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The Discrete Poisson-Sujatha Distribution

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Abstract In this paper, Poisson-Sujatha distribution has been obtained by compounding Poisson distribution with Sujatha distribution introduced by Shanker (2015 a). The first four moments about origin and the moments about mean has been obtained. The expression for coefficient of variation, skewness and kurtosis has been obtained. Important mathematical and statistical properties of the distribution have been derived and discussed. The estimation of its parameter has been discussed using both maximum likelihood estimation and the method of moments. The proposed distribution has been fitted using maximum likelihood estimate to some count data - sets to test its goodness of fit and the fit is compared with that obtained using other one parameter discrete distributions.

Keywords Sujatha distribution, Compounding, Moments, Mathematical and statistical properties, Estimation of parameter, Goodness of fit

1. Introduction

The probability density function (p.d.f) of one parameter Sujatha distribution introduced by Shanker (2015 a) for modeling real lifetime data sets from engineering and biomedical science is given by

$$f(x;\theta)=\frac{\theta^3}{\theta^2+\theta+2}\left(1+x+x^2\right)e^{-\theta x}\quad ;x>0,\ \theta>0 \tag{1.1}$$

Its corresponding cumulative distribution function (c.d.f) is given by

$$F(x)=1-\left[1+\frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x};x>0,\theta>0 \tag{1.2}$$

It has been shown by Shanker (2015 a) that Sujatha distribution is a three component mixture of an exponential distribution with scale parameter θ , a gamma distribution having shape parameter 2 and a scale parameter θ, and a gamma distribution having shape parameter 3 and a scale parameter θ with their mixing proportions $\frac{\theta^2}{\theta^2+\theta+2}$, $\frac{\theta}{\theta^2+\theta+2}$ and $\frac{2}{\theta^2+\theta+2}$ respectively. Shanker (2015 a) has discussed its various mathematical and statistical properties including its shape, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, distribution of order statistics, Bonferroni and Lorenz curves, Renyi entropy measure, stress-strength reliability , some amongst others. Further, Shanker (2015 a) has also discussed its estimation of parameter using maximum likelihood estimation and method of moments along with applications for modeling lifetime data and observed that it gives much closer fit than Akash and Shanker distributions introduced by Shanker (2015 b, 2015 c), Lindley (1958) and exponential distributions. It would be recalled that Shanker (2015 b, 2015 c) has proposed Akash and Shanker distributions along with their various interesting mathematical and statistical properties to model lifetime data arising from engineering and biomedical sciences and showed that these distributions provide much closer fit than Lindley and exponential distributions.

In the present paper, a Poisson mixture of Sujatha distribution introduced by Shanker (2015 a) has been obtained. The first four moments about origin and the moments about mean have been obtained and thus the expression for coefficient of variation, skewness and kurtosis has been given. The estimation of its parameter has been discussed using maximum likelihood estimation and method of moments. The distribution has been fitted using maximum likelihood estimate to some count data sets to test its goodness of fit over Poisson distribution and Poisson-Lindley distribution (PLD), a Poisson-mixture of Lindley distribution (1958), introduced by Sankaran (1970).

2. Poisson-Sujatha Distribution

Suppose the parameter λ of the Poisson distribution follows Sujatha distribution (1.1). Then the Poisson mixture of Sujatha distribution (1.1) can be obtained as

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$$P(X=x)=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\cdot\frac{\theta^3}{\theta^2+\theta+2}\left(1+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda \tag{2.1}$$

$$=\frac{\theta^3}{\left(\theta^2+\theta+2\right)x!}\int_0^\infty \lambda^x\left(1+\lambda+\lambda^2\right)e^{-(\theta+1)\lambda}d\lambda$$

$$=\frac{\theta^3}{\theta^2+\theta+2}\cdot\frac{x^2+(\theta+4)x+\left(\theta^2+3\theta+4\right)}{(\theta+1)^{x+3}};x=0,1,2,...,\theta>0. \tag{2.2}$$

We name this distribution "Poisson-Sujatha distribution (PSD)". The graphs of the probability mass function (pmf) of PSD for varying values of its parameter θ are shown in the figure 1.

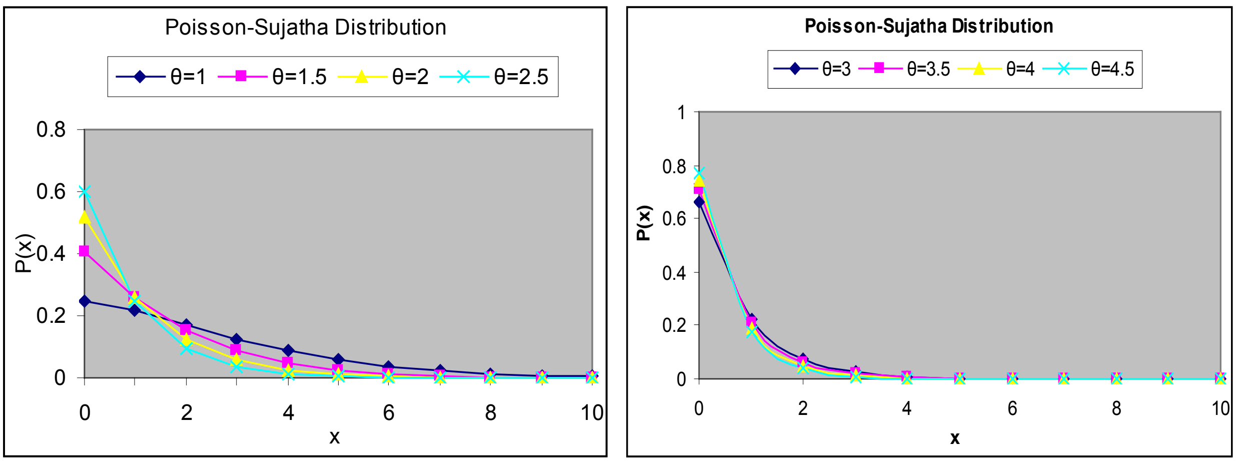

Figure 1. Graphs of probability mass function of PSD for varying values of the parameter θ

3. Moments and Related Measures

The r th factorial moment about origin of Poisson-Sujatha distribution (2.2) can be obtained as

$$\mu_{(r)}'=E\left[E\left(X^{(r)}|\lambda\right)\right], \text{ where } X^{(r)}=X(X-1)(X-2)...(X-r+1)$$

Using (2.1) the r th moment about origin of PSD (2.2) can be obtained as

$$\mu_{(r)}'=E\left[E\left(X^{(r)}|\lambda\right)\right]=\frac{\theta^3}{\theta^2+\theta+2}\int_0^\infty\left[\sum_{x=0}^\infty x^{(r)}\frac{e^{-\lambda}\lambda^x}{x!}\right]\left(1+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

$$=\frac{\theta^3}{\theta^2+\theta+2}\int_0^\infty\left[\lambda^r\sum_{x=r}^\infty\frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right]\left(1+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

Taking $x+r$ in place of x within bracket, we get

$$\mu_{(r)}'=\frac{\theta^3}{\theta^2+\theta+2}\int_0^\infty\lambda^r\left[\sum_{x=0}^\infty\frac{e^{-\lambda}\lambda^x}{x!}\right]\left(1+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

The expression within the bracket is clearly unity and hence we have

$$\mu_{(r)}'=\frac{\theta^3}{\theta^2+\theta+2}\int_0^\infty\lambda^r\left(1+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

Using gamma integral and some algebraic simplification, we get finally a general expression for the r th factorial moment of PSD (2.2) as

$$\mu_{(r)}' = \frac{r!\left[\theta^2+(r+1)\theta+(r+1)(r+2)\right]}{\theta^r\left(\theta^2+\theta+2\right)}\;; r=1,2,3,.... \tag{3.1}$$

Substituting $r = 1,2,3,\text{and } 4$ in (3.1), the first four factorial moments can be obtained and using the relationship between factorial moments and moments about origin, the first four moments about origin of the PSD (2.2) are obtained as

$$\mu_1' = \frac{\theta^2+2\theta+6}{\theta\left(\theta^2+\theta+2\right)}$$

$$\mu_2' = \frac{\theta^3+4\theta^2+12\theta+24}{\theta^2\left(\theta^2+\theta+2\right)}$$

$$\mu_3' = \frac{\theta^4+8\theta^3+30\theta^2+96\theta+120}{\theta^3\left(\theta^2+\theta+2\right)}$$

$$\mu_4' = \frac{\theta^5+16\theta^4+84\theta^3+336\theta^2+840\theta+720}{\theta^4\left(\theta^2+\theta+2\right)}$$

Using the relationship between moments about mean and the moments about origin, the moments about mean of the PSD (2.2) are thus given by

$$\mu_2 = \sigma^2 = \frac{\theta^5+4\theta^4+14\theta^3+28\theta^2+24\theta+12}{\theta^2\left(\theta^2+\theta+2\right)^2}$$

$$\mu_3 = \frac{\theta^8+7\theta^7+32\theta^6+110\theta^5+228\theta^4+300\theta^3+240\theta^2+144\theta+48}{\theta^3\left(\theta^2+\theta+2\right)^3}$$

$$\mu_4 = \frac{\begin{pmatrix}\theta^{11}+15\theta^{10}+99\theta^9+488\theta^8+1682\theta^7+4016\theta^6+7008\theta^5+9016\theta^4\\ +8784\theta^3+6240\theta^2+2880\theta+720\end{pmatrix}}{\theta^4\left(\theta^2+\theta+2\right)^4}$$

The coefficient of variation $(C.V)$, coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis $\left(\beta_2\right)$ and index of dispersion (γ) of the PSD (2.2) are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^5+4\theta^4+14\theta^3+28\theta^2+24\theta+12}}{\theta^2+2\theta+6}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\theta^8+7\theta^7+32\theta^6+110\theta^5+228\theta^4+300\theta^3+240\theta^2+144\theta+48}{\left(\theta^5+4\theta^4+14\theta^3+28\theta^2+24\theta+12\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\begin{pmatrix}\theta^{11}+15\theta^{10}+99\theta^9+488\theta^8+1682\theta^7+4016\theta^6+7008\theta^5\\ +9016\theta^4+8784\theta^3+6240\theta^2+2880\theta+720\end{pmatrix}}{\left(\theta^5+4\theta^4+14\theta^3+28\theta^2+24\theta+12\right)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^5 + 4\theta^4 + 14\theta^3 + 28\theta^2 + 24\theta + 12}{\theta\left(\theta^2 + \theta + 2\right)\left(\theta^2 + 2\theta + 6\right)}$$

To study the nature and behavior of $\mu_1', \mu_2, \text{C.V}, \sqrt{\beta_1}, \beta_2$ and γ of PSD (2.2), numerical values of these characteristics for varying values of the parameter θ have been presented in table 1. It is clear that μ_1', μ_2, and γ are decreasing whereas C.V., $\sqrt{\beta_1}$, β_2 are increasing for increasing values of the parameter θ.

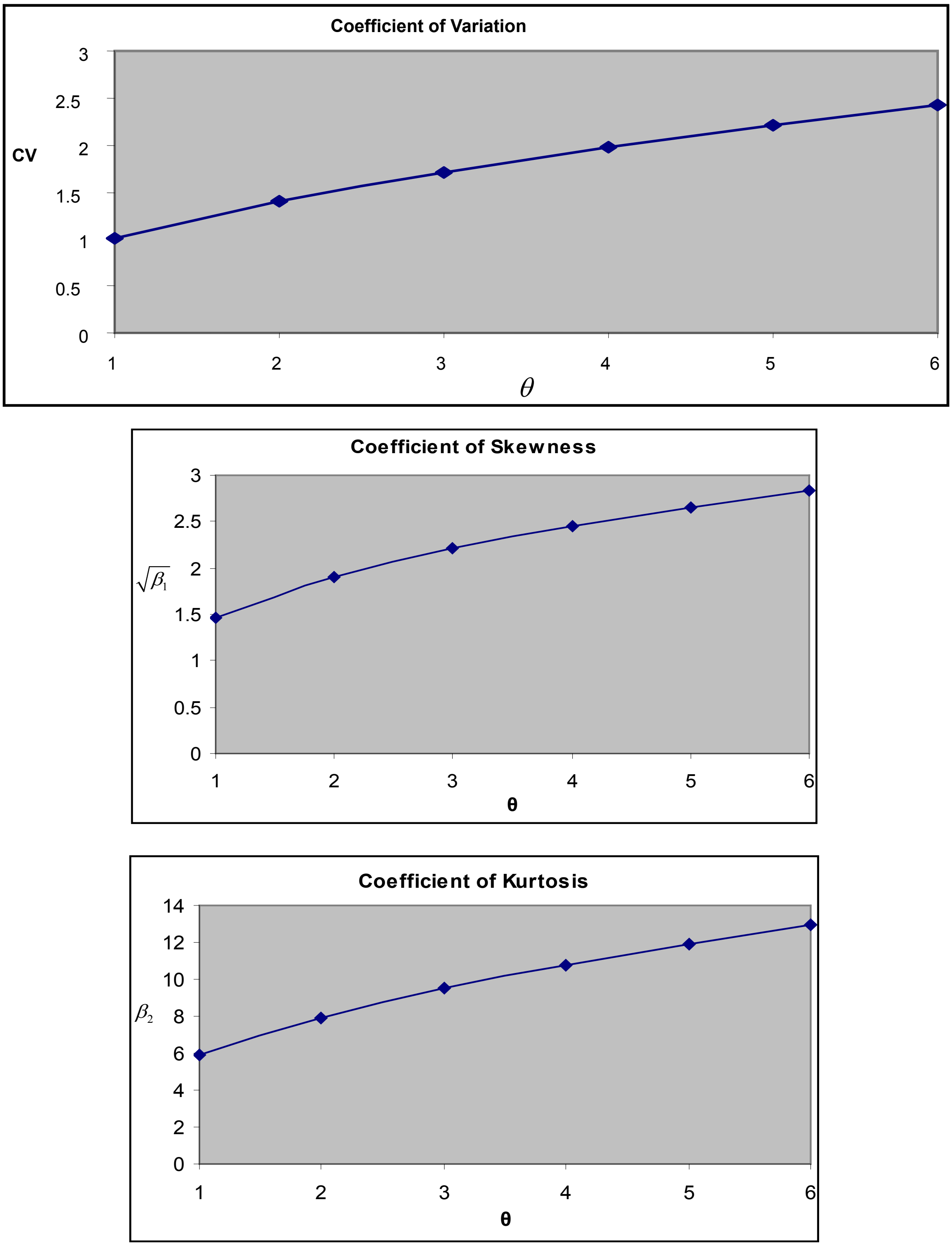

Figure 2. Graphs of coefficient of variation, coefficient of skewness and coefficient of kurtosis of PSD for varying values of the parameter θ

Table 1

	Values of θ for Poisson-Sujatha Distribution					
	1	2	3	4	5	6
μ_1'	2.25	0.875	0.5	0.340909	0.25625	0.204545
μ_2	5.1875	1.484375	0.726190	0.451963	0.320586	0.246039
CV	1.012270	1.392399	1.704336	1.972026	2.209573	2.425006
$\sqrt{\beta_1}$	1.467931	1.898599	2.205880	2.443548	2.646990	2.831109
β_2	5.944113	7.864044	9.479441	10.777366	11.910905	12.969309
γ	2.305555	1.696428	1.452381	1.325757	1.251067	1.202862

Graphs to study the nature and behavior of coefficient of variation, coefficient of skewness, and coefficient of kurtosis of PSD (2.2) for varying values of the parameter θ have been shown in the figure 2. It is obvious from the graphs that the coefficient of variation, coefficient of skewness, and coefficient of kurtosis of PSD are increasing for increasing values of the parameter θ.

4. Mathematical and Statistical Properties

4.1. Over-Dispersion

The PSD (2.2) is always over- dispersed $\left(\sigma^2 > \mu\right)$. We have

$$\sigma^2 = \frac{\theta^5+4\theta^4+14\theta^3+28\theta^2+24\theta+12}{\theta^2\left(\theta^2+\theta+2\right)^2}$$

$$= \frac{\theta^2+2\theta+6}{\theta\left(\theta^2+\theta+2\right)}\left[\frac{\theta^5+4\theta^4+14\theta^3+28\theta^2+24\theta+12}{\theta\left(\theta^2+\theta+2\right)\left(\theta^2+2\theta+6\right)}\right]$$

$$= \frac{\theta^2+2\theta+6}{\theta\left(\theta^2+\theta+2\right)}\left[1+\frac{\theta^4+4\theta^3+18\theta^2+12\theta+12}{\theta\left(\theta^2+\theta+2\right)\left(\theta^2+2\theta+6\right)}\right]$$

$$= \mu\left[1+\frac{\theta^4+4\theta^3+18\theta^2+12\theta+12}{\theta\left(\theta^2+\theta+2\right)\left(\theta^2+2\theta+6\right)}\right] > \mu .$$

This shows that PSD (2.2) is always over-dispersed.

4.2. Increasing Hazard Rate (IHR) and Unimodality

The PSD (2.2) has an increasing hazard rate and unimodal. Since

$$\frac{P(x+1;\theta)}{P(x;\theta)} = \frac{1}{\theta+1}\left[\frac{(x+1)^2+(\theta+4)(x+1)+\left(\theta^2+3\theta+4\right)}{x^2+(\theta+4)x+\left(\theta^2+3\theta+4\right)}\right]$$

$$= \frac{1}{\theta+1}\left[1+\frac{2x+\theta+5}{x^2+(\theta+4)x+\left(\theta^2+3\theta+4\right)}\right]$$

is decreasing function in x, $P(x;\theta)$ is log-concave. Therefore, the PSD has an increasing hazard rate and unimodal. The interrelationship between log-concavity, unimodality and increasing hazard rate (IHR) of discrete distributions has been discussed in Grandell (1997).

4.3. Generating Functions

The probability generating function of the PSD (2.2) can be obtained as

$$P_X(t)=\frac{\theta^3}{(\theta^2+\theta+2)(\theta+1)^3}\left[\sum_{x=0}^{\infty}x^2\left(\frac{t}{\theta+1}\right)^x+(\theta+4)\sum_{x=0}^{\infty}x\left(\frac{t}{\theta+1}\right)^x+(\theta^2+3\theta+4)\sum_{x=0}^{\infty}\left(\frac{t}{\theta+1}\right)^x\right]$$

$$=\frac{\theta^3}{(\theta^2+\theta+2)(\theta+1)^3}\left[\frac{t(\theta+1)(\theta+1+t)}{(\theta+1-t)^3}+\frac{t(\theta+4)(\theta+1)}{(\theta+1-t)^2}\frac{(\theta^2+3\theta+4)(\theta+1)}{(\theta+1-t)}\right]$$

$$=\frac{\theta^3}{(\theta^2+\theta+2)(\theta+1)^2}\left[\frac{t(\theta+1+t)}{(\theta+1-t)^3}+\frac{t(\theta+4)}{(\theta+1-t)^2}\frac{(\theta^2+3\theta+4)}{(\theta+1-t)}\right]$$

The moment generating function of the PSD (2.2) is thus obtained as

$$M_X(t)=\frac{\theta^3}{(\theta^2+\theta+2)(\theta+1)^2}\left[\frac{e^t(\theta+1+e^t)}{(\theta+1-e^t)^3}+\frac{e^t(\theta+4)}{(\theta+1-e^t)^2}\frac{(\theta^2+3\theta+4)}{(\theta+1-e^t)}\right]$$

5. Estimation of the Parameter

5.1. Maximum Likelihood Estimate (MLE) of the Parameter

Let $(x_1,x_2,...,x_n)$ be a random sample of size n from the PSD (2.2) and let f_x be the observed frequency in the sample corresponding to $X=x$ $(x=1,2,3,...,k)$ such that $\sum_{x=1}^{k}f_x=n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the PSD (2.2) is given by

$$L=\left(\frac{\theta^3}{\theta^2+\theta+2}\right)^n\frac{1}{(\theta+1)^{\sum_{x=1}^{k}f_x(x+3)}}\prod_{x=1}^{k}\left[x^2+(\theta+4)x+(\theta^2+3\theta+4)\right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L=n\log\left(\frac{\theta^3}{\theta^2+\theta+2}\right)-\sum_{x=1}^{k}f_x(x+3)\log(\theta+1)+\sum_{x=1}^{k}f_x\log\left[x^2+(\theta+4)x+(\theta^2+3\theta+4)\right]$$

The first derivative of the log likelihood function is given by

$$\frac{d\log L}{d\theta}=\frac{n(\theta^2+2\theta+6)}{\theta(\theta^2+\theta+2)}-\frac{n(\bar{x}+3)}{\theta+1}+\sum_{x=1}^{k}\frac{\left[x+(2\theta+3)\right]f_x}{\left[x^2+(\theta+4)x+(\theta^2+3\theta+4)\right]}$$

where $\bar{x}$ is the sample mean.

The maximum likelihood estimate (MLE), $\hat{\theta}$ of θ of PSD (2.2) is the solution of the equation $\frac{d\log L}{d\theta}=0$ and is given by the solution of the non-linear equation

$$\frac{n\left(\theta^2+2\theta+6\right)}{\theta\left(\theta^2+\theta+2\right)}-\frac{n\left(\overline{x}+3\right)}{\theta+1}+\sum_{x=1}^{k}\frac{\left[x+\left(2\theta+3\right)\right]f_x}{\left[x^2+\left(\theta+4\right)x+\left(\theta^2+3\theta+4\right)\right]}=0$$

This non-linear equation can be solved by any numerical iteration methods such as Newton- Raphson, Bisection method, Regula –Falsi method etc.

5.2. Method of Moment Estimate (MOME) of the Parameter

Let $\left(x_1,x_2,...,x_n\right)$ be a random sample of size n from the PSD (2.2). Equating the first moment about origin to the corresponding sample moment, the MOME $\tilde{\theta}$ of θ of PSD (2.2) is the solution of the following cubic equation $\overline{x}\theta^3+\left(\overline{x}-1\right)\theta^2+2\left(\overline{x}-1\right)\theta-6=0$, where $\overline{x}$ is the sample mean.

Table 1. Distribution of mistakes in copying groups of random digits

No. of errors per group	Observed Frequency	Expected Frequency		
		PD	PLD	PSD
0	35	27.4	33.0	32.9
1	11	21.5	15.3	15.3
2	8	8.4	6.8	6.8
3	4	2.2	2.9	2.9
4	2	0.5	2.0	2.1
Total	60	60.0	60.0	60.0
ML estimate		$\hat{\theta}=0.7833$	$\hat{\theta}=1.7434$	$\hat{\theta}=2.167811$
χ^2		7.98	2.20	1.63
d.f.		1	1	2
p-value		0.0047	0.1380	0.4426

Table 2. Distribution of Pyrausta nublilalis in 1937

No. of insects	Observed Frequency	Expected Frequency		
		PD	PLD	PSD
0	33	26.4	31.5	31.5
1	12	19.8	14.2	14.2
2	6	7.4	6.1	6.1
3	3	1.8	2.5	2.6
4	1	0.3	1.0	1.0
5	1	0.3	0.7	0.6
Total	56	56.0	56.0	56.0
ML estimate		$\hat{\theta}=0.7500$	$\hat{\theta}=1.8081$	$\hat{\theta}=2.241454$
χ^2		4.87	0.53	0.45
d.f.		1	1	1
p-value		0.0273	0.4666	0.5023

Table 3. Accidents to 647 women working on high explosive shells

No. of accidents	Observed Frequency	Expected Frequency		
		PD	PLD	PSD
0	447	406	439.5	439.8
1	132	189	142.8	142.1
2	42	45	45.0	45.0
3	21	7	13.9	13.9
4	3	1	4.2 }	4.2 }
≥ 5	2	0.1	1.3 }	2.0 }
Total	647	647.0	647.0	647.0
ML estimate		$\hat{\theta} = 0.465$	$\hat{\theta} = 2.729$	$\hat{\theta} = 3.168063$
χ^2		61.08	4.82	2.75
d.f.		1	3	3
p-value		0.0273	0.1855	0.4318

Table 4. Number of outbreaks of strike in U.K during 1948-1959, Consul (1989)

No. of outbreaks of strikes	Observed Frequency	Expected Frequency		
		PD	PLD	PSD
0	110	103.5	110.2	110.3
1	33	42.5	32.7	32.5
2	9	8.7 }	9.4 }	9.4 }
3	3	1.2 }	2.7 }	2.7 }
4	1	0.1 }	1.0 }	1.1 }
Total	156	156.0	156.0	156.0
ML estimate		$\hat{\theta} = 0.410256$	$\hat{\theta} = 3.039932$	$\hat{\theta} = 3.473748$
χ^2		3.431	0.001	0.01
d.f.		1	1	1
p-value		0.0640	0.9748	0.9203

Table 5. Number of outbreaks of strike in U.K during 1948-1959, Consul (1989)

No. of outbreaks of strikes	Observed Frequency	Expected Frequency		
		PD	PLD	PSD
0	117	112.5	117.3	117.4
1	29	36.8	29.3	29.1
2	9	6.0 }	7.1 }	7.2 }
3	0	0.6 }	1.7 }	1.7 }
4	1	0.1 }	0.6 }	0.6 }
Total	156.0	156.0	156.0	156.0
ML estimate		$\hat{\theta} = 0.326923$	$\hat{\theta} = 3.801247$	$\hat{\theta} = 4.128452$
χ^2		3.458	0.04	0.03
d.f.		1	1	1
p-value		0.0629	0.8415	0.8625

6. Applications and Goodness of Fit

The PSD has been fitted to a number of count data - sets to test its goodness of fit over Poisson distribution (PD) and Poisson-Lindley distribution (PLD. The maximum likelihood estimate (MLE) has been used to fit the PSD. Five observed data-sets, for which the PD, PLD and PSD has been fitted, are presented. The first data-set is due to Kemp and kemp (1965) regarding the distribution of mistakes in copying groups of random digits, the second data-set is due to Beall (1940) regarding the distribution of Pyrausta nublilalis, the third data-set is the number of accidents to 647 women working on high explosive shells in 5 weeks, available in Sankaran (1970), the fourth and fifth data-sets are relating to the number of outbreaks of strike in U.K during 1948-1959 available in Consul (1989).

It is obvious from the fitting of PD, PLD, and PSD that the PSD gives much closer fit than PD and PLD in tables 1,2,3,and 5, whereas in table 4 PLD gives better fit than PD and PSD. It is to be noted that the fitting of PSD (2.2) in table 1 is also better than the generalized Poisson-Lindley distribution of Mahmoudi and Zakerzadeh (2010). Further, the fitting of PSD in table 3 is better than the fitting of negative binomial distribution (NBD) and the generalized Poisson-Lindley distribution of Mahmoudi and Zakerzadeh (2010).

7. Conclusions

In the present paper Poisson-Sujatha distribution (PSD) has been introduced by compounding Poisson distribution with Sujatha distribution proposed by Shanker (2015 a). Expression for r th factorial moment has been derived and the first four moments about origin and the moments about mean has been obtained. The expression for coefficient of variation, skewness and kurtosis has also been given. The estimation of its parameter has been discussed using maximum likelihood estimation and the method of moments. The PSD has been fitted using maximum likelihood estimate to some count data - sets to test its goodness of fit over Poisson distribution (PD) and Poisson-Lindley distribution (PLD) and it is found that Poisson-Sujatha distribution (PSD) is better than Poisson and Poisson-Lindley distributions for most of the count data-sets.

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