A Nonparametric Test for Umbrella Alternate Scale Problem Using Ranked-set Data

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Abstract   One of the commonly observed one-factor response patterns in practice is the umbrella ordering in which the response variable increases with an increase in the treatment up to a point, then decreases with further increase in the treatment level. In this article, a nonparametric test for several sample scale problem against umbrella alternative with at least one strict inequality, when peak of the umbrella is known, is proposed using ranked-set data. The proposed statistic has the advantage of not requiring the several distribution functions to have a common median, but rather any common quantile of order α, $0 \leq \alpha \leq 1$, (not necessarily ½) which is assumed to be known. The proposed test statistic is based on weighted linear combination of statistic proposed by Ozturk and Deshpande[11]. The distribution of the test statistic and optimal weights has been calculated. It is shown that the new test is uniformly more efficient in terms of Pitman asymptotic relative efficiency than its simple random sampling analog and its Mack-Wolfe (Mack and Wolfe (1981)) version as well.

Keywords   Umbrella Ordering, *U*-statistic, Test for Scale, Ranked-set Sample

1. Introduction

The ranked-set sampling (RSS) approach was first proposed by McIntyre[9] for estimating mean pasture yields. This approach is useful where making measurements on experimental units is either expensive, time-consuming or destructive, but the mechanism for either informally or formally ranking a set of sample units is relatively easy and reliable. RSS utilizes large number of informal measurements for deciding which expensive units should be fully measured. RSS generates a set of observations (data) with smaller sampling variation than for the simple random sampling. Barnett and Moore[1], Kaur, Patil, Sinha, and Taillie[6] and Patil[13] have given eloquent account on the settings where RSS technique has found applications. However, a brief introduction to the concept of RSS for completeness is given below for completeness.

Suppose *X* is a random variable with density function *f(x)*. First, the items in each of *k* independent random samples of size *k* from *f(x)* are separately subjected to ordering on the attribute of interest via some ranking process by visual inspection or based on a concomitant variable. RSS involves selecting one unit among every ranked set consisting of *k* units for quantification, with the other *k*-1 units not being investigated further. One may select the unit with rank 1 from the first set, the unit with rank 2 from the second set,

and so on, i.e., from the rth sample, r = 1, 2,....., k, the item judged to be the rth smallest is retained and measured. The first cycle is completed when the unit with rank *k* is selected from the kth set. The selected rank order can be any permutation of 1, 2,....., k based on the type of ranking, whether perfect or imperfect. This entire process is repeated for *m* independent cycles. Each cycle involves k^2 units and among which only *k* units will be selected for quantification. Finally, the RSS data consists of *mk* independent observations, with one item retained and measured from each of the *mk* independent samples. When the judgment ranking is perfect, then the observations retained from the ith cycle are denoted by $\mathbf{X}_i = X_{(1)i}, ..., X_{(k)i}$; i = 1, 2,....., m, and the entire ranked-set sample is denoted by $\mathbf{X}_1, ..., \mathbf{X}_m = X_{(1)1}, ..., X_{(k)1} ,....., X_{(1)m}, ..., X_{(k)m}$. In case the judgment ranking is not perfect, the round brackets in the subscript are replaced by square brackets.

There has been a lot of work on testing for equality of location parameters in ranked-set sampling, but very little work on testing for equality of scale parameters problem. Stokes and Sager[17] were the first researchers to consider a nonparametric setting with RSS data. They developed important properties of the empirical distribution function from a RSS and compared these properties with those of the empirical distribution of a simple random sample. Bohn and Wolfe[2] developed a nonparametric test based on RSS data for testing the difference between two treatments. They compared this procedure with the Mann-Whitney test for simple random sample data. Bohn and Wolfe[3] proposed a the test for ranked-set two-sample location problem under imperfect judgment ranking. Ozturk[10] provided

two-sample inference based on the RSS sign statistic. Ozturk and Wolfe[12] proposed an improved ranked-set two-sample Mann-Whitney-Wilcoxon test. Flinger and MacEachern[4] proposed a test for two-sample location problem based on the ranks in the RSS which led to tests for the centers of distributions. Ozturk and Deshpande[11] proposed a test of equality of the scale parameters of the two populations based on RSS. But, to our knowledge, no test of equality of homogeneity of scale parameters against umbrella alternatives based on RSS is available.

A nonparametric test for homogeneity of scale parameters against umbrella alternative, with at least one strict inequality has been tackled as a testing problem in the following context. Let $X_{i1}, X_{i2}, ..., X_{in_i}$; $i = 1, 2,......., k$, be independent random samples of size n_i from absolutely continuous cumulative distribution functions $F_i(x) = F\left(\dfrac{x}{\theta_i}\right), i = 1, 2,......., k$, where $\theta_i (\theta_i > 0)$ is the scale parameter. It is assumed that these distribution functions have zero as the common quantile of order α $(0 \le \alpha \le 1)$, i.e., $F_i(0) = \alpha$ for $i = 1, 2,......, k$. It is also assumed that $F_i(x)$, $i = 1, 2,......, k$, are identical in all respects except possibly their scale parameters. The hypothesis which is of interest in this paper can be formally stated as follows:

$$H_0 : \theta_1 = \theta_2 = ... = \theta_k$$

against the umbrella alternative

$$H_1 : \theta_1 \le \theta_2 \le .... \le \theta_{h-1} \le \theta_h \ge \theta_{h+1} \ge .... \ge \theta_{k-1} \ge \theta_k$$

with at least one strict inequality and h, the peak of the umbrella, is known.

For some earlier work on this problem see Gaur et al.[5] and Singh and Liu[16]. Singh and Liu[16] proposed a test statistics for homogeneity of scale parameters against umbrella alternative with at least one strict inequality based isotonic estimator of scale parameter. They also provided one-sided simultaneous confidence intervals for all the ordered pairwise scale ratios, and critical points for two parameter exponential probability distribution. Recently, Gaur et al.[5] provided three test statistics based on linear combination of two-sample U-statistics for testing homogeneity of scale parameters against umbrella alternative, with at least one strict inequality, when the peak of the umbrella, h is known. Gaur et al.[5] test require that the different distribution functions have common quantile of order α, $0 \le \alpha \le 1$, (not necessarily ½) which is assumed to be known.

In this paper, we extend the concept of ranked-set samples to the k-sample scale setting when the scale parameters follow an umbrella pattern, with peak of umbrella, h to be known. The test is given in section 2 and distribution of the test statistic is discussed in section 3. Section 4 is devoted to optimal choice of weights. In section 5, we consider the

performance of the proposed test statistic against its simple random sample analog. It is shown that the proposed test has higher Pitman efficiency for the same number of fully measured observations.

2. The Proposed Test

To construct a multi-sample umbrella alternative problem, we select n_v independent ranked-set samples from absolutely continuous distribution $F_v; v = 1, 2,......, k$, then apply the ranked-set concept and repeat the process for u cycles, where $u = 1, 2,......, m_v$. Let $X_{(t)u,v}$; $t = 1, 2,......., n_v$; $u = 1, 2,......, m_v$; $v = 1, 2,......., k$, be k independent ranked-set samples of size n_v*m_v. We assume that the distribution functions have zero as the common quantile of order α, $(0 \le \alpha \le 1)$, i.e., $F_i(0) = \alpha$ for $i = 1, 2,......, k$.

First we consider the two-sample U-statistic, proposed by Ozturk and Deshpande[11] where the assumption of the common quantile of order α, $(0 \le \alpha \le 1)$ is made and then extend it to the k sample umbrella alternative problem. Define for $i<j$; $i,j=1,2,...,k$,

$\varphi_{ij}(X_{(p)r,i}, X_{(q)s,j}) =$

$$\begin{cases} 1, & \text{if } 0 \le X_{(p)r,i} \le X_{(q)s,j} \ \ or \ \ X_{(q)s,j} \le X_{(p)r,i} < 0 \ , \\ -1, & \text{if } 0 \le X_{(q)s,j} \le X_{(p)r,i} \ \ or \ \ X_{(p)r,i} \le X_{(q)s,j} < 0 \ , \\ 0, & otherwise \ . \end{cases}$$

The two-sample U-statistic corresponding to the kernel ϕ_{ij} is

$$U_{ij} = \frac{1}{m_i n_i m_j n_j} \sum_{p=1}^{n_i} \sum_{r=1}^{m_i} \sum_{q=1}^{n_j} \sum_{s=1}^{m_j} \varphi_{ij}(X_{(p)r,i}, X_{(q)s,j}).$$

The statistic U_{ij} is obviously a U-statistic (Lehmann[7]) corresponding to the kernel ϕ_{ij}. It can be seen that the kernel takes non-zero value only when both $X_{(p)r,i}$'s and $X_{(q)s,j}$'s have the same sign.

For testing H_0 against H_1, with $F_i(0) = \alpha$ for $i = 1, 2,......, k$, we propose the test statistics

$$T = T_1 + T_2,$$

where, $T_1 = \sum_{i=1}^{h-1} a_i U_{i, i+1}$, $T_2 = \sum_{i=h}^{k-1} a_i U_{i+1, i}$,

where $(a_1, a_2,........., a_{k-1})$ are some real positive constants to be chosen suitably and $U_{i,i+1}$ is the ranked-set two-sample statistic proposed by Ozturk and Deshpande[11]. For each set of values $(a_1, a_2,........., a_{k-1})$, we get a distinct member of this class of test statistics. Large values of T are

significant for testing H_0 against H_1. When $a_i = 1$, ($i = 1, 2, \ldots, k-1$), we obtain Mack-Wolfe (Mack and Wolfe[8]) version of T as

$$T_M = \sum_{i=1}^{h-1} U_{i,\,i+1} + \sum_{i=h}^{k-1} U_{i+1,\,i} \,.$$

3. Distribution of the Proposed Test

$$E(T) = \sum_{i=1}^{h-1} a_i \mu_{i,i+1} + \sum_{i=h}^{k-1} a_i \mu_{i+1,i} \text{, where} \quad (3.1)$$

$$\mu_{i,i+1} = E(U_{i,i+1}) \text{ and } \mu_{i+1,i} = E(U_{i+1,i}). \quad (3.2)$$

Under H_0, $E(T) = E(T_M) = 0.$ (3.3)

Using the results of Lehmann[7], the proof of the following theorem follows from the transformation theorem (see Serfling[15], page 122) immediately

Theorem 3.1: The asymptotic null distribution of $\sqrt{N}\,[T - E(T)]$, as $N \to \infty$ in such a way that $\dfrac{n_i}{N} \to p_i,\ 0 < p_i < 1,\ i = 1, 2, \ldots, k,$ is normal with mean 0 and variance η, where

$$\eta = Var(T) = \eta_1 + \eta_2 + 2\eta_{12}, \quad (3.4)$$

$$\eta_1 = Var(T_1) = \underline{a_1'} \Sigma_1 \underline{a_1}\,,\quad \eta_2 = Var(T_2) = \underline{a_2'} \Sigma_2 \underline{a_2}\,,\quad \eta_{12} = Cov(T_1, T_2) \quad (3.5)$$

$$\Sigma_1 = ((\sigma_{1,ij})),\ \ \Sigma_2 = ((\sigma_{2,ij})),$$

$$\underline{a_1'} = (a_1, a_2, \ldots, a_{h-1}),\quad \underline{a_2'} = (a_h, a_{h+1}, \ldots, a_{k-1}),$$

$$\sigma_{1,ij} = \begin{cases} \dfrac{1}{p_i}\xi_{i,i+1;i,i+1}^{(i)} + \dfrac{1}{p_{i+1}}\xi_{i,i+1;i,i+1}^{(i+1)} & \text{for } i = j = 1,2,\ldots,h\text{-}1, \\[2ex] \dfrac{1}{p_{i+1}}\xi_{i,i+1;i+1,i+2}^{(i+1)} & \text{for } j = i+1; i = 1,2,\ldots,h\text{-}2, \\[2ex] \dfrac{1}{p_i}\xi_{i-1,i;i,i+1}^{(i)} & \text{for } j = i-1; i = 2,3,\ldots,h\text{-}1, \\[2ex] 0 & \text{otherwise}, \end{cases}$$

$$\sigma_{2,ij} = \begin{cases} \dfrac{1}{p_i}\xi_{i+1,i;i+1,i}^{(i)} + \dfrac{1}{p_{i+1}}\xi_{i+1,i;i+1,i}^{(i+1)} & \text{for } i = j = h, h+1, \ldots, k\text{-}1, \\[2ex] \dfrac{1}{p_{i+1}}\xi_{i+2,i+1;i+1,i}^{(i+1)} & \text{for } j = i+1; i = h, h+1, \ldots, k\text{-}2, \\[2ex] \dfrac{1}{p_i}\xi_{i+1,i;i,i-1}^{(i)} & \text{for } j = i-1; i = h+1, h+2, \ldots, k\text{-}1, \\[2ex] 0 & \text{otherwise}, \end{cases}$$

where,

$$\xi_{i,i+1;i,i+1}^{(i)} = E\!\left[\left\{\psi_{i,i+1}^{(i)}(x)\right\}^2\right] - E^2\!\left[U_{i,i+1}\right],$$

$$\xi_{i,i+1;i,i+1}^{(i+1)} = E\!\left[\left\{\psi_{i,i+1}^{(i+1)}(x)\right\}^2\right] - E^2\!\left[U_{i,i+1}\right],$$

$$\xi_{i,i+1;i+1,i+2}^{(i+1)} = E\!\left[\left\{\psi_{i,i+1}^{(i+1)}(x)\right\}\psi_{i+1,i+2}^{(i+1)}(x)\right] - E\!\left[U_{i,i+1}\right]E\!\left[U_{i+1,i+2}\right]$$

$$\psi_{i,j}^{(i)}(x) = E\!\left[\phi_{ij}(x, X_{(q)s,j})\right],$$

$$\psi_{i,j}^{(j)}(x) = E\!\left[\phi_{ij}(X_{(p)r,i}, x)\right].$$

After involved computations, it can be seen that under H_0,

$$\sigma_{1,ij} = \begin{cases} \dfrac{A(n_i,\alpha)+C(n_i,\alpha)}{p_i} + \dfrac{A(n_{i+1},\alpha)+C(n_{i+1},\alpha)}{p_{i+1}} & \text{for } i=j=1,2,...,h\text{-}1, \\[2ex] -\dfrac{A(n_{i+1},\alpha)+C(n_{i+1},\alpha)}{p_{i+1}} & \text{for } j=i+1;\ i=1,2,.....,h\text{-}2, \\[2ex] -\dfrac{A(n_i,\alpha)+C(n_i,\alpha)}{p_i} & \text{for } j=i\text{-}1;\ i=2,3,.....,\ h\text{-}1, \\[2ex] 0 & \text{otherwise,} \end{cases} \tag{3.6}$$

$$\sigma_{2,ij} = \begin{cases} \dfrac{A(n_i,\alpha)+C(n_i,\alpha)}{p_i} + \dfrac{A(n_{i+1},\alpha)+C(n_{i+1},\alpha)}{p_{i+1}} & \text{for } i=j=h,h+1,...,k\text{-}1, \\[2ex] -\dfrac{A(n_{i+1},\alpha)+C(n_{i+1},\alpha)}{p_{i+1}} & \text{for } j=i+1;\ i=h,h+1,.....,k\text{-}2, \\[2ex] -\dfrac{A(n_i,\alpha)+C(n_i,\alpha)}{p_i} & \text{for } j=i\text{-}1;\ i=h+1,h+2,.....,k\text{-}1, \\[2ex] 0 & \text{otherwise,} \end{cases} \tag{3.7}$$

where, $A(r,\alpha)=[1/3-\alpha(1-\alpha)]/r$,

$$C(r,\alpha)=-\frac{1}{r^2}\sum_{h=1}^{r}\left\{(1+\alpha)(1-B_{h,r+1-h}(\alpha))-\alpha B_{h,r+1-h}(\alpha)+\frac{2h}{r+1}(2B_{h+1,r+1-h}(\alpha)-1)\right\}^2,$$

$$B_{i,j}(\alpha)=\int_0^{\alpha}\frac{(i+j-1)!}{(i-1)!(j-1)!}u^{i-1}(1-u)^{j-1}\,du.$$

Also, $\eta_{12}=Cov(T_1,T_2)=a_{h-1}a_h Cov(U_{h-1,h},U_{h+1,h})=((a_{h-1}a_h)/p_h)\xi_{k-1,k}$, $\qquad$ (3.8)

$$\xi_{k-1,k}=A(n_{k-1},\alpha)+C(n_{k-1},\alpha),\ \ N=\sum_{i=1}^{k}n_i.$$

In case all the sample sizes are equal i.e., $p_1=p_2=...=p_k=\dfrac{1}{k}$ and $n_i=n$, then

$$\sigma_{1,ij} = \begin{cases} 2k(A(n,\alpha)+C(n,\alpha)) & \text{for } i=j=1,2,...,\ h\text{-}1, \\ -k(A(n,\alpha)+C(n,\alpha)) & \text{for } j=i+1;\ i=1,2,.....,\ h\text{-}2, \\ -k(A(n,\alpha)+C(n,\alpha)) & \text{for } j=i-1;\ i=2,3,.....,\ h\text{-}1, \\ 0 & \text{otherwise,} \end{cases} \tag{3.9}$$

$$\sigma_{2,ij} = \begin{cases} 2k(A(n,\alpha)+C(n,\alpha)) & \text{for } i=j=h,h+1,...,\ k\text{-}1, \\ -k(A(n,\alpha)+C(n,\alpha)) & \text{for } j=i+1;\ i=h,h+1,.....,\ k\text{-}2, \\ -k(A(n,\alpha)+C(n,\alpha)) & \text{for } j=i-1;\ i=h+1,h+2,.....,\ k\text{-}1, \\ 0 & \text{otherwise,} \end{cases} \tag{3.10}$$

$$\xi=\xi_{i,i+1}=\xi_{j,j+1}=A(n,\alpha)+C(n,\alpha);\ i=1,\ 2,\ .....,\ h\text{-}1\ ;\ j=h,\ h+1,\ .....,\ k\text{-}1. \tag{3.11}$$

Also, substituting (3.5), (3.8), (3.9), (3.10) and (3.11) in (3.4), we get

$$\eta=2k\xi\left[\sum_{i=1}^{h-1}a_i^2-\sum_{i=1}^{h-2}a_ia_{i+1}+\sum_{i=h}^{k-1}a_i^2-\sum_{i=h}^{k-2}a_ia_{i+1}+a_{h-1}a_h\right]$$

$$=2k\xi\left[\sum_{i=1}^{k-1}a_i^2-\sum_{i=1}^{k-2}a_ia_{i+1}+2a_{h-1}a_h\right] \tag{3.12}$$

Similarly, the asymptotic null distribution of $\sqrt{N}\left[T_M - E(T_M)\right]$ is normal with mean zero and variance $\eta_M = 6k\xi$.

4. Optimal Choice of Weights

Under the sequence of Pitman alternatives, the square of the efficacy of test T is given by

$$e(T) = \frac{I^2 \left(\sum_{i=1}^{k} a_i \right)^2}{\eta},$$

where, $I = 2\left\{ \int_{-\infty}^{\infty} |x| f(x) dF(x) \right\}$

For efficiency comparisons, we consider the equal sample size and equally spaced alternatives of the type

$$\theta_i = \begin{cases} i\theta & for\ i = 1,\ 2,....,\ h, \\ (2h-i)\theta & for\ i = h+1,\ ....,\ k. \end{cases}$$
$$\theta > 0. \tag{4.1}$$

Making use of the results due to Rao[14] (page 60) for determining optimal weights, we obtain the optimal weights a_i^* for which T has maximum efficiency. For odd k and $h=(k+1)/2$,

$$a_i^* = \begin{cases} (i/k)(k(h-i)-(h-1)) & for\ i = 1,\ 2,....,\ h\text{-}1, \\ ((k-i)/k)(k(i+1-h)-(h-1)) & for\ i = h,\ ....,\ k\text{-}1. \end{cases} \tag{4.2}$$

The square of the efficacy of tests T with optimal choice of weights in (3.3) is given by

$$e^*(T) = \frac{I^2 \left[(k^4 + 2k^2 - 3)/4k^2 \right]}{12\xi}. \tag{4.3}$$

And the square of the efficacy of tests T_M is given by

$$e^*(T_M) = \frac{I^2 (k-1)^2}{6k\xi}. \tag{4.4}$$

5. Asymptotic Relative Efficiencies

In this section, we compute and compare the Pitman asymptotic relative efficiency of T with respect to its Mack-Wolfe version, T_M, also with its competitor test (the test A given by Gaur et al.[5]) in simple random sampling. Using Ozturk and Deshpande[11] and Gaur et al.[5], efficacy of test A of Gaur et al.[5], T_{SRS} test under multivariate setting can be given by

$$e(T_{SRS}) = \frac{I^2 \left[(k^4 + 2k^2 - 3)/4k^2 \right]}{12A(n,\alpha)}.$$

Then the asymptotic relative efficiency (ARE) of the T test with respect to T_{SRS} test can be computed form the ratio of the Pitman efficacies, and the ARE for different values of n are given in Table 1.

$$ARE(T_k, T_{SRS}) = \frac{A(n,\alpha)}{A(n,\alpha)+C(n,\alpha)} \geq 1. \tag{5.1}$$

Also, $ARE\ (T,\ T_M) = \dfrac{(k^4 + 2k^2 - 3)}{8k(k-1)^2}$ and the values of ARE for different values of k are given in Table 2.

It is straight forward that these asymptotic relative efficiencies are independent from underlying distribution F. The inequality in equation (5.1) follows from the fact that $C(n,\alpha)$ is negative. The asymptotic relative efficiency (ARE) of the T test with respect to T_{SRS} test has been calculated for certain value of n and α and presented in Table 1.

6. Conclusions

The AREs in Table 1 and Table 2, immediately show that test for homogeneity of scale parameters against umbrella alternative based on ranked-set sample, T, is always better than the simple random sample test, T_{SRS}, with respect to pitman efficiency.

Also, the proposed test T is better than its Mack-Wolfe version, T_M irrespective of underlying distribution. As k increases, the proposed test T become more efficient as compared to its Mack-Wolfe version, T_M.

Table 1. ARE of the T with respect to T_{SRS}

n	α								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2	1.139	1.195	1.099	1.028	1	1.028	1.099	1.195	1.139
3	1.589	1.386	1.242	1.142	1.103	1.142	1.242	1.386	1.589
4	1.853	1.573	1.396	1.277	1.231	1.277	1.396	1.573	1.853
5	2.092	1.756	1.552	1.418	1.367	1.418	1.552	1.756	2.092
6	2.319	1.934	1.708	1.561	1.506	1.561	1.708	1.934	2.319

Table 2. ARE of T with respect to T_M with different values of k

k	3	5	7	9	11	13	15	17	19
ARE(T,T$_M$)	1.000	1.050	1.238	1.458	1.691	1.929	2.171	2.415	2.661

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