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Stress and Strain Controlled Hysteresis of Rubbers

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Abstract In this paper, the authors obtained the hysteresis loop curves in the case of imposed strains and stresses respectively introducing a nonlinear constitutive integral equation with singular kernels. The Mullins effect was taken into account using a damping function related with the initial damage by large strains. The nonlinearity due to these strains was taken into account using the Ogden equation. Comparisons between theory and experiments were made.

Keywords Hysteresis, Rubbers, Mullins effect, Large strains, Strain & stress controlled laws

1. Introduction

Rubbers are increasingly used in modern industry [1-3]. Resinous materials and rubbers are elastoviscous solids. They are very deformable and possess non-linear viscous behavior. Their creep and stress relaxation is non-linear according to the applied stresses (strains) [2-4]. The last non-linearity can be observed excluding the time factor from the creep curves (the so-called isochrones). Thus, rubbers by large strains require identification and description of nonlinear elastoviscosity. One of the most important results to describe the vibration attenuation capability of rubbers is the hysteresis loop. In the general nonlinear viscoelastic case, this loop can be obtained from the solution of the constitutive mechanical stress-strain equation. It is well-known that the Boltzmann hereditary theory using integral equations of Volterra [3-6] can well describe the creep and stress relaxation of different viscoelastic solids. In the case of large strains one needs to take into account both the physical and geometrical nonlinearities. In this study, we propose an analytical approach to obtain the hysteresis loop of rubbers as a function of the imposed strain or stress amplitude and frequency. Here we examine a Polyisoprene rubber with included fillers (cinders) as an example described in [7].

2. General Framework

Assuming similarity to the isochrones of the stress relaxation curves let's introduce the following integral equation in the case of nonlinear elastoviscous behavior to describe the mechanical behavior of such a rubbers [4-6]

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$$\sigma(t) = \varphi(\varepsilon(\mathrm{t})) - \int_0^t R(t-\tau)\varphi(\varepsilon(\tau))d\tau \,. \quad (1)$$

Here $\sigma(t)$ is the stress as a function of the time t, $\varepsilon(t)$ is the imposed strain and $R(t-\tau)$ is the relaxation kernel which can be found from stress relaxation tests and $\varphi(\varepsilon(t))$ is the instantaneous stress-strain curve. To well describe this curve one can apply the Ogden relation [3], which in the uniaxial (traction) test can be expressed as

$$\varphi(\varepsilon) = \sum_{i=1}^{3} \mu_i (\lambda(\varepsilon)^{\kappa_i - 1} - \lambda(\varepsilon)^{-\frac{\kappa_i}{2} - 1}) \,. \quad (2)$$

Here $\mu_1, \mu_2, \mu_3, \kappa_1, \kappa_2, \kappa_3$ are parameters obtained from instantaneous stress-strain test and the stretch λ is related with the engineering strain as follows $\lambda(\varepsilon) = 1 + \varepsilon$. Equation (1) with (2) represent the stress response in the case of imposed strain law.

The solution of equation (1) can be represented as follows [4-6]

$$\varphi(\varepsilon(t)) = \sigma(t) + \int_0^t K(t-\tau)\sigma(\tau))d\tau = y(t) \,. \quad (3)$$

Here $K(t-\tau)$ represents the resolving creep kernel.

To obtain the strain curve (nonlinear creep) one should use the inverse function [4, 6]

$$\varepsilon(t) = \varphi^{-1}(y(t)) \,. \quad (4)$$

Equation (4) with (3) represents the strain response in the case of imposed stress law.

In this paper, we will employ the following stress relaxation kernel

$$R(t) = \sum_{i=1}^{3} A_i \frac{e^{-\beta_i t}}{t^{\alpha_i}} \quad , \quad (5)$$

In this case the solution has the form (3) with the following creep (resolving) kernel [6]

$$K(t) = \frac{e^{-\beta_i t}}{t} \sum_{n=1}^{\infty} A_i \Gamma(\alpha_i)^n t^{\alpha_i n} / \Gamma(\alpha_i n) \ . \quad (6)$$

Here $\Gamma(\alpha)$ is the gamma function. In equations (2, 3) the kernel $R(t-\tau)$ can be identified from stress relaxation tests.

To take into account the Mullins effect (stress softening after the first cycle) we will introduce the so-called normalized damping function in the nonlinear integral equation related as mentioned in [8] with the initial damage by large strains

$$g_o(\varepsilon) = 1 - C\frac{1}{1+\varepsilon}, \quad (7)$$

Here C is a damping parameter which should take into account the damage at the end of the first cycle which is related with some physical parameters. As mentioned in [8] this damping function should remain constant when unloading. Thus, our nonlinear constitutive integral equation (the stress response by imposed strains) becomes

$$\sigma(t) = \chi(\varepsilon(t)) - \int_0^t R(t-\tau)\chi(\varepsilon(\tau))d\tau, \quad (8)$$

where $\chi(\varepsilon(t)) = \varphi(\varepsilon_{imp}(t))g(\varepsilon_{imp}(t))$ and

$$g(\varepsilon_{imp}(t)) = g_o(\varepsilon_{imp}(t))H(T/2 - t) + g_o(\varepsilon_{imp}(T/2))H(t - T/2) \ . \quad (9)$$

In this equation H(t) is the Heaviside function, which help us to retain the damping function constant after the first loading from 0 to the half period T/2 (the loading time at the end of the first cycle). In the case of sinusoidal impulse loading, the imposed period T is related with the imposed angular frequency ω as $T = 2\pi/\omega$ - see equation (16)).

In the case of stress controlled test we need the solution of the nonlinear equation (8). Thus, using the resolving kernel (6) to the strain response we can write (see equations (3,4)

$$\varepsilon(t) = \chi^{-1}(y(t)), \quad (10)$$

with $$y(t) = \sigma_{imp}(t) + \int_0^t K(t-\tau)\sigma_{imp}(\tau)d\tau \ .$$

Here $\chi^{-1}(y(t))$ is the inverse function of $\chi(y)$ and $K(t-\tau)$ is the creep kernel which has the form (6). The inverse function can be approximated introducing a parabolic function of arbitrary degree.

To obtain the damping parameter C we need to calculate the damage d after the first and the second cycle. This damage can be defined as [9]

$$d = 1 - \frac{U_2}{U_1}, \quad (11)$$

where U_1 and U_2 are the stored energies after the first and the second cycle respectively. In the case of strain controlled test these energies can be expressed as the energy stored upon loading from zero to maximum strain [10]

$$U = \int_0^{\varepsilon_{max}} \sigma d\varepsilon \ . \quad (12)$$

In the case of sinusoidal pulsations for the *n-th* cycle we can write

$$U = \int_{(n-1)T}^{(n-1)T+T/2} \sigma(t)\dot{\varepsilon}_{imp}(t)dt \ . \quad (13)$$

In the case of imposed stresses (stress controlled test) we should use another definition concerning the stored energy [10], namely the energy stored upon loading from zero to maximum stress

$$U = \int_0^{\sigma_{max}} \sigma d\varepsilon \ \text{ or } \ U = \int_{(n-1)T}^{(n-1)T+T/2} \sigma_{imp}(t)\dot{\varepsilon}(t)dt \ . \quad (14)$$

Note that in the case of small strains (linear stress-strain relation) these energies coincide [10].

From equations (8, 9, 10, 13 and 14) one can see that these energies depend on the damping parameter C. In order to identify this parameter we need an independent test to obtain the damage mentioned in equation (11). In [11, 12] the authors propose a method to obtain such a damage as a function of the strain rate. In the case of imposed stresses, one need to proceed in the same manner but introducing the stress rate. These experimental damages at the end of the loading process (the end of the half period in the case of sinusoidal pulsations) should correspond to the damage from equation (11) in the case of imposed strains (stresses) respectively. Thus, the damping parameter C should be obtained from the following equation

$$d_{exp} = 1 - \frac{U_2(C)}{U_1(C)} \ . \quad (15)$$

In the strain (stress) controlled case we should use equation (13) or (14) respectively. Thus, for n = 1,2 to the stored energies after the first and the second cycle in the case of imposed strains we have

$$U_1 = \int_0^{T/2} \sigma(t)\dot{\varepsilon}_{imp}(t)dt \ ,$$

$$U_2 = \int_T^{3T/2} \sigma(t)\dot{\varepsilon}_{imp}(t)dt \ . \quad (16)$$

Identifying C we obtain $\chi(y)$ and its inverse function and from equations (8,10) we arrive to the stress and strain responses respectively.

3. Experimental Comparisons

In this work, we used a polyisoprene rubber [7] produced at UCTM-Sofia. The kernel parameters were respectively

$A_1 = 0.031, A_2 = 0.001, A_3 = 0.007, \alpha_1 = 0.19, \alpha_2 = 0.77,$
$\alpha_3 = 0.43, \beta_1 = 0.0062, \beta_2 = 0.08, \beta_3 = 0.039.$

The Ogden parameters were as follows:

$\mu_1 = 0.13, \mu_2 = -1.46, \mu_3 = 6.2x10^{-3},$

$\kappa_1 = 2.35, \kappa_2 = -0.991, \kappa_3 = 5.74$

and the imposed strain and stress laws were:

$$\varepsilon_{imp}(t) = \varepsilon_o (1 + \sin(\omega t - \pi/2)),$$

$$\sigma_{imp}(t) = \sigma_o (1 + \sin(\omega t - \pi/2)). \quad (16)$$

Here $\varepsilon_o = 0.35$, $\sigma_o = 0.32$ [MPa], $\omega = 0.5$ [Hz]. The damping parameter was obtained as $C = 0.3$. Note that in the first equation (16) we added a small constant initial strain in order to avoid buckling phenomenon (negative contraction stresses)-see figure 1a.

In the first figure we illustrated the stress (strain) responses in the case of imposed strains (stresses) according to equations (8) and (10) respectively.

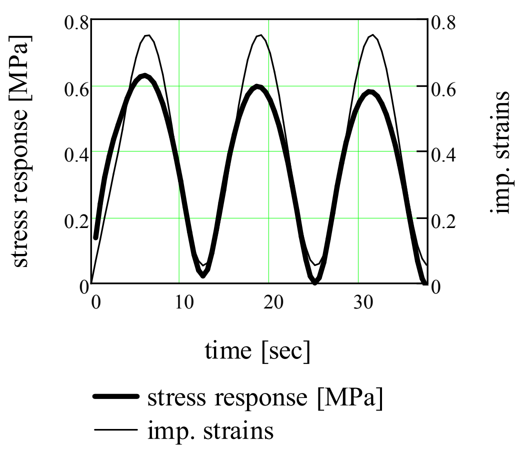

Figure 1a. Imposed strain and stress response

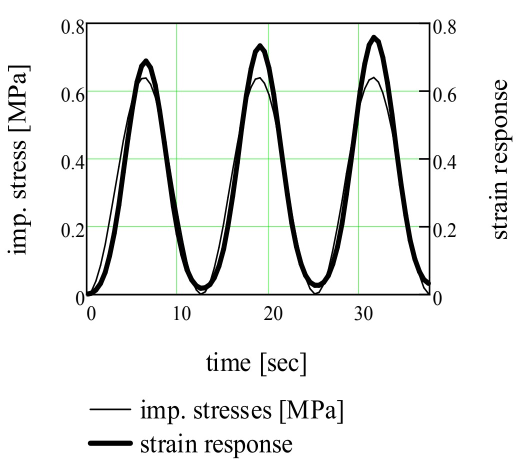

Figure 1b. Imposed stress and strain response

In the second figure 2 one can see the averaged curve of the damage evolution using the method proposed in [10, 11]. The end of the half period (the end of the first cycle loading) was marked with vertical dotted line. The cross point between this line and the averaged damage curve (with thick line) gives us the experimental damage at the end of the first cycle (the horizontal line). This damage $d_{exp} = 0.135$ should be introduced in equations (15, 16) in order to obtain the damping parameter *C*.

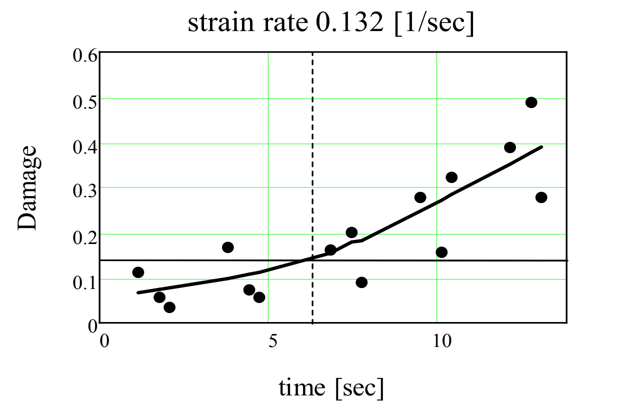

Figure 2. Damage evolution and experimental damage $d_{exp} = 0.135$

Note that the averaged damage evolution curve was obtained by smoothing the experimental data using the MathCad averaging software procedure in the case of data scattered along a band whose width fluctuates considerably.

Finally, in figure 3, we illustrated the hysteresis loops for our polyisoprene rubber according to equations (8, 10).

The respective experimental curves were obtained using the device discussed in [13, 14].

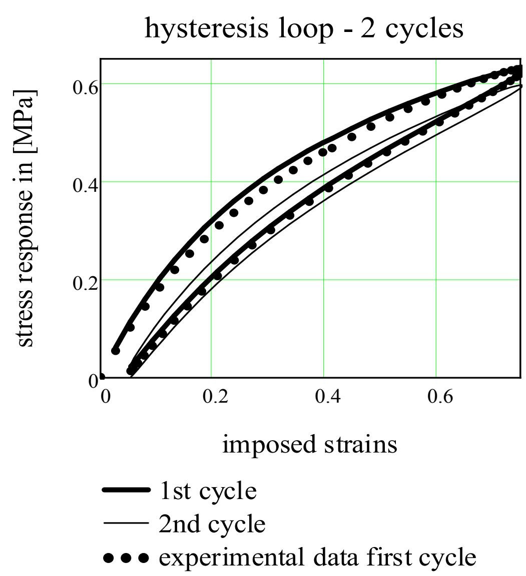

Figure 3a. Hysteresis loop for polyisoprene rubber: imposed strains

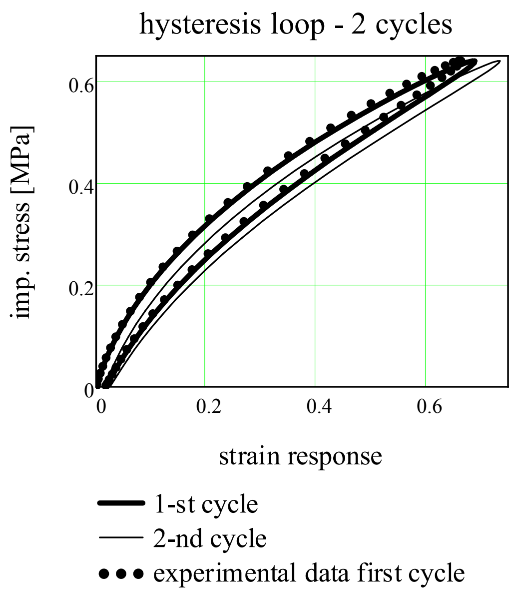

Figure 3b. Hysteresis loop for polyisoprene rubber: imposed stresses

4. Conclusions

Using nonlinear integral equations with three singular kernels and a damping function whose parameter can be obtained from independent experimentation, we have obtained the stress (strain) responses in the case of imposed strains (stresses) taking into account the Mullins softening effect. The experimental hysteresis curves agree with the theoretical ones obtained from the stress (strain) responses by imposing large sinusoidal pulsations for the strain (stress) law. Comparing the two figures 3a and 3b, one can conclude that the Mullins effect (the first cycle hysteresis area greater than the second one) is more pronounced in the case of imposed strains.

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