallmatrix}\right)$ . Performing the two cases we get the number of GB members as

$$GB\left(\begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M & U \\ V & X \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N & U \\ V & Y \end{smallmatrix}\right).$$

**Remarks:** (i)  $B^1\left(\begin{smallmatrix} M & 0 \\ V & 0 \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$  (2.2)

(ii)  $B^2\left(\begin{smallmatrix} N & 0 \\ V & 0 \end{smallmatrix}\right) = B^2\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$  (2.3)

**Proofs:** Let B first number is for number of homogenations then

$$H\left(\begin{smallmatrix} M & 0 \\ V & 0 \end{smallmatrix}\right) = 0^{(0)} E_1^V(M) = 1.M^V = M^V = H\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$$

Again let B first number is for number of formations then

\* Corresponding author:

philosclub@yahoo.com (Deapon Biswas)

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$$F\left(\begin{smallmatrix} M & 0 \\ V & 0 \end{smallmatrix}\right) = C\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) C\left(\begin{smallmatrix} M-0+V-1 \\ V-0 \end{smallmatrix}\right) = 1 \times C\left(\begin{smallmatrix} M+V-1 \\ V \end{smallmatrix}\right) \\ = C\left(\begin{smallmatrix} M+V-1 \\ V \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$$

Now for the B second number let it is represent for number of permutations then

$$P\left(\begin{smallmatrix} N & 0 \\ V & 0 \end{smallmatrix}\right) = V! C\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) C\left(\begin{smallmatrix} N-0 \\ V-0 \end{smallmatrix}\right) = V! \times 1 \times C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right) = P\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$$

Again let B second number is represent for number of combinations then

$$C\left(\begin{smallmatrix} N & 0 \\ V & 0 \end{smallmatrix}\right) = C\left(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right) C\left(\begin{smallmatrix} N-0 \\ V-0 \end{smallmatrix}\right) = 1 \times C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right) = C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$$

**Example 2.1:** Find the number of GB members of the GB space  $GB\left\{\begin{smallmatrix} 3 & 5 & 3 \\ 4 & 2 \end{smallmatrix}\right\}$  chosen first way arranged and second way selected and 2 is observed in the first way.

**Solution:** From the theorem 2.1 we get

$$GB\left(\begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M & U \\ V & X \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N & U \\ V & Y \end{smallmatrix}\right)$$

As X is observed in the first way so we get

$$GB\left(\begin{smallmatrix} 3 & 5 & 3 \\ 4 & 2 \end{smallmatrix}\right) = H\left(\begin{smallmatrix} 3 & 3 \\ 4 & 2 \end{smallmatrix}\right) C\left(\begin{smallmatrix} 5 & 0 \\ 4 & 0 \end{smallmatrix}\right) = 42 \times 5 = 210.$$

## 2.2. EB Theorem

**Theorem 2.2 EB theorem:** The number of B members of the experiment M sided N dice taken V at a time, denoted by  $EB\left(\begin{smallmatrix} M & N \\ V \end{smallmatrix}\right)$  is

$$EB\left(\begin{smallmatrix} M & N \\ V \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right) \quad (2.4)$$

where  $B^1\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$  called B first number is the number of homogenations or formations of M sided V dice experiment and  $B^2\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$  called B second number is the number of permutations or combinations of N dice (things) taken V at a time.

**Proof:** Let M sided 1 dice is tossed, then we get the number of B members is  $B^1\left(\begin{smallmatrix} M \\ 1 \end{smallmatrix}\right)$ . In the same why M sided 2 dice is tossed then we get the number of B members is  $B^1\left(\begin{smallmatrix} M \\ 2 \end{smallmatrix}\right)$ . In the same why M sided 3 dice is tossed then we get the number of B members is  $B^1\left(\begin{smallmatrix} M \\ 3 \end{smallmatrix}\right)$ . Similarly for M sided V dice experiment we get the number of B members is  $B^1\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$ . In the other hand, N dice (or things) taken 1 at a time then we get the number of B members is  $B^2\left(\begin{smallmatrix} N \\ 1 \end{smallmatrix}\right)$ . Again N dice (or things) taken 2 at a time then we get the number of B members is  $B^2\left(\begin{smallmatrix} N \\ 2 \end{smallmatrix}\right)$ . In the same way N dice (or things) taken 3 at a time then we get the number of B members is  $B^2\left(\begin{smallmatrix} N \\ 3 \end{smallmatrix}\right)$ . Similarly for N dice (or things) taken V at a time then we get the number of B members is  $B^2\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$ . When the two case performed in the same time then we get the number

of B members of M sided N dice taken V at a time is  $B^1\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$  i.e.,  $EB\left(\begin{smallmatrix} M & N \\ V \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$ .

Hence the proof.

**Example 2.2:** Find the number of B members of the experiment 6 sided 10 dice taken 5 at a time where B first number is the number of homogenations and B second number is the number of combinations.

**Solution:** The numbers of B members is

$$EB\left(\begin{smallmatrix} 6 & 10 \\ 5 \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} 6 \\ 5 \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} 10 \\ 5 \end{smallmatrix}\right) = H\left(\begin{smallmatrix} 6 \\ 5 \end{smallmatrix}\right) C\left(\begin{smallmatrix} 10 \\ 5 \end{smallmatrix}\right) \\ = 7776 \times 36 = 279936.$$

**Theorem 2.3 ESB theorem:** The number of ESB members of the experiment M sided N dice taken V at a time, denoted by  $ESB\left(\begin{smallmatrix} M & N \\ V \end{smallmatrix}\right)$  is

$$ESB\left(\begin{smallmatrix} M & N \\ V \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right) \quad (2.5)$$

where  $F\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$  is the number of formations of M sided V dice experiment and  $C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$  is the number of combinations of N dice (things) taken V at a time.

**Proof:** The experiment gives the outcomes selected i.e., ESB members. Firstly M sided 1 die is tossed, then the B first number gives the number of formations is  $F\left(\begin{smallmatrix} M \\ 1 \end{smallmatrix}\right)$ . Secondly M sided 2 dice is tossed then the B first number i.e., the number of formations is  $F\left(\begin{smallmatrix} M \\ 2 \end{smallmatrix}\right)$ . Thirdly M sided 3 dice is tossed then the B first number i.e., the number of formations is  $F\left(\begin{smallmatrix} M \\ 3 \end{smallmatrix}\right)$ . Similarly for M sided V dice experiment we get the number of formations is  $F\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$ . Again for N dice (things) taken 1 at a time then the B second number gives the number of combinations is  $C\left(\begin{smallmatrix} N \\ 1 \end{smallmatrix}\right)$ . Secondly N dice (things) taken 2 at a time then the B second number i.e., number of combinations is  $C\left(\begin{smallmatrix} N \\ 2 \end{smallmatrix}\right)$ . Thirdly for the same reason when N dice (things) taken 3 at a time then the B second number i.e., the number of combinations is  $C\left(\begin{smallmatrix} N \\ 3 \end{smallmatrix}\right)$ . Similarly we get for N dice (things) taken V at a time the number is  $C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$ . Now for the two case performed in the same time the number of ESB members of M sided N dice taken V at a time is  $F\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) \times C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$  i.e.,  $ESB\left(\begin{smallmatrix} M & N \\ V \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$ .

**Example 2.3:** Find the number of ESB members of the experiment 3 sided 5 dice taken 4 at a time.

**Solution:** The number of ESB members is

$$ESB\left(\begin{smallmatrix} 3 & 5 \\ 4 \end{smallmatrix}\right) = F\left(\begin{smallmatrix} 3 \\ 4 \end{smallmatrix}\right) C\left(\begin{smallmatrix} 5 \\ 4 \end{smallmatrix}\right) = 15 \times 5 = 75.$$

**Corollary 2.1:** The number of ESB members of the experiment 1 sided N dice taken V at a time is the same as the numbers of combinations of N things taken V at a time i.e.,

$$\text{ESB} \left( \begin{smallmatrix} 1 & N \\ & V \end{smallmatrix} \right) = C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.6)$$

**Corollary 2.2:** The number of ESB members of the experiment M sided V dice taken all at a time is the same as the number of formations of M sided V dice experiment i.e.,

$$\text{ESB} \left( \begin{smallmatrix} M & V \\ & V \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) \quad (2.7)$$

**Theorem 2.4 EAB theorem:** The number of EAB members of the experiment M sided N dice taken V at a time, denoted by  $\text{EAB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right)$  is

$$\text{EAB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.8)$$

where  $H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right)$  is the number of homogenations of M sided V dice experiment and  $P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$  is the number of permutations of N dice (things) taken V at a time.

**Proof:** Here the experiment results all the B members arranged i.e., EAB members. Now M sided 1 die is tossed then the B first number gives the number of homogenations is  $H \left( \begin{smallmatrix} M \\ 1 \end{smallmatrix} \right)$ . Again M sided 2 dice is tossed then the B first number i.e., number of homogenations is  $H \left( \begin{smallmatrix} M \\ 2 \end{smallmatrix} \right)$ . For the same reason when M sided 3 dice is tossed then the B first number i.e., the number of homogenations is  $H \left( \begin{smallmatrix} M \\ 3 \end{smallmatrix} \right)$ . Similarly for M sided V dice experiment we get the number of homogenations is  $H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right)$ . On the other hand when N dice (things) taken 1 at a time then the B second number gives the number of permutations is  $P \left( \begin{smallmatrix} N \\ 1 \end{smallmatrix} \right)$ . Again N dice (things) taken 2 at a time then the B second number i.e., the number of permutations is  $P \left( \begin{smallmatrix} N \\ 2 \end{smallmatrix} \right)$ . For the same reason when N dice (things) taken 3 at a time then the B second number i.e., the number of permutations is  $P \left( \begin{smallmatrix} N \\ 3 \end{smallmatrix} \right)$ . Similarly for N dice (things) taken V at a time we get the number of permutations is  $P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$ . When the two cases performed in the same time then we get the number of EAB members of M sided N dice taken V at a time is  $H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) \times P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$  i.e.,

$$\text{EAB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$$

**Example 2.4:** Find the number of EAB members of the experiment 3 sided 8 dice taken 4 at a time.

**Solution:** The number of AB members is

$$\text{EAB} \left( \begin{smallmatrix} 3 & 8 \\ & 4 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 3 \\ 4 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 8 \\ 4 \end{smallmatrix} \right) = 81 \times 1680 = 136080.$$

**Corollary 2.3:** The number of EAB members of the experiment 1 sided N dice taken V at a time is the same as the number of permutations of N taken things V at a time i.e.,

$$\text{EAB} \left( \begin{smallmatrix} 1 & N \\ & V \end{smallmatrix} \right) = P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.9)$$

**Theorem 2.5 ESAB theorem:** The number of ESAB

members of the experiment M sided N dice taken V at a time, denoted by  $\text{ESAB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right)$  is

$$\text{ESAB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.10)$$

where  $F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right)$  is the number of formations of M sided V dice experiment and  $P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$  is the number of permutations of N dice (things) taken V at a time.

**Proof:** The above experiment gives the B members first way selected and second way arranged. We now firstly M sided 1 die is tossed, then the B first number gives the number of formations is  $F \left( \begin{smallmatrix} M \\ 1 \end{smallmatrix} \right)$ . Secondly M sided 2 dice is tossed then the B first number i.e., the number of formations is  $F \left( \begin{smallmatrix} M \\ 2 \end{smallmatrix} \right)$ . Thirdly M sided 3 dice is tossed then the B first number i.e., the number of formations is  $F \left( \begin{smallmatrix} M \\ 3 \end{smallmatrix} \right)$ . In the same way when M sided V dice tossed then we get the number of formations is  $F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right)$ . In the other hand N dice (things) taken 1 at a time then the B second number gives the number of permutations is  $P \left( \begin{smallmatrix} N \\ 1 \end{smallmatrix} \right)$ . Secondly when N dice (things) taken 2 at a time the B second number i.e., the number of permutations is  $P \left( \begin{smallmatrix} N \\ 2 \end{smallmatrix} \right)$ . Thirdly when N dice (things) taken 3 at a time then the B second number i.e., the number of permutations is  $P \left( \begin{smallmatrix} N \\ 3 \end{smallmatrix} \right)$ . In the same way we get for N dice (things) taken V at a time the number of permutations is  $P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$ . Performing the two cases at a time we get the number of ESAB members of N sided N dice taken V at a time is

$$F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) \times P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \text{ i.e., } \text{ESAB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$$

**Example 2.5:** Find the number of ESAB members of the experiment 6 sided 4 dice taken 2 at a time.

**Solution:** The number of ESAB members is

$$\text{ESAB} \left( \begin{smallmatrix} 6 & 4 \\ & 2 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 6 \\ 2 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \right) = 21 \times 12 = 252.$$

**Corollary 2.4:** The number of ESAB members of the experiment 1 sided N dice taken V at a time is the same as the number of permutations of N things taken V at a time i.e.,

$$\text{ESAB} \left( \begin{smallmatrix} 1 & N \\ & V \end{smallmatrix} \right) = P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.11)$$

**Theorem 2.6 EASB theorem:** The number of EASB members of the experiment M sided N dice taken V at a time, denoted by  $\text{EASB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right)$  is

$$\text{EASB} \left( \begin{smallmatrix} M & N \\ & V \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.12)$$

where  $H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right)$  is the number of homogenations of M sided V dice experiment and  $C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right)$  is the number of combinations of N dice (things) taken V at a time.

**Proof:** The experiment gives the outcomes first way

arranged and second way selected. Now in first M sided 1 die is tossed then the B first number gives the number of homogenations is  $H\left(\begin{smallmatrix} M \\ 1 \end{smallmatrix}\right)$ . In second M sided 2 dice is tossed, then the B first number i.e., the number of homogenations is  $H\left(\begin{smallmatrix} M \\ 2 \end{smallmatrix}\right)$ . In third M sided 3 dice is tossed, then the B first number i.e., the number of homogenations is  $H\left(\begin{smallmatrix} M \\ 3 \end{smallmatrix}\right)$ . Similarly when M sided V dice tossed then we get the number of homogenations is  $H\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right)$ . On the other hand N dice (things) taken 1 at a time then the B second number i.e., the number of combinations is  $C\left(\begin{smallmatrix} N \\ 1 \end{smallmatrix}\right)$ . In second when N dice (things) taken 2 at a time then the B second number i.e., the number of combinations is  $C\left(\begin{smallmatrix} N \\ 2 \end{smallmatrix}\right)$ . In third when N dice (things) taken 3 at a time then the B second number i.e., the number of combinations is  $C\left(\begin{smallmatrix} N \\ 3 \end{smallmatrix}\right)$ . Similarly we get for N dice (things) taken V at a time the number of combinations is  $C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$ . Now performing the two cases altogether we get the number of EASB members of M sided N dice taken V at a time is  $H\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) \times C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$  i.e.,  $EASB\left(\begin{smallmatrix} M N \\ V \end{smallmatrix}\right) = H\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right)$ .

**Example 2.6:** Find the number of EASB members of the experiment 3 sided 10 dice taken 6 at a time.

**Solution:** The number of EASB members is

$$EASB\left(\begin{smallmatrix} 3 \ 10 \\ 6 \end{smallmatrix}\right) = H\left(\begin{smallmatrix} 3 \\ 6 \end{smallmatrix}\right) C\left(\begin{smallmatrix} 10 \\ 6 \end{smallmatrix}\right) = 729 \times 210 = 153090.$$

**Corollary 2.5:** The number EASB members of the experiment 1 sided N dice taken V at a time is the same as the number of combinations of N things taken V at a time i.e.,

$$EASB\left(\begin{smallmatrix} 1 \ N \\ V \end{smallmatrix}\right) = C\left(\begin{smallmatrix} N \\ V \end{smallmatrix}\right) \quad (2.13)$$

**Corollary 2.6:** The number of EASB members of the experiment M sided V dice taken all at a time is the same as the number of homogenations of M sided V dice experiment i.e.,

$$EASB\left(\begin{smallmatrix} M \ N \\ V \end{smallmatrix}\right) = H\left(\begin{smallmatrix} M \\ V \end{smallmatrix}\right) \quad (2.14)$$

### 2.3. IB Theorem

**Theorem 2.7 IB theorem:** The number of IB members of the B event  $IB\left\{\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right\}$  denoted by  $IB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right)$  is

$$IB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \quad (2.15)$$

where  $M'$ ,  $N'$  and  $V'$  are to be called affected M, N and V respectively and  $B^1$  is the B first number is to be holds the number of homogenations or formations and  $B^2$  is the B second number is to be holds the number of permutations or combinations.

**Proof:** As the IB members are identified so the parameters M, N, V are to be affected as possible. Let M sided V dice are tossed then as there are v dice identified in the first way or second way or both way so the parameters M and V may be

affected as  $M'$  and  $V'$  respectively. Hence the first number is to be  $B^1\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right)$ . Again when N dice (objects) taken V at time then for the same reason the parameters may be affected as  $N'$  and  $V'$  respectively. So the B second number is to be  $B^2\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$ . Performing the two cases at a time we get the number of IB members is  $B^1\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) \times B^2\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$  i.e.,

$$IB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right).$$

**Example 2.7:** Find the number of IB members of the B event  $IB\left\{\begin{smallmatrix} 6 \ 5 \\ 4/2K \end{smallmatrix}\right\}$  where B first number is the number of formations and B second number is the number of combinations.

**Solution:** From the theorem 2.7 we get

$$IB\left(\begin{smallmatrix} 6 \ 5 \\ 4/2K \end{smallmatrix}\right) = B^1\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) B^2\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \text{ where,}$$

$$\begin{aligned} M' &= M - m_2 + 1 \\ &= 6 - 2 + 1 = 5 \end{aligned}$$

$$\begin{aligned} N' &= N - n_2 \\ &= 5 - 2 = 3 \end{aligned}$$

$$\begin{aligned} V' &= V - v \\ &= 4 - 2 = 2 \end{aligned}$$

$$\text{Thus, } IB\left(\begin{smallmatrix} 6 \ 5 \\ 4/2K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} 5 \\ 2 \end{smallmatrix}\right) C\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) = 15 \times 3 = 45.$$

**Corollary 2.7:** The number of IB members of the B events  $IB\left\{\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right\}$  is the same as the number of B members of the B space  $B\left\{\begin{smallmatrix} M' \ N' \\ V' \end{smallmatrix}\right\}$  i.e.,

$$IB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right) = B\left(\begin{smallmatrix} M' \ N' \\ V' \end{smallmatrix}\right) \quad (2.16)$$

**Theorem 2.8 ISB theorem:** The number of ISB members of the SB event  $ISB\left\{\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right\}$  denoted by  $ISB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right)$  is

$$ISB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) C\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \quad (2.17)$$

where  $M'$ ,  $N'$  and  $V'$  are to be called affected M, N and V respectively.

**Proof:** As the ISB members are identified so the parameters M, N and V are to be affected as possible. Now M sided V dice are tossed in which first v dice are identified in the first way or second way or both way, the parameters M and V may be affected as  $M'$  and  $V'$  respectively. So the number of formations is to be  $F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right)$ . Again when N dice (objects) taken V at a time then for the same reason the parameters may be affected as  $N'$  and  $V'$  respectively. So the number of combinations is to be  $C\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$ . Performing the two cases at the same time we get the number of ISB members is  $F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) \times C\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$  i.e.,  $ISB\left(\begin{smallmatrix} M \ N \\ V/V'K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) C\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$ .

**Example 2.8:** Find the number of ISB members of the SB

event  $ISB\left\{\begin{smallmatrix} 3 & 5 \\ 4/2K \end{smallmatrix}\right\}$  where  ${}^2K = (b_{11}, b_{12})$ .

**Solution:** From the theorem 2.8 we get

$$ISB\left(\begin{smallmatrix} 3 & 5 \\ 4/2K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) C\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \text{ where,}$$

$$M' = M - m_2 + 1$$

$$= 3 - 1 + 1 = 3$$

$$N' = N - n_2 = 5 - 2 = 3$$

$$V' = V - v = 4 - 2 = 2$$

$$\text{Thus, } ISB\left(\begin{smallmatrix} 3 & 5 \\ 4/2K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) C\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) = 6 \times 3 = 18.$$

**Corollary 2.8:** The number of ISB members of the SB event  $ISB\left\{\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right\}$  is the same as the number of SB members of the SB space  $SB\left\{\begin{smallmatrix} M' & N' \\ V' \end{smallmatrix}\right\}$  i.e.,

$$ISB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = SB\left(\begin{smallmatrix} M' & N' \\ V' \end{smallmatrix}\right) \quad (2.18)$$

**Theorem 2.9 IAB theorem:** The number of IAB members of the AB event  $IAB\left\{\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right\}$  denoted by  $IAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right)$  is

$$IAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = H\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \quad (2.19)$$

where  $M'$ ,  $N'$  and  $V'$  are to be called affected  $M$ ,  $N$  and  $V$  respectively.

**Proof:** As the IAB members are identified so the parameters  $M$ ,  $N$ ,  $V$  are to be affected as possible. Here the  $B$  first number is to be number of homogenations and the  $B$  second number is to be number of permutations. Now  $M$  sided  $V$  dice are tossed in which first  $v$  dice identified in the first way or second way or both way, then the parameters  $M$  and  $V$  may be affected as  $M'$  and  $V'$  respectively. So the number of homogenations is to be  $H\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right)$ . Again when  $N$  dice (object) taken  $V$  at a time then for the same reason the parameters may be affected as  $N'$  and  $V'$  respectively. So the number of permutations is to be  $P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$ . Performing the two cases at a time the number of IAB members becomes  $H\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) \times P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$  i.e.,

$$IAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = H\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right).$$

**Example 2.9:** Find the number of IAB members of the AB event  $IAB\left\{\begin{smallmatrix} 3 & 6 \\ 5/3K \end{smallmatrix}\right\}$  where  ${}^3K = (b_{21}, b_{22}, b_{13})$ .

**Solution:** From the theorem 2.9 we get

$$IAB\left(\begin{smallmatrix} 3 & 6 \\ 5/3K \end{smallmatrix}\right) = H\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \text{ where,}$$

$$M' = M = 3$$

$$N' = N - n_3 = 6 - 3 = 3$$

$$V' = V - v = 5 - 3 = 2$$

$$\text{Thus, } IAB\left(\begin{smallmatrix} 3 & 6 \\ 5/3K \end{smallmatrix}\right) = H\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) P\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) = 9 \times 6 = 54.$$

**Corollary 2.9:** The number of IAB members of the AB event  $IAB\left\{\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right\}$  is the same as the number of AB members of the AB space  $AB\left\{\begin{smallmatrix} M' & N' \\ V' \end{smallmatrix}\right\}$  i.e.,

$$IAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = AB\left(\begin{smallmatrix} M' & N' \\ V' \end{smallmatrix}\right) \quad (2.20)$$

**Theorem 2.10 ISAB theorem:** The number of ISAB members of the SAB event  $ISAB\left\{\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right\}$  denoted by  $ISAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right)$  is

$$ISAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \quad (2.21)$$

where  $M'$ ,  $N'$  and  $V'$  are to be called affected  $M$ ,  $N$  and  $V$  respectively.

**Proof:** As the ISAB members are identified so the parameters  $M$ ,  $N$  and  $V$  are to be affected as possible. Let  $M$  sided  $V$  dice are tossed in which first  $v$  dice identified in the first way or second way or both way then the parameters  $M$  and  $V$  may be affected as  $M'$  and  $V'$  respectively. Thus the number of formations is to be  $F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right)$ . Again when  $N$  dice (objects) taken  $V$  at a time then for the same reason the parameters possibly affected as  $N'$  and  $V'$  respectively. Hence the number of permutations is to be  $P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$ . Performing the two cases at the same time we get the number of ISAB members is  $F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) \times P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$  i.e.,  $ISAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right)$ .

**Example 2.10 :** Find the number of ISAB members of the SAB event  $ISAB\left\{\begin{smallmatrix} 4 & 6 \\ 5/3K \end{smallmatrix}\right\}$  where  ${}^3K = (b_{11}, b_{13}, b_{22})$ .

**Solution:** From the theorem 2.10 we get

$$ISAB\left(\begin{smallmatrix} 4 & 6 \\ 5/3K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} M' \\ V' \end{smallmatrix}\right) P\left(\begin{smallmatrix} N' \\ V' \end{smallmatrix}\right) \text{ where,}$$

$$M' = M - m_3 + 1$$

$$= 4 - 2 + 1 = 3$$

$$N' = N - n_3 = 6 - 3 = 3$$

$$V' = V - v = 5 - 3 = 2$$

$$\text{Thus } ISAB\left(\begin{smallmatrix} 4 & 6 \\ 5/3K \end{smallmatrix}\right) = F\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) P\left(\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right) = 6 \times 6 = 36.$$

**Corollary 2.10:** The number of ISAB members of the SAB event  $ISAB\left\{\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right\}$  is the same as the number of SAB members of the SAB space  $SAB\left\{\begin{smallmatrix} M' & N' \\ V' \end{smallmatrix}\right\}$  i.e.,

$$ISAB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right) = SAB\left(\begin{smallmatrix} M' & N' \\ V' \end{smallmatrix}\right) \quad (2.22)$$

**Theorem 2.11 IASB theorem:** The number of IASB members of the ASB event  $IASB\left\{\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right\}$  denoted by  $IASB\left(\begin{smallmatrix} M & N \\ V/vK \end{smallmatrix}\right)$  is

$$\text{IASB}\left(\frac{M N}{V/vK}\right) = H\left(\frac{M'}{V'}\right) C\left(\frac{N'}{V'}\right) \quad (2.23)$$

where  $M'$ ,  $N'$  and  $V'$  are to be called affected  $M$ ,  $N$  and  $V$  respectively.

**Proof:** As the IASB members are identified so the parameters  $M$ ,  $N$  and  $V$  are to be affected as possible. Let  $M$  sided  $V$  dice are tossed in which first  $v$  dice are identified in the first way or second way or both way, the parameters  $M$  and  $V$  may be affected as  $M'$  and  $V'$  respectively. So the number of homogenations is to be  $H\left(\frac{M'}{V'}\right)$ . Again when  $N$  dice (objects) taken  $V$  at a time then for the same reason the parameters may be effected as  $N'$  and  $V'$  respectively. Thus the number of combinations is to be  $C\left(\frac{N'}{V'}\right)$ . Performing the two cases at the same time we get the number of IASB members is  $H\left(\frac{M'}{V'}\right) \times C\left(\frac{N'}{V'}\right)$  i.e.,  $\text{IASB}\left(\frac{M N}{V/vK}\right) = H\left(\frac{M'}{V'}\right) C\left(\frac{N'}{V'}\right)$ .

**Example 2.11:** Find the number of IASB members of the ASB event  $\text{IASB}\left\{\frac{56}{4/2K}\right\}$  where  ${}^2K = (b_{21}, b_{13})$ .

**Solution:** From the theorem 2.11 we get

$$\text{IASB}\left(\frac{56}{4/2K}\right) = H\left(\frac{M'}{V'}\right) C\left(\frac{N'}{V'}\right) \text{ where,}$$

$$M' = M - 5$$

$$N' = N - n_2 = 6 - 3 = 3$$

$$V' = V - v = 4 - 2 = 2$$

$$\text{Thus } \text{IASB}\left(\frac{56}{4/2K}\right) = H\left(\frac{5}{2}\right) C\left(\frac{3}{2}\right) = 25 \times 3 = 75.$$

**Corollary 2.11:** The number of IASB members of the ASB event  $\text{IASB}\left\{\frac{M N}{V/vK}\right\}$  is the same as the number of ASB members of the ASB space  $\text{ASB}\left\{\frac{M' N'}{V'}\right\}$  i.e.,

$$\text{IASB}\left(\frac{M N}{V/vK}\right) = \text{ASB}\left(\frac{M' N'}{V'}\right) \quad (2.24)$$

Now we describe three kinds of IB theorems i.e., first way IB theorem, second way IB theorem and both way IB theorem and their different forms.

**Theorem 2.12 First way IB theorem:** The number of IB members of the first way B event  $\text{IB}_1\left\{\frac{M N}{V/v_1K}\right\}$  denoted by  $\text{IB}_1\left(\frac{M N}{V/v_1K}\right)$  is

$$\text{IB}_1\left(\frac{M N}{V/v_1K}\right) = B^1\left(\frac{M'}{V'}\right) B^2\left(\frac{N}{V}\right) \quad (2.25)$$

where  $M'$  and  $V'$  are to be called affected  $M$  and  $V$  respectively and  $B^1$  is called to be B first number to be holds the number homogenations or formations and  $B^2$  is called to be B second number to be holds the number of permutations or combinations.

**Proof:** Here the IB members are identified in the first way so the parameters hold to the B first number possibly affected and the parameters hold to the B second number do not be affected. Now let  $M$  sided  $V$  dice tossed in which first  $v$  dice

identified in the first way so the parameters  $M$  and  $V$  be affected as  $M'$  and  $V'$  respectively. So the B first number is to be  $B^1\left(\frac{M'}{V'}\right)$ . Again let  $N$  dice (objects) taken  $V$  at a time in which first  $v$  dice are not identified so the parameters  $N$  and  $V$  not be affected. Hence the B second number is to be  $B^2\left(\frac{N}{V}\right)$ . Performing the two cases at a time we get the number of IB members is  $B^1\left(\frac{M'}{V'}\right) \times B^2\left(\frac{N}{V}\right)$  i.e.,

$$\text{IB}_1\left(\frac{M N}{V/v_1K}\right) = B^1\left(\frac{M'}{V'}\right) B^2\left(\frac{N}{V}\right).$$

**Example 2.12:** Find the number of IB members of the B event  $\text{IB}_1\left\{\frac{66}{5/3K}\right\}$  where B first number is the number of formations and B second number is the number of combinations and first 3 identified  $m$ 's are 2, 3 and 4.

**Solution:** From the theorem 2.12 we get

$$\text{IB}_1\left(\frac{66}{5/3K}\right) = B^1\left(\frac{M'}{V'}\right) B^2\left(\frac{N}{V}\right) \text{ where,}$$

$$M' = M - m_3 + 1$$

$$= 6 - 4 + 1 = 3$$

$$V' = V - v = 5 - 3 = 2$$

$$N = 6$$

$$V = 5$$

$$\text{Thus } \text{IB}_1\left(\frac{66}{5/3K}\right) = F\left(\frac{3}{2}\right) C\left(\frac{6}{5}\right) = 6 \times 6 = 36.$$

**Theorem 2.13 Second way IB theorem:** The number of IB members of the second way B event  $\text{IB}_2\left\{\frac{M N}{V/v_2K}\right\}$  denoted by  $\text{IB}_2\left(\frac{M N}{V/v_2K}\right)$  is

$$\text{IB}_2\left(\frac{M N}{V/v_2K}\right) = B^1\left(\frac{M}{V}\right) B^2\left(\frac{N'}{V'}\right) \quad (2.26)$$

where  $N'$  and  $V'$  are to be called affected  $N$  and  $V$  respectively and  $B^1$  is called to be B first number holds the number of homogenations or formations and  $B^2$  is called to be B second number holds the number of permutations or combinations.

**Proof:** Here the IB members are identified in the second way so the parameters hold to the B first number do not affected and the parameters hold to the B second number possibly affected. Let  $M$  sided  $V$  dice tossed in which first  $v$  dice not identified so the parameters  $M$  and  $V$  are not be affected. Thus the B first number is to be  $B^1\left(\frac{M}{V}\right)$ . Again let  $N$  dice (objects) taken  $V$  at a time in which first  $v$  dice identified in the second way so the parameters  $N$  and  $V$  be affected as  $N'$  and  $V'$  respectively. So the B second number is to be  $B^2\left(\frac{N'}{V'}\right)$ . Performing the two cases at the same time we get the number of IB members is  $B^1\left(\frac{M}{V}\right) \times B^2\left(\frac{N'}{V'}\right)$  i.e.,

$$\text{IB}_2\left(\frac{M N}{V/v_2K}\right) = B^1\left(\frac{M}{V}\right) B^2\left(\frac{N'}{V'}\right).$$

**Example 2.13:** Find the number of IB members of the B

event  $IB_2 \left\{ \begin{smallmatrix} 3 & 7 \\ 5 & 3/2 K \end{smallmatrix} \right\}$  where B first number is the number of formations and B second number is the number of combinations and first 3 identified n's are 1, 3 and 5.

**Solution:** From the theorem 2.13 we get

$$IB_2 \left( \begin{smallmatrix} 3 & 7 \\ 5 & 3/2 K \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N' \\ V' \end{smallmatrix} \right) \text{ where,}$$

$$M = 3$$

$$V = 5$$

$$N' = N - n_3 = 7 - 5 = 2$$

$$V' = V - v = 5 - 3 = 2$$

$$\text{Thus } IB_2 \left( \begin{smallmatrix} 3 & 7 \\ 5 & 3/2 K \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 3 \\ 5 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \right) = 21 \times 1 = 21.$$

**Theorem 2.14 Both way IB theorem:** The number of IB members of the both way B event  $IB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $IB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$IB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M' \\ V' \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N' \\ V' \end{smallmatrix} \right) \quad (2.27)$$

where  $M'$ ,  $N'$  and  $V'$  are to be called affected  $M$ ,  $N$  and  $V$  respectively and  $B^1$  is the B first number to be holds the number of homogenations or formations and  $B^2$  is the B second number to be holds the number of permutations or combinations.

**Proof:** Here the IB members are identified in the both way so the parameters hold to the both B first number and B second number possibly affected. Now let  $M$  sided  $V$  dice tossed in which first  $v$  dice identified in the first way so the parameters  $M$  and  $V$  be affected as  $M'$  and  $V'$  respectively. Thus the B first number is to be  $B^1 \left( \begin{smallmatrix} M' \\ V' \end{smallmatrix} \right)$ . Again let  $N$  dice (objects) taken  $V$  at a time in which first  $v$  dice identified in the second way so the parameters  $N$  and  $V$  be affected as  $N'$  and  $V'$  respectively. Thus the B second number is to be  $B^2 \left( \begin{smallmatrix} N' \\ V' \end{smallmatrix} \right)$ . Performing the two cases at the same time we get the number of IB members is  $B^1 \left( \begin{smallmatrix} M' \\ V' \end{smallmatrix} \right) \times B^2 \left( \begin{smallmatrix} N' \\ V' \end{smallmatrix} \right)$  i.e.,

$$IB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M' \\ V' \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N' \\ V' \end{smallmatrix} \right).$$

**Example 2.14:** Find the number of IB members of the B event  $IB_{12} \left\{ \begin{smallmatrix} 4 & 6 \\ 5 & 1/2 K \end{smallmatrix} \right\}$  where B first number is the number of formations and B second number is the number of permutations and first 3 identified m's are 1, 2, 2 and n's are 2, 3, 4.

**Solution:** From the theorem 2.14 we get

$$IB_{12} \left( \begin{smallmatrix} 4 & 6 \\ 5 & 1/2 K \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M' \\ V' \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N' \\ V' \end{smallmatrix} \right) \text{ where,}$$

$$M' = M - m_3 + 1$$

$$= 4 - 2 + 1 = 3$$

$$N' = N - v = 6 - 3 = 3$$

$$V' = V - v = 5 - 3 = 2$$

$$\text{Thus } IB_{12} \left( \begin{smallmatrix} 4 & 6 \\ 5 & 1/2 K \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 3 \\ 5 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \right) = 6 \times 6 = 36.$$

**Theorem 2.15 First way ISB theorem:** The number of ISB members of the first way SB event  $ISB_1 \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $ISB_1 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$ISB_1 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M - m_v - 1 \\ V - v \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.28)$$

**Theorem 2.16 Second way ISB theorem:** The number of ISB members of the second way SB event  $ISB_2 \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $ISB_2 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$ISB_2 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) C \left( \begin{smallmatrix} N - n_v \\ V - v \end{smallmatrix} \right). \quad (2.29)$$

**Theorem 2.17 Both way ISB theorem:** The number of ISB members of the both way SB event  $ISB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $ISB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$ISB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M - m_v + 1 \\ V - v \end{smallmatrix} \right) C \left( \begin{smallmatrix} N - n_v \\ V - v \end{smallmatrix} \right) \quad (2.30)$$

**Example 2.15:** Find the number of ISB members of the SB event  $ISB_{12} \left\{ \begin{smallmatrix} 6 & 7 \\ 4 & 1/2 K \end{smallmatrix} \right\}$  where first 3 m's are 1, 2, 4 and n's are 1, 2, 4.

**Solution:** From the theorem 2.17 we get

$$ISB_{12} \left( \begin{smallmatrix} 6 & 7 \\ 4 & 1/2 K \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 6 - 4 + 1 \\ 4 - 3 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 7 - 4 \\ 4 - 3 \end{smallmatrix} \right) = 3 \times 3 = 9.$$

**Corollary 2.12:** The number of ISB members of the SB event  $ISB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  is the same as the number of SB members of the SB space  $SB \left\{ \begin{smallmatrix} M - m_v + 1 & N - n_v \\ V - v \end{smallmatrix} \right\}$  i.e.,

$$ISB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = SB \left( \begin{smallmatrix} M - m_v + 1 & N - n_v \\ V - v \end{smallmatrix} \right) \quad (2.31)$$

**Theorem 2.18 First way IAB theorem:** The number of IAB members of the first way AB event  $IAB_1 \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $IAB_1 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$IAB_1 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V - v \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.32)$$

**Theorem 2.19 Second way IAB theorem:** The number of IAB members of the second way AB event  $IAB_2 \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $IAB_2 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$IAB_2 \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N - v \\ V - v \end{smallmatrix} \right) \quad (2.33)$$

**Theorem 2.20 Both way IAB theorem:** The number of IAB members of the both way AB event  $IAB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right\}$  denoted by  $IAB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right)$  is

$$IAB_{12} \left( \begin{smallmatrix} M & N \\ V & 1/2 K \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V - v \end{smallmatrix} \right) P \left( \begin{smallmatrix} N - v \\ V - v \end{smallmatrix} \right) \quad (2.34)$$

**Example 2.16:** Find the number of IAB members of the AB event  $IAB_{12} \left\{ \begin{smallmatrix} 5 & 4 \\ 3 & 12 \end{smallmatrix} K \right\}$  where first m's are 4, 3 and n's are 2, 4.

**Solution:** From the theorem 23.20 we get

$$IAB_{12} \left( \begin{smallmatrix} 5 & 4 \\ 3 & 12 \end{smallmatrix} K \right) = H \left( \begin{smallmatrix} 5 \\ 3-2 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 4-2 \\ 3-2 \end{smallmatrix} \right) = 5 \times 2 = 10.$$

**Corollary 2.13:** The number of IAB members of the AB event  $IAB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right\}$  is the same as the number of AB members of the AB space  $AB \left\{ \begin{smallmatrix} M & N - v \\ V - v \end{smallmatrix} \right\}$  i.e.,

$$IAB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right) = AB \left( \begin{smallmatrix} M & N - v \\ V - v \end{smallmatrix} \right) \quad (2.35)$$

**Theorem 2.21 First way ISAB theorem:** The number of ISAB members of the first way SAB event  $ISAB_1 \left\{ \begin{smallmatrix} M & N \\ V & 1 \end{smallmatrix} K \right\}$  denoted by  $ISAB_1 \left( \begin{smallmatrix} M & N \\ V & 1 \end{smallmatrix} K \right)$  is

$$ISAB_1 \left( \begin{smallmatrix} M & N \\ V & 1 \end{smallmatrix} K \right) = F \left( \begin{smallmatrix} M - m_v + 1 \\ V - v \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right). \quad (2.36)$$

**Theorem 2.22 Second way ISAB theorem:** The number of ISAB members of the second way SAB event  $ISAB_2 \left\{ \begin{smallmatrix} M & N \\ V & 2 \end{smallmatrix} K \right\}$  denoted by  $ISAB_2 \left( \begin{smallmatrix} M & N \\ V & 2 \end{smallmatrix} K \right)$  is

$$ISAB_2 \left( \begin{smallmatrix} M & N \\ V & 2 \end{smallmatrix} K \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N - v \\ V - v \end{smallmatrix} \right) \quad (2.37)$$

**Theorem 2.23 Both way ISAB theorem:** The number of ISAB members of the both way SAB event  $ISAB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right\}$  denoted by  $ISAB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right)$  is

$$ISAB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right) = F \left( \begin{smallmatrix} M - m_v + 1 \\ V - v \end{smallmatrix} \right) P \left( \begin{smallmatrix} N - v \\ V - v \end{smallmatrix} \right) \quad (2.38)$$

**Example 2.17:** Find the number of ISAB members of the SAB event  $ISAB_{12} \left\{ \begin{smallmatrix} 8 & 7 \\ 6 & 12 \end{smallmatrix} K \right\}$  where first 4 identified m's are 1, 3, 4, 6 and n's are 2, 3, 4, 6.

**Solution:** From the theorem 2.23 we get

$$ISAB_{12} \left( \begin{smallmatrix} 8 & 7 \\ 6 & 12 \end{smallmatrix} K \right) = F \left( \begin{smallmatrix} 8 - 6 + 1 \\ 6 - 4 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 7 - 4 \\ 6 - 4 \end{smallmatrix} \right) = 6 \times 6 = 36.$$

**Corollary 2.14:** The number of ISAB members of the SAB event  $ISAB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right\}$  is the same as the number of SAB members of the SAB space  $SAB \left\{ \begin{smallmatrix} M - m_v + 1 & N - v \\ V - v \end{smallmatrix} \right\}$  i.e.,

$$ISAB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right) = SAB \left( \begin{smallmatrix} M - m_v + 1 & N - v \\ V - v \end{smallmatrix} \right) \quad (2.39)$$

**Theorem 2.24 First way IASB theorem:** The number of IASB members of the first way ASB event  $IASB_1 \left\{ \begin{smallmatrix} M & N \\ V & 1 \end{smallmatrix} K \right\}$  denoted by  $IASB_1 \left( \begin{smallmatrix} M & N \\ V & 1 \end{smallmatrix} K \right)$  is

$$IASB_1 \left( \begin{smallmatrix} M & N \\ V & 1 \end{smallmatrix} K \right) = H \left( \begin{smallmatrix} M \\ V - v \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.40)$$

**Theorem 2.25 Second way IASB theorem:** The number of IASB members of the second way ASB event  $IASB_2 \left\{ \begin{smallmatrix} M & N \\ V & 2 \end{smallmatrix} K \right\}$  denoted by  $IASB_2 \left( \begin{smallmatrix} M & N \\ V & 2 \end{smallmatrix} K \right)$  is

$$IASB_2 \left( \begin{smallmatrix} M & N \\ V & 2 \end{smallmatrix} K \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) C \left( \begin{smallmatrix} N - n_v \\ V - v \end{smallmatrix} \right) \quad (2.41)$$

**Theorem 2.26 Both way IASB theorem:** The number of IASB members of the both way ASB event  $IASB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right\}$  denoted by  $IASB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right)$  is

$$IASB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right) = H \left( \begin{smallmatrix} M \\ V - v \end{smallmatrix} \right) C \left( \begin{smallmatrix} N - n_v \\ V - v \end{smallmatrix} \right) \quad (2.42)$$

**Example 2.18:** Find the number of IASB members of the ASB event  $IASB_{12} \left\{ \begin{smallmatrix} 7 & 6 \\ 5 & 12 \end{smallmatrix} K \right\}$  where first 4 identified m's are 6, 7, 3, 4 and n's are 2, 3, 4, 5.

**Solution:** From the theorem 2.26 we get

$$IASB_{12} \left( \begin{smallmatrix} 7 & 6 \\ 5 & 12 \end{smallmatrix} K \right) = H \left( \begin{smallmatrix} 7 \\ 5 - 4 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 6 - 5 \\ 5 - 4 \end{smallmatrix} \right) = 7 \times 1 = 7.$$

**Corollary 2.15:** The number of IASB members of the ASB event  $IASB_{12} \left\{ \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right\}$  is the same as the number of ASB members of the ASB space  $ASB \left\{ \begin{smallmatrix} M & N - n_v \\ V - v \end{smallmatrix} \right\}$  i.e.,

$$IASB_{12} \left( \begin{smallmatrix} M & N \\ V & 12 \end{smallmatrix} K \right) = ASB \left( \begin{smallmatrix} M & N - n_v \\ V - v \end{smallmatrix} \right) \quad (2.43)$$

## 2.4. CB Theorem

**Theorem 2.27 CB theorem:** The number of B events of the CB space  $CB \left\{ \begin{smallmatrix} M & N \\ V & v \end{smallmatrix} \right\}$  denoted by  $CB \left( \begin{smallmatrix} M & N \\ V & v \end{smallmatrix} \right)$  is

$$CB \left( \begin{smallmatrix} M & N \\ V & v \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.44)$$

where  $B^1$  is the B first number to be holds the number of homogenations or formations and  $B^2$  is the B second number to be holds the number of permutations or combinations.

**Proof:** From the theorem 2.2 we get the B number is equal to the product of B first number and B second number. We get more the B first number is the number of events of homogenations or formations and the B second number is the number of events of permutations and combination. Now the B members are characterized at the  $v^{\text{th}}$  die, so the B first number is to be  $B^1 \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right)$  and B second number is to be  $B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right)$ . Hence the theorem.

$$\text{Remarks: (i) } B^1 \left( \begin{smallmatrix} M \\ V/0 \end{smallmatrix} \right) = 1 \quad (2.45)$$

$$\text{and (ii) } B^2 \left( \begin{smallmatrix} N \\ V/0 \end{smallmatrix} \right) = 1 \quad (2.46)$$

**Explanations:** The number of B members of M sided V dice experiment in which no dice characterized is the B space itself of M sided V dice experiment. Again for the second remark the number of B members of N objects taken V at a



time in which no objects characterized is the B space itself of N objects taken V at a time.

**Example 2.19:** Find the number of B events of the CB space  $CB_{4/2}^{\{4\ 6\}}$  where B first number is the number of events of formations and second die characterized in the first way.

**Solution:** From the theorem 2.27 we get

$$CB_{4/2}^{\{4\ 6\}} = F_{4/2}^{\{4\}} B^2_{4/0}^{\{6\}} = 10 \times 1 = 10.$$

**Theorem 2.28 CSB theorem:** The number of SB events of the CSB space  $CSB_{V/v}^{\{M\ N\}}$  denoted by  $CSB_{V/v}^{\{M\ N\}}$  is

$$CSB_{V/v}^{\{M\ N\}} = F_{V/v}^{\{M\}} C_{V/v}^{\{N\}} \quad (2.47)$$

**Proof:** From the theorem 2.3 we get the SB number is equal to the product of number of formations and number of combinations. Now the SB members are characterized at the  $v^{\text{th}}$  die, so the number of events of formations is  $F_{V/v}^{\{M\}}$  and the number of events of combinations is  $C_{V/v}^{\{N\}}$ . Hence the number of events of SB members is to be  $F_{V/v}^{\{M\}} C_{V/v}^{\{N\}}$ .

**Example 2.20:** Find the number of SB events of the CSB space  $CSB_{4/3}^{\{3\ 5\}}$  where third die characterized in the second way.

**Solution:** From the theorem 2.28 we get

$$CSB_{4/3}^{\{3\ 5\}} = F_{4/0}^{\{3\}} C_{4/3}^{\{5\}} = 1 \times 4 = 4.$$

**Theorem 2.29 CAB theorem:** The number of AB events of the CAB space  $CAB_{V/v}^{\{M\ N\}}$  denoted by  $CAB_{V/v}^{\{M\ N\}}$  is

$$CAB_{V/v}^{\{M\ N\}} = H_{V/v}^{\{M\}} P_{V/v}^{\{N\}} \quad (2.48)$$

**Proof:** From the theorem 2.4 we get the AB number is equal to the product of number of homogenations and number of permutation. Now the AB members are characterized at the  $v^{\text{th}}$  die, so the number of events of homogenations is  $H_{V/v}^{\{M\}}$  and the number of event of permutations is  $P_{V/v}^{\{N\}}$ . Hence the number of events of AB members is  $H_{V/v}^{\{M\}} P_{V/v}^{\{N\}}$ .

**Example 2.21:** Find the number of AB events of the CAB space  $CAB_{2/1}^{\{3\ 4\}}$  where first die characterized in the first way.

**Solution:** From the theorem 2.29 we get

$$CAB_{2/1}^{\{3\ 4\}} = H_{2/1}^{\{3\}} P_{2/1}^{\{4\}} = 3 \times 1 = 3.$$

**Theorem 2.30 CSAB theorem:** The number of SAB events of the CSAB space  $CSAB_{V/v}^{\{M\ N\}}$  denoted by  $CSAB_{V/v}^{\{M\ N\}}$  is

$$CSAB_{V/v}^{\{M\ N\}} = F_{V/v}^{\{M\}} P_{V/v}^{\{N\}} \quad (2.49)$$

**Proof:** From the theorem 2.5 we get the SAB number is equal to the product of number of formations and number of permutations. Now the SAB members are characterized at the  $v^{\text{th}}$  die, so the number of events of formations is  $F_{V/v}^{\{M\}}$  and the number of events of permutations is  $P_{V/v}^{\{N\}}$ . Hence the number of events of SAB members is to be  $F_{V/v}^{\{M\}} P_{V/v}^{\{N\}}$ .

**Example 2.22:** Find the numbers of SAB events of the CSAB space  $CSAB_{4/2}^{\{6\ 7\}}$  where second die characterized in the first way.

**Solution:** From the theorem 2.30 we get

$$CSAB_{4/2}^{\{6\ 7\}} = F_{4/2}^{\{6\}} P_{4/0}^{\{7\}} = 21 \times 1 = 21.$$

**Theorem 2.31 CASB theorem:** The number of ASB events of the CASB space  $CASB_{V/v}^{\{M\ N\}}$  denoted by  $CASB_{V/v}^{\{M\ N\}}$  is

$$CASB_{V/v}^{\{M\ N\}} = H_{V/v}^{\{M\}} C_{V/v}^{\{N\}} \quad (2.50)$$

**Proof:** From the theorem 2.6 we get the ASB number is equal to the product of number of homogenations and number of combinations. Now the ASB members are characterized at the  $v^{\text{th}}$  die, so the number of event of homogenations is  $H_{V/v}^{\{M\}}$  and the number of events of combinations is  $C_{V/v}^{\{N\}}$ . Hence the number of events of ASB members is  $H_{V/v}^{\{M\}} C_{V/v}^{\{N\}}$ .

**Example 2.23:** Find the number of ASB events of the CASB space  $CASB_{5/3}^{\{6\ 7\}}$  where third die characterized in the second way.

**Solution:** From the theorem 2.31 we get

$$CASB_{5/3}^{\{6\ 7\}} = H_{5/0}^{\{6\}} C_{5/3}^{\{7\}} = 1 \times 10 = 10.$$

Here we describe three kinds of CB theorems i.e., first way CB theorem, second way CB theorem and both way CB theorem and their different forms.

**Theorem 2.32 First way CB theorem:** The number of B events of the first way CB space  $CB_1_{V/v}^{\{M\ N\}}$  denoted by  $CB_1_{V/v}^{\{M\ N\}}$  is

$$CB_1_{V/v}^{\{M\ N\}} = B^1_{V/v}^{\{M\}} \quad (2.51)$$

where  $B^1$  is the B first number to be holds the number of homogenations or formations.

**Proof:** From the theorem 2.2 we get the B number is equal to the product of B first number and B second number. As the

$v^{\text{th}}$  die characterized in the first way so from equation (2.46) the B second number is equal to one and the B first number as the number of B members characterized at the  $v^{\text{th}}$  die is  $B^1 \binom{M}{V/v}$ . Hence the theorem.

**Example 2.24:** Find the number of B events of the first way CB space  $CB_1 \left\{ \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right\}$  where B first number is the number of events of homogenations.

**Solution:** From the theorem 2.32 we get

$$CB_1 \left( \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 3 \\ 3/2 \end{smallmatrix} \right) = 9.$$

**Theorem 2.33 Second way CB theorem:** The number of B events of the second way CB space  $CB_2 \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CB_2 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CB_2 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.52)$$

where  $B^2$  is the B second number to be holds the number of permutations or combinations.

**Proof:** From the theorem 2.2 we get the B number is equal to the product of B first number and B second number. As the  $v^{\text{th}}$  die characterized in the second way so from equation (2.45) the B first number is equal to one and the B second number as the number of B members characterized at the  $v^{\text{th}}$  die is  $B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right)$ . Hence the theorem.

**Example 2.25:** Find the number of B events of the second way CB space  $CB_2 \left\{ \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right\}$  where B second number is the number of events of permutations.

**Solution:** From the theorem 2.33 we get

$$CB_2 \left( \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right) = P \left( \begin{smallmatrix} 4 \\ 3/2 \end{smallmatrix} \right) = 12.$$

**Theorem 2.34 Both way CB theorem:** The number of B events of the both way CB space  $CB_{12} \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CB_{12} \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CB_{12} \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.53)$$

Where  $B^1$  is the B first number to be holds the number of events of homogenations or formations and  $B^2$  is the B second number to be holds number of events of permutations or combinations.

**Proof:** From the theorem 2.2 we get the B number is equal to the product of B first number and B second number. As the  $v^{\text{th}}$  die characterized in the both way so the B first number is  $B^1 \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right)$  and the B second number is  $B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right)$ . Hence the number of B events is  $B^1 \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right)$ .

**Example 2.26:** Find the number of B events of the both way CB space  $CB_{12} \left\{ \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right\}$  where B first number is the number of events of homogenations and B second number is

the number of events of permutations.

**Solution:** From the theorem 2.34 we get

$$CB_{12} \left( \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 3 \\ 3/2 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 4 \\ 3/2 \end{smallmatrix} \right) = 9 \times 12 = 108.$$

**Theorem 2.35 First way CSB theorem:** The number of SB events of the first way CSB space  $CSB_1 \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CSB_1 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CSB_1 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) \quad (2.54)$$

**Theorem 2.36 Second way CSB theorem:** The number of SB events of the second way CSB space  $CSB_2 \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CSB_2 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CSB_2 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = C \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.55)$$

**Theorem 2.37 Both way CSB theorem:** The number of SB events of the both way CSB space  $CSB_{12} \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CSB_{12} \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CSB_{12} \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.56)$$

**Example 2.27:** Find the number of SB events of the both way CSB space  $CSB_{12} \left\{ \begin{smallmatrix} 3 & 5 \\ 4/3 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.37 we get

$$CSB_{12} \left( \begin{smallmatrix} 3 & 5 \\ 4/3 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 3 \\ 4/3 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 5 \\ 4/3 \end{smallmatrix} \right) = 10 \times 4 = 40.$$

**Theorem 2.38 First way CAB theorem:** The number of AB events of the first way CAB space  $CAB_1 \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CAB_1 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CAB_1 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) \quad (2.57)$$

**Theorem 2.39 Second way CAB theorem:** The number of AB events of the second way CAB space  $CAB_2 \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CAB_2 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CAB_2 \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = P \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.58)$$

**Theorem 2.40 Both way CAB theorem:** The number of AB events of the both way CAB space  $CAB_{12} \left\{ \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right\}$  denoted by  $CAB_{12} \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right)$  is

$$CAB_{12} \left( \begin{smallmatrix} M & N \\ V/v \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V/v \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V/v \end{smallmatrix} \right) \quad (2.59)$$

**Example 2.28:** Find the number of AB events of the both way CAB space  $CAB_{12} \left\{ \begin{smallmatrix} 3 & 4 \\ 3/2 \end{smallmatrix} \right\}$ .

**Solution:** The solution is the same as the solution of example 2.26 i.e.,  $CAB_{12} \left( \begin{smallmatrix} 3 & 4 \\ 3 & 2 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 3 \\ 3/2 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 4 \\ 3/2 \end{smallmatrix} \right) = 9 \times 12 = 108$ .

**Theorem 2.41 First way CSAB theorem:** The number of SAB events of the first way CSAB space  $CSAB_1 \left\{ \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right\}$  denoted by  $CSAB_1 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right)$  is

$$CSAB_1 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V/V \end{smallmatrix} \right) \quad (2.60)$$

**Theorem 2.42 Second way CSAB theorem:** The number of SAB events of the second way CSAB space  $CSAB_2 \left\{ \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right\}$  denoted by  $CSAB_2 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right)$  is

$$CSAB_2 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right) = P \left( \begin{smallmatrix} N \\ V/V \end{smallmatrix} \right) \quad (2.61)$$

**Theorem 2.43 Both way CSAB theorem:** The number of SAB events of the second way CSAB space  $CSAB_{12} \left\{ \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right\}$  denoted by  $CSAB_{12} \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right)$  is

$$CSAB_{12} \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V/V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V/V \end{smallmatrix} \right) \quad (2.62)$$

**Example 2.29:** Find the number of SAB events of the both way CSAB space  $CSAB_{12} \left\{ \begin{smallmatrix} 3 & 5 \\ 4 & 2 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.43 we get

$$CSAB_{12} \left( \begin{smallmatrix} 3 & 5 \\ 4 & 2 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 3 \\ 4/2 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 5 \\ 4/2 \end{smallmatrix} \right) = 6 \times 20 = 120.$$

**Theorem 2.44 First way CASB theorem:** The number of ASB events of the first way CASB space  $CASB_1 \left\{ \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right\}$  denoted by  $CASB_1 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right)$  is

$$CASB_1 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V/V \end{smallmatrix} \right) \quad (2.63)$$

**Theorem 2.45 Second way CASB theorem:** The number of ASB events of the second way CASB space  $CASB_2 \left\{ \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right\}$  denoted by  $CASB_2 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right)$  is

$$CASB_2 \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right) = C \left( \begin{smallmatrix} N \\ V/V \end{smallmatrix} \right) \quad (2.64)$$

**Theorem 2.46 Both way CASB theorem:** The number of ASB events of the both way CASB space  $CASB_{12} \left\{ \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right\}$  denoted by  $CASB_{12} \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right)$  is

$$CASB_{12} \left( \begin{smallmatrix} M & N \\ V & V \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V/V \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V/V \end{smallmatrix} \right) \quad (2.65)$$

**Example 2.30:** Find the number of ASB events of the both way CASB space  $CASB_{12} \left\{ \begin{smallmatrix} 4 & 6 \\ 3 & 1 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.46 we get

$$CASB_{12} \left( \begin{smallmatrix} 4 & 6 \\ 3 & 1 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 4 \\ 3/1 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 6 \\ 3/1 \end{smallmatrix} \right) = 4 \times 4 = 16.$$

## 2.5. GB Theorem

**Theorem 2.47 GB theorem:** The number of GB members of the GB space  $GB \left\{ \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right\}$  denoted by  $GB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right)$  is

$$GB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M & U \\ V & X \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N & U \\ V & Y \end{smallmatrix} \right) \quad (2.66)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

And  $B^1$  is the B first number to be hold the number of homogenations or formations and  $B^2$  is the B second number to be hold the number of permutations or combinations.

**Proof:** This theorem is the Biswas theorem itself. So the proof is the same as the theorem of Biswas theorem.

**Theorem 2.48 GSB theorem:** The number of GSB members of the GSB space  $GSB \left\{ \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right\}$  denoted by  $GSB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right)$  is

$$GSB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ V & Y \end{smallmatrix} \right) \quad (2.67)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Proof:** We know the B number is to be product of B first number and B second number. As the space is a GSB space so B first number get from general formations theorem and B second number get from generation combination theorem. Thus the two numbers are  $F \left( \begin{smallmatrix} M & U \\ V & X \end{smallmatrix} \right)$  and  $C \left( \begin{smallmatrix} N & U \\ V & Y \end{smallmatrix} \right)$  respectively. Now multiplying the two numbers we get the desired GSB number.

**Example 2.31:** Find the number of GSB members of the GSB space  $GSB \left\{ \begin{smallmatrix} 3 & 7 & 4 \\ 4 & 3 \end{smallmatrix} \right\}$  where 3 is observed in the second way.

**Solution:** From the theorem 2.48 we get

$$GSB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ V & Y \end{smallmatrix} \right)$$

As Y is observed in the second way so we get

$$GSB \left( \begin{smallmatrix} 3 & 7 & 4 \\ 4 & 3 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 3 & 0 \\ 4 & 0 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 7 & 4 \\ 4 & 3 \end{smallmatrix} \right) = 15 \times 12 = 180.$$

**Theorem 2.49 GAB theorem:** The number of GAB members of the GAB space  $GAB \left\{ \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right\}$  denoted by  $GAB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right)$  is

$$GAB \left( \begin{smallmatrix} M & N & U \\ V & X & Y \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M & U \\ V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ V & Y \end{smallmatrix} \right) \quad (2.68)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way

respectively.

**Proof:** We know the B number is to be product of B first number and B second number. As the space is a GAB space so B first number get from general homogenation theorem and B second number get from general permutation theorem. Thus the two numbers are  $H\binom{M}{V}U$  and  $P\binom{N}{V}U$  respectively. Multiplying the two numbers we get the desired GAB number.

**Example 2.32:** Find the number of GAB members of the GAB space  $GAB\left\{\begin{smallmatrix} 4 & 5 & 3 \\ & 4 & 2 \end{smallmatrix}\right\}$  where 2 is observed in the second way.

**Solution:** From the theorem 2.49 we get

$$GAB\binom{M}{V}N U = H\binom{M}{V}U P\binom{N}{V}U$$

As Y is observed in the second way so we get

$$GAB\binom{4}{4}5 3 = H\binom{4}{4}0 P\binom{5}{4}3 = 256 \times 72 = 18432.$$

**Theorem 2.50 GSAB theorem:** The number of GSAB members of the GSAB space  $GSAB\left\{\begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix}\right\}$  denoted by  $GSAB\binom{M}{V}N U$  is

$$GSAB\binom{M}{V}N U = F\binom{M}{V}U P\binom{N}{V}U \quad (2.69)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Proof:** We know the B number is to be product of B first number and B second number. As the space is a GSAB space so B first number get from general formation theorem and B second number get from general permutation theorem. Thus the two numbers are  $F\binom{M}{V}U$  and  $P\binom{N}{V}U$  respectively. Now multiplying the two numbers we get the desired GSAB number.

**Example 2.33:** Find the number of GSAB members of the GSAB space  $GSAB\left\{\begin{smallmatrix} 4 & 6 & 5 \\ & 3 & 1 & 2 \end{smallmatrix}\right\}$  where 1 is observed in the first way and 2 in the second way.

**Solution:** From the theorem 2.50 we get

$$GSAB\binom{M}{V}N U = F\binom{M}{V}U P\binom{N}{V}U$$

As 1 is observed in the first way and 2 in the second way and  $M \geq U$  for first way we take  $U = 4$ , Then

$$GSAB\binom{4}{3}6 5 = F\binom{4}{3}4 P\binom{6}{3}5 = 4 \times 60 = 240.$$

**Theorem 2.51 GASB theorem:** The number of GASB members of the GASB space  $GASB\left\{\begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix}\right\}$  denoted by  $GASB\binom{M}{V}N U$  is

$$GASB\binom{M}{V}N U = H\binom{M}{V}U C\binom{N}{V}U \quad (2.70)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Proof:** We know the B number is to be product of B first number and B second number. As the space is a GASB space so B first number get from general homogenation theorem and B second number get from general combination theorem. Thus the two numbers are  $H\binom{M}{V}U$  and  $C\binom{N}{V}U$  respectively. Now multiplying the two numbers we get the desired GASB number.

**Example 2.34:** Find the number of GASB members of the GASB space  $GASB\left\{\begin{smallmatrix} 2 & 5 & 3 \\ & 4 & 2 \end{smallmatrix}\right\}$  where 2 is observed in the both way.

**Solution:** From the theorem 2.51 we get

$$GASB\binom{M}{V}N U = H\binom{M}{V}U C\binom{N}{V}U$$

As X, Y are observed in the both way and for  $M \geq U$  we take  $U = 2$  for the first way

$$GASB\binom{2}{4}5 3 = H\binom{2}{4}2 C\binom{5}{4}3 = 14 \times 3 = 42.$$

Here we describe three kinds of GB numbers i.e., first way GB theorem, second way GB theorem and both way GB theorem and their different forms.

**Theorem 2.52 First way GB theorem:** The number of GB members of the first way GB space  $GB_1\left\{\begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix}\right\}$  denoted by  $GB_1\binom{M}{V}N U$  is

$$GB_1\binom{M}{V}N U = B^1\binom{M}{V}U B^2\binom{N}{V}U \quad (2.71)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
and  $M \geq U$

And  $B^1$  is the B first number to be hold the number of homogenations or formations and  $B^2$  is the B second number to be hold the number of permutations or combinations.

**Proof:** We know the B number is to be product of B first number and B second number. As we consider a first way GB experiment so we get the B first number from general homogenation theorem or general formation theorem and the number is  $B^1\binom{M}{V}U$ . Again B second number get from elementary permutation theorem or elementary combination theorem and the number is  $B^2\binom{N}{V}U$ . Now multiplying the two numbers we get the desired first way GB number.

**Example 2.35:** Find the number of GB members of the first way GB space  $GB_1\left\{\begin{smallmatrix} 5 & 6 & 4 \\ & 3 & 3 \end{smallmatrix}\right\}$  chosen first way selected and second way arranged.

**Solution:** From the theorem 2.52 we get

$$GB_1\binom{M}{V}N U = B^1\binom{M}{V}U B^2\binom{N}{V}U \\ \Rightarrow GB_1\binom{5}{3}6 4 = F\binom{5}{3}4 P\binom{6}{3}3 = 4 \times 120 = 480.$$

**Theorem 2.53 Second way GB theorem:** The number of GB members of the second way GB space  $GB_2\left\{\begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix}\right\}$

denoted by  $GB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right)$  is

$$GB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.72)$$

where,  $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$   
and  $N \geq U$

and  $B^1$  is the B first number to be hold the number of homogenations or formations and  $B^2$  is the B second number to be hold the number of permutations or combinations.

**Proof:** We know the B number is to be product of B first number and B second number. As we consider a second way GB experiment so we get the B first number from elementary homogenation theorem or elementary formation theorem and the number is  $B^1 \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right)$ . Again B second number get from general permutation theorem or general combination theorem and the number is  $B^2 \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$ . Now multiplying the two numbers we get the desired second way GB number.

**Example 2.36:** Find the number of GB members of the second way GB space  $GB_2 \left\{ \begin{smallmatrix} 6 & 5 & 4 \\ & 4 & 2 \end{smallmatrix} \right\}$  chosen both way selected.

**Solution:** From the theorem 2.53 we get

$$GB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$$

$$\Rightarrow GB_2 \left( \begin{smallmatrix} 6 & 5 & 4 \\ & 4 & 2 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 6 \\ 4 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 5 & 4 \\ & 4 & 2 \end{smallmatrix} \right) = 126 \times 0 = 0$$

Here we get  $C \left( \begin{smallmatrix} 5 & 4 \\ & 4 & 2 \end{smallmatrix} \right) = 0$  as X ranges from 3 to 4

$$\text{So, } C \left( \begin{smallmatrix} 5 & 4 \\ & 4 & 2 \end{smallmatrix} \right) = C \left( \begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 5-4 \\ 4-2 \end{smallmatrix} \right) = C \left( \begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \right) = 6 \times 0 = 0$$

**Theorem 2.54 Both way GB theorem:** The number of GB members of the both way GB space  $GB_{12} \left\{ \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right\}$  denoted by  $GB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right)$  is

$$GB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.73)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

And  $B^1$  is the B first number to be hold the number of homogenations or formations and  $B^2$  is the B second number to be hold the number of permutations or combinations.

**Proof:** We know the B number is to be product of B first number and B second number. As we consider a both way GB experiment so we get the B first number from general homogenation theorem or general formation theorem and the number is  $B^1 \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right)$ . Again B second number get from generation permutation theorem or general combination theorem and the number is  $B^2 \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$ . Now multiplying the two numbers we get the desired both way GB number.

**Example 2.37:** Find the number of GB members of the

both way GB space  $GB_{12} \left\{ \begin{smallmatrix} 3 & 5 & 4 \\ & 4 & 1 & 3 \end{smallmatrix} \right\}$  chosen both way arranged.

**Solution:** From the theorem 2.54 we get

$$GB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = B^1 \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) B^2 \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$$

As 1 is observed in the first way and 3 is observed in the second way and  $M \geq U$  for first way we take  $U = 3$ . Then

$$GB_{12} \left( \begin{smallmatrix} 3 & 5 & 4 \\ & 4 & 1 & 3 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 3 & 3 \\ & 4 & 1 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 5 & 4 \\ & 4 & 3 \end{smallmatrix} \right) = 3 \times 96 = 288.$$

**Theorem 2.55 First way GSB theorem:** The number of GSB members of the first way GSB space  $GSB_1 \left\{ \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right\}$

denoted by  $GSB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right)$  is

$$GSB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.74)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
and  $M \geq U$

**Theorem 2.56 Second way GSB theorem:** The number of GSB members of the second way GSB space  $GSB_2 \left\{ \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right\}$  denoted by  $GSB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right)$  is

$$GSB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.75)$$

where,  $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$   
and  $N \geq U$

**Theorem 2.57 Both way GSB theorem:** The number of GSB members of the both way GSB space  $GSB_{12} \left\{ \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right\}$  denoted by  $GSB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right)$  is

$$GSB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.76)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Example 2.38:** Find the number of GSB members of the both way GSB space  $GSB_{12} \left\{ \begin{smallmatrix} 4 & 5 & 4 \\ & 3 & 1 & 2 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.57 we get

$$GSB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$$

$$\Rightarrow GSB_{12} \left( \begin{smallmatrix} 4 & 5 & 4 \\ & 3 & 1 & 2 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 4 & 4 \\ & 3 & 1 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 5 & 4 \\ & 3 & 2 \end{smallmatrix} \right) = 4 \times 6 = 24.$$

**Theorem 2.58 First way GAB theorem:** The number of GAB members of the first way GAB space  $GAB_1 \left\{ \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right\}$

denoted by  $GAB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right)$  is

$$GAB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.77)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
and  $M \geq U$

**Theorem 2.59 Second way GAB theorem:** The number of GAB members of the second way GAB space  $GAB_2 \left\{ \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right\}$  denoted by  $GAB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right)$  is

$$GAB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.78)$$

where,  $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$   
and  $N \geq U$

**Theorem 2.60 Both way GAB theorem:** The number of GAB members of the both way GAB space  $GAB_{12} \left\{ \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right\}$  denoted by  $GAB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right)$  is

$$GAB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.79)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Example 2.39:** Find the number of GAB members of the both way GAB space  $GAB_{12} \left\{ \begin{smallmatrix} 4 & 6 & 3 \\ & 4 & 3 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.60 we get

$$\begin{aligned} GAB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) &= H \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \\ \Rightarrow GAB_{12} \left( \begin{smallmatrix} 4 & 6 & 3 \\ & 4 & 3 \end{smallmatrix} \right) &= H \left( \begin{smallmatrix} 4 & 3 \\ & 4 & 3 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 6 & 3 \\ & 4 & 3 \end{smallmatrix} \right) = 60 \times 72 = 4320. \end{aligned}$$

**Theorem 2.61 First way GSAB theorem:** The number of GSAB members of the first way GSAB space  $GSAB_1 \left\{ \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right\}$  denoted by  $GSAB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right)$  is

$$GSAB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.80)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
and  $M \geq U$

**Theorem 2.62 Second way GSAB theorem:** The number of GSAB members of the second way GSAB space  $GSAB_2 \left\{ \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right\}$  denoted by  $GSAB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right)$  is

$$GSAB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.81)$$

where,  $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$   
and  $N \geq U$

**Theorem 2.63 Both way GSAB theorem:** The number of GSAB members of the both way GSAB space  $GSAB_{12} \left\{ \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right\}$  denoted by  $GSAB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right)$  is

$$GSAB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.82)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Example 2.40:** Find the number of GSAB members of the both way GSAB space  $GSAB_{12} \left\{ \begin{smallmatrix} 5 & 6 & 3 \\ & 4 & 3 & 2 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.63 we get

$$GSAB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = F \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) P \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$$

As 3 is observed in the first way and 2 is observed in the second way so we get

$$GSAB_{12} \left( \begin{smallmatrix} 5 & 6 & 3 \\ & 4 & 3 & 2 \end{smallmatrix} \right) = F \left( \begin{smallmatrix} 5 & 3 \\ & 4 & 3 \end{smallmatrix} \right) P \left( \begin{smallmatrix} 6 & 3 \\ & 4 & 2 \end{smallmatrix} \right) = 5 \times 216 = 1080.$$

**Theorem 2.64 First way GASB theorem:** The number of GASB members of the first way GASB space  $GASB_1 \left\{ \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right\}$  denoted by  $GASB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right)$  is

$$GASB_1 \left( \begin{smallmatrix} M & N & U \\ & V & X \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N \\ V \end{smallmatrix} \right) \quad (2.83)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
and  $M \geq U$

**Theorem 2.65 Second way GASB theorem:** The number of GASB members of the second way GASB space  $GASB_2 \left\{ \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right\}$  denoted by  $GASB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right)$  is

$$GASB_2 \left( \begin{smallmatrix} M & N & U \\ & V & Y \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M \\ V \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.84)$$

where,  $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$   
and  $N \geq U$

**Theorem 2.66 Both way GASB theorem:** The number of GASB members of the both way GASB space  $GASB_{12} \left\{ \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right\}$  denoted by  $GASB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right)$  is

$$GASB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right) \quad (2.85)$$

where,  $\min(M, U) - M + 1 \leq X \leq \min(M, U, V)$   
 $\min(N, U) - N + V \leq Y \leq \min(N, U, V)$

$M \geq U$  and  $N \geq U$  for first way and second way respectively.

**Example 2.41:** Find the number of GASB members of the both way GASB space  $GASB_{12} \left\{ \begin{smallmatrix} 3 & 6 & 4 \\ & 4 & 3 \end{smallmatrix} \right\}$ .

**Solution:** From the theorem 2.66 we get

$$GASB_{12} \left( \begin{smallmatrix} M & N & U \\ & V & X & Y \end{smallmatrix} \right) = H \left( \begin{smallmatrix} M & U \\ & V & X \end{smallmatrix} \right) C \left( \begin{smallmatrix} N & U \\ & V & Y \end{smallmatrix} \right)$$

Using  $M \geq U$  for first way we take  $U = 3$ , thus

$$\Rightarrow GASB_{12} \left( \begin{smallmatrix} 3 & 6 & 4 \\ & 4 & 3 \end{smallmatrix} \right) = H \left( \begin{smallmatrix} 3 & 3 \\ & 4 & 3 \end{smallmatrix} \right) C \left( \begin{smallmatrix} 6 & 4 \\ & 4 & 3 \end{smallmatrix} \right) = 36 \times 8 = 288.$$

### 3. Conclusions

First of all we get from this paper an interesting Biswas theorem and its related theorems. The related theorems are EB theorems, IB theorems, CB related theorems and GB theorems. We get combination theorem, permutation theorem, formation theorem and homogenation theorem from these related theorems.

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