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Estimation of the Generalized Power Weibull Distribution Parameters Using Progressive Censoring Schemes

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Abstract In this paper, we compared between the maximum likelihood estimator (MLE) and Bayesian estimation for the shape parameters α and θ of the generalized power Weibull (GPW) distribution based on complete censoring data, type-II censoring scheme and type-II progressive censoring scheme. The Bayesian estimates for the GPW parameters are obtained based on binary loss function and squared error loss function. The optimal censoring scheme has been suggested using two different optimality criteria (mean squared of error (MSE) and Bias). This comparison was done by using simulation study and application on real life of data. We discussed the method of obtaining the optimal type-II progressive censoring schemes.

Keywords Generalized power Weibull, Type-II progressive censoring, Maximum likelihood estimation, Bayesian estimation

1. Introduction

The Weibull distribution is one of the most popular distributions in analyzing lifetime data. This weibull family, which was presented at first by Bagdonavicius and Nikulin (2001), contains four shapes of the hazard function and it is mostly used in the reliability and survival analysis domains. This Weibull family is often used for constructing accelerated failures times (AFT) models which describe dependence of the lifetime distribution on explanatory variables. Pham, and Lai, (2007) introduced, the generalized power Weibull (GPW) distribution as a another extension of the Weibull family.

Nikulin and Haghighi (2007), introduced the cumulative distribution function of the generalized power Weibull (GPW) family is

$$F(x;\theta,\alpha) = 1 - e^{1-(1+x^{\alpha})^{\theta}}; \quad x \geq 0, \alpha, \theta > 0 \tag{1}$$

and its the corresponding probability density function is

$$f(x;\theta,\alpha) = \theta\alpha x^{\alpha-1}(1+x^{\alpha})^{\theta-1}e^{1-(1+x^{\alpha})^{\theta}} \tag{2}$$

The quantile function of the generalized power Weibull family is

$$x_u = \left((1-\ln(1-u))^{\frac{1}{\theta}} - 1\right)^{\frac{1}{\alpha}} \quad ; 0 < u < 1 \tag{3}$$

The survival function is

$$S(t;\theta,\alpha) = e^{1-(1+x^{\alpha})^{\theta}} \tag{4}$$

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Particular cases of the generalized power Weibull distribution are:

$\theta = 1$: the family of Weibull distributions;
$\theta = 1$ and $\alpha = 1$: the family of exponential distributions.

A sample is said to be censored if while it is drawn from a complete population, the item values of some of its members are unknown. Such sample members are themselves called censored. Kundu and Pradhan (2009) discussed the two most common censoring schemes are termed as type-I and type-II censoring schemes. Briefly, they can be described as follows; consider n items under observation in a particular experiment. In the conventional type-I censoring scheme, the experiment continues up to a pre-specified time T. On the other hand the conventional type-II censoring scheme requires the experiment to continue until a pre-specified number of failures $R \leq n$ occur. Kim and Han (2009) discussed, progressively type II censored sampling is an important method of obtaining data in lifetime studies. Live units removed early can be readily used in other tests, thereby saving costs to the experimenter, and a compromise can be achieved between time consumption and the observation of some extreme values. Balakrishnan and Chen (2004) introduced, a progressively type II censored sample as follows. Suppose that *n* experimental units are placed on a life test, and suppose that the experimenter decides to have only *m* of these *n* units fail. At the time of the first failure, it is decided that R_1 of the $n-1$ surviving units are randomly removed from the life testing experiment. Continuing on, at the time of the second failure, R_2 of the $n-R_1-2$ surviving units are randomly removed. Finally, at the time of the $m-th$ failure, all of the remaining $n-m-R_1-\cdots-R_{m-1}=R_m$ surviving units are withdrawn from the experiment. For more examples, see Dey et al (2014), Dey et al (2016), see for instance the book by Balakrishnan and Aggrawalla (2000), and an excellent review article by Balakrishnan et al (2007).

2. Maximum Likelihood Estimation for the Parameter of the GPW Distribution

The estimation problem of the unknown parameters of the GPW distribution has been discussed by many authors using complete samples, Type II censored sample and Progressive Type-II Censoring Scheme.

2.1. Complete Sample

Balakrishnan et al (2007), introduced the maximum likelihood estimators (MLE) of the parameters assuming a cumulative exposure model with lifetimes being exponentially distributed. Consider a random sample from the Generalized Power Weibull distribution (2). The likelihood function is given by

$$L(x;\theta,\alpha) = \theta^n\alpha^n \prod_{i=1}^n x_i^{\alpha-1}(1+x_i^{\alpha})^{\theta-1}\, e^{\sum_{i=1}^n\left(1-(1+x_i^{\alpha})^{\theta}\right)} \quad (5)$$

The natural logarithm of the likelihood function equation can be obtained as follows

$$\ln L = n\ln\theta + n\ln\alpha + (\alpha-1)\sum_{i=1}^n \ln x_i + (\theta-1)\sum_{i=1}^n \ln(1+x_i^{\alpha}) + \sum_{i=1}^n\left(1-(1+x_i^{\alpha})^{\theta}\right) \quad (6)$$

The estimates of all parameters are obtained by differentiating the log-likelihood function in (6) with respect to θ and α and equating them to zero. Basing on this, differentiating the log-likelihood function with respect to θ is given as

$$\frac{\partial \ln L}{\partial\theta} = \frac{n}{\theta} + \sum_{i=1}^n \ln(1+x_i^{\alpha}) - \sum_{i=1}^n (1+x_i^{\alpha})^{\theta}\ln(1+x_i^{\alpha}) \quad (7)$$

Differentiating the log-likelihood function in (6) with respect to α is given as

$$\frac{\partial \ln L}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^n \ln x_i + \sum_{i=1}^n \frac{(\theta-1)\,x_i^{\alpha}}{(1+x_i^{\alpha})}\ln x_i - \sum_{i=1}^n \theta x_i^{\alpha}\ln x_i\,(1+x_i^{\alpha})^{\theta-1} \quad (8)$$

But the three equation has to be performed numerically using a nonlinear optimization algorithm.

2.2. Type II Censored Sample

A life test is terminated after a specified number of failures have been occurred is known as "failure censoring" or "type II censoring". The likelihood function can be written as:

$$L = \frac{n!}{(n-r)!}\prod_{i=1}^r f(x_{(i)})\left(1-F(x_{(r)})\right)^{n-r} \quad (9)$$

Considering the Equation (9), the log likelihood function of a generalized power Weibull distribution used type II censored is defined as

$$L(x;\theta,\alpha) = \frac{n!}{(n-r)!}\,\theta^r\alpha^r\prod_{i=1}^r\left(x_i^{\alpha-1}(1+x_i^{\alpha})^{\theta-1}\right) e^{\sum_{i=1}^r 1-(1+x_i^{\alpha})^{\theta}}\left(e^{1-(1+x_r^{\alpha})^{\theta}}\right)^{n-r} \quad (10)$$

the natural logarithm of the likelihood function is

$$\ln L = \ln\frac{n!}{(n-r)!} + r\ln\theta + r\ln\alpha + (\alpha-1)\sum_{i=1}^r \ln x_i + (\theta-1)\sum_{i=1}^r \ln(1+x_i^{\alpha}) + \sum_{i=1}^r\left(1-(1+x_i^{\alpha})^{\theta}\right) + (n-r)\left(1-(1+x_r^{\alpha})^{\theta}\right) \quad (11)$$

to obtain the normal equations for the unknown parameters, we differentiate (11) partially with respect to the parameters θ and α and equate them to zero. The estimators for θ and α can be obtained as the solution of the following equations.

Differentiating the log-likelihood function in (11) with respect to α is given as

$$\frac{\partial \ln L}{\partial\theta} = \frac{r}{\theta} + \sum_{i=1}^r \ln(1+x_i^{\alpha}) - \sum_{i=1}^r (1+x_i^{\alpha})^{\theta}\ln(1+x_i^{\alpha}) - (n-r)(1+x_r^{\alpha})^{\theta}\ln(1+x_r^{\alpha}) \quad (12)$$

and

$$\frac{\partial \ln L}{\partial\alpha} = \frac{r}{\alpha} + \sum_{i=1}^r \ln x_i + \sum_{i=1}^r \frac{(\theta-1)\,x_i^{\alpha}}{(1+x_i^{\alpha})}\ln x_i + \sum_{i=1}^r \theta x_i^{\alpha}\ln x_i\,(1+x_i^{\alpha})^{\theta-1} - (n-r)\theta x_r^{\alpha}\ln x_r\,(1+x_r^{\alpha})^{\theta-1} \quad (13)$$

But the three equation has to be performed numerically using a nonlinear optimization algorithm.

Dey et al (2014) introduced among the different censoring schemes, Type-II are the most popular censoring schemes. Unfortunately, in any of these censoring schemes, it is not possible to withdraw live items during the experiment. In this paper, we consider a generalization of the classical Type-II censoring scheme, where it is possible to withdraw live items during the experiment, and it is known as progressive Type-II censoring scheme.

2.3. Progressive Type-II Censoring Scheme

Dey et al (2016) discussed Progressive Type-II censoring scheme can be described as follows: Suppose n units are placed on a life test and the experimenter decides beforehand the quantity m, the number of failures to be observed. Now at the time of the first failure, R_1 of the remaining $n-1$ surviving units are randomly removed from the experiment. At the time of the second failure, R_2 of the remaining $n - R_1 - 1$ units are randomly removed from the experiment. Finally, at the time of the m-th failure, all the remaining surviving units $R_m = n - m - R_1 - \cdots - R_{m-1}$ are removed from the experiment.

Therefore, a progressive Type-II censoring scheme consists of m, and $R_1,\cdots,R_m$, such that

$$R_1 + \cdots + R_m = n - m.$$

The m failure times obtained from a progressive Type-II censoring scheme will be denoted by $t_1,\cdots,t_m$. We observed the data $\{(t_1,R_1),\ldots,(t_m,R_m)\}$ in a progressive censoring scheme. Although we have included $R_1,\ldots,R_m$ as part of the data, these are known in advance.

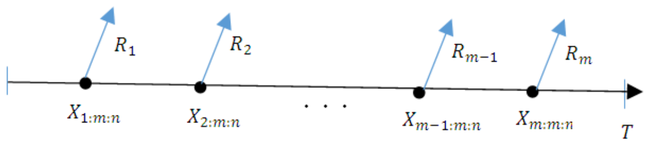


Figure 1. Plot the progressive type-II censoring scheme

Based on the observed sample $x_1 < \cdots < x_m$ from a progressive Type-II censoring scheme, $R_1, \ldots, R_m$ the likelihood function can be written as

$$L = A\prod_{i=1}^{m} f(t_i;\, \theta, \alpha)\,(1 - F(t_i;\, \theta, \alpha)^{R_i} \;\; ; \;\; \theta, \alpha > 0 \quad (14)$$

where

$$A = n(n - R_1 - 1)\ldots\left(n - \sum_{i=1}^{m-1} R_i - (m-1)\right)$$

The likelihood function can be written as

$$L = A\;\; \theta^m \alpha^m e^{\sum_{i=1}^{m}(1-(1+x_i{}^{\alpha})^{\theta})^{R_i+1}} \prod_{i=1}^{m}\left[x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right] \quad (15)$$

the natural logarithm of the likelihood function is

$$\ln L = \ln A + m\ln\theta + m\ln\alpha + (\alpha - 1)\sum_{i=1}^{m} \ln x_i + (\theta - 1)\sum_{i=1}^{m} \ln(1 + x_i{}^{\alpha}) + \sum_{i=1}^{m}(R_i + 1)\left(1 - (1 + x_i{}^{\alpha})^{\theta}\right) \quad (16)$$

To obtain the normal equations for the unknown parameters, we differentiate (16) partially with respect to the parameters θ and α and equate them to zero. The estimators for θ and α can be obtained as the solution of the following equations.

$$\frac{\partial \ln L}{\partial \theta} = \frac{m}{\theta} + \sum_{i=1}^{m} \ln(1 + x_i^{\alpha}) - \sum_{i=1}^{m}(1 + x_i{}^{\alpha})^{\theta} \ln(1 + x_i{}^{\alpha}) - \sum_{i=1}^{m}(R_i + 1)\,(1 + x_i{}^{\alpha})^{\theta}\, \ln(1 + x_i{}^{\alpha}) \quad (17)$$

Differentiating the log-likelihood function in (16) with respect to α is given as

$$\frac{\partial \ln L}{\partial \alpha} = \frac{m}{\alpha} + \sum_{i=1}^{m} \ln x_i + \sum_{i=1}^{m} \frac{(\theta-1)\,x_i{}^{\alpha}}{(1+x_i{}^{\alpha})} \ln x_i + \sum_{i=1}^{m} \theta x_i{}^{\alpha} \ln x_i\,(1 + x_i{}^{\alpha})^{\,\theta-1} - \sum_{i=1}^{m} \theta x_i{}^{\alpha}\,(R_i + 1)\ln x_i\,(1 + x_i{}^{\alpha})^{\,\theta-1} \quad (18)$$

The MLE $\hat{\alpha}, \hat{\theta}$ can be obtained by solving simultaneously the likelihood equations

$$\left.\frac{\partial L}{\partial \theta}\right|_{\theta=\hat{\theta}} = 0, \qquad \left.\frac{\partial L}{\partial \alpha}\right|_{\alpha=\hat{\alpha}} = 0$$

But the equation (17), (18) has to be performed numerically using a nonlinear optimization algorithm.

3. Bayesian Estimation of the GPW Distribution

In this section we consider the Bayesian estimation for the parameters of the GPW distribution under the assumption that the random variables α and θ have an independent gamma distribution is a conjugate prior to the GPW distributions. Assumed that $\theta \sim Gamma(a, b)$ and $\alpha \sim Gamma(c, d)$, then, the joint prior density of α and θ can be written as

$$g(\alpha, \theta) \propto \theta^{a-1}\, e^{-\theta b}\, \alpha^{c-1}\, e^{-\alpha d} \;;\; a, b, c, and\; d > 0 \quad (19)$$

here all the hyper parameters a, b, c and d are known and non-negative.

3.1. Bayesian Estimation in Complete Data

Based on the likelihood function (5) and the joint prior density (19), the joint posterior of complete (c) data (GPW) density of α and θ is

$$g(\alpha, \theta|x)_{\mathrm{c}} = K\theta^{n+a-1}\alpha^{n+c-1}e^{-\alpha d-\theta b+\sum_{i=1}^{n}(1-(1+x_i{}^{\alpha})^{\theta})} \prod_{i=1}^{n}\left(x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right) \quad (20)$$

here the normalizing constant K is

$$K = \left[\iint_0^{\infty} \frac{\theta^{n+a-1}\alpha^{n+c-1}e^{-\alpha d-\theta b+\sum_{i=1}^{n}(1-(1+x_i{}^{\alpha})^{\theta})}}{\prod_{i=1}^{n}\left(x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right)}\, d\theta\, d\alpha\right]^{-1} \quad (21)$$

under binary loss function it should be the estimation of all parameters are obtained by differentiating the posterior function in (20) with respect to θ and α and equating them to zero.

$$\hat{\alpha} = \frac{\partial g(\alpha,\theta|x)}{\partial \alpha}\;,\; \hat{\theta} = \frac{\partial g(\alpha,\theta|x)}{\partial \theta} \quad (22)$$

under square error loss function it should be the estimation of all parameters are obtained by

$$\hat{\theta} = \iint_0^{\infty} \frac{K\theta^{n+a}\alpha^{n+c-1}e^{-\alpha d-\theta b+\sum_{i=1}^{n}(1-(1+x_i{}^{\alpha})^{\theta})}}{\prod_{i=1}^{n}\left(x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right)}\, d\theta\, d\alpha \quad (23)$$

and

$$\hat{\alpha} = \iint_0^{\infty} \frac{K\theta^{n+a-1}\alpha^{n+c}e^{-\alpha d-\theta b+\sum_{i=1}^{n}(1-(1+x_i{}^{\alpha})^{\theta})}}{\prod_{i=1}^{n}\left(x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right)}\, d\theta d\alpha \quad (24)$$

but the equations (22, 23 and 24) has to be performed numerically using a nonlinear optimization algorithm.

3.2. Bayesian Estimation in Type II Censored

Based on the likelihood function (10) and the joint prior density (19), the joint posterior of Type II censored of α and θ is

$$g(\alpha, \theta|x)_{\text{Type II}} = K\theta^{r+a-1}\alpha^{r+c-1}e^{-\alpha d-\theta b+\sum_{i=1}^{r} 1-(1+x_i{}^{\alpha})^{\theta}} \left(e^{1-(1+x_r{}^{\alpha})^{\theta}}\right)^{n-r} \prod_{i=1}^{r}\left(x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right) \quad (25)$$

here the normalizing constant K is

$$K = \left[\iint_0^{\infty} \frac{\theta^{r+a-1}\alpha^{r+c-1}e^{-\alpha d-\theta b+\sum_{i=1}^{r} 1-(1+x_i{}^{\alpha})^{\theta}}}{\left(e^{1-(1+x_r{}^{\alpha})^{\theta}}\right)^{n-r} \prod_{i=1}^{r}\left(x_i{}^{\alpha-1}(1 + x_i{}^{\alpha})^{\theta-1}\right)}\, d\theta\, d\alpha\right]^{-1} \quad (26)$$

under binary loss function it should be the estimation of all parameters are obtained by differentiating the posterior function in (25) with respect to θ and α and equating them to zero.

$$\hat{\alpha}_{\text{Type II}} = \frac{\partial g(\alpha,\theta|x)}{\partial \alpha} \tag{27}$$

$$\hat{\theta}_{\text{Type II}} = \frac{\partial g(\alpha,\theta|x)}{\partial \theta} \tag{28}$$

under square error loss function it should be the estimation of all parameters are obtained by

$$\hat{\theta}_{\text{Type II}} = \iint_0^\infty \frac{K\theta^{r+a}\alpha^{r+c-1}e^{-\alpha d-\theta b+\sum_{i=1}^{r}1-(1+x_i{}^{\alpha})^{\theta}}}{\prod_{i=1}^{r}\left[x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right]\left(e^{1-(1+x_r{}^{\alpha})^{\theta}}\right)^{n-r}}\, d\theta\, d\alpha \tag{29}$$

and

$$\hat{\alpha}_{\text{Type II}} = \iint_0^\infty \frac{K\theta^{r+a-1}\alpha^{r+c}e^{-\alpha d-\theta b+\sum_{i=1}^{r}1-(1+x_i{}^{\alpha})^{\theta}}}{\prod_{i=1}^{r}\left[x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right]\left(e^{1-(1+x_r{}^{\alpha})^{\theta}}\right)^{n-r}}\, d\theta\, d\alpha \tag{30}$$

but the equation (28), (29) and (30) has to be performed numerically using a nonlinear optimization algorithm.

3.3. Bayesian Estimation in Progressive Type-II Censoring Scheme

Based on the likelihood function (15) and the joint prior density (19), the joint posterior of Progressive (P) Type-II censored of α and θ is

$$g(\alpha,\theta|x)_p = K\theta^{m+a-1}\alpha^{m+c-1}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}} \prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right) \tag{31}$$

here the normalizing constant K is

$$K = \left[\iint_0^\infty \frac{\theta^{m+a-1}\alpha^{m+c-1}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}}}{\prod_{i=1}^{m}\left(x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right)}\, d\theta\, d\alpha\right]^{-1} \tag{32}$$

under binary loss function it should be the estimation of all parameters are obtained by differentiating the posterior function in (31) with respect to θ and α and equating them to zero.

$$\hat{\alpha}_p = \frac{\partial g(\alpha,\theta|x)}{\partial \alpha}\ ,\ \hat{\theta}_p = \frac{\partial g(\alpha,\theta|x)}{\partial \theta} \tag{33}$$

under square error loss function it should be the estimation of all parameters are obtained by

$$\hat{\theta}_p = \iint_0^\infty \frac{K\theta^{m+a}\alpha^{m+c-1}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}}}{\prod_{i=1}^{m}\left[x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right]}\, d\theta\, d\alpha \tag{34}$$

and

$$\hat{\alpha}_p = \iint_0^\infty \frac{K\theta^{m+a-1}\alpha^{m+c}e^{-\alpha d-\theta b}\, e^{\sum_{i=1}^{m}\left(1-(1+x_i{}^{\alpha})^{\theta}\right)^{R_i+1}}}{\prod_{i=1}^{m}\left[x_i{}^{\alpha-1}(1+x_i{}^{\alpha})^{\theta-1}\right]}\, d\theta\, d\alpha \tag{35}$$

and equating them to zero. But the equation (33, 34 and 35) has to be performed numerically using a nonlinear optimization algorithm.

4. Simulation Study

In this section; Monte Carlo simulation is done for comparison between estimation methods based on censoring in complete data, Type-II Censoring Scheme and Progressive Type-II Censoring Scheme. For estimating GPW Distribution in life time by R language.

4.1. Simulation Algorithm Scheme

Monte Carlo experiments were carried out based on the following data- generated form Generalized Power Weibull Distribution, where x are distributed as GPW, shape parameter $\theta = 2$ and shape parameter $\alpha = 1.5$, for different sample size $n = 20, 25, and\ 30$, different effective sample sizes (m), and set of different samples schemes. The parameters of prior distribution are $(a, b) = (1.1, 2)$ and $(c, d) = (1.2, 1.5)$. All computations are obtained based on the R language. The Balakrishnan and Aggarwala (2000) introduced optimal censoring schemes: we could define the best scheme as the scheme which minimizes the mean squared error (MSE) of the estimator. That is, the objective function (to be minimized in this case) would be

$$MSE = Mean(\hat{\delta} - \delta)^2 \tag{36}$$

where $\hat{\delta}$ is the estimated value of $\delta = (\theta, \alpha)$.

$$Bias = \hat{\delta} - \delta \tag{37}$$

The simulation methods are compared using MSE and Bias for the parameters using GPW distribution, and this assess their performance through a Monte Carlo simulation study, we restricted the number of repeated-samples to 1000.

In table 1, (n, m)=(20,0), the scheme (0) is special case of progressive type II censoring (complete sample), and (n, m) = (20, 5), the scheme (0*4, 15) is special case of progressive type II censoring (type II censoring), in other schemes they are could progressive type II censoring, we note that the Bias and the MSE in progressive type II censoring is lower than the type II censoring. In general the progressive type II to censoring scheme is more efficient than type II censoring. We note the best method is Bayesian estimation based on binary loss function, since the MSE of binary loss function is less than MSE of square errors. We note the estimation of the complete sample has the least MSE, where it consist of full observation. In general the Bayesian estimation based on binary loss function is the best method in small sample (20, 25, and 30), but in the large sample the Bayesian estimation based on square error is the best method.

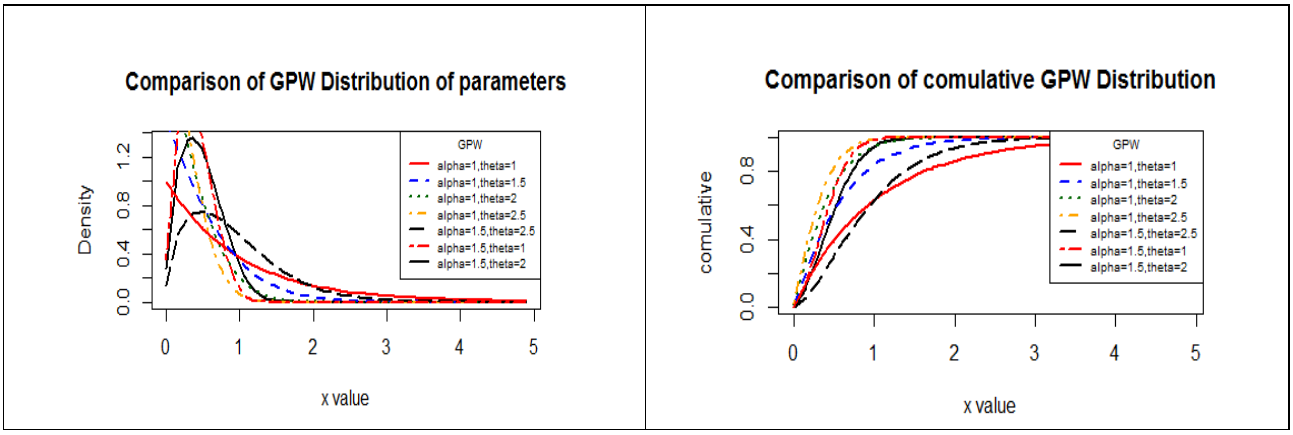


Figure 2. Plot of GPW distribution

Table 1. Estimation to Progressive Type-II Censoring Scheme when n=20

			MLE		Bayesian estimation			
					binary		square error	
(n, m)	scheme		bias	MSE	bias	MSE	bias	MSE
(20, 0)	complete	$\hat{\theta}$	-1.2989	1.6870	-0.1004	0.0685	-0.6829	0.5273
		$\hat{\alpha}$	-0.2921	0.1416	-0.0648	0.0629	-0.4629	0.3179
(20, 5)	(0*4, 15) type 2	$\hat{\theta}$	-1.4150	2.0037	-1.1922	1.4382	-1.2874	1.6715
		$\hat{\alpha}$	-0.5873	0.3864	-0.5519	0.3407	-0.7693	0.6247
	(15, 0*4)	$\hat{\theta}$	-1.2889	1.6614	-0.4703	0.2378	-0.7905	0.6295
		$\hat{\alpha}$	-0.6275	0.4328	-0.4933	0.2838	-0.7796	0.6434
	(3*5)	$\hat{\theta}$	-1.3570	1.8418	-0.9575	0.9252	-1.1100	1.2392
		$\hat{\alpha}$	-0.5919	0.3903	-0.5058	0.2922	-0.7585	0.6101
(20, 10)	(0*9, 10) type 2	$\hat{\theta}$	-1.3578	1.8438	-0.6418	0.4604	-0.9715	0.9934
		$\hat{\alpha}$	-0.5721	0.3614	-0.3319	0.1621	-0.6384	0.4749
	(10, 0*9)	$\hat{\theta}$	-1.2921	1.6696	0.6121	0.4154	-0.1541	0.0730
		$\hat{\alpha}$	-0.6002	0.3926	-0.2596	0.1236	-0.5927	0.4327
	(1*10)	$\hat{\theta}$	-1.3258	1.7579	-0.1253	0.0623	-0.5752	0.3840
		$\hat{\alpha}$	-0.5661	0.3543	-0.2341	0.1095	-0.5620	0.4012
	(2*5, 0*5)	$\hat{\theta}$	-1.3080	1.7109	0.3432	0.1516	-0.2624	0.1180
		$\hat{\alpha}$	-0.5704	0.3588	-0.1947	0.0944	-0.5297	0.3775
	(0*5, 2*5)	$\hat{\theta}$	-1.3435	1.8051	-0.4148	0.2229	-0.7789	0.6594
		$\hat{\alpha}$	-0.5620	0.3500	-0.2644	0.1231	-0.5801	0.4148
(20, 15)	(0*14, 5) types 2	$\hat{\theta}$	-1.3233	1.7512	-0.3010	0.1560	-0.7795	0.6879
		$\hat{\alpha}$	-0.4788	0.2675	-0.1826	0.0933	-0.5336	0.3908
	(5, 0*14)	$\hat{\theta}$	-1.2937	1.6735	0.5900	0.4698	-0.1097	0.1730
		$\hat{\alpha}$	-0.4858	0.2741	-0.0733	0.0741	-0.4134	0.3404
	(1*5, 0*10)	$\hat{\theta}$	-1.2976	1.6839	0.5707	0.4412	-0.0837	0.1841
		$\hat{\alpha}$	-0.4692	0.2593	-0.0229	0.0723	-0.3541	0.3344
	(0*10, 1*5)	$\hat{\theta}$	-1.3181	1.7374	-0.1158	0.0966	-0.6024	0.4764
		$\hat{\alpha}$	-0.4699	0.2598	-0.1121	0.0774	-0.4525	0.3468

Table 2. Estimation to Progressive Type-II Censoring Scheme when n=25

			MLE		Bayesian estimation			
					binary		square error	
(n, m)	scheme		Bias	MSE	Bias	MSE	Bias	MSE
(25, 0)	complete	$\hat{\theta}$	-1.2984	1.6859	-0.0716	0.0562	-0.6750	0.5044
		$\hat{\alpha}$	-0.3023	0.1321	-0.0455	0.0485	-0.4686	0.2835
(25, 5)	(0*4, 20) type 2	$\hat{\theta}$	-1.4442	2.0873	-1.2637	1.6112	-1.3308	1.7834
		$\hat{\alpha}$	-0.5946	0.3865	-0.5633	0.3479	-0.7830	0.6368
	(20, 0*4)	$\hat{\theta}$	-1.3002	1.6908	-0.5162	0.2884	-0.8191	0.6762
		$\hat{\alpha}$	-0.6416	0.4432	-0.5056	0.2893	-0.7983	0.6616
	(4*5)	$\hat{\theta}$	-1.3793	1.9026	-1.0301	1.0680	-1.1547	1.3391
		$\hat{\alpha}$	-0.6009	0.3926	-0.5176	0.2981	-0.7741	0.6233
	(5*4, 0)	$\hat{\theta}$	-1.3614	1.8537	-0.9369	0.8849	-1.0858	1.1834
		$\hat{\alpha}$	-0.6032	0.3951	-0.5032	0.2833	-0.7712	0.6190
	(0, 5*4)	$\hat{\theta}$	-1.3965	1.9508	-1.0995	1.2179	-1.2012	1.4500
		$\hat{\alpha}$	-0.5911	0.3811	-0.5198	0.2998	-0.7687	0.6150
	(10*2, 0*3)	$\hat{\theta}$	-1.3227	1.7497	-0.6904	0.4869	-0.9215	0.8524
		$\hat{\alpha}$	-0.6208	0.4163	-0.4923	0.2740	-0.7802	0.6331
	(0*3, 10*2)	$\hat{\theta}$	-1.4291	2.0435	-1.2165	1.4930	-1.2906	1.6766
		$\hat{\alpha}$	-0.5907	0.3813	-0.5467	0.3289	-0.7755	0.6253
(25, 10)	(0*9, 15) type 2	$\hat{\theta}$	-1.3808	1.9070	-0.7381	0.5882	-1.0290	1.0969
		$\hat{\alpha}$	-0.6029	0.3886	-0.3568	0.1687	-0.6687	0.4865
	(15, 0*9)	$\hat{\theta}$	-1.2970	1.6823	0.7281	0.5581	-0.1133	0.0337
		$\hat{\alpha}$	-0.6451	0.4399	-0.2953	0.1311	-0.6364	0.4500
	(3*5, 0*5)	$\hat{\theta}$	-1.3188	1.7393	0.3383	0.1383	-0.2826	0.1033
		$\hat{\alpha}$	-0.6086	0.3946	-0.2281	0.0955	-0.5780	0.3840
	(0*5, 3*5)	$\hat{\theta}$	-1.3639	1.8604	-0.5170	0.3094	-0.8460	0.7522
		$\hat{\alpha}$	-0.5942	0.3780	-0.2940	0.1281	-0.6198	0.4283
(25, 15)	(0*14, 10) type 2	$\hat{\theta}$	-1.3431	1.8040	-0.4038	0.2295	-0.8375	0.7697
		$\hat{\alpha}$	-0.5536	0.3316	-0.2221	0.0999	-0.5759	0.3915
	(10, 0*14)	$\hat{\theta}$	-1.2949	1.6767	0.9873	1.0905	0.0944	0.1023
		$\hat{\alpha}$	-0.5774	0.3575	-0.1255	0.0697	-0.4662	0.3046
	(0*10, 2*5)	$\hat{\theta}$	-1.3359	1.7848	-0.1991	0.1172	-0.6529	0.5092
		$\hat{\alpha}$	-0.5452	0.3226	-0.1531	0.0756	-0.5050	0.3276
	(2*5, 0*10)	$\hat{\theta}$	-1.3025	1.6966	0.9319	0.9630	0.1210	0.1130
		$\hat{\alpha}$	-0.5519	0.3295	-0.0478	0.0585	-0.3833	0.2515
	(1*10, 0*5)	$\hat{\theta}$	-1.3108	1.7181	0.6810	0.5542	0.0028	0.1009
		$\hat{\alpha}$	-0.5431	0.3202	-0.0279	0.0569	-0.3675	0.2405
	(0*5, 1*10)	$\hat{\theta}$	-1.3275	1.7622	0.0649	0.0873	-0.4344	0.2822
		$\hat{\alpha}$	-0.5396	0.3166	-0.0918	0.0622	-0.4434	0.2822
(25, 20)	(0*19, 5) type 2	$\hat{\theta}$	-1.3181	1.7373	-0.1960	0.1040	-0.7262	0.6001
		$\hat{\alpha}$	-0.4621	0.2425	-0.1232	0.0678	-0.5082	0.3317
	(5, 0*19)	$\hat{\theta}$	-1.2950	1.6771	0.6352	0.5361	-0.0688	0.1649
		$\hat{\alpha}$	-0.4671	0.2472	0.0011	0.0604	-0.3645	0.2485
	(1*5, 0*15)	$\hat{\theta}$	-1.2973	1.6830	0.6423	0.5468	-0.0173	0.1988
		$\hat{\alpha}$	-0.4537	0.2355	0.0411	0.0647	-0.3233	0.2343
	(0*15, 1*5)	$\hat{\theta}$	-1.3148	1.7288	-0.0421	0.0838	-0.5731	0.4309
		$\hat{\alpha}$	-0.4551	0.2366	-0.0585	0.0604	-0.4324	0.2807

Table 3. Estimation to Progressive Type-II Censoring Scheme when $n = 30$

			MLE		Bayesian estimation			
					binary		square error	
(n, m)	scheme		bias	MSE	bias	MSE	bias	MSE
(30, 0)	complete	$\hat{\theta}$	-1.2977	1.6841	-0.0514	0.0624	-0.6665	0.4897
		$\hat{\alpha}$	-0.3063	0.1230	-0.0266	0.0341	-0.4661	0.2612
(30, 5)	(0*4, 25) type 2	$\hat{\theta}$	-1.4666	2.1528	-1.3104	1.7323	-1.3591	1.8589
		$\hat{\alpha}$	-0.5963	0.3851	-0.5681	0.3508	-0.7901	0.6447
	(25, 0*4)	$\hat{\theta}$	-1.3107	1.7183	-0.5608	0.3398	-0.8454	0.7212
		$\hat{\alpha}$	-0.6470	0.4467	-0.5094	0.2903	-0.8079	0.6733
	(5*5)	$\hat{\theta}$	-1.3970	1.9520	-1.0822	1.1774	-1.1862	1.4122
		$\hat{\alpha}$	-0.6036	0.3923	-0.5234	0.3013	-0.7829	0.6334
(30, 10)	(0*9, 20) type 2	$\hat{\theta}$	-1.4011	1.9633	-0.8181	0.7092	-1.0767	1.1925
		$\hat{\alpha}$	-0.6167	0.4021	-0.3740	0.1769	-0.6867	0.5037
	(20, 0*9)	$\hat{\theta}$	-1.3021	1.6954	0.7781	0.6368	-0.0961	0.0221
		$\hat{\alpha}$	-0.6688	0.4678	-0.3099	0.1338	-0.6582	0.4691
	(2*10)	$\hat{\theta}$	-1.3555	1.8376	-0.2797	0.1102	-0.6807	0.4898
		$\hat{\alpha}$	-0.6182	0.4035	-0.2861	0.1191	-0.6330	0.4373
	(5*4, 0*6)	$\hat{\theta}$	-1.3225	1.7490	0.4309	0.2020	-0.2462	0.0748
		$\hat{\alpha}$	-0.6336	0.4221	-0.2537	0.1018	-0.6104	0.4113
	(0*6, 5*4)	$\hat{\theta}$	-1.3868	1.9235	-0.6625	0.4789	-0.9478	0.9304
		$\hat{\alpha}$	-0.6101	0.3940	-0.3280	0.1444	-0.6527	0.4606
	(10*2, 0*8)	$\hat{\theta}$	-1.3095	1.7148	0.6795	0.4796	-0.1352	0.0306
		$\hat{\alpha}$	-0.6523	0.4459	-0.2830	0.1172	-0.6333	0.4380
	(0*8, 10*2)	$\hat{\theta}$	-1.3965	1.9505	-0.7710	0.6346	-1.0366	1.1076
		$\hat{\alpha}$	-0.6142	0.3990	-0.3585	0.1656	-0.6749	0.4885
(30, 15)	(0*14, 15) type 2	$\hat{\theta}$	-1.3603	1.8504	-0.4846	0.2982	-0.8830	0.8414
		$\hat{\alpha}$	-0.5905	0.3694	-0.2470	0.1066	-0.6009	0.4110
	(15, 0*14)	$\hat{\theta}$	-1.2972	1.6827	1.2408	1.6628	0.1931	0.0983
		$\hat{\alpha}$	-0.6273	0.4130	-0.1609	0.0701	-0.4842	0.3086
	(5*3, 0*12)	$\hat{\theta}$	-1.3029	1.6976	1.2239	1.5868	0.2155	0.1082
		$\hat{\alpha}$	-0.6072	0.3885	-0.1087	0.0584	-0.4268	0.2640
	(0*12, 5*3)	$\hat{\theta}$	-1.3560	1.8387	-0.3850	0.2172	-0.7950	0.6995
		$\hat{\alpha}$	-0.5864	0.3645	-0.2137	0.0914	-0.5692	0.3773
	(3*5, 0*10)	$\hat{\theta}$	-1.3080	1.7108	1.1366	1.3666	0.1991	0.1025
		$\hat{\alpha}$	-0.5964	0.3757	-0.0753	0.0529	-0.3985	0.2432
	(0*10,3*5)	$\hat{\theta}$	-1.3516	1.8268	-0.2749	0.1482	-0.6966	0.5594
		$\hat{\alpha}$	-0.5828	0.3604	-0.1814	0.0788	-0.5396	0.3477
(30, 20)	(0*19, 10) type 2	$\hat{\theta}$	-1.3332	1.7776	-0.2660	0.1424	-0.7594	0.6564
		$\hat{\alpha}$	-0.5349	0.3079	-0.1551	0.0729	-0.5365	0.3505
	(10, 0*19)	$\hat{\theta}$	-1.2952	1.6775	1.1047	1.3882	0.2801	0.2879
		$\hat{\alpha}$	-0.5546	0.3287	-0.0303	0.0539	-0.4181	0.2602
	(1*10, 0*10)	$\hat{\theta}$	-1.3044	1.7016	1.0344	1.2159	0.3420	0.3521
		$\hat{\alpha}$	-0.5242	0.2969	0.0748	0.0632	-0.2892	0.2068
	(0*10, 1*10)	$\hat{\theta}$	-1.3236	1.7521	0.1456	0.1268	-0.3738	0.2785
		$\hat{\alpha}$	-0.5222	0.2950	-0.0223	0.0546	-0.3893	0.2462
	(5*2, 0*18)	$\hat{\theta}$	-1.2964	1.6806	1.1198	1.4219	0.3089	0.3160
		$\hat{\alpha}$	-0.5469	0.3204	-0.0095	0.0540	-0.3951	0.2482
	(0*18, 5*2)	$\hat{\theta}$	-1.3321	1.7745	-0.2219	0.1255	-0.7175	0.6013
		$\hat{\alpha}$	-0.5331	0.3060	-0.1378	0.0685	-0.5178	0.3342

(30, 25)	(0*24, 5) type 2	$\hat{\theta}$	-1.3138	1.7262	-0.1342	0.0798	-0.6955	0.5517
		$\hat{\alpha}$	-0.4492	0.2272	-0.0868	0.0561	-0.4923	0.3088
	(5, 0*24)	$\hat{\theta}$	-1.2950	1.6770	0.6167	0.5052	-0.0619	0.1828
		$\hat{\alpha}$	-0.4526	0.2303	0.0409	0.0566	-0.3389	0.2245
	(0*20, 1*5)	$\hat{\theta}$	-1.3116	1.7203	-0.0052	0.0755	-0.5654	0.4141
		$\hat{\alpha}$	-0.4436	0.2225	-0.0288	0.0534	-0.4232	0.2644
	(1*5, 0*20)	$\hat{\theta}$	-1.2965	1.6808	0.6282	0.5225	-0.0287	0.1964
		$\hat{\alpha}$	-0.4418	0.2211	0.0714	0.0620	-0.3016	0.2140

Table 4. Estimation to Progressive Type-II Censoring Scheme when $n = 50$

			MLE		Bayesian estimation			
					binary		square error	
(n, m)	bias	MSE	bias	MSE	bias	MSE	bias	MSE
(50,0)	complete	$\hat{\theta}$	-1.2931	1.6721	-0.0272	0.0324	-0.0231	0.0337
		$\hat{\alpha}$	-0.3234	0.1239	-0.0119	0.0259	0.0080	0.0268
(50, 5)	(0*4, 45) type 2	$\hat{\theta}$	-1.5272	2.3347	-1.4312	2.0597	-1.0986	1.2298
		$\hat{\alpha}$	-0.5993	0.3807	-0.5864	0.3654	-0.4114	0.1959
	(45, 0*4)	$\hat{\theta}$	-1.3408	1.7985	-0.6865	0.5015	-0.2669	0.0988
		$\hat{\alpha}$	-0.6579	0.4543	-0.5331	0.3071	-0.4394	0.2223
(50, 10)	(0*9, 40) type 2	$\hat{\theta}$	-1.4619	2.1379	-1.0257	1.0867	-1.2033	1.4745
		$\hat{\alpha}$	-0.6318	0.4141	-0.4183	0.2024	-0.7300	0.5519
	(40, 0*9)	$\hat{\theta}$	-1.3193	1.7407	0.7769	0.6709	-0.1062	0.0163
		$\hat{\alpha}$	-0.7087	0.5166	-0.3389	0.1435	-0.7712	0.6119
(50, 15)	(0*14, 35) type 2	$\hat{\theta}$	-1.4135	1.9983	-0.7069	0.5531	-0.4813	0.3108
		$\hat{\alpha}$	-0.6412	0.4239	-0.3098	0.1271	-0.2128	0.0802
	(35, 0*14)	$\hat{\theta}$	-1.3076	1.7098	1.7146	3.0981	1.6550	2.7537
		$\hat{\alpha}$	-0.7128	0.5203	-0.2672	0.1008	-0.2622	0.0993
(50, 20)	(0*19, 30) type 2	$\hat{\theta}$	-1.3807	1.9065	-0.4743	0.2938	-0.8812	0.8359
		$\hat{\alpha}$	-0.6298	0.4082	-0.2251	0.0837	-0.6078	0.4011
	(30, 0*19)	$\hat{\theta}$	-1.3012	1.6931	1.6262	3.5168	0.4849	0.2685
		$\hat{\alpha}$	-0.6902	0.4872	-0.0880	0.0281	-0.6395	0.4316
(50, 25)	(0*24, 25) type 2	$\hat{\theta}$	-1.3575	1.8428	-0.3262	0.1814	-0.1935	0.1377
		$\hat{\alpha}$	-0.6121	0.3865	-0.1719	0.0667	-0.1038	0.0520
	(25, 0*24)	$\hat{\theta}$	-1.2970	1.6823	-0.9995	0.9991	1.8761	3.5915
		$\hat{\alpha}$	-0.6595	0.4460	-0.5001	0.2501	-0.0692	0.0370
50, 30)	(0*29, 20) type 2	$\hat{\theta}$	-1.3392	1.7936	-0.2144	0.1142	-0.7426	0.6185
		$\hat{\alpha}$	-0.5800	0.3470	-0.1228	0.0493	-0.5385	0.3286
	(20, 0*29)	$\hat{\theta}$	-1.2946	1.6761	-1.0000	0.9999	0.5750	0.4359
		$\hat{\alpha}$	-0.6136	0.3865	-0.5000	0.2500	-0.4389	0.2318
(50, 35)	(0*34, 15) Type 2	$\hat{\theta}$	-1.3251	1.7559	-0.1864	0.0998	-0.0856	0.0826
		$\hat{\alpha}$	-0.5420	0.3065	-0.1140	0.0480	-0.0477	0.0412
	(15, 0*34)	$\hat{\theta}$	-1.2929	1.6716	-0.9999	0.9999	1.4443	2.2510
		$\hat{\alpha}$	-0.5636	0.3301	-0.5000	0.2500	0.1112	0.0533
(50, 40)	(0*39, 10) Type 2	$\hat{\theta}$	-1.3131	1.7242	-0.3371	0.2378	-0.6827	0.5111
		$\hat{\alpha}$	-0.4898	0.2521	-0.1862	0.0763	-0.5003	0.2853
	(10, 0*39)	$\hat{\theta}$	-1.2920	1.6693	-1.0000	0.9999	0.1903	0.1965
		$\hat{\alpha}$	-0.4993	0.2613	-0.5000	0.2500	-0.3009	0.1562
(50, 45)	(0*44, 5) type 2	$\hat{\theta}$	-1.3028	1.6974	-0.7269	0.6611	-0.6712	0.4848
		$\hat{\alpha}$	-0.4250	0.1946	-0.3815	0.1752	-0.4894	0.2740
	(5, 0*44)	$\hat{\theta}$	-1.2918	1.6687	-0.9999	0.9998	-0.1957	0.1253
		$\hat{\alpha}$	-0.4246	0.1943	-0.5000	0.2500	-0.3340	0.1685

5. Application of Real Data

The data set from Chhikara, and Folks, (1989). This data represents the repair-times correspond to maintenance data on active repair times in hours for an airborne communications transceiver. A vector containing 46 observations and also several data sets related to life test are available in the “reliaR” package, which have been taken from the literature.

Table 5. Estimation for Complete data

	MLE	Bayesian estimation	
		binary	square error
$\hat{\theta}$	0.3553	0.3722	0.5458
$\hat{\alpha}$	1.5661	1.4802	0.9368

Table 6. Goodness of Fit data

One-sample Kolmogorov-Smirnov test (D)			
	MLE	Bayesian estimation	
		binary	square error
D	0.1328	0.1241	0.1838
p-value	0.3912	0.4777	0.0893

The empirical distribution function is defined as

$$\tilde{F}(x) = \frac{\sum_{i=1}^{n} I(X_i \le x)}{n+1} \tag{38}$$

Table 7. Estimation for Progressive Type-II Censoring Scheme of real data

(n, m)	scheme		MLE	Bayesian estimation	
				binary	square error
(46, 5)	(0*4,41) type 2	$\hat{\theta}$	1.3855	0.3637	0.5680
		$\hat{\alpha}$	3.6167	1.7240	2.1647
	(41, 0*4)	$\hat{\theta}$	29.8982	1.3911	1.7961
		$\hat{\alpha}$	4.9477	2.0984	2.2984
	(1, 10*4)	$\hat{\theta}$	2.1127	0.4392	0.6755
		$\hat{\alpha}$	3.8932	1.8190	2.2280
	(10*4, 1)	$\hat{\theta}$	3.1563	0.5475	0.8131
		$\hat{\alpha}$	4.0692	1.8930	2.2579
(46, 10)	(0*9, 36) type 2	$\hat{\theta}$	0.7163	0.5151	0.5984
		$\hat{\alpha}$	2.8840	2.0651	2.3175
	(36, 0*9)	$\hat{\theta}$	5.8395	2.4282	2.6525
		$\hat{\alpha}$	3.8658	2.5491	2.7058
(46, 15)	(0*14, 31) type 2	$\hat{\theta}$	0.4814	0.4602	0.4764
		$\hat{\alpha}$	2.2443	1.9144	2.0149
	(31, 0*14)	$\hat{\theta}$	2.1117	0.5092	1.8308
		$\hat{\alpha}$	3.0028	1.5019	2.7054
(46, 20)	(0*19, 26) type 2	$\hat{\theta}$	0.4024	0.4082	0.4136
		$\hat{\alpha}$	1.8522	1.6792	1.7269
	(26, 0*19)	$\hat{\theta}$	1.2418	0.5201	1.1944
		$\hat{\alpha}$	2.5852	1.5051	2.4499
(46, 30)	(0*29, 16) type 2	$\hat{\theta}$	0.3722	0.3860	0.3889
		$\hat{\alpha}$	1.6555	1.5484	1.5789
	(16, 0*29)	$\hat{\theta}$	0.6361	0.6496	0.6471
		$\hat{\alpha}$	2.0376	1.9332	1.9653

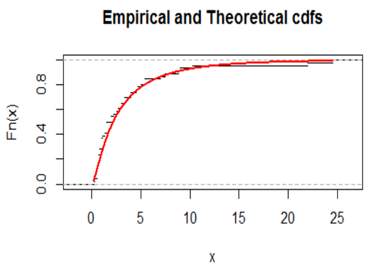
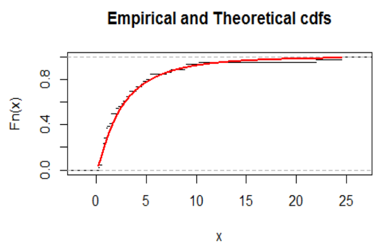
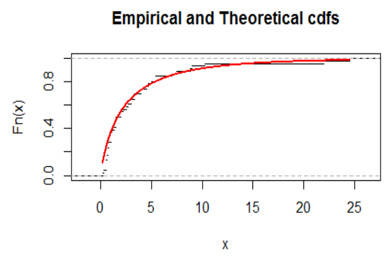
MLE	Bayesian estimation	
	binary	square error
		

Figure 3. Plot the maximum distance between the empirical and theoretical CDF

6. Conclusions

In this paper, we discussed the estimation problem of the unknown parameters of the GPW distribution based on progressive type-II censoring scheme, type-II censoring data and complete censoring data. We used MLE and Bayesian estimation methods to estimate the unknown parameters. The performances of the maximum likelihood estimators are also quite satisfactory. We obtained the Bayes estimates based on binary loss function and square error loss function under the assumption of independent gamma priors. A real data set is used to show how the scheme works in practice. The performance of the different estimator's optimal censoring schemes is compared based on simulation study to determine the optimal censoring schemes by using MSE and Bias. It is observed that Bayesian estimates with respect to the gamma priors behave quite similarly with the corresponding MLE in terms of mean squared errors. We note that the complete censoring is a special case of the progressive type II censoring scheme that can be obtained simply by taking $R_1 = \cdots = R_m = 0$, and note that the usual type II censoring scheme is a special case of the progressive type II censoring scheme that can be obtained simply by taking $R_1 = \cdots = R_{m-1} = 0$. We note that, the Bayesian estimation is better method, where Bias and MSE decreases in Bayesian estimation than the maximum likelihood estimation method. In small sample, we note that the binary loss function is the best method, but in large sample the square error loss function is the best method under the assumption of independent gamma priors.

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