

Estimation of Inverse Rayleigh Distribution Parameters for Type II Singly and Doubly Censored Data Based on Lower Record Values

Summaira Manzoor^{1,2,*}, Ahmed Zogo Memon², Maryam Firdos¹, Tahira Kanwal¹, Rabia Manzoor³

¹Department of Statistics, University of Azad Jammu and Kashmir, Muzaffarabad, Pakistan

²Department of Statistics, College of Statistical and Actuarial Sciences, University of the Punjab, Lahore, Pakistan

³Department of Soil Science and SWC, PAMS Aried Agriculture University, Rawalpindi, Pakistan

Abstract In this paper Lloyd's method is applied for the estimation of scale and location parameters of Inverse Rayleigh distribution for Type II singly and doubly censored data based on a sample size up to 15. Some further analysis is done to determine the variance range for the censored estimators of the scale parameter when the same number of observations is included in a censored sample. Lloyd's estimators are compared for the scale parameter in situations where one or both Inverse Rayleigh distribution parameters are unknown.

Keywords Type II censoring, Efficiency, Best linear unbiased estimator

1. Introduction

Ordered random variables such as record values have gained a lot of importance during the past years. Record values are widely used in many situations of daily life as well as in statistical modelling and inference; to describe random variables arranged in order of magnitude.

Inverse Rayleigh distribution (IRD) is used in the area of acceptance sampling plan in which life time of a product follows the IRD. The software developed by Wood [1] is used for fitting data to this distribution. The IRD is considered as a model for a life time random variable, and has been assumed by Rosiah and Kantam [2]. The probability density function of two parameter IRD with location parameter θ and scale parameter λ is

$$f(x) = \frac{2\lambda^2}{(x-\theta)^3} e^{-\frac{\lambda^2}{(x-\theta)^2}} \quad \theta < x < \infty, \lambda > 0, \quad (1)$$

Hirai [3] estimated the location and scale parameters of Rayleigh distribution by quadratic coefficients method. Dyer and Whisenand [4] derived the best linear unbiased estimator (BLUE) of the parameter of the Rayleigh distribution using order statistics in a Type II censored sample. The concept of record values from normal distribution was discussed by Balakrishnan and Chan [5]. They computed mean, variance and covariance's for the record values from normal

distribution and gave the BLUEs of location and scale parameter for the first n record values. Aleem [6] found concomitants of order statistics for bivariate IRD and their moments. Hirai and Akhter [7] discussed cubic coefficients estimates and asymptotic properties of the estimates from the two parameters Rayleigh distribution. Shawky and Bakoban [8] considered order statistics from an exponentiated Gamma distribution. Estimates of the scale parameter of exponentiated Gamma distribution under Type II censoring were obtained. Akhter and Hirai [9] estimates the scale parameter based on order statistics from Rayleigh distribution using type II singly and doubly censored samples by BLUE and Bloom's method for sample size up to 8. Azaz and Akhter [10] discussed the estimation of location and scale parameters using upper record values of Rayleigh distribution by BLUE and alternative best linear unbiased estimates (ABBLUE) from Type II singly and doubly censored samples. Tables of coefficients for BLUE and ABLUE of location and scale parameters were presented for various choices of censoring for $n \leq 8$. Feroze and Aslam [11] discussed the Bayesian analysis of IRD under singly and doubly Type II censored data. The Bayes estimators and corresponding risks have been derived under the assumption of non-informative priors and using symmetric and asymmetric loss functions. Jabeen et al. [12] discussed the estimation of location and scale parameters of Weibull distribution using concept of generalized order statistics by the BLUE and ABLUE from Type II singly and doubly censored samples.

This paper is designed for estimation of parameters of IRD for Type II singly and doubly censored data based on lower

* Corresponding author:

ssummaira@gmail.com (Summaira Manzoor)

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record values for a sample size up to 15. Moments of Record values are discussed in section 2. The estimation of location and scale parameters of IRD using Type II singly and doubly censored data under lower record values is described in section 3. Estimation of location and scale parameters of IRD under censoring with 10 missing observations are presented in section 4. Finally, conclusions are reported in Section 5.

2. Moments of Record Values Relating IRD

The i^{th} moment of $X_{L(r)}$ is given by

$$E(X_{L(r)}^i) = \frac{\lambda^i}{\Gamma(r)} \left[r - \frac{i}{2} \right], \quad (2)$$

For $i=1$ and 2 .

$$E(X_{L(r)}) = \frac{\lambda}{\Gamma(r)} \left[r - \frac{1}{2} \right] \text{ and } E(X_{L(r)}^2) = \frac{\lambda^2}{r-1}$$

The product moment of $X_{L(r)}$ and $X_{L(s)}$ is ($r < s$)

$$E(X_{L(r)}, X_{L(s)}) = \frac{\sqrt{r-1/2} \sqrt{s-1}}{\Gamma(r) \Gamma(s-1/2)}, \quad (3)$$

Variance of the r^{th} lower record value is given as:

$$\text{var}(X_{L(r)}) = \lambda^2 \left[\frac{1}{(r-1)} - \left(\frac{\sqrt{r-1/2}}{\Gamma(r)} \right)^2 \right], \quad (4)$$

and covariance is:

$$\text{cov}(X_{L(r)}, X_{L(s)}) = \lambda^2 \frac{\sqrt{r-1/2}}{\Gamma(r)} \left[\frac{\sqrt{s-1}}{\Gamma(s-1/2)} - \frac{\sqrt{s-1/2}}{\Gamma(s)} \right]. \quad (5)$$

Tables 1 and 2 provides the means and the variance-covariance of the r^{th} lower record values for the IRD with $\lambda=1$ for r up to 15.

Table 1. Means of lower record values for the Inverse Rayleigh distribution for $2 \leq r \leq 15$

r	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Means	0.8862	0.6647	0.5539	0.4847	0.4362	0.3998	0.3713	0.3481	0.3287	0.3123	0.2981	0.2857	0.2747	0.2649

Table 2. Variance-Covariance of Lower Record values from the Inverse Rayleigh distribution $2 \leq r \leq s \leq 15$

s/r	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	0.2146													
3	0.0776	0.0582												
4	0.0425	0.0318	0.0265											
5	0.0276	0.0207	0.0173	0.0151										
6	0.0198	0.01483	0.0124	0.0108	0.0097									
7	0.0151	0.0111	0.0094	0.0082	0.0074	0.0068								
8	0.0120	0.0090	0.0075	0.0065	0.0059	0.0054	0.0050							
9	0.0098	0.0073	0.0061	0.0054	0.0048	0.0044	0.0041	0.0038						
10	0.0082	0.0062	0.0051	0.0045	0.0040	0.0037	0.0034	0.0032	0.0030					
11	0.0070	0.0053	0.0044	0.0038	0.0034	0.0032	0.0029	0.0028	0.0026	0.0025				
12	0.0061	0.0046	0.0040	0.0033	0.0030	0.0027	0.0025	0.0024	0.0023	0.0021	0.0020			
13	0.0053	0.0040	0.0033	0.0029	0.0026	0.0024	0.0022	0.0021	0.0020	0.0019	0.0018	0.0017		
14	0.0047	0.0035	0.0030	0.0026	0.0023	0.0021	0.0020	0.0019	0.0018	0.0017	0.0016	0.0015	0.0015	
15	0.0042	0.0032	0.0026	0.0023	0.0021	0.0019	0.0018	0.0017	0.0016	0.0015	0.0014	0.0014	0.0013	0.0013

3. Type II Singly and Doubly Censored Data under Lower Record Values

Lloyd's method using lower record values is used to the censored samples. The BLUE's of θ and λ are given by

$$\hat{\theta} = \sum_{i=m_1+1}^{r-m_1-m_2} a_i X_{L(i)} \quad (6)$$

$$\hat{\lambda} = \sum_{i=m_1+1}^{r-m_1-m_2} b_i X_{L(i)} \quad (7)$$

where

$$a = \frac{\alpha' (V_L^{(n)})^{-1} \alpha 1' (V_L^{(n)})^{-1} - \alpha' (V_L^{(n)})^{-1} 1 \alpha' (V_L^{(n)})^{-1}}{(\alpha' (V_L^{(n)})^{-1} \alpha) (1' (V_L^{(n)})^{-1} 1) - (\alpha' (V_L^{(n)})^{-1} 1)^2}$$

$$b = \frac{1' (V_L^{(n)})^{-1} 1 \alpha' (V_L^{(n)})^{-1} - 1' (V_L^{(n)})^{-1} \alpha 1' (V_L^{(n)})^{-1}}{(\alpha' (V_L^{(n)})^{-1} \alpha) (1' (V_L^{(n)})^{-1} 1) - (\alpha' (V_L^{(n)})^{-1} 1)^2}$$

$$a = (a_{m_1+1}, a_{m_1+2}, a_{m_1+3}, \dots, a_{r-m_1-m_2}),$$

$$b = (b_{m_1+1}, b_{m_1+2}, b_{m_1+3}, \dots, b_{r-m_1-m_2})$$

$$\alpha = (\alpha_{m_1+1}, \alpha_{m_1+2}, \alpha_{m_1+3}, \dots, \alpha_{r-m_1-m_2})'$$

$$1 = (1, 1, 1, \dots, 1)'$$

and $V_L^{(n)}$ is the variance covariance matrix of $r-m_1-m_2$ record values. Where m_1 and m_2 indicate the number of missing observations from left and right side respectively. Furthermore, the variance and covariance of these BLUE's are given by

$$Var(\hat{\theta}) = \lambda^2 \left[\frac{\alpha' (V_L^{(n)})^{-1} \alpha}{(\alpha' (V_L^{(n)})^{-1} \alpha) (1' (V_L^{(n)})^{-1} 1) - (\alpha' (V_L^{(n)})^{-1} 1)^2} \right], \quad (8)$$

$$Var(\hat{\lambda}) = \lambda^2 \left[\frac{1' (V_L^{(n)})^{-1} 1}{(\alpha' (V_L^{(n)})^{-1} \alpha) (1' (V_L^{(n)})^{-1} 1) - (\alpha' (V_L^{(n)})^{-1} 1)^2} \right], \quad (9)$$

$$Cov(\hat{\theta}, \hat{\lambda}) = \lambda^2 \left[\frac{-1' (V_L^{(n)})^{-1} \alpha}{(\alpha' (V_L^{(n)})^{-1} \alpha) (1' (V_L^{(n)})^{-1} 1) - (\alpha' (V_L^{(n)})^{-1} 1)^2} \right]. \quad (10)$$

The values of coefficients a_i and b_i of Lloyd's estimators of θ and λ are obtained when record numbers are 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15 for all possible choice of

censoring. Variances and covariance's of the these estimators are provided in Appendix-I.

4. Estimation of Location and Scale Parameters of IRD under Censoring with 10 Missing Observations

We consider the estimation of scale and location parameters of IRD when a total of 10 observations are missing on left or on right side for the case $r=15$. The purpose is to investigate the impact of different combinations of m_1 and m_2 on variances of $\hat{\theta}$ and $\hat{\lambda}$ such that $m_1+m_2=10$.

Table 3. Variances of scale and location parameters for censored data with 10 missing observations

r	m_1	m_2	$Var(\hat{\theta})$	$Var(\hat{\lambda})$
15	1	9	0.099990	0.576777
	2	8	0.0714291	0.489284
	3	7	0.0555570	0.439353
	4	6	0.0454548	0.406896
	5	5	0.0384481	0.383966
	6	4	0.0333371	0.367103
	7	3	0.0294159	0.353983
	8	2	0.0263172	0.343491
	9	1	0.0238006	0.334828
	10	0	0.0217351	0.327795

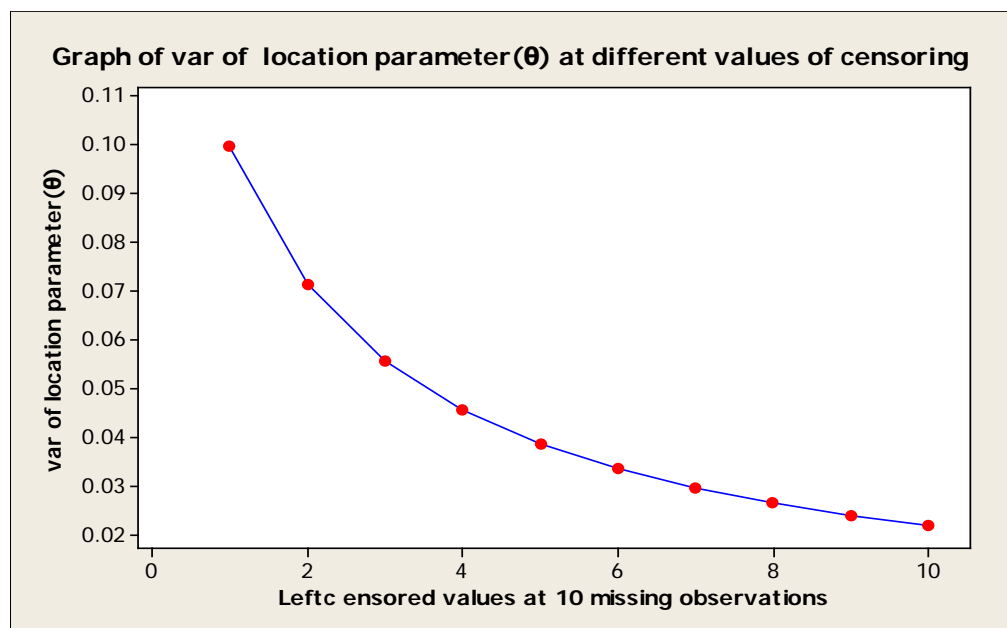
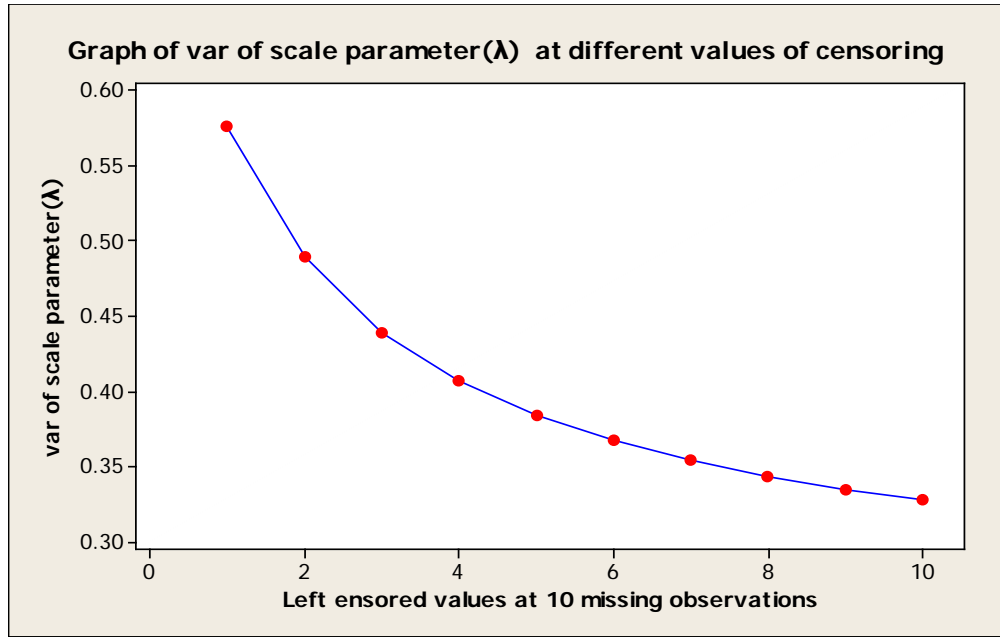


Figure 1. Variance of θ (10 missing observations)

Figure 2. Variance of λ (10 missing observations)

From figures 1 and 2 when Type II singly and doubly censoring done by Lloyd's method we may conclude:

- Left censoring provides much more precise estimators of λ and θ than the right censoring when the same number of observations is not available in the sample.
- For the same sample size r when m_1 increases the estimator of λ and θ gradually loses their efficiency.
- Also, for the same r when m_2 increases the estimator of λ and θ again loses their efficiency.
- When m_1 and m_2 both increase within the same sample size r efficiency of these estimators decreases.

So the variance of the estimators of the scale and location parameters under censoring depends on whether the censoring is left, or right, or double censoring. Also, it depends on the number of observations censored. For example, if only 5 observations are to be included in a sample of size 7 we can have two ways of doing it when $m_1=1$, $m_2=1$ and $m_1=2$, $m_2=0$. The corresponding censored estimators have the variances 0.576777 and 0.489284 respectively.

For different combinations of m_1 and m_2 with m_1+m_2 remaining fixed within sample size r the variances of the estimators change e.g. with $m_1=1$, $m_2=1$ and $m_1=2$, $m_2=0$ the range of variances of scale parameter is 0.087493. We have developed below a table that provides such information based on different sample sizes.

Thus a censored sample including the same number of observations may yield a large range. The range decreases when the number of available observations increases for the same sample size. But it increases when the sample size increases including the same number of available observations. The above information is useful for planning a sampling scheme for the estimation of location and scale parameters.

Table 4. Variance range for censored estimators of the scale parameter

R	(5 observations included)	(7 observations included)	(9 observations included)
7	0.087493		
8	0.137424		
9	0.169881	0.044357	
10	0.192811	0.073261	
11	0.209674	0.093558	0.026818
12	0.222794	0.108721	0.045768
13	0.233286	0.120476	0.059891
14	0.241949	0.129866	0.070863
15	0.248982	0.137558	0.079630

5. Conclusions

It is interesting to discover that left censoring provides much more precise estimators of λ and θ than the right censoring when the same number of observations is missing in the sample. Furthermore, as the sample size increases but the number of censored observations remains constant, the precision of these estimates increases exponentially. Covariance's of the above estimators also display a noteworthy behaviour. Firstly, these covariance's are negative. Secondly, right censoring provides larger covariance's of Lloyd's estimators of λ and θ than the left censoring. Also as the sample size increases but the number of censored observations remains constant, these covariance's approach zero rapidly. A censored sample including the same number of observations may yield a large range. The range decreases when the number of available observations increases for the same sample size. But it increases when the sample size increases including

the same number of available observations. This aspect therefore must not be ignore when planning a censored sampling scheme for the estimation of location and scale parameters.

APPENDIX-I

Appendix-I. Variances and covariance's of BLUEs for location and scale based on singly and doubly censored samples

r	m_1	m_2	$\text{Var}(\hat{\theta})$	$\text{Var}(\hat{\lambda})$	$\text{Cov}(\hat{\theta}, \hat{\lambda})$
4	1	1	0.999998	2.39530	-1.504503
	2	0	0.500004	1.71626	-0.90271
5	1	1	0.33333	1.173000	-0.601805
	1	2	0.999998	2.39530	-1.504503
	2	0	0.199999	0.91578	-0.412663
	2	1	0.500004	1.71626	-0.90271
6	3	0	0.333335	1.48343	-0.687777
	1	1	0.166667	0.773872	-0.343886
	1	2	0.33333	1.173000	-0.601805
	1	3	0.999998	2.39530	-1.504503
	2	0	0.111111	0.635176	-0.254731
	2	1	0.199999	0.91578	-0.412663
	2	2	0.500004	1.71626	-0.90271
	3	0	0.142855	0.802024	-0.327508
	3	1	0.333335	1.48343	-0.687777
	4	0	0.250001	1.36518	-0.573148
7	1	1	0.09999	0.576777	-0.229258
	1	2	0.166667	0.773872	-0.343886
	1	3	0.33333	1.173000	-0.601805
	1	4	0.999998	2.39530	-1.504503
	2	0	0.0714281	0.489284	-0.178642
	2	1	0.111111	0.635176	-0.254731
	2	2	0.199999	0.91578	-0.412663
	2	3	0.500004	1.71626	-0.90271
	3	0	0.0833322	0.56374	-0.208414
	3	1	0.142855	0.802024	-0.327508
	3	2	0.333335	1.48343	-0.687777
	4	0	0.111111	0.737495	-0.277888
	4	1	0.250001	1.36518	-0.573148
	5	0	0.199984	1.29351	-0.500159
	1	1	0.0666662	0.459494	-0.166732
	1	2	0.09999	0.576777	-0.229258
8	1	3	0.166667	0.773872	-0.343886
	1	4	0.33333	1.173000	-0.601805
	1	5	0.999998	2.39530	-1.504503
	2	0	0.05000012	0.399049	-0.134672
	2	1	0.0714281	0.489284	-0.178642
	2	2	0.111111	0.635176	-0.254731
	2	3	0.199999	0.91578	-0.412663
	2	4	0.500004	1.71626	-0.90271
	3	0	0.0555571	0.439353	-0.149636
	3	1	0.0833322	0.56374	-0.208414
	3	2	0.142855	0.802024	-0.327508
	3	3	0.333335	1.48343	-0.687777
	4	0	0.0666681	0.519956	-0.179562

	4	1	0.111111	0.737495	- 0.277888
	4	2	0.250001	1.36518	- 0.573148
	5	0	0.090915	0.69584	-0.244867
	5	1	0.199984	1.29351	- 0.500159
	6	0	0.166664	1.24535	-0.448888
9	1	1	0.0476201	0.381775	-0.128258
	1	2	0.0666662	0.459494	-0.166732
	1	3	0.09999	0.576777	-0.229258
	1	4	0.166667	0.773872	-0.343886
	1	5	0.33333	1.173000	-0.601805
	1	6	0.999998	2.39530	-1.504503
	2	0	0.037037	0.337418	-00.106405
	2	1	0.05000012	0.399049	-0.134672
	2	2	0.0714281	0.489284	- 0.178642
	2	3	0.111111	0.635176	- 0.254731
	2	4	0.199999	0.91578	-0.412663
	2	5	0.500004	1.71626	-0.90271
	3	0	0.0399999	0.361874	-0.114918
	3	1	0.0555571	0.439353	-0.149636
	3	2	0.0833322	0.56374	-0.208414
	3	3	0.142855	0.802024	-0.327508
	3	4	0.333335	1.48343	-0.687777
	4	0	0.0454548	0.406896	-0.130589
	4	1	0.0666681	0.519956	- 0.179562
	4	2	0.111111	0.737495	- 0.277888
	4	3	0.250001	1.36518	- 0.573148
	5	0	0.055534	0.49025	-0.159602
	5	1	0.090915	0.69584	-0.244867
	5	2	0.199984	1.29351	- 0.500159
	6	0	0.0769223	0.666622	-0.220993
	6	1	0.166664	1.24535	-0.448888
	7	0	0.142881	1.21102	-0.410486
10	1	1	0.0357143	0.326500	-0.102605
	1	2	0.0476201	0.381775	-0.128258
	1	3	0.0666662	0.459494	-0.166732
	1	4	0.09999	0.576777	-0.229258
	1	5	0.166667	0.773872	-0.343886
	1	6	0.333333	1.173000	-0.601805
	1	7	0.999998	2.39530	-1.504503
	2	0	0.0285644	0.292497	-0.0868963
	2	1	0.037037	0.337418	-00.106405
	2	2	0.05000012	0.399049	-0.134672
	2	3	0.0714281	0.489284	- 0.178642
	2	4	0.111111	0.635176	- 0.254731
	2	5	0.199999	0.91578	-0.412663
	2	6	0.500004	1.71626	-0.90271
	3	0	0.0302951	0.308514	-0.0921611
	3	1	0.0399999	0.361874	-0.114918
	3	2	0.0555571	0.439353	-0.149636
	3	3	0.0833322	0.56374	-0.208414
	3	4	0.142855	0.802024	-0.327508
	3	5	0.333335	1.48343	-0.687777
	4	0	0.033324	0.336545	-0.101376
	4	1	0.0454548	0.406896	-0.130589

	4	2	0.0666681	0.519956	- 0.179562
	4	3	0.111111	0.737495	- 0.277888
	4	4	0.250001	1.36518	- 0.573148
	5	0	0.0384481	0.383966	-0.116964
	5	1	0.055534	0.49025	-0.159602
	5	2	0.090915	0.69584	-0.244867
	5	3	0.199984	1.29351	- 0.500159
	6	0	0.047602	0.468676	-0.14481
	6	1	0.0769223	0.666622	-0.220993
	6	2	0.166664	1.24535	-0.448888
	7	0	0.0666331	0.644799	-0.202705
	7	1	0.142881	1.21102	-0.410486
	8	0	0.124848	1.18355	-0.379802
11	1	1	0.027771	0.285156	-0.084483
	1	2	0.0357143	0.326500	-0.102605
	1	3	0.0476201	0.381775	-0.128258
	1	4	0.0666662	0.459494	-0.166732
	1	5	0.099999	0.576777	-0.229258
	1	6	0.166667	0.773872	-0.343886
	1	7	0.333333	1.173000	-0.601805
	1	8	0.999998	2.39530	-1.504503
	2	0	0.0227287	0.258338	-0.0727774
	2	1	0.0285644	0.292497	-0.0868963
	2	2	0.037037	0.337418	-0.106405
	2	3	0.05000012	0.399049	-0.134672
	2	4	0.0714281	0.489284	- 0.178642
	2	5	0.111111	0.635176	- 0.254731
	2	6	0.199999	0.91578	-0.412663
	2	7	0.500004	1.71626	-0.90271
	3	0	0.0238112	0.269437	-0.0762437
	3	1	0.0302951	0.308514	-0.0921611
	3	2	0.0399999	0.361874	-0.114918
	3	3	0.0555571	0.439353	-0.149636
	3	4	0.0833322	0.56374	-0.208414
	3	5	0.142855	0.802024	-0.327508
	3	6	0.333335	1.48343	-0.687777
	4	0	0.0256429	0.288217	-0.0821087
	4	1	0.033324	0.336545	-0.101376
	4	2	0.0454548	0.406896	-0.130589
	4	3	0.0666681	0.519956	- 0.179562
	4	4	0.111111	0.737495	- 0.277888
	4	5	0.250001	1.36518	- 0.573148
	5	0	0.0285735	0.318265	-0.0914927
	5	1	0.0384481	0.383966	-0.116964
	5	2	0.055534	0.49025	-0.159602
	5	3	0.090915	0.69584	-0.244867
	5	4	0.199984	1.29351	- 0.500159
	6	0	0.0333371	0.367103	-0.106745
	6	1	0.047602	0.468676	-0.14481
	6	2	0.0769223	0.666622	-0.220993
	6	3	0.166664	1.24535	-0.448888
	7	0	0.0416736	0.452575	-0.133439
	7	1	0.0666331	0.644799	-0.202705
	7	2	0.142881	1.21102	-0.410486

	8	0	0.0588266	0.628451	-0.188364
	8	1	0.124848	1.18355	-0.379802
	9	0	0.111033	1.16369	-0.355525
12	1	1	0.0222235	0.253159	-0.07116
	1	2	0.027771	0.285156	-0.084483
	1	3	0.0357143	0.326500	-0.102605
	1	4	0.0476201	0.381775	-0.128258
	1	5	0.0666662	0.459494	-0.166732
	1	6	0.09999	0.576777	-0.229258
	1	7	0.166667	0.773872	-0.343886
	1	8	0.33333	1.173000	-0.601805
	1	9	0.999998	2.39530	-1.504503
	2	0	0.0187904	0.234421	-0.0630323
	2	1	0.0227287	0.258338	-0.0727774
	2	2	0.0285644	0.292497	-0.0868963
	2	3	0.037037	0.337418	-0.106405
	2	4	0.05000012	0.399049	-0.134672
	2	5	0.0714281	0.489284	-0.178642
	2	6	0.111111	0.635176	-0.254731
	2	7	0.199999	0.91578	-0.412663
	2	8	0.500004	1.71626	-0.90271
	3	0	0.0192318	0.239388	-0.0645129
	3	1	0.0238112	0.269437	-0.0762437
	3	2	0.0302951	0.308514	-0.0921611
	3	3	0.0399999	0.361874	-0.114918
	3	4	0.0555571	0.439353	-0.149636
	3	5	0.0833322	0.56374	-0.208414
	3	6	0.142855	0.802024	-0.327508
	3	7	0.333335	1.48343	-0.687777
	4	0	0.0204094	0.252639	-0.0684633
	4	1	0.0256429	0.288217	-0.0821087
	4	2	0.033324	0.336545	-0.101376
	4	3	0.0454548	0.406896	-0.130589
	4	4	0.0666681	0.519956	-0.179562
	4	5	0.111111	0.737495	-0.277888
	4	6	0.250001	1.36518	-0.573148
	5	0	0.0222236	0.273054	-0.074549
	5	1	0.0285735	0.318265	-0.0914927
	5	2	0.0384481	0.383966	-0.116964
	5	3	0.055534	0.49025	-0.159602
	5	4	0.090915	0.69584	-0.244867
	5	5	0.199984	1.29351	-0.500159
	6	0	0.0250015	0.304312	-0.0838673
	6	1	0.0333371	0.367103	-0.106745
	6	2	0.047602	0.468676	-0.14481
	6	3	0.0769223	0.666622	-0.220993
	6	4	0.166664	1.24535	-0.448888
	7	0	0.0294159	0.353983	-0.0986751
	7	1	0.0416736	0.452575	-0.133439
	7	2	0.0666331	0.644799	-0.202705
	7	3	0.142881	1.21102	-0.410486
	8	0	0.0370404	0.439783	-0.124252
	8	1	0.0588266	0.628451	-0.188364
	8	2	0.124848	1.18355	-0.379802

	9	0	0.0526102	0.61498	-0.17648
	9	1	0.111033	1.16369	-0.355525
	10	0	0.100059	1.14889	-0.335645
13	1	1	0.0181827	0.227583	-0.0609938
	1	2	0.0222235	0.253159	-0.07116
	1	3	0.027771	0.285156	-0.084483
	1	4	0.0357143	0.326500	-0.102605
	1	5	0.0476201	0.381775	-0.128258
	1	6	0.0666662	0.459494	-0.166732
	1	7	0.099999	0.576777	-0.229258
	1	8	0.166667	0.773872	-0.343886
	1	9	0.333333	1.173000	-0.601805
	1	10	0.999998	2.39530	-1.504503
	2	0	0.0153857	0.209554	-0.0538546
	2	1	0.0187904	0.234421	-0.0630323
	2	2	0.0227287	0.258338	-0.0727774
	2	3	0.0285644	0.292497	-0.0868963
	2	4	0.037037	0.337418	-0.106405
	2	5	0.05000012	0.399049	-0.134672
	2	6	0.0714281	0.489284	-0.178642
	2	7	0.1111111	0.635176	-0.254731
	2	8	0.1999999	0.91578	-0.412663
	2	9	0.500004	1.71626	-0.90271
	3	0	0.0158741	0.215538	-0.0555642
	3	1	0.0192318	0.239388	-0.0645129
	3	2	0.0238112	0.269437	-0.0762437
	3	3	0.0302951	0.308514	-0.0921611
	3	4	0.0399999	0.361874	-0.114918
	3	5	0.0555571	0.439353	-0.149636
	3	6	0.0833322	0.56374	-0.208414
	3	7	0.142855	0.802024	-0.327508
	3	8	0.333335	1.48343	-0.687777
	4	0	0.016668	0.225265	-0.058343
	4	1	0.0204094	0.252639	-0.0684633
	4	2	0.0256429	0.288217	-0.0821087
	4	3	0.033324	0.336545	-0.101376
	4	4	0.0454548	0.406896	-0.130589
	4	5	0.0666681	0.519956	-0.179562
	4	6	0.1111111	0.737495	-0.277888
	4	7	0.250001	1.36518	-0.573148
	5	0	0.0178586	0.239853	-0.0625106
	5	1	0.0222236	0.273054	-0.074549
	5	2	0.0285735	0.318265	-0.0914927
	5	3	0.0384481	0.383966	-0.116964
	5	4	0.055534	0.49025	-0.159602
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	6	0	0.0196089	0.261299	-0.0686375
	6	1	0.0250015	0.304312	-0.0838673
	6	2	0.0333371	0.367103	-0.106745
	6	3	0.047602	0.468676	-0.14481
	6	4	0.0769223	0.666622	-0.220993
	6	5	0.166664	1.24535	-0.448888
	7	0	0.0222233	0.293331	-0.0777887

	7	1	0.0294159	0.353983	-0.0986751
	7	2	0.0416736	0.452575	-0.133439
	7	3	0.0666331	0.644799	-0.202705
	7	4	0.142881	1.21102	-0.410486
	8	0	0.0263172	0.343491	-0.0921186
	8	1	0.0370404	0.439783	-0.124252
	8	2	0.0588266	0.628451	-0.188364
	8	3	0.124848	1.18355	-0.379802
	9	0	0.0333244	0.429342	-0.116646
	9	1	0.0526102	0.61498	-0.17648
	9	2	0.111033	1.16369	-0.355525
	10	0	0.0476251	0.604563	-0.166703
	10	1	0.100059	1.14889	-0.335645
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	1	2	0.0181827	0.227583	-0.0609938
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	1	4	0.027771	0.285156	-0.084483
	1	5	0.0357143	0.326500	-0.102605
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	1	7	0.0666662	0.459494	-0.166732
	1	8	0.099999	0.576777	-0.229258
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	1	11	0.999998	2.39530	-1.504503
	2	0	0.0129868	0.191518	-0.047277
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	4	4	0.033324	0.336545	-0.101376
	4	5	0.0454548	0.406896	-0.130589
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	5	0	0.0147054	0.214293	-0.0535332
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	4	2	0.016668	0.225265	-0.058343
	4	3	0.0204094	0.252639	-0.0684633
	4	4	0.0256429	0.288217	-0.0821087
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	5	5	0.0384481	0.383966	-0.116964
	5	6	0.055534	0.49025	-0.159602
	5	7	0.090915	0.69584	-0.244867
	5	8	0.199984	1.29351	-0.500159
	6	0	0.0131565	0.205526	-0.0496689
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	6	4	0.0333371	0.367103	-0.106745
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	6	6	0.0769223	0.666622	-0.220993
	6	7	0.166664	1.24535	-0.448888
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	8	1	0.0199995	0.284452	-0.0728057

	8	2	0.0263172	0.343491	-0.0921186
	8	3	0.0370404	0.439783	-0.124252
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	9	0	0.0181761	0.277067	-0.068619
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	9	3	0.0526102	0.61498	-0.17648
	9	4	0.111033	1.16369	-0.355525
	10	0	0.0217351	0.327795	-0.0820556
	10	1	0.0303061	0.421035	-0.110325
	10	2	0.0476251	0.604563	-0.166703
	10	3	0.100059	1.14889	-0.335645
	11	0	0.0277773	0.413899	-0.104865
	11	1	0.0434818	0.595639	-0.158289
	11	2	0.0909682	1.13559	-0.318415
	12	0	0.0399881	0.587952	-0.150966
	12	1	0.0833455	1.12394	-0.303409
	13	0	0.076952	1.11474	-0.290509

(Each value is to be multiplied by λ^2)

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