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The Discrete Poisson-Akash Distribution

Rama Shanker

Department of Statistics, Eritrea Institute of Technology, Asmara, Eritrea

Abstract A Poisson-Akash distribution has been obtained by compounding Poisson distribution with Akash distribution introduced by Shanker (2015). A general expression for the r th factorial moment has been derived and hence the first four moments about origin and the moments about mean has been obtained. The expressions for its coefficient of variation, skewness and kurtosis have been obtained. Its statistical properties including generating function, increasing hazard rate and unimodality and over-dispersion have been discussed. The maximum likelihood estimation and the method of moments for estimating its parameter have been discussed. The goodness of fit of the proposed distribution using maximum likelihood estimation has been given for some count data-sets and the fit is compared with that obtained by other distributions.

Keywords Akash distribution, Compounding, Moments, Skewness, Kurtosis, Estimation of parameter, Statistical properties, Goodness of fit

1. Introduction

A lifetime distribution named, "Akash distribution" having probability density function

$$f\left(x,\theta\right)=\frac{\theta^3}{\theta^2+2}\left(1+x^2\right)e^{-\theta x}\ ;x>0,\theta>0 \qquad (1.1)$$

has been proposed by Shanker (2015) for modeling lifetime data from biomedical science and engineering and shown that the proposed distribution is a two-component mixture of exponential distribution having scale parameter θ and a gamma distribution having shape parameter 3 and scale parameter θ with their mixing proportions $\frac{\theta^2}{\theta^2+2}$ and $\frac{2}{\theta^2+2}$ respectively. Various mathematical and statistical properties of Akash distribution including its shape, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic ordering, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure and stress-strength reliability have been discussed by Shanker (2015). It has been shown by Shanker (2015) that Akash distribution provides much closer fit than Lindley and exponential distributions for modeling lifetime data from medical science and engineering. Further, Shanker *et al* (2016) have done a detailed comparative study on Akash, Lindley and exponential distributions for modeling different types of lifetime data from engineering and medical science and concluded that Akash distribution has some advantage over Lindley and exponential distributions, Lindley distribution has some advantage over Akash and exponential distributions and exponential distribution has some advantage over Akash and Lindley distributions due to their over-dispersion, equi-dispersion, and under-dispersion for various values of their parameters. Recently, Shanker and Shukla (2016) has introduced a two-parameter weighted Akash distribution (WAD) andstudied its various mathematical and statistical properties, estimation of parameter and application for modeling lifetime data. Shanker (2016) has also proposed a two-parameter quasi Akash distribution and studied its statistical and mathematical properties, estimation of parameters using maximum likelihood estimation and method of moments along with its application for modeling lifetime data from engineering and biomedical sciences.

In the present paper, a Poisson mixture of Akash distribution introduced by Shanker (2015) named, "Poisson-Akash distribution (PAD) has been proposed. Its various mathematical and statistical properties including its shape, moments, coefficient of variation, skewness, and kurtosis have been discussed. The estimation of its parameter has been discussed using maximum likelihood estimation and method of moments. The goodness of fit of PAD along with Poisson distribution and Poisson-Lindley distribution (PLD), a Poisson mixture of Lindley (1958) distribution and introduced by Sankaran (1970), has been given with some count data-sets.

2. Poisson-Akash Distribution

Suppose the parameter λ of Poisson distribution follows Akash distribution (1.1). Then the Poisson mixture of Akash

* Corresponding author:
shankerrama2009@gmail.com (Rama Shanker)
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distribution (1.1) can be obtained as

$$P(X=x)=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\cdot\frac{\theta^3}{\theta^2+2}\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda \tag{2.1}$$

$$=\frac{\theta^3}{\left(\theta^2+2\right)x!}\int_0^\infty e^{-(\theta+1)\lambda}\left[\lambda^x+\lambda^{x+2}\right]d\lambda$$

$$=\frac{\theta^3}{\theta^2+2}\cdot\frac{x^2+3x+\left(\theta^2+2\theta+3\right)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0\,.\tag{2.2}$$

We name this distribution "Poisson-Akash distribution (PAD)". Shanker *et al* (2016) have detailed study on applications of PAD to model count data from different fields of knowledge.

It would be recalled that Sankaran (1970) obtained Poisson-Lindley distribution (PLD) having probability mass function (p.m.f)

$$P(X=x)=\frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0 \tag{2.3}$$

by compounding Poisson distribution with Lindley distribution, introduced by Lindley (1958) having probability density function (p.d.f)

$$f(x,\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{\theta x}\;;x>0,\theta>0 \tag{2.4}$$

Ghitany *et al* (2008) have discussed various interesting properties of Lindley distribution, estimation of parameter and application for modeling waiting time data from a bank. Shanker *et al* (2015) have detailed study on modeling of lifetime data using exponential and Lindley distributions. Further, Shanker *et al* (2016) have discussed the comparative applications of Akash, Lindley and exponential distributions for modeling lifetime data from biomedical sciences and engineering. Shanker and Hagos (2015) have detailed and critical study on applications of PLD to model count data from biological sciences.

The graphs of the pmf of Poisson-Akash distribution (PAD) and Poisson-Lindley distribution (PLD) for different values of their parameter are shown in the figure 1.

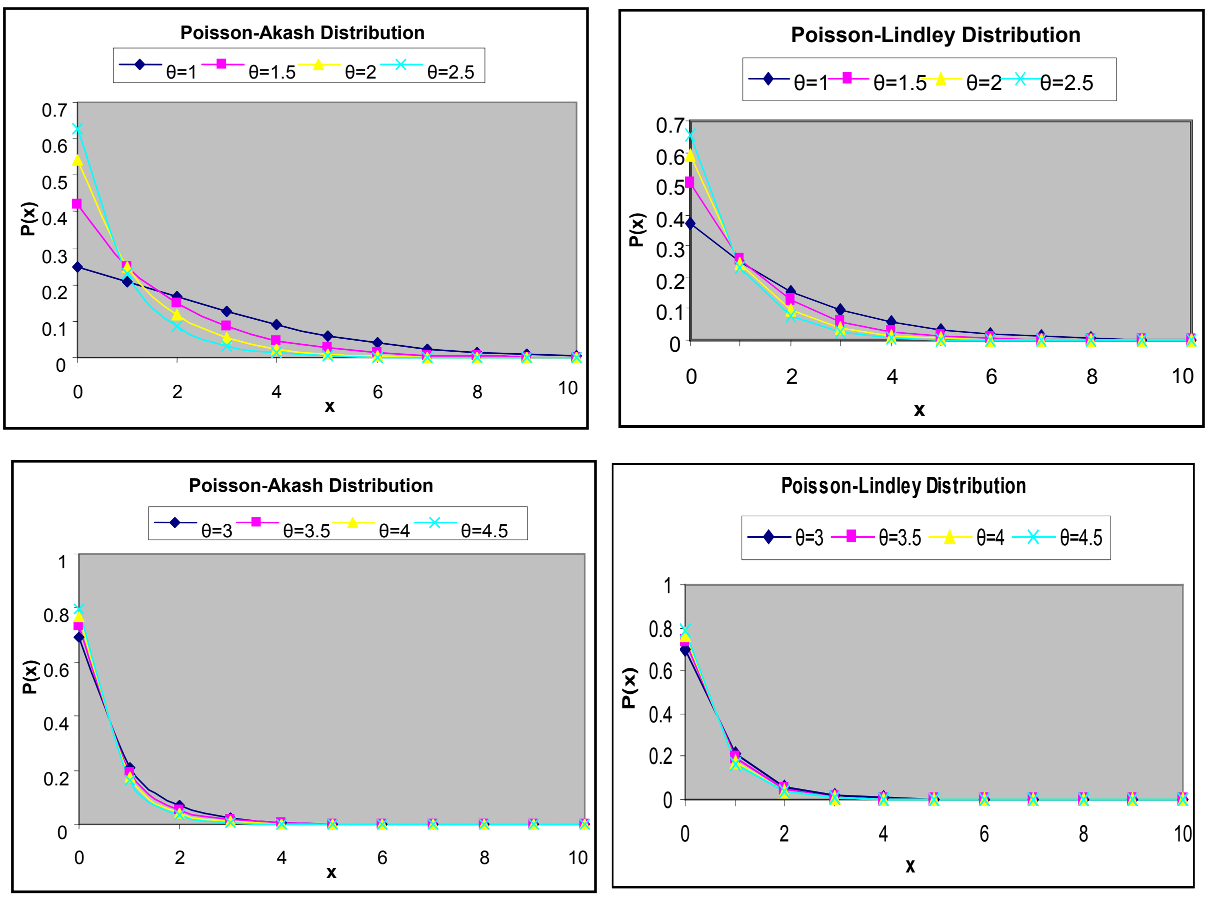


Figure 1. Graphs of probability mass function of PAD and PLD for different values of the parameter θ

3. Moments and Associated Measures

The r th factorial moment about origin of Poisson-Akash distribution (PAD) (2.2) can be obtained as

$$\mu_{(r)}' = E\left[E\left(X^{(r)} \mid \lambda\right)\right], \text{ where}$$

$$X^{(r)} = X\left(X-1\right)\left(X-2\right)...\left(X-r+1\right)$$

Using (2.1), the r th factorial moment about origin of PAD (2.2) can be obtained as

$$\mu_{(r)}' = E\left[E\left(X^{(r)} \mid \lambda\right)\right]$$

$$= \frac{\theta^3}{\theta^2+2}\int_0^\infty \left[\sum_{x=0}^{\infty} x^{(r)} \frac{e^{-\lambda}\lambda^x}{x!}\right]\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

$$= \frac{\theta^3}{\theta^2+2}\int_0^\infty \lambda^r \left[\sum_{x=r}^{\infty} \frac{e^{-\lambda}\lambda^{x-r}}{\left(x-r\right)!}\right]\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

Taking $x+r$ in place of x within the bracket, we get

$$\mu_{(r)}' = \frac{\theta^3}{\theta^2+2}\int_0^\infty \lambda^r \left[\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{x!}\right]\left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

The expression within the bracket is clearly unity and hence we have

$$\mu_{(r)}' = \frac{\theta^3}{\theta^2+2}\int_0^\infty \lambda^r \left(1+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

Using gamma integral and a little algebraic simplification, we get finally, a general expression for the r th factorial moment of PAD (2.2) as

$$\mu_{(r)}' = \frac{r!\left[\theta^2+\left(r+1\right)\left(r+2\right)\right]}{\theta^r\left(\theta^2+2\right)};r=1,2,3,.... \quad (3.1)$$

Substituting $r=1,2,3,\text{and } 4$ in (3.1), the first four factorial moments about origin can be obtained and using the relationship between factorial moments about origin and moments about origin, the first four moment about origin of the PAD (2.2) are obtained as

$$\mu_1' = \frac{\theta^2+6}{\theta\left(\theta^2+2\right)}$$

$$\mu_2' = \frac{\theta^3+2\theta^2+6\theta+24}{\theta^2\left(\theta^2+2\right)}$$

$$\mu_3' = \frac{\theta^4+6\theta^3+12\theta^2+72\theta+120}{\theta^3\left(\theta^2+2\right)}$$

$$\mu_4' = \frac{\theta^5+14\theta^4+42\theta^3+192\theta^2+720\theta+720}{\theta^4\left(\theta^2+2\right)}$$

Using the relationship between moments about mean and the moments about origin, the moments about mean of the PAD (2.2) are obtained as

$$\mu_2 = \sigma^2 = \frac{\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12}{\theta^2\left(\theta^2+2\right)^2}$$

$$\mu_3 = \frac{\theta^8+3\theta^7+12\theta^6+54\theta^5+88\theta^4+132\theta^3+96\theta^2+72\theta+48}{\theta^3\left(\theta^2+2\right)^3}$$

$$\mu_4 = \frac{\begin{pmatrix}\theta^{11}+10\theta^{10}+30\theta^9+197\theta^8+576\theta^7+1208\theta^6+2144\theta^5+2584\theta^4\\+2928\theta^3+2496\theta^2+1440\theta+720\end{pmatrix}}{\theta^4\left(\theta^2+2\right)^4}$$

The coefficient of variation $\left(C.V\right)$, coefficient of skewness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis $\left(\beta_2\right)$, and index of dispersion $\left(\gamma\right)$ of the PAD (2.2) are thus given by

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12}}{\theta^2+6}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\theta^8+3\theta^7+12\theta^6+54\theta^5+88\theta^4+132\theta^3+96\theta^2+72\theta+48}{\left(\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2} = \frac{\begin{pmatrix} \theta^{11} + 10\theta^{10} + 30\theta^9 + 197\theta^8 + 576\theta^7 + 1208\theta^6 + 2144\theta^5 \\ +2584\theta^4 + 2928\theta^3 + 2496\theta^2 + 1440\theta + 720 \end{pmatrix}}{\left(\theta^5 + \theta^4 + 8\theta^3 + 16\theta^2 + 12\theta + 12\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^5 + \theta^4 + 8\theta^3 + 16\theta^2 + 12\theta + 12}{\theta\left(\theta^2 + 2\right)\left(\theta^2 + 6\right)}$$

The expressions for μ_1', μ_2, C.V, $\sqrt{\beta_1}$, β_2 and γ of PLD (2.3) obtained by Sankaran (1970) and Ghitany and Al-Mutairi (2009) are given by

$$\mu_1' = \frac{\theta + 2}{\theta\left(\theta + 1\right)}, \qquad \mu_2 = \frac{\theta^3 + 4\theta^2 + 6\theta + 2}{\theta^2\left(\theta + 1\right)^2}$$

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^3 + 4\theta^2 + 6\theta + 2}}{\theta + 2}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{{\mu_2}^{3/2}} = \frac{\theta^5 + 7\theta^4 + 22\theta^3 + 32\theta^2 + 18\theta + 4}{\left(\theta^3 + 4\theta^2 + 6\theta + 2\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2} = \frac{\left(\theta^7 + 15\theta^6 + 87\theta^5 + 258\theta^4 + 406\theta^3 + 338\theta^2 + 144\theta + 24\right)}{\left(\theta^3 + 4\theta^2 + 6\theta + 2\right)^2}$$

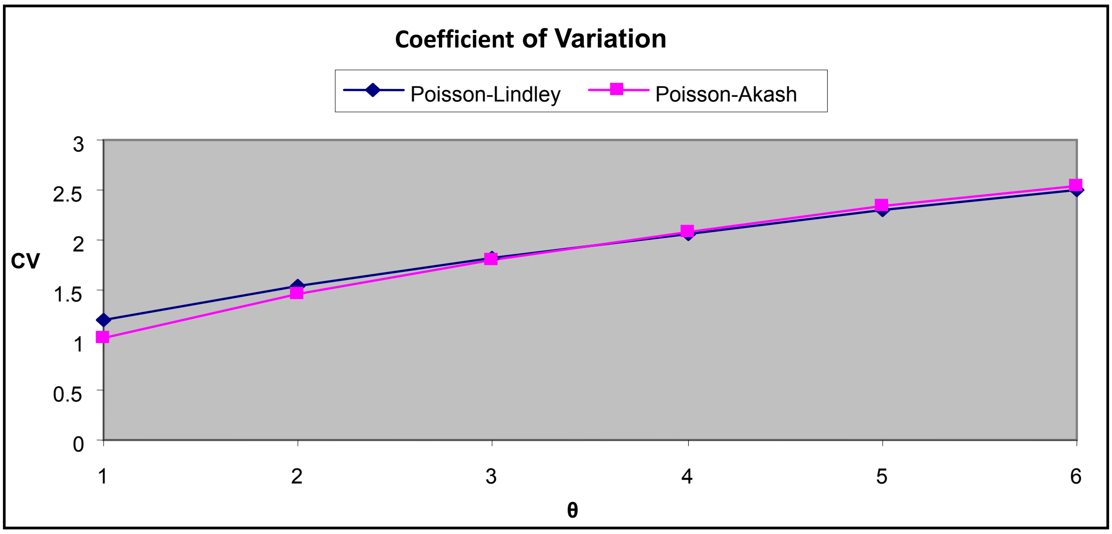
$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^3 + 4\theta^2 + 6\theta + 2}{\theta\left(\theta + 1\right)\left(\theta + 2\right)}$$

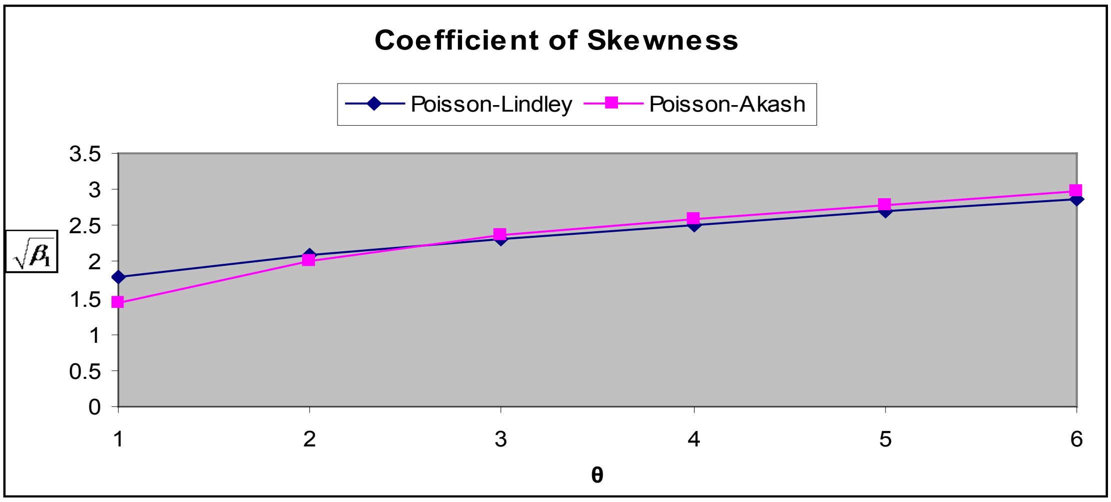
Table 1. Values of μ_1', μ_2, C.V, $\sqrt{\beta_1}$, β_2 and γ of PAD and PLD for different values of θ

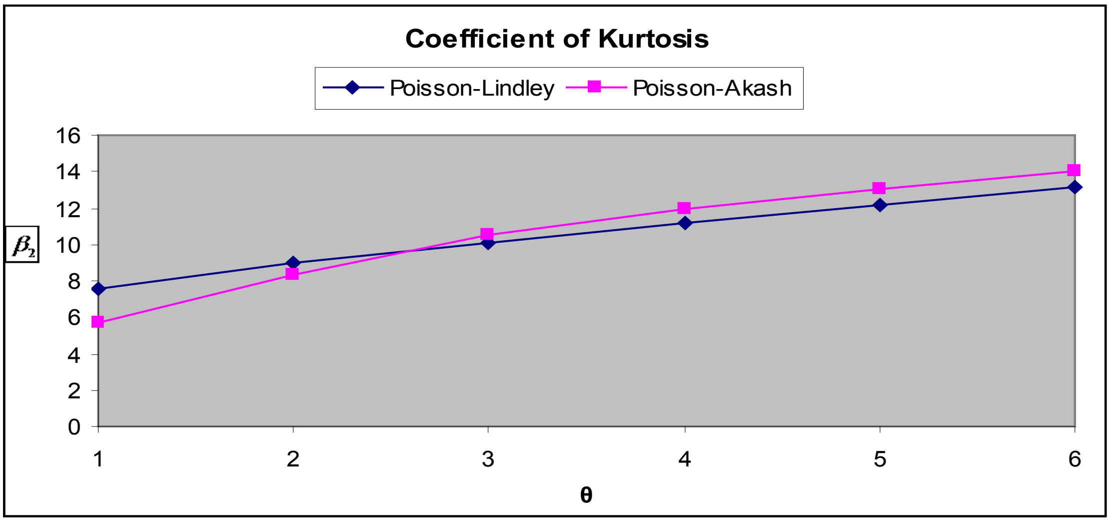
	Values of θ for Poisson-Akash Distribution					
	1	**2**	**3**	**4**	**5**	**6**
μ_1'	2.333333	0.833333	0.454545	0.305555	0.229629	0.184210
μ_2	5.555556	1.472222	0.672176	0.406636	0.286529	0.220452
CV	1.010152	1.456022	1.803699	2.086953	2.331078	2.548843
$\sqrt{\beta_1}$	1.431184	2.000799	2.363690	2.603557	2.797221	2.971690
β_2	5.7336	8.367746	10.503158	11.911543	13.005753	14.002950
γ	2.380952	1.766667	1.478788	1.330808	1.247789	1.196742
	Values of θ for Poisson-Lindley Distribution					
	1	**2**	**3**	**4**	**5**	**6**
μ_1'	1.5	0.666667	0.416667	0.3	0.233333	0.190476
μ_2	3.25	1.055556	0.576389	0.385	0.285556	0.225624
CV	1.20185	1.541104	1.822087	2.068279	2.290174	2.493742
$\sqrt{\beta_1}$	1.792108	2.083265	2.314307	2.517935	2.704839	2.87957
β_2	7.532544	8.941828	10.10611	11.17187	12.19654	13.203
γ	2.166667	1.583333	1.383333	1.283333	1.22381	1.184524

To study the behavior of μ_1', μ_2, C.V, $\sqrt{\beta_1}$, β_2 and γ of PAD and PLD, values of these characteristics for different values of parameter θ have been computed and presented in table 1.

The graphs of coefficient of variation, coefficient of skewness, coefficient of kurtosis and index of dispersion of PAD and PLD for different values of the parameter θ have been shown in the figure 2.







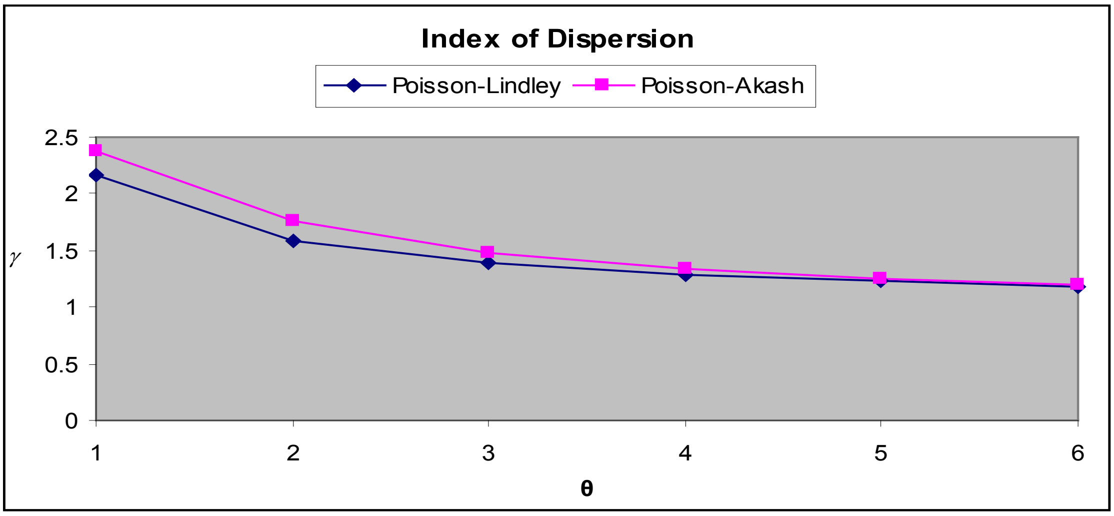


Figure 2. Graphs of coefficient of variation, coefficient of skewness, coefficient of kurtosis and Index dispersion of PAD and PLD for different values of the parameter θ

4. Mathematical and Statistical Properties

4.1. Increasing Hazard Rate and Unimodality

The PAD (2.2) has an increasing hazard rate (IHR) and unimodal. Since

$$\frac{P(x+1;\theta)}{P(x;\theta)}=\frac{1}{\theta+1}\left[1+\frac{2(x+2)}{x^2+3x+\left(\theta^2+2\theta+3\right)}\right]$$

is decreasing function in x, $P(x;\theta)$ is log-concave. Therefore, the PAD has an increasing hazard rate and unimodal. A detailed discussion about interrelationship between log-concavity, unimodality and increasing hazard rate (IHR) of discrete distributions can be seen in Grandell (1997).

4.2. Generating Functions

The probability generating function of PAD (2.2) can be obtained as

$$P_X(t)=\frac{\theta^3}{\left(\theta^2+2\right)(\theta+1)^3}\left[\sum_{x=0}^{\infty}x^2\left(\frac{t}{\theta+1}\right)^x+3\sum_{x=0}^{\infty}x\left(\frac{t}{\theta+1}\right)^x+\left(\theta^2+2\theta+3\right)\sum_{x=0}^{\infty}\left(\frac{t}{\theta+1}\right)^x\right]$$

$$=\frac{\theta^3}{\left(\theta^2+2\right)(\theta+1)^2}\left[\frac{2t(\theta+1)}{(\theta+1-t)^3}+\frac{4t}{(\theta+1-t)^2}+\frac{\left(\theta^2+2\theta+3\right)}{(\theta+1-t)}\right]$$

$$=\frac{\theta^3}{\left(\theta^2+2\right)(\theta+1)^2}\left[\frac{\theta^2+2\theta+3}{(\theta+1-t)}+\frac{2t\left\{3(\theta+1)-2t\right\}}{(\theta+1-t)^3}\right]$$

The moment generating function of the PAD (2.2) can thus be obtained as

$$M_X(t)=\frac{\theta^3}{\left(\theta^2+2\right)(\theta+1)^2}\left[\frac{\theta^2+2\theta+3}{\left(\theta+1-e^t\right)}+\frac{2e^t\left\{3(\theta+1)-2e^t\right\}}{\left(\theta+1-e^t\right)^3}\right]$$

4.3. Over-dispersion

The PAD (2.2) is always over-dispersed $\left(\sigma^2>\mu\right)$. We have

$$\sigma^2=\frac{\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12}{\theta^2\left(\theta^2+2\right)^2}$$
$$=\frac{\theta^2+6}{\theta\left(\theta^2+2\right)}\left[\frac{\theta^5+\theta^4+8\theta^3+16\theta^2+12\theta+12}{\theta\left(\theta^2+2\right)\left(\theta^2+6\right)}\right]$$
$$=\frac{\theta^2+6}{\theta\left(\theta^2+2\right)}\left[1+\frac{\theta^4+16\theta^2+12}{\theta\left(\theta^2+2\right)\left(\theta^2+6\right)}\right]$$
$$=\mu\left[1+\frac{\theta^4+16\theta^2+12}{\theta\left(\theta^2+2\right)\left(\theta^2+6\right)}\right]>\mu\cdot$$

This shows that PAD (2.2) is always over dispersed.

5. Estimation of the Parameter

5.1. Maximum Likelihood Estimate (MLE)

Let $\left(x_1,x_2,...,x_n\right)$ be a random sample of size n from the PAD (2.2) and let f_x be the observed frequency in the sample corresponding to $X=x\ (x=1,2,3,...,k)$ such that $\sum_{x=1}^{k} f_x=n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the PAD (2.2) is given by

$$L=\left(\frac{\theta^3}{\theta^2+2}\right)^n\frac{1}{\left(\theta+1\right)^{\sum_{x=1}^{k}(x+3)f_x}}\prod_{x=1}^{k}\left[x^2+3x+\left(\theta^2+2\theta+3\right)\right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L=n\log\left(\frac{\theta^3}{\theta^2+2}\right)-\sum_{x=1}^{k}\left(x+3\right)f_x\log\left(\theta+1\right)+\sum_{x=1}^{k}f_x\log\left[x^2+3x+\left(\theta^2+2\theta+3\right)\right]$$

The first derivative of the log likelihood function is given by

$$\frac{d\log L}{d\theta}=\frac{n\left(\theta^2+6\right)}{\theta\left(\theta^2+2\right)}-\frac{n\left(\bar{x}+3\right)}{\theta+1}+\sum_{x=1}^{k}\frac{2\left(\theta+1\right)f_x}{\left[x^2+3x+\left(\theta^2+2\theta+3\right)\right]}$$

where $\bar{x}$ is the sample mean.

The maximum likelihood estimate (MLE), $\hat{\theta}$ of θ of PAD (2.2) is the solution of the equation $\frac{d\log L}{d\theta}=0$ and is given by the solution of the following non-linear equation

$$\frac{n\left(\theta^2+6\right)}{\theta\left(\theta^2+2\right)}-\frac{n\left(\bar{x}+3\right)}{\theta+1}+\sum_{x=1}^{k}\frac{2\left(\theta+1\right)f_x}{\left[x^2+3x+\left(\theta^2+2\theta+3\right)\right]}=0$$

This non-linear equation can be solved by any numerical iteration methods such as Newton- Raphson, Bisection method, Regula –Falsi method etc

5.2. Method of Moment Estimate (MOME)

Let $\left(x_1,x_2,...,x_n\right)$ be a random sample of size n from the PAD (2.2). Equating the population mean to the corresponding sample mean, the MOME $\tilde{\theta}$ of θ of PAD (2.2) is the solution of the following cubic equation

$$\bar{x}\,\theta^3-\theta^2+2\bar{x}\,\theta-6=0$$

where $\bar{x}$ is the sample mean.

6. Goodness of Fit

The PAD has been fitted to a number of data - sets to test its goodness of fit with Poisson distribution (PD) and Poisson-Lindley distribution (PLD. The maximum likelihood estimate (MLE) has been used to fit the PAD. Five examples of observed count data-sets, for which the PAD, PD, and PLD has been fitted, are presented. The first data-set is due to Kemp and kemp (1965) regarding the distribution of mistakes in copying groups of random digits, the second data-set is due to Beall (1940) regarding the distribution of Pyrausta nublilalis, the third data-set is the number of accidents to 647 women working on high explosive shells in 5 weeks, available in Sankaran (1970), the fourth data-set is the distribution of red mites per leaf on apple leaves, available in Fisher et al (1943), and the fifth data-set is distribution of number of Chromatid aberrations, available in Loeschke and Kohler (1976) and Janardan and Schaeffer (1977).

Table 6.1. Distribution of mistakes in copying groups of random digits

No. of errors per group	Observed Frequency	Expected Frequency		
		PD	PLD	PAD
0	35	27.4	33.0	33.5
1	11	21.5	15.3	14.7
2	8	8.4	6.8	6.6
3	4	2.2	2.9	2.9
4	2	0.5	2.0	2.3
Total	60	60.0	60.0	60.0
ML estimate		$\hat{\theta}=0.7833$	$\hat{\theta}=1.7434$	$\hat{\theta}=2.077978$
χ^2		7.98	2.20	1.40
d.f.		1	1	2
p-value		0.0047	0.1380	0.4966

Table 6.2. Distribution of Pyrausta nublilalis

No. of insects	Observed Frequency	Expected Frequency		
		PD	PLD	PAD
0	33	26.4	31.5	32.0
1	12	19.8	14.2	13.6
2	6	7.4	6.1	5.9
3	3	1.8	2.5	2.6
4	1	0.3	1.0	1.1
5	1	0.3	0.7	0.8
Total	56	56.0	56.0	56.0
ML estimate		$\hat{\theta}=0.7500$	$\hat{\theta}=1.8081$	$\hat{\theta}=2.144578$
χ^2		4.87	0.53	0.24
d.f.		1	1	1
p-value		0.0273	0.4666	0.6242

Table 6.3. Accidents to 647 women working on high explosive shells

No. of accidents	Observed Frequency	Expected Frequency		
		PD	PLD	PAD
0	447	406	439.5	443.3
1	132	189	142.8	137.7
2	42	45	45.0	44.5
3	21	7	13.9	14.5
4	3	1	4.2	4.7
≥ 5	2	0.1	1.3	2.3
Total	647	647.0	647.0	647.0
ML estimate		$\hat{\theta}=0.465$	$\hat{\theta}=2.729$	$\hat{\theta}=2.951190$
χ^2		61.08	4.82	3.88
d.f.		1	3	3
p-value		0.0273	0.1855	0.2747

Table 6.4. Distribution of number of red mites on Apple leaves, Fisher et al (1943)

Number of red mites per leaf	Observed Frequency	Expected Frequency		
		PD	PLD	PAD
0	38	25.3	35.8	36.3
1	17	29.1	20.7	20.1
2	10	16.7	11.4	11.2
3	9	6.4	6.0	6.1
4	3	1.8	3.1	3.2
5	2	0.4	1.6	1.6
6	1	0.2	0.8	0.8
7+	0	0.1	0.6	0.7
Total	80	80.0	80.0	80.0
ML estimate		$\hat{\theta}=1.15$	$\hat{\theta}=1.255891$	$\hat{\theta}=1.620588$
χ^2		18.27	2.47	2.07
d.f.		2	3	3
p-value		0.0001	0.4807	0.5580

Table 6.5. Distribution of number of Chromatid aberrations (0.2 g chinon 1, 24 hours)

No. of Chromatid aberrations	Observed Frequency	Expected Frequency		
		PD	PLD	PAD
0	268	231.3	257.0	260.4
1	87	126.7	93.4	89.7
2	26	34.7	32.8	32.1
3	9	6.3	11.2	11.5
4	4	0.8	3.8	4.1
5	2	0.1	1.2	1.4
6	1	0.1	0.4	0.5
7+	3	0.1	0.2	0.3
Total	400	400.0	400.0	400.0
ML estimate		$\hat{\theta}=0.5475$	$\hat{\theta}=2.380442$	$\hat{\theta}=2.659408$
χ^2		38.21	6.21	4.17
d.f.		2	3	3
p-value		0.0000	0.1018	0.2437

7. Concluding Remarks

In this paper Poisson-Akash distribution (PAD) has been obtained by compounding Poisson distribution with Akash distribution introduced by Shanker (2015). The expression for the r th factorial moment has been derived and hence the first four moments about origin and the moments about mean has been given. The expression for coefficient of variation, skewness and kurtosis has been obtained. The maximum likelihood estimation and the method of moments for estimating its parameter have been discussed. The distribution has been fitted using maximum likelihood estimate to some data - sets to test its goodness of fit over Poisson distribution (PD) and Poisson-Lindley distribution (PLD) and it is clear from the fit of PAD that PAD gives much closer fit than PD and PLD in almost all data-sets.

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