

Exact Solutions of the BBM and MBBM Equations by the Generalized (G'/G)-expansion Method Equations

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Abstract In this article, we establish exact solutions for the BBM and the MBBM equations by using a generalized (G'/G)-expansion method. The generalized (G'/G)-expansion method was used to construct solitary wave and periodic wave solutions of nonlinear evolution equations. This method is developed for searching exact travelling wave solutions of nonlinear partial differential equations. It is shown that the (G'/G)-expansion method is straightforward and powerful mathematical tool for solving nonlinear problems.

Keywords Generalized (G'/G)-Expansion Method, BBM, MBBM Equations, Solitary Wave, Periodic Wave

1 Introduction

Nonlinear phenomena play a fundamental role in applied mathematics and physics. Recently, the study of nonlinear partial differential equations in modelling physical phenomena has become an important tool. The investigation of the travelling wave solutions plays an important role in nonlinear sciences. A variety of powerful methods has been presented, such as the inverse scattering transform[1], Hirota's bilinear method[2], sine-cosine method[3], homotopy perturbation method[4], homotopy analysis method[5, 6], variational iteration method[7, 8, 9], tanh-function method[10], Backlund transformation[11], Exp-function method[12, 13, 14, 15, 16] and so on. The basic idea of the (G'/G)-expansion method was first proposed by Wang's[17]. A new method is to look for travelling wave solutions of nonlinear evolution equations (NLEEs) and systematically illustrated in Refs.[17, 18, 19, 20]. Zhang et al.[18] have examined the generalized (G'/G)-expansion method and its applications. Authors of[19] have used to mKdV equation with variable coefficients using the (G'/G)-expansion method. Also, Bekir[20] has used to application of the (G'/G)-expansion method for nonlinear evolution equations (NLEEs). In this article, we used the generalized (G'/G)-expansion method to investigate the Benjamin-Bona-Mahony (BBM) equation[22, 23, 24]

$$u_t + u_x - a(u^2)_x - bu_{xxx} = 0, \quad (1.1)$$

is well known in physical applications. This equation models long waves in a nonlinear dispersive system, and as a result, the BBM equation incorporates dispersive effects. The

solutions of the BBM equation exhibit definite soliton-like behavior that is not explainable by any known theory[21]. Both KdV and BBM equations cover cases of the following type[22]: surface waves of long wavelength in liquids, acoustic-gravity waves in compressible fluids, hydromagnetic waves in cold plasma, acoustic waves in anharmonic crystals. Motivated by the rich treasure of the Benjamin-Bona-Mahony equation in science, we will study nonlinear dispersive the modified Benjamin-Bona-Mahony equation (MBBM)[24]

$$u_t + u_x + au^2u_x + u_{xxx} = 0, \quad (1.2)$$

which was first derived to describe an approximation for surface long waves in nonlinear dispersive media[25]. The equation can also characterize the hydromagnetic waves in cold plasma, acoustic waves in anharmonic crystals and acoustic-gravity waves in compressible fluids[26, 27]. The article is organized as follows: In Section 2, first we briefly give the steps of the method and apply it to solve nonlinear partial differential equations. In Sections 3 and 4, we employ this technique to the BBM and the MBBM equations. Also a conclusion is given in Section 5.

2. Basic Idea of (G'/G)-expansion Method

We give the detailed description of method which first presented by Wang[20].

Step 1. For a given nonlinear partial differential equations (NLPDEs) with independent variables $X = (x, y, z, t)$ and dependent variable u

$$P(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, u_{xy}, u_{xt}, u_{xt}, u_{ty}, u_{tz}, \dots) = 0, \quad (2.1)$$

can be converted to an ODE

$$M(u, -cu', u', u', u', u'', \dots), \quad (2.2)$$

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which transformation $\xi = x + y - ct$ is wave variable. Also, c is constant to be determined later.

Step 2. We seek its solutions in the more general polynomial form as follows

$$u(\xi) = a_0 + \sum_{k=1}^m a_k \left(\frac{G'(\xi)}{G(\xi)} \right)^k, \quad (2.3)$$

where $G(\xi)$ satisfies the second order linear ordinary differential equation (LODE) in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \quad (2.4)$$

where $a_0, a_k (k = 1, 2, \dots, m), \lambda$ and μ are constants to be determined later, $a_m = 0$, but the degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (2.2).

Step 3. Substituting (2.3) and Eq. (2.4) into Eq. (2.2) with the value of m obtained in Step 1. Collecting the coefficients of $\left(\frac{G'(\xi)}{G(\xi)} \right)^k (k = 1, 2, \dots, m)$, then setting each coefficient to zero, we can get a set of over-determined partial differential equations for $a_0, a_i (i = 1, 2, \dots, m), \lambda, c$ and μ with the aid of symbolic computation Maple 12.

Step 4. Solving the algebraic equations in Step 3, then substituting a_i, \dots, a_m, c and general solutions of Eq. (2.4) into (2.3) we can obtain a series of fundamental solutions of Eq. (2.1) depending of the solution $G(\xi)$ of Eq. (2.4).

3. The Benjamin–Bona–Mahony (BBM) Equation

In this section we employ the (G'/G) -expansion method to the BBM equation as follows

$$u_t + u_x - a(u^2)_x - bu_{xxt} = 0, \quad (3.1)$$

and by using the wave variable $\xi = x - ct$ reduces it to an ODE

$$-cu + u - au^2 + bcu'' = 0, \quad (3.2)$$

obtained by integrating the resulting equation and by considering each constant of integration to be zero. In order to determine value of m , we balance the linear term of the highest order u'' with the highest order nonlinear term u^2 in Eq. (3.2), along with (2.3) we get

$$u^2(\xi) = a_m^2 \left(\frac{G'(\xi)}{G(\xi)} \right)^{2m} + \dots, \quad (3.3)$$

$$u_{\xi}(\xi) = -ma_m \left(\frac{G'(\xi)}{G(\xi)} \right)^{m+1} + \dots,$$

$$u_{\xi\xi}(\xi) = m(m+1)a_m \left(\frac{G'(\xi)}{G(\xi)} \right)^{m+2} + \dots.$$

Balancing u'' with u^2 in Eq. (3.2), based on (3.3) we

required that $2m = m + 2 \Rightarrow m = 2$. We can suppose that the solutions of Eq. (3.1) is as follows

$$u(\xi) = a_0 + a_1 \left(\frac{G'(\xi)}{G(\xi)} \right) + a_2 \left(\frac{G'(\xi)}{G(\xi)} \right)^2, \quad a_2 \neq 0. \quad (3.4)$$

and therefore

$$u^2(\xi) = a_2^2 \left(\frac{G'(\xi)}{G(\xi)} \right)^4 + 2a_1a_2 \left(\frac{G'(\xi)}{G(\xi)} \right)^3 + (a_1^2 + 2a_0a_2) \left(\frac{G'(\xi)}{G(\xi)} \right)^2 + 2a_0a_1 \left(\frac{G'(\xi)}{G(\xi)} \right) + a_0^2, \quad (3.5)$$

$$u_{\xi\xi}(\xi) = 6a_2 \left(\frac{G'(\xi)}{G(\xi)} \right)^3 + (2a_1 + 10a_2\lambda) \left(\frac{G'(\xi)}{G(\xi)} \right)^2 + (8a_2\mu + 3a_1\lambda + 4a_2\lambda^2) \left(\frac{G'(\xi)}{G(\xi)} \right) + (6a_2\lambda\mu + 2a_1\mu + a_1\lambda^2) \left(\frac{G'(\xi)}{G(\xi)} \right) + 2a_2\mu^2 + a_1\lambda\mu. \quad (3.6)$$

Substituting (3.4)–(3.6) in Eq. (3.3), and by using the well-known Maple software, we obtain the system of the following results

$$a_0 = \frac{bc(\lambda^2 + 2\mu)}{a}, \quad a_1 = \frac{6bc\lambda}{a}, \quad a_2 = \frac{6bc}{a}, \quad c = \frac{1}{1 + b(\lambda^2 - 4\mu)}, \quad (3.7)$$

or

$$a_0 = \frac{6b\mu}{a}, \quad a_1 = \frac{6bc\lambda}{a}, \quad a_2 = \frac{6bc}{a}, \quad c = \frac{1}{1 - b(\lambda^2 - 4\mu)}, \quad (3.8)$$

where λ and μ are arbitrary constants. Substituting (3.7) and (3.8) into expression (3.4), can be written as

$$u(\xi) = \frac{b(\lambda^2 + 2\mu)}{1 + b(\lambda^2 - 4\mu)} + \frac{6b\lambda}{1 + b(\lambda^2 - 4\mu)} \left(\frac{G'(\xi)}{G(\xi)} \right) + \frac{6b}{1 + b(\lambda^2 - 4\mu)} \left(\frac{G'(\xi)}{G(\xi)} \right)^2, \quad (3.9)$$

$$\xi = x - \frac{1}{1 + b(\lambda^2 - 4\mu)} t,$$

$$u(\xi) = \frac{6b\mu}{1 - b(\lambda^2 - 4\mu)} + \frac{6b\lambda}{1 - b(\lambda^2 - 4\mu)} \left(\frac{G'(\xi)}{G(\xi)} \right) + \frac{6b}{1 - b(\lambda^2 - 4\mu)} \left(\frac{G'(\xi)}{G(\xi)} \right)^2, \quad (3.10)$$

or

$$\xi = x - \frac{1}{1 - b(\lambda^2 - 4\mu)} t.$$

Substituting the general solutions of Eq. (2.4) into (3.9) and (3.10) we have three types of exact solutions of Eq. (3.1).

I. When $\lambda^2 - 4\mu > 0$, we obtain hyperbolic function solution

$$u_1(\xi) = \frac{3}{2a + ab(\lambda^2 - 4\mu)} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right)} \right)^2 \quad (3.11)$$

$$- \frac{1}{2a + ab(\lambda^2 - 4\mu)}, \quad \xi = x - \frac{1}{1 + b(\lambda^2 - 4\mu)} t,$$

$$u_2(\xi) = \frac{3}{2a - ab(\lambda^2 - 4\mu)} \left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right)} \right)^2 \quad (3.12)$$

$$- \frac{3}{2a - ab(\lambda^2 - 4\mu)}, \quad \xi = x - \frac{1}{1 - b(\lambda^2 - 4\mu)} t.$$

II. When $\lambda^2 - 4\mu < 0$, we have trigonometric function solution

$$u_3(\xi) = \frac{3}{2a + ab(\lambda^2 - 4\mu)} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right) + C_2 \cos\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right)}{C_1 \cos\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right) + C_2 \sin\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right)} \right)^2 \quad (3.13)$$

$$- \frac{1}{2a + ab(\lambda^2 - 4\mu)}, \quad \xi = x - \frac{1}{1 + b(\lambda^2 - 4\mu)} t,$$

$$u_4(\xi) = \frac{3}{2a-ab(\lambda^2-4\mu)} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right) + C_2 \cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}{C_1 \cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right) + C_2 \sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)} \right)^2 - \frac{3}{2a-ab(\lambda^2-4\mu)} \frac{b(\lambda^2-4\mu)}{1-b(\lambda^2-4\mu)} t, \quad \xi = x - \frac{1}{1-b(\lambda^2-4\mu)} t. \quad (3.14)$$

III. When $\lambda^2 - 4\mu = 0$, we get rational solution

$$u_5(\xi) = \frac{6b}{a} \frac{C_2^2}{(C_1 + C_2\xi)^2}, \quad \xi = x - t. \quad (3.15)$$

(I) The first set: If $C_1 \neq 0, C_2 = 0, \lambda > 0, \mu = 0$, then solution (3.11) and (3.12), respectively, give:

$$u_6(x, t) = \frac{b\lambda^2}{2a + 2ab\lambda^2} \left\{ 3 \tanh^2 \left[\frac{\lambda}{2} \left(x - \frac{1}{1+b\lambda^2} t \right) \right] - 1 \right\}, \quad (3.16)$$

$$u_7(x, t) = -\frac{3b\lambda^2}{2a - 2ab\lambda^2} \operatorname{sech}^2 \left[\frac{\lambda}{2} \left(x - \frac{1}{1-b\lambda^2} t \right) \right]. \quad (3.17)$$

Also, if $C_1 = 0, C_2 \neq 0, \lambda > 0, \mu = 0$, then (3.11) and (3.12) give respectively

$$u_8(x, t) = \frac{b\lambda^2}{2a + 2ab\lambda^2} \left\{ 3 \coth^2 \left[\frac{\lambda}{2} \left(x - \frac{1}{1+b\lambda^2} t \right) \right] - 1 \right\}, \quad (3.18)$$

$$u_9(x, t) = \frac{3b\lambda^2}{2a - 2ab\lambda^2} \operatorname{csch}^2 \left[\frac{\lambda}{2} \left(x - \frac{1}{1-b\lambda^2} t \right) \right]. \quad (3.19)$$

(II) The second set: If $C_1 \neq 0, C_2 = 0, \lambda = 0, \mu > 0$, then (3.13) and (3.14) can be written as respectively

$$u_{10}(x, t) = \frac{2b\mu}{a - 4ab\mu} \left\{ 3 \tan^2 \left[\sqrt{\mu} \left(x - \frac{1}{1-4b\mu} t \right) \right] + 1 \right\}, \quad (3.20)$$

$$u_{11}(x, t) = \frac{6b\mu}{a + 4ab\mu} \sec^2 \left[\sqrt{\mu} \left(x - \frac{1}{1+4b\mu} t \right) \right]. \quad (3.21)$$

In particular, if $C_1 = 0, C_2 \neq 0, \lambda = 0, \mu > 0$, then (3.13) and (3.14) can be written as respectively

$$u_{12}(x, t) = \frac{2b\mu}{a - 4ab\mu} \left\{ 3 \cot^2 \left[\sqrt{\mu} \left(x - \frac{1}{1-4b\mu} t \right) \right] + 1 \right\}, \quad (3.22)$$

$$u_{13}(x, t) = \frac{6b\mu}{a + 4ab\mu} \csc^2 \left[\sqrt{\mu} \left(x - \frac{1}{1+4b\mu} t \right) \right]. \quad (3.23)$$

(III) The third set: If $C_1 \neq 0, C_2 = 0, \lambda = 0, \mu < 0$, then (3.11) and (3.12) give respectively

$$u_{14}(x, t) = \frac{2b\mu}{a - 4ab\mu} \left\{ 2 - 3 \tanh^2 \left[\sqrt{-\mu} \left(x - \frac{1}{1-4b\mu} t \right) \right] \right\}, \quad (3.24)$$

$$u_{15}(x, t) = \frac{6b\mu}{a + 4ab\mu} \operatorname{sech}^2 \left[\sqrt{-\mu} \left(x - \frac{1}{1+4b\mu} t \right) \right]. \quad (3.25)$$

But, if $C_1 = 0, C_2 \neq 0, \lambda = 0, \mu < 0$, then (3.11) and (3.12) give respectively

$$u_{16}(x, t) = \frac{2b\mu}{a - 4ab\mu} \left\{ 2 - 3 \coth^2 \left[\sqrt{-\mu} \left(x - \frac{1}{1-4b\mu} t \right) \right] \right\}, \quad (3.26)$$

$$u_{17}(x, t) = \frac{6b\mu}{a + 4ab\mu} \operatorname{csch}^2 \left[\sqrt{-\mu} \left(x - \frac{1}{1+4b\mu} t \right) \right]. \quad (3.27)$$

4 The Modified Benjamin–Bona–Mahony (MBBM) Equation

We consider the (G'/G)-expansion method to the modified

Benjamin–Bona–Mahony (MBBM) as follows

$$u_t + u_x + au^2 u_x + u_{xxx} = 0, \quad (4.1)$$

and by using the wave variable $\xi = x - ct$ reduces it to an ODE

$$-cu + u + \frac{a}{3} u^3 + u''' = 0, \quad (4.2)$$

obtained by integrating the resulting equation and by considering each constant of integration to be zero. Balancing the linear term of the highest order u''' with the highest order nonlinear term u^3 in Eq. (4.2), and by using Eq. (2.3) we will have

$$u^3(\xi) = a_m^3 \left(\frac{G'(\xi)}{G(\xi)} \right)^{3m} + \dots, \quad (4.3)$$

$$u_\xi(\xi) = -ma_m \left(\frac{G'(\xi)}{G(\xi)} \right)^{m+1} + \dots,$$

$$u_{\xi\xi}(\xi) = m(m+1)a_m \left(\frac{G'(\xi)}{G(\xi)} \right)^{m+2} + \dots.$$

Balancing u''' with u^3 in Eq. (4.2), based on (4.3) we required that $3m = m + 2 \Rightarrow m = 1$. We can suppose that the solutions of Eq. (4.1) is as follows

$$u(\xi) = a_0 + a_1 \left(\frac{G'(\xi)}{G(\xi)} \right), \quad a_1 \neq 0, \quad (4.4)$$

and therefore

$$u^3(\xi) = a_1^3 \left(\frac{G'(\xi)}{G(\xi)} \right)^3 + 3a_1^2 a_0 \left(\frac{G'(\xi)}{G(\xi)} \right)^2 + 3a_0^2 a_1 \left(\frac{G'(\xi)}{G(\xi)} \right) + a_0^3, \quad (4.5)$$

$$u_{\xi\xi}(\xi) = 2a_1 \left(\frac{G'(\xi)}{G(\xi)} \right)^3 + 3a_1 \lambda \left(\frac{G'(\xi)}{G(\xi)} \right)^2 + (2a_1 \mu + a_1 \lambda^2) \left(\frac{G'(\xi)}{G(\xi)} \right) + a_1 \lambda \mu. \quad (4.6)$$

Substituting (4.4)–(4.6) in Eq. (4.2), and by using the well-known Maple software, we will have

$$a_0 = \pm \frac{3\lambda}{\sqrt{6a}} i, \quad a_1 = \pm \frac{6}{a} i, \quad c = 1 - \frac{\lambda^2 - 4\mu}{2}, \quad (4.7)$$

where λ and μ are arbitrary constants. Substituting (4.7) into expression (4.4), can be written as

$$u(\xi) = \pm \frac{3\lambda}{\sqrt{6a}} i + \pm \frac{6}{a} i \left(\frac{G'(\xi)}{G(\xi)} \right), \quad \xi = x - \frac{2 - \lambda^2 + 4\mu}{2} t. \quad (4.8)$$

Substituting the general solutions of Eq. (2.4) into (4.8) we have three types of exact solutions of Eq. (4.1). I. When $\lambda^2 - 4\mu > 0$ we obtain hyperbolic function solution

$$u_1(\xi) = \pm \sqrt{\frac{3(\lambda^2 - 4\mu)}{2a}} i \left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2}\right)} \right) \pm \frac{3\lambda}{\sqrt{6a}} i, \quad (4.9)$$

$$\text{where } \xi = x - \frac{2 - \lambda^2 + 4\mu}{2} t.$$

II. When $\lambda^2 - 4\mu < 0$, we have trigonometric function solution

$$u_2(\xi) = \pm \sqrt{\frac{3(4\mu - \lambda^2)}{2a}} i \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right) + C_2 \cos\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right)}{C_1 \cos\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right) + C_2 \sin\left(\frac{\sqrt{4\mu - \lambda^2}\xi}{2}\right)} \right) \pm \frac{3\lambda}{\sqrt{6a}} i, \quad (4.10)$$

where $\xi = x - \frac{2 - \lambda^2 + 4\mu}{2}t$.

III. When $\lambda^2 - 4\mu = 0$, we get rational solution

$$u_3(x, t) = \pm \sqrt{\frac{6}{a}} i \frac{C_2}{C_1 + C_2(x - t)} \pm \frac{3\lambda}{\sqrt{6a}} i. \quad (4.11)$$

(I) The first set: If $(C_1 \neq 0, C_2 = 0, \lambda > 0, \mu = 0)$ or $(C_1 = 0, C_2 \neq 0, \lambda > 0, \mu = 0)$, then (4.9) gives

$$u_4(x, t) = \pm \sqrt{\frac{3}{2a}} i \lambda \tanh \left[\frac{\lambda}{2} \left(x - \frac{2 - \lambda^2}{2} t \right) \right] \pm \frac{3\lambda}{\sqrt{6a}} i, \quad (4.12)$$

$$u_5(x, t) = \pm \sqrt{\frac{3}{2a}} i \lambda \coth \left[\frac{\lambda}{2} \left(x - \frac{2 - \lambda^2}{2} t \right) \right] \pm \frac{3\lambda}{\sqrt{6a}} i. \quad (4.13)$$

(II) The second set: If $(C_1 \neq 0, C_2 = 0, \lambda = 0, \mu > 0)$ or $(C_1 = 0, C_2 \neq 0, \lambda = 0, \mu > 0)$, then (4.10) becomes respectively

$$u_6(x, t) = \mp \sqrt{\frac{6\mu}{a}} i \tan[\sqrt{\mu}(x - (1 + 2\mu)t)], \quad (4.14)$$

$$u_7(x, t) = \mp \sqrt{\frac{6\mu}{a}} i \cot[\sqrt{\mu}(x - (1 + 2\mu)t)]. \quad (4.15)$$

(III) The third set: If $(C_1 \neq 0, C_2 = 0, \lambda = 0, \mu < 0)$ or $(C_1 = 0, C_2 \neq 0, \lambda = 0, \mu < 0)$, then (4.9) gives respectively

$$u_8(x, t) = \mp \sqrt{\frac{6\mu}{a}} i \tanh[\sqrt{-\mu}(x - (1 + 2\mu)t)], \quad (4.16)$$

$$u_9(x, t) = \pm \sqrt{\frac{6\mu}{a}} i \coth[\sqrt{-\mu}(x - (1 + 2\mu)t)]. \quad (4.17)$$

5. Conclusions

In this article, we investigated the BBM and the modified BBM equations using the generalized (G'/G)-expansion method. This method is applied for finding travelling wave solutions of nonlinear evolution equations. This method has been successfully applied to obtain some new generalized solitary solutions to the BBM and the MBBM equations. These exact solutions include three types hyperbolic function solution, trigonometric function solution and rational solution. The generalized (G'/G)-expansion method is more powerful in searching for exact solutions of NLPDEs. Can be seen that the results are the same, with comparing results[24]. Also, new results are formally developed in this article. It can be concluded that the this method is a very powerful and efficient technique in finding exact solutions for wide classes of problems.that the this method is a very powerful and efficient technique in finding exact solutions for wide classes of problems.

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