International Journal of Probability and Statistics 2013, 2(2): 35-41
DOI: 10.5923/j.ijps.20130202.04

Computing Ruin Probability in Generalized Risk Processes under Constant Interest Force

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Abstract The aim of this paper is to build an exact formula for ruin probability of generalized risk processes under constant interest force with sequences of random variables such that these sequences are usually assumed to be positive integer – valued random variables. An exact formula for finite time ruin (non-ruin) probabilities are derive by using technique of classical probability. A numerical example is given to illustrate results.

Keywords Ruin Probability, Non –ruin Probability, Homogeneous Markov Chain

1. Introduction

Claude Lefèvre and Stéphane Loisel[1] studied the problem of ruin in the classical compound binomial and compound Poisson risk models. Their primary purpose is to extend those models which is an exact formula derived by Picard and Lefèvre[9] for the probability of (non-ruin) ruin within finite time. Hong N.T.T. (see [7]) recently built an exact formula for finite time ruin (non-ruin) probability for model:

$$U_t = u + \sum_{i=1}^{t} X_i - \sum_{i=1}^{t} Y_i$$

With u, t, X_i, Y_i are positive integer number.

However, Claude Lefèvre and Stéphane Loisel[1] did not provide an exact formula for ruin probability of generalized risk processes under constant interest force with sequences of random variables such that these sequences are usually assumed to be positive integer – valued random variables, with surplus process $\{U_t\}_{t\geq 1}$ written as

$$U_t = u(1+r)^t + \sum_{i=1}^{t} X_i (1+r)^{t-i+1} - \sum_{i=1}^{t} Y_i (1+r)^{t-i} \quad (1.1)$$

where $U_o = u$ is initial surplus ($u > 0$), r is constant interest ($r > 0$), u and t are positive integer numbers, $X = \{X_i\}_{i\geq 1}$ and $Y = \{Y_j\}_{j\geq 1}$ take values in a finite set of positive integer numbers; X and Y are assumed to be independent.

With these assumptions, the aim of this paper is to build an exact formula for finite time ruin (non-ruin) probability of model (1.1). In our study, we extended the result of Hong N. T. T for model (1.1) with any r > 0. This is the first time that gives an exact formula for ruin (non-ruin) probability for model (1.1) whose exact formula for finite time ruin (non-ruin) probability are derived by using technique of classical probability.

The paper is organized as follows; in Section 2, we build an exact formula for ruin (non-ruin) probability for model (1.1) with $X = \{X_i\}_{i\geq 1}$ and $Y = \{Y_j\}_{j\geq 1}$ being independent and identically distributed positive integer – valued random variables, X and Y are assumed to be independent. An extended result in Section 2 with X and Y being homogeneous Markov chains is given in Section 3. A numerical example is give to illustrate these results in Section 4. Finally, we conclude our paper in Section 5.

2. Computing Ruin Probability of Generalized Risk Processes under Constant Interest Force with Sequences of Independent and Identically Distributed Random Variables

Let model (1.1). We assume that:

Assumption 2.1. u and t are positive integer numbers.

Assumption 2.2 $X = \{X_n\}_{n\geq 1}$ is a sequence of independent and identically distributed random variables, X_n take values in a finite set of positive integer numbers

$E_X = \{1, 2, ..., M\}$ with $p_k = P(X_1 = k) (k \in E_X)$,

$0 \leq p_k \leq 1, \sum_{k=1}^{M} p_k = 1$.

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Published online at http://journal.sapub.org/ijps

Assumption 2.3 $Y=\{Y_n\}_{n\geq 1}$ is a sequence of independent and identically distributed random variables, Y_n take values in a finite set of positive integer numbers $E_Y=\{1,2,...,N\}$ with $q_k=P(Y_1=k)(k\in E_Y)$,

$$0\leq q_k\leq 1,\sum_{k=1}^{N}q_k=1.$$

Assumption 2.4 X and Y are assumed to be independent.

From (1.1), we have:

$$U_t=u(1+r)^t+\sum_{i=1}^{t}X_i(1+r)^{t-i+1}-\sum_{i=1}^{t-1}Y_i(1+r)^{t-i}-Y_t. \tag{2.1}$$

Supposing that the ruin time is defined by $T_u=\inf\{j:U_j<0\}$, where $\inf\phi=\infty$.

We define the finite time ruin (non-ruin) probabilities of model (1.1) with Assumption 2.1 to Assumption 2.4, respectively, by

$$\psi_t^{(1)}(u)=P(T_u\leq t)=P\left(\bigcup_{j=1}^{t}(U_j<0)\right), \tag{2.2}$$

$$\varphi_t^{(1)}(u)=1-\psi_t^{(1)}(u)$$
$$=P(T_u\geq t+1)=P\left(\bigcap_{j=1}^{t}(U_j\geq 0)\right). \tag{2.3}$$

Throughout this paper, we denote $A\overset{as}{=}B$ if $P\big((A\setminus B)\cup(B\setminus A)\big)=0$

To establish a formula for $\psi_t^{(1)}(u),\varphi_t^{(1)}(u)$, we first proof the following Lemma.

Lemma 2.1. Any u and $\{x_i\}_{i=1}^t,\{y_i\}_{i=1}^t$ are positive integer numbers.

With *p* being a positive integer number and $1\leq p\leq t-1$ satisfies:

$$y_p\leq u(1+r)^p+\sum_{k=1}^{p}x_k(1+r)^{p+1-k}-\sum_{k=1}^{p-1}y_k(1+r)^{p-k}, \tag{2.4}$$

then

$$u(1+r)^{p+1}+\sum_{k=1}^{p+1}x_k(1+r)^{p+2-k}-\sum_{k=1}^{p}y_k(1+r)^{p+1-k}\geq 1. \tag{2.5}$$

Proof.

From (2.4), we have

$$y_p\leq u(1+r)^p+\sum_{k=1}^{p}x_k(1+r)^{p+1-k}-\sum_{k=1}^{p-1}y_k(1+r)^{p-k}$$

$$\Leftrightarrow -y_p\geq -u(1+r)^p-\sum_{k=1}^{p}x_k(1+r)^{p+1-k}+\sum_{k=1}^{p-1}y_k(1+r)^{p-k}$$

Implies

$$u(1+r)^{p+1}+\sum_{k=1}^{p+1}x_k(1+r)^{p+2-k}-\sum_{k=1}^{p}y_k(1+r)^{p+1-k}$$
$$=u(1+r)^{p+1}+\sum_{k=1}^{p+1}x_k(1+r)^{p+2-k}-\sum_{k=1}^{p-1}y_k(1+r)^{p+1-k}-y_p(1+r)$$
$$\geq u(1+r)^{p+1}+\sum_{k=1}^{p+1}x_k(1+r)^{p+2-k}-\sum_{k=1}^{p-1}y_k(1+r)^{p+1-k}$$
$$-\left(u(1+r)^p+\sum_{k=1}^{p}x_k(1+r)^{p+1-k}-\sum_{k=1}^{p-1}y_k(1+r)^{p-k}\right)(1+r)$$
$$=x_{p+1}(1+r)\geq 1.$$

Hence (2.5) holds.

This completes the proof.

Next, we give an exact formula for finite time non-ruin (ruin) probability of model (1.1).

Theorem 2.1. Let model (1.1) satisfy Assumption 2.1 to Assumption 2.4, then finite time non-ruin probability of model (1.1) is defined by

$$\varphi_t^{(1)}(u)=\sum_{x_1,x_2,...,x_t=1}^{M}p_{x_1}p_{x_2}...p_{x_t}\left(\sum_{1\leq y_1\leq g_1}\sum_{1\leq y_2\leq g_2}...\sum_{1\leq y_t\leq g_t}q_{y_1}q_{y_2}...q_{y_t}\right), \tag{2.6}$$

where

$$g_1=\min\{[u(1+r)+x_1(1+r)],N\},$$

$$g_2=\min\left\{\left[u(1+r)^2+\sum_{k=1}^{2}x_k(1+r)^{3-k}-\sum_{k=1}^{1}y_k(1+r)^{2-k}\right],N\right\},$$

…

$$g_t=\min\left\{\left[u(1+r)^t+\sum_{k=1}^{t}x_k(1+r)^{t+1-k}-\sum_{k=1}^{t-1}y_k(1+r)^{t-k}\right],N\right\},$$

In addition, $[u(1+r)+x_1(1+r)]$ is integer part of the $u(1+r)+x_1(1+r)$.

Proof.

Firstly, we have

$$A:=\bigcap_{j=1}^{t}(U_j\geq 0)=(U_1\geq 0)\cap(U_2\geq 0)\cap...\cap(U_t\geq 0)$$
$$=(Y_1\leq u(1+r)+X_1(1+r))\cap$$
$$\left(Y_2\leq u(1+r)^2+\sum_{k=1}^{2}X_k(1+r)^{3-k}-\sum_{k=1}^{1}Y_k(1+r)^{2-k}\right)\cap$$
$$\left(Y_3\leq u(1+r)^3+\sum_{k=1}^{3}X_k(1+r)^{4-k}-\sum_{k=1}^{2}Y_k(1+r)^{3-k}\right)\cap$$
$$...\cap\left(Y_t\leq u(1+r)^t+\sum_{k=1}^{t}X_k(1+r)^{t+1-k}-\sum_{k=1}^{t-1}Y_k(1+r)^{t-k}\right) \tag{2.7}$$

By Assumption 2.2, we let $X_1 = x_1, X_2 = x_2, ..., X_t = x_t$ with $x_1, x_2, ..., x_t$ being positive integer numbers and satisfying: $1 \le x_1, x_2, ..., x_t \le M$.

Let

$$A_{x_1 x_2 ... x_t} = (X_1 = x_1) \cap (X_2 = x_2) \cap ... \cap (X_t = x_t).$$

Since $X = \{X_n\}_{n \ge 1}$ is a sequence of independent random variables then

$$P(A_{x_1 x_2 ... x_t}) = P\left[(X_1 = x_1) \cap (X_2 = x_2) \cap ... \cap (X_t = x_t)\right] = P(X_1 = x_1).P(X_2 = x_2)...P(X_t = x_t) = p_{x_1} p_{x_2} ... p_{x_t} \quad (2.8)$$

Hence, (2.7) is given

$$A \overset{as}{=} \bigcup_{x_1, x_2, .., x_t = 1}^{M} \left(\{(X_1 = x_1) \cap (X_2 = x_2) \cap ... \cap (X_t = x_t)\} \cap B_{x_1 x_2 ... x_t}\right) = \bigcup_{x_1, x_2, .., x_t = 1}^{M} \left(A_{x_1 x_2 ... x_t} \cap B_{x_1 x_2 ... x_t}\right), \quad (2.9)$$

where

$$\begin{aligned} B_{x_1 x_2 ... x_t} \overset{as}{=} & \left(Y_1 \le u(1+r) + x_1(1+r)\right) \cap \left(Y_2 \le u(1+r)^2 + \sum_{k=1}^{2} x_k (1+r)^{3-k} - \sum_{k=1}^{1} Y_k (1+r)^{2-k}\right) \cap \\ & \left(Y_3 \le u(1+r)^3 + \sum_{k=1}^{3} x_k (1+r)^{4-k} - \sum_{k=1}^{2} Y_k (1+r)^{3-k}\right) \cap ... \\ & \cap \left(Y_t \le u(1+r)^t + \sum_{k=1}^{t} x_k (1+r)^{t+1-k} - \sum_{k=1}^{t-1} Y_k (1+r)^{t-k}\right) \end{aligned} \quad (2.10)$$

By Assumption 2.3, we let $Y_1 = y_1, Y_2 = y_2, ..., Y_{t-1} = y_{t-1}$ with $y_1, y_2, ..., y_{t-1}$ being positive integer numbers andsatisfying: $1 \le y_1, y_2, ..., y_t \le N$.

Let

$$g_1 = \min\left\{\left[u(1+r) + x_1(1+r)\right], N\right\},$$

$$g_2 = \min\left\{\left[u(1+r)^2 + \sum_{k=1}^{2} x_k (1+r)^{3-k} - \sum_{k=1}^{1} y_k (1+r)^{2-k}\right], N\right\},$$

…

$$g_t = \min\left\{\left[u(1+r)^t + \sum_{k=1}^{t} x_k (1+r)^{t+1-k} - \sum_{k=1}^{t-1} y_k (1+r)^{t-k}\right], N\right\},$$

In addition, $\left[u(1+r) + x_1(1+r)\right]$ is integer part of $u(1+r) + x_1(1+r)$,

By using Lemma 2.1, $u(1+r) + x_1(1+r)$, $r(1+r)^2 + \sum_{k=1}^{2} x_k (1+r)^{3-k} - \sum_{k=1}^{1} y_k (1+r)^{2-k}$, …,

$r(1+r)^t + \sum_{k=1}^{t} x_k (1+r)^{t+1-k} - \sum_{k=1}^{t-1} y_k (1+r)^{t-k}$ are integer numbers.

Thus, (2.10) is written as

$$\begin{aligned} B_{x_1 x_2 ... x_t} \overset{as}{=} & \bigcup_{1 \le y_1 \le g_1} (Y_1 = y_1) \cap \Bigg(\bigcup_{1 \le y_2 \le g_2} (Y_2 = y_2) \cap ... \cap \\ & \cap \Bigg(\bigcup_{1 \le y_3 \le g_3} (Y_3 = y_3) \cap ... \cap \left[Y_t \le u(1+r)^t + \sum_{k=1}^{t} x_k (1+r)^{t+1-k} - \sum_{k=1}^{t-1} y_k (1+r)^{t-k} \right] \Bigg) ... \Bigg) \Bigg) \end{aligned} \quad (2.11)$$

As Assumption 2.3, we let $Y_t = y_t$ with y_t is positive integer number then $1 \le y_t \le N$. Combining Assumption 2.3, (2.10) and formulas define $g_1, g_2, ..., g_t$, we have $1 \le y_j \le g_j \ (j = \overline{1, t})$.

Therefore, (2.11) can be rearranged as

$$B_{x_1x_2...x_t} \overset{as}{=} \bigcup_{1\le y_1\le g_1}\bigcup_{1\le y_2\le g_2}...\bigcup_{1\le y_t\le g_t}\{(Y_1=y_1)\cap(Y_2=y_2)\cap...\cap(Y_t=y_t)\} \tag{2.12}$$

For the reason that $Y=\{Y_n\}_{n\ge1}$ is a sequence of independent random variables then

$$P[(Y_1=y_1)\cap(Y_2=y_2)\cap...\cap(Y_t=y_t)] = P(Y_1=y_1).P(Y_2=y_2)...P(Y_t=y_t) = q_{y_1}q_{y_2}...q_{y_t}$$

In the other hand, system of events $\{(Y_1=y_1)\cap(Y_2=y_2)\cap...\cap(Y_t=y_t)\}_{1\le y_j\le g_j(j=\overline{1,t})}$ in (2.12) is incompatible then

$$P(B_{x_1x_2...x_t}) = \sum_{1\le y_1\le g_1}\sum_{1\le y_2\le g_2}...\sum_{1\le y_t\le g_t} q_{y_1}q_{y_2}...q_{y_t} \tag{2.13}$$

Next, we consider

$$A_{x_1x_2...x_t} \overset{as}{=} (X_1=x_1)\cap(X_2=x_2)\cap...\cap(X_t=x_t)$$
$$(x_1,x_2,...,x_t\in E_X)$$

and

$$B_{x_1x_2...x_t} \overset{as}{=} \bigcup_{1\le y_1\le g_1}\bigcup_{1\le y_2\le g_2}...\bigcup_{1\le y_t\le g_t}\{(Y_1=y_1)\cap(Y_2=y_2)\cap...\cap(Y_t=y_t)\}$$

By using, X and Y are independent, if $x_1,x_2,...,x_t\in E_X$ and $x_1,x_2,...,x_n$ hold then $A_{x_1x_2...x_t}$ and $B_{x_1x_2...x_t}$ are independent events.

In addition, system of events $\left\{A_{x_1x_2...x_t}\cap B_{x_1x_2...x_t}\right\}_{x_1,x_2,...,x_t\in E_X}$ in (2.9) is incompatible.

Therefore, combining (2.8) and (2.13), we have

$$\varphi_t^{(1)}(u) = P(A) = \sum_{x_1,x_2,...,x_t=1}^{M} P\left\{A_{x_1x_2...x_t}\cap B_{x_1x_2...x_t}\right\} = \sum_{x_1,x_2,...,x_t=1}^{M} P\left(A_{x_1x_2...x_t}\right)P\left(B_{x_1x_2...x_t}\right)$$
$$= \sum_{x_1,x_2,...,x_t=1}^{M} p_{x_1}p_{x_2}...p_{x_t}\left(\sum_{1\le y_1\le g_1}\sum_{1\le y_2\le g_2}...\sum_{1\le y_t\le g_t} q_{y_1}q_{y_2}...q_{y_t}\right) \tag{2.14}$$

This completes the proof.

Corollary 2.1. Let model (1.1) satisfy Assumption 2.1 to Assumption 2.4, then finite time ruin probability of model (1.1) is defined by

$$\psi_t^{(1)}(u) = 1-\varphi_t^{(1)}(u) = 1-\sum_{x_1,x_2,...,x_t=1}^{M} p_{x_1}p_{x_2}...p_{x_t}\left(\sum_{1\le y_1\le g_1}\sum_{1\le y_2\le g_2}...\sum_{1\le y_t\le g_t} q_{y_1}q_{y_2}...q_{y_t}\right) \tag{2.15}$$

Remark 2.1. Formula (2.6) (or (2.15)) gives a method to compute exactly finite time non-ruin (ruin) probability of model (1.1) which $X=\{X_n\}_{n\ge1}$ and $Y=\{Y_n\}_{n\ge1}$ are sequences of independent and identically distributed random variables, and they take values in a finite set of positive integer numbers.

3. Computing Ruin Probability of Generalized Risk Processes under Constant Interest Force with Homogeneous Markov Chains

Let model (1.1). We assume that:

Assumption 3.1. u, t are positive integer numbers.

Assumption 3.2. $X=\{X_n\}_{n\ge1}$ is a homogeneous Markov chain, X_n take values in a finite set of positive integer numbers $E_X = \{1,2,...,M\}$ with $p_{ij} = P\left(X_{m+1} = j \middle| X_m = i\right), (m \in N^* = N \setminus \{0\}, i \in E_X, j \in E_X)$ where $0 \le p_{ij} \le 1, \sum_{j=1}^{M} p_{ij} = 1$. In addition,

$$P(X_1 = i) = p_i \, (i \in E_X), 0 \le p_i \le 1, \sum_{i=1}^{M} p_i = 1 .$$

Assumption 3.3. $Y = \{Y_n\}_{n \ge 1}$ is a homogeneous Markov chain, Y_n take values in a finite set of positive integer numbers $E_Y = \{1,2,...,N\}$ with $q_{rs} = P\left(Y_{m+1} = s \middle| Y_m = r\right), (m \in N^* = N \setminus \{0\}, r \in E_Y, s \in E_Y)$ where $0 \le q_{rs} \le 1, \sum_{s=1}^{N} q_{rs} = 1$. In addition,

$$P(Y_1 = i) = q_i \, (i \in E_Y), \ 0 \le q_i \le 1, \sum_{i=1}^{N} q_i = 1 .$$

Assumption 3.4. X and Y are assumed to be independent.

Supposing that the ruin time is defined by $T_u = \inf\left\{ j : U_j < 0 \right\}$ where $\inf \phi = \infty$.

We define the finite time ruin (non-ruin) probability of model (1.1) using Assumption 3.1 to Assumption 3.4, respectively, by

$$\psi_t^{(2)}(u) = P(T_u \le t) = P\left(\bigcup_{j=1}^{t} (U_j < 0) \right), \tag{3.1}$$

$$\varphi_t^{(2)}(u) = 1 - \psi_t^{(2)}(u) = P(T_u \ge t+1) = P\left(\bigcap_{j=1}^{t} (U_j \ge 0) \right). \tag{3.2}$$

Similar to Theorem 2.1, we have

Theorem 3.1. Let model (1.1) satisfy Assumption 3.1 to Assumption 3.4, then finite time non-ruin probability of model (1.1) is defined by

$$\varphi_t^{(2)}(u) = \sum_{x_1, x_2, ..., x_t = 1}^{M} p_{x_1} p_{x_1 x_2} \cdots p_{x_{t-1} x_t} \left(\sum_{1 \le y_1 \le g_1} \sum_{1 \le y_2 \le g_2} \cdots \sum_{1 \le y_t \le g_t} q_{y_1} q_{y_1 y_2} \cdots q_{y_{t-1} y_t} \right), \tag{3.3}$$

where, $g_1, g_2, ..., g_t$ is defined in the same way with Theorem 2.1.

Proof.

We proof similarly as Theorem 2.1, where,

(2.8) replaced by

$$\begin{aligned} P(A_{x_1 x_2 \ldots x_t}) &= P\left[(X_1 = x_1) \cap (X_2 = x_2) \cap \ldots \cap (X_t = x_t) \right] \\ &= P(X_1 = x_1).P\left(X_2 = x_2 \middle| X_1 = x_1\right) \ldots P\left(X_t = x_t \middle| X_{t-1} = x_{t-1}\right) \\ &= p_{x_1} p_{x_1 x_2} \cdots p_{x_{t-1} x_t} \end{aligned}$$

In the other hand, we have

$$\begin{aligned} & P\left[(Y_1 = y_1) \cap (Y_2 = y_2) \cap \ldots \cap (Y_t = y_t) \right] \\ &= P(Y_1 = y_1).P\left(Y_2 = y_2 \middle| Y_1 = y_1\right) \ldots P\left(Y_t = y_t \middle| Y_{t-1} = y_{t-1}\right) \\ &= q_{y_1} q_{y_1 y_2} \ldots q_{y_{t-1}\, y_t} \end{aligned}$$

In addition, (2.13) substituted by

$$P(B_{x_1x_2\dots x_t}) = \sum_{1\le y_1\le g_1}\sum_{1\le y_2\le g_2}\dots\sum_{1\le y_t\le g_t} q_{y_1}q_{y_1y_2}\dots q_{y_{t-1}y_t} .$$

By using the same method to prove Theorem 2.1, we have formula (3.3).

This completes the proof.

Corollary 3.1. Let model (1.1) satisfy Assumption 3.1 to Assumption 3.4, then finite time ruin probability of model (1.1) is defined by

$$\psi_t^{(2)}(u) = 1-\varphi_t^{(2)}(u) = 1-\sum_{x_1,x_2,\dots,x_t=1}^{M} p_{x_1}p_{x_1x_2}\dots p_{x_{t-1}x_t}\left(\sum_{1\le y_1\le g_1}\sum_{1\le y_2\le g_2}\dots\sum_{1\le y_t\le g_t} q_{y_1}q_{y_1y_2}\dots q_{y_{t-1}y_t}\right) \quad (3.4)$$

Remark 3.1. Formula (3.3) (or (3.4)) gives a method to compute exactly finite time non-ruin (ruin) probability of model (1.1) which $X=\{X_n\}_{n\ge1}$ and $Y=\{Y_n\}_{n\ge1}$ are homogeneous Markov chains and they take values in a finite set of positive integer numbers.

4. Numerical Illustration

4.1. Numerical Illustration for $\psi_t^{(1)}(u)$

Let $X=\{X_n\}_{n\ge1}$ be a sequence of independent and identically distributed random variables, X_n take values in a finite set of positive integer numbers $E_X=\{1,2,3,4,5\}$ with X_1 having a distribution:

X_1	1	2	3	4	5
P	0,687918	0,107263	0,027260	0,044032	0,133522

Let $Y=\{Y_n\}_{n\ge1}$ be a sequence of independent and identically distributed random variables, Y_n take values in a finite set of positive integer numbers $E_Y=\{1,2,3,4,5\}$ with Y_1 having a distribution:

Y_1	1	2	3	4	5
P	0,693655	0,234842	0,034024	0,022141	0,015337

By using the C progaram, the $\psi_t^{(1)}(u)$ is calculated with the assumptions above of random variables X_1, Y_1.

Table 4.1 shows $\psi_t^{(1)}(u)$ for a range of value of u.

Table 4.1. Ruin probabilities of model (1.1) with Assumption 2.1- 2.4 and r = 0,15

	t = 3	t = 5	t = 7
u = 1,5	0,451331	0,731346	0,872909
u = 2,5	0,421682	0,720987	0,868336
u= 3,5	0,369372	0,713196	0,862931
u = 4,5	0,350304	0,703524	0,859522
u= 5,5	0,345694	0,691736	0,856952
u= 6,5	0,340705	0,687219	0,853781
u = 7,5	0,332709	0,684591	0,850476

4.2. Numerical Illustration for $\psi_t^{(2)}(u)$

Let $X=\{X_n\}_{n\ge1}$ be a homogeneous Markov chain, X_n take values in a finite set of positive integer numbers $E_X=\{1,2,3,4,5\}$,with X_1 having a distribution

X_1	1	2	3	4	5
P	0,412732	0,143721	0,201232	0,112731	0,129584

In addition, matrix $P=\left[p_{ij}\right]_{5x5}$ is given by

$$P=\begin{bmatrix} 0{,}755119 & 0{,}169668 & 0{,}046277 & 0{,}019325 & 0{,}009610 \\ 0{,}469955 & 0{,}225771 & 0{,}074864 & 0{,}205732 & 0{,}023678 \\ 0{,}585528 & 0{,}072188 & 0{,}072098 & 0{,}241161 & 0{,}029025 \\ 0{,}376690 & 0{,}076737 & 0{,}230476 & 0{,}003048 & 0{,}313049 \\ 0{,}003357 & 0{,}621674 & 0{,}312923 & 0{,}053181 & 0{,}008866 \end{bmatrix}$$

Let $Y=\{Y_n\}_{n\ge1}$ be a homogeneous Markov chain, Y_n take values in a finite set of positive integer numbers $E_Y=\{1,2,3,4,5\}$, with Y_1 having a distribution

Y_1	1	2	3	4	5
P	0,713095	0,060022	0,118444	0,075802	0,032637

In addition, matrix $Q = \left[q_{ij} \right]_{5x5}$ is given by

$$Q = \begin{bmatrix} 0,764641 & 0,105781 & 0,030568 & 0,039239 & 0,059771 \\ 0,728355 & 0,183338 & 0,031809 & 0,009616 & 0,046882 \\ 0,319773 & 0,068527 & 0,406201 & 0,199290 & 0,006209 \\ 0,422742 & 0,220847 & 0,270579 & 0,062131 & 0,023701 \\ 0,458144 & 0,073241 & 0,313488 & 0,040222 & 0,114905 \end{bmatrix}$$

By using the C progaram, the $\psi_t^{(2)}(u)$ is calculated with the assumptions above of random variables X_1, Y_1 and matrixs P, Q.

Table 4.2 shows $\psi_t^{(2)}(u)$ for a range of value of u.

Table 4.2. Ruin probabilities of model (1.1) with Assumption 3.1-3.4 and r = 0,15

	t = 3	t = 5	t = 7
u = 1,5	0,438056	0,670702	0,806759
u = 2,5	0,414305	0,660951	0,802202
u= 3,5	0,387413	0,651153	0,798389
u = 4,5	0,372796	0,642734	0,795026
u= 5,5	0,368182	0,636163	0,792125
u= 6,5	0,363660	0,633552	0,789425
u = 7,5	0,357099	0,631753	0,786936

5. Conclusions

By using technique of classical probability with u , t , claims, premiums all are positive integer numbers and r is a positive number, this paper constructed an exact formula for ruin (non-ruin) probability of model (1.1), where sequences of claims and premiums are independent and identically distributed random variables or homogeneous Markov chains. Our main results in this paper not only prove Theorem 2.1 and Theorem 3.1 but also give numerical examples to illustrate for Theorem 2.1 and Theorem 3.1. These results proof for the suitability of theoretical results and practical examples. It also mean that:

When initial u is increasing then $\psi_t^{(1)}(u)$, $\varphi_t^{(2)}(u)$ are decreasing,

With u being unchanged, when t is increasing then $\psi_t^{(1)}(u)$, $\varphi_t^{(2)}(u)$ are increasing.

ACKNOWLEDGMENTS

The authors are thankful to the referee for providing valuable suggestions to improve the quality of the pape.

In addition, the author would like to thank my advisor, Professor Bui Khoi Dam for his patient guidance, encouragement and advice.

REFERENCES

[1] Claude Lefèvre, Stéphane Loisel, *On finité - time ruina probabilities for classical models*, Scandinavian Actuarial Journal, Volume 2008, Issue 1, (2008), 56-68.

[2] De Vylder, F. E., *La formule de Picard et Lefèvre pour la probabilité de ruine en temps fini*, Bulletin Francais d'Actuariat, 1, (1997), 31-40.

[3] De Vylder, F. E., *Numerical finite – time ruin probabilities by the Picard – Lefèvre formula*. Scandinavian Actual Journal, 2, (1999), 97-105.

[4] De Vylder, F. E. and Goovaerts, M. J., *Recursive calculation of finite – time ruin probabilities,* Insurance: Mathematics and Economics, 7, (1998), 1-7.

[5] De Vylder, F. E. an d Goovaerts, M.J., *Explicit finite – time and infinite – time ruin probabilities in the continuous case*. Insurance: Mathematics and Economics,24, (1999)155-172.

[6] Gerber, H.U., *An Introduction to Mathematical Risk Theory*. S. S. Huebner Foundation Monograph, University of Philadelphia: Philadelphia. *Insurance: Mathematics and Economics,* 24, (1979),155-172.

[7] Hong, N.T.T, *On finite – time ruin probabilities for general risk models.* East-West Journal of Mathematics: Vol.15, No1 (2013), pp.86-101.

[8] Ignatov, Z.G., Kaishev, V. K. and Krachunov, R. S., *An improved finite – time ruin probability formula and its Mathematica implementation*. Insurance: Mathematics and Economics, 29, (2001), 375-386.

[9] Ignatov, Z.G., Kaishev, V. K., *A finite – time ruin probability formula for continuous claim severities*. Journal of Applied Probability, 41,(2004), 570-578.

[10] Picard, Ph. and Lefèvre, Cl., *The probability of ruin in finite time with discrete claim size distribution*. Scandinavian Actuarial, (1997), 58 – 69.

[11] Rullière, D. and Loisel, St., *Another look at the Picard – Lefèvre formula for finite – time ruin probabilities*. Insurance: Mathematics and Economics,35,(2004),187-203.