International Journal of Electromagnetics and Applications 2019, 9(1): 14-34
DOI: 10.5923/j.ijea.20190901.03

Optimization of Optical Parametric Amplification Efficiency in a Microresonator Under Ultrashort Pump Wave Excitation

Ö züm Emre Aşırım*, Mustafa Kuzuoğlu

Department of Electrical and Electronics Engineering, Middle East Technical University, Ankara, Turkey

Abstract This paper aims to computationally show that it is possible to achieve wideband high-gain optical parametric amplification in a very small low-loss microcavity. Our model involves numerical modeling of the charge polarization density in terms of the nonlinear electron cloud motion. Through a series of finite difference time domain simulations, we have determined the pump wave frequencies that maximize the electric energy density inside the microcavity. These pump wave frequencies that maximize the energy density are then selected for stimulus (input) wave amplification via nonlinear energy coupling. The achieved amplification factors are tabulated in terms of the pump wave frequency, stored electric energy density, and the intracavity charge polarization density. It is found that unlike what the current literature on nonlinear wave mixing suggests, micrometer-scale achievement of wideband high-gain optical parametric amplification is possible by choosing the optimum pump wave frequency that maximizes the stored electric energy density.

Keywords Optical amplification, Nonlinear wave mixing, Optical microcavity, Parametric amplifier, Optimization

1. Introduction

Electromagnetic wave amplification by nonlinear wave mixing has been studied extensively in the context of nonlinear optics [1]. It has been well established that we can amplify a relatively low power *input wave*, by using another wave of high intensity, which is called the *pump wave*, in a nonlinear medium [2]. The theory of electromagnetic wave amplification via nonlinear wave mixing involves the transfer of energy from the pump wave to the input wave as a result of nonlinear coupling in a medium that exhibits a strong nonlinear response under excitation [3]. The nonlinear wave mixing theory has been mostly investigated experimentally rather than computationally, especially for wave amplification purposes. The most important reason behind this trend is that, this concept is almost always studied in the micrometer or nanometer wavelength range, and yet the required medium length to observe any significant nonlinear effect is in the milimeter or centimeter range, which requires an extremely high computational cost for obtaining meaningful results, particularly for the purpose of wave amplification.

* Corresponding author:
ozumasirim88@gmail.com (Ö züm Emre Aşırım)
Published online at http://journal.sapub.org/ijea

Furthermore, based on the current concept of wave amplification via nonlinear wave mixing a.k.a *parametric amplification*, it is impossible to achieve a non-negligible wave amplification in a few micron sized gain medium or a cavity. The required gain medium length for significant parametric amplification is mostly on the order of centimeters [4]. For this reason, the concept of parametric amplification is not feasible to be used for optical microsystems. In this paper, we have carried out a computational analysis that aims to show that it is possible to amplify a low power stimulus wave by mixing it with an intense pump wave of very short duration, in a wide range of frequencies and with a very large gain coefficient, inside a low-loss micro-cavity of several micrometers of length, by maximizing the stored pump wave energy in the cavity.

We will first discuss about high energy storage in low-loss cavities. A low-loss cavity has a high quality factor, which is a measure of the energy storage capability of a cavity. Then, we will discuss about wave propagation in nonlinear dispersive media, and formulate our problem along with it's finite difference time domain implementation. Finally, we will present our results through a set of simulations and we will try to depict that a very high gain optical parametric amplification is possible in a low-loss micro-cavity, when the correct pump wave frequencies that maximize the stored intracavity energy are chosen.

2. Intracavity Energy Density and the Cavity Quality (Q) Factor

The Q factor is a measure of the energy storage capability of a cavity. A high Q value means that high energy can be stored in the cavity. It depends on the length of the cavity, the values of the reflection coefficients of the cavity walls, the frequency of the wave that is propagating inside the cavity, the total absorption coefficient of the medium between the cavity walls, and any kind of diffraction or scattering loss that can occur inside a cavity [14-16]. The Q factor of a cavity is defined as

$$CAVITY\ QUALITY\ (Q) FACTOR$$
$$= 2\pi \frac{Stored\ \ energy}{Energy\ \ dissipated\ \ per\ round\ \ trip} = fT_{rt}\frac{2\pi}{\zeta} = \frac{4fL\pi}{\zeta c}. \quad (1)$$

T_{rt}: *Cavity round trip time*
f: *Wave frequency*
ζ: *Fractional power loss per round trip*
c: *Speed of light*
L: *Cavity length*

The stored electric energy density in a medium depends on both the intensity of the electric field excitation, and the electric charge polarization density of the medium. Assuming a one dimensional analysis, the stored electric energy density in an isotropic medium is defined as [3]

$$W_e = Stored\ energy\ density$$
$$= \frac{1}{2}ED = \frac{1}{2}E(\varepsilon_\infty E + P) = \frac{1}{2}\varepsilon_\infty E^2 + \frac{1}{2}EP. \quad (2)$$

D: *Electric flux density*
P: *Electric charge polarization density*
E: *Electric field intensity*
ε_∞: *Background (infinite spectral band) permittivity*

As an example case in which a very high energy can be stored inside a cavity, involves a dispersive medium, which has a frequency dependent permittivity as stated below [5]

$$\varepsilon(\omega) = 1 + \chi + \frac{Ne^2}{m\varepsilon_0}\left[\frac{1}{\omega^2 - \omega_0{}^2 - i\gamma\omega}\right]. \quad (3)$$

N: *Electron density*
χ: *Linear electric field susceptibility*
γ: *Natural damping coefficient*
ω_0: *Angular resonance frequency*

If $\omega\gamma \ll (\omega^2 - \omega_0{}^2)$, then we can write [2]

$$\varepsilon(\omega) = 1 + \chi + \frac{Ne^2}{m\varepsilon_0}\left[\frac{1}{\omega^2 - \omega_0{}^2}\right]. \quad (4)$$

Assume that there are two monochromatic waves propagating inside a dispersive medium that is placed between two highly reflective walls, $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$. Let us assume that the angular frequency ω of $\boldsymbol{E_1}$ is almost equal to the resonance frequency of the medium ($\omega \approx \omega_0$), then the stored energy in this cavity becomes extremely high. Now assume that $\boldsymbol{E_2}$ has a significantly different frequency than the resonance frequency of the medium. Unless it has a very high power, $\boldsymbol{E_2}$ will not yield a noticeable energy increase in the cavity as it's frequency is not almost equal to the resonance frequency. Furthermore, $\boldsymbol{E_2}$ will not be able to absorb any energy from the energized cavity as there is no energy coupling mechanism, simply because the medium is linear. To account for an energy transfer from an energized cavity to a low power wave, we need a nonlinear wave propagation analysis between the cavity walls, and for nonlinearity to arise, either the medium itself must be highly nonlinear, or at least one of the waves that propagate inside the cavity must have a very high amplitude.

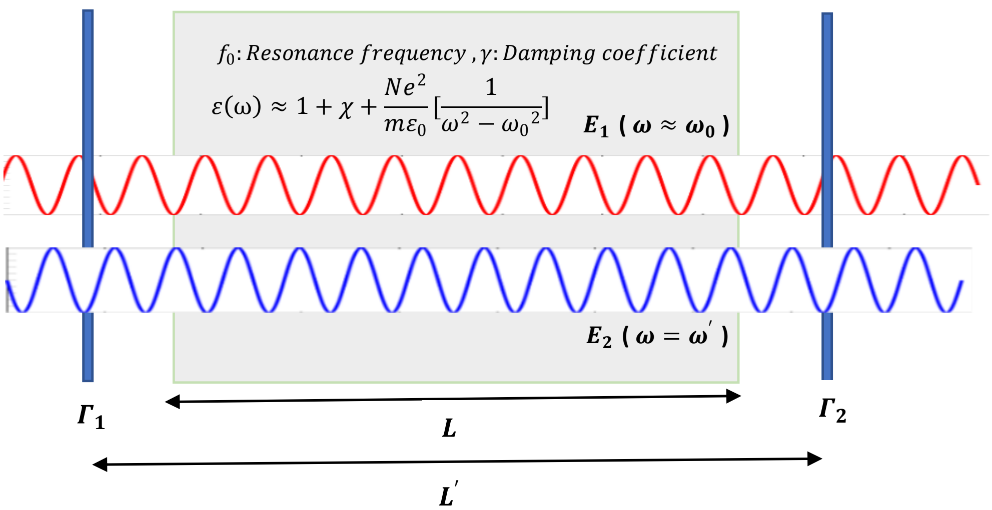

Figure 1. A cavity with a high electric energy density (due to $\boldsymbol{E_1}$) and two propagating waves

3. Wave Propagation in Nonlinear Dispersive Media

Nonlinearity arises when at least one of the waves that propagate in a medium have a very high intensity. Such high intensities are only possible with very short duration pulses [6,8], such as the pulses of a mode locked laser, which have durations on the scale of picoseconds or femtoseconds [7,9,10]. When high intensity pulses have very short durations, which is almost always the case, we have to account for the wave dispersion, as the impulse response of the charge polarization density of many dielectric media last much longer in duration than the pulse durations of such high intensity pulses.

One can solve the wave equation in parallel with the nonlinear equation of motion of the electron cloud for a nonlinear dispersive medium. This is because the parameters such as the resonance frequency, damping coefficient, atom density, and atomic diameter have known values for most solids, which makes the simulation results much more meaningful and precise. In order to determine the time variation of the electric field in a nonlinear dispersive medium, we need to solve the following two equations [1]:

$$\nabla^2 E - \mu_0 \varepsilon_\infty \frac{\partial^2 E}{\partial t^2} = \mu_0 \sigma \frac{\partial E}{\partial t} + \mu_0 \frac{\partial^2 P}{\partial t^2}. \tag{5}$$

$$\frac{\partial^2 P}{\partial t^2} + \gamma \frac{\partial P}{\partial t} + \omega_0{}^2 P - \frac{\omega_0{}^2}{Ned} P^2 - \frac{\omega_0{}^2}{N^2 e^2 d^2} P^3 = \frac{Ne^2}{m} E. \tag{6}$$

$P: Charge\ polarization\ density\ (Coulomb/m^2),\ e: Electron\ charge, m: Electron\ mass$

In Eq. (6) we have made an expansion up to the third order of the nonlinear charge polarization density as higher order terms will be negligibly small. For a dielectric medium, the electrical conductivity can be assumed as negligible ($\sigma \approx 0$). Some typical values for solid dielectric media are [1]

$Resonance\ frequency: f_0 = 1.1 \times 10^{15} Hz,\ Damping\ rate: \gamma = 1 \times 10^9 Hz$

$Electron\ density: N = 3.5 \times 10^{28}/m^3,\ Atomic\ diameter: d = 0.3\ nanometers$

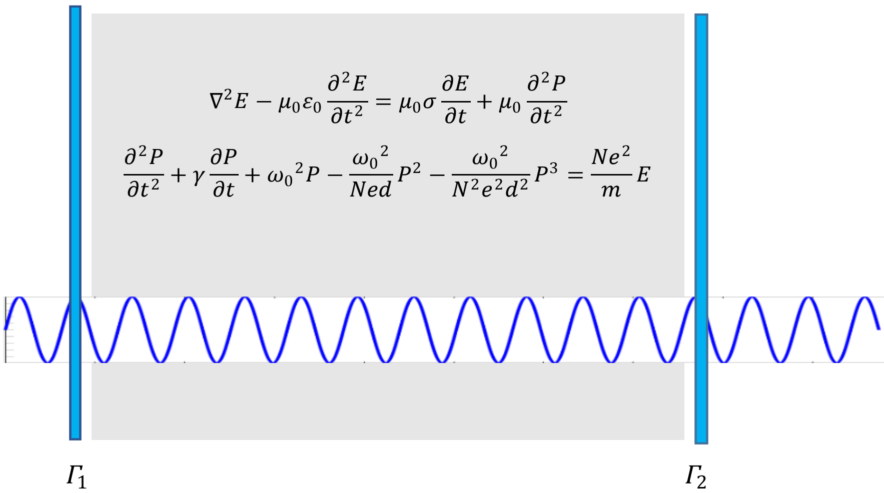

Figure 2. A nonlinear dipersive medium placed in a cavity

Consider the wave E_2 that has a very high amplitude, without the presence of the low amplitude wave E_1, the pair of equations that describe the propagation of E_2 in a nonlinear dispersive medium is given as

$$\nabla^2 (E_2) - \mu_0 \varepsilon_\infty \frac{\partial^2 (E_2)}{\partial t^2} = \mu_0 \sigma \frac{\partial (E_2)}{\partial t} + \mu_0 \frac{\partial^2 P_2}{\partial t^2}. \tag{7a}$$

$$\frac{\partial^2 P_2}{\partial t^2} + \gamma \frac{\partial P_2}{\partial t} + \omega_0{}^2 (P_2) - \frac{\omega_0{}^2}{Ned} (P_2)^2 - \frac{\omega_0{}^2}{N^2 e^2 d^2} (P_2)^3 = \frac{Ne^2}{m} (E_2). \tag{7b}$$

$P_2: Charge\ polarization\ density\ due\ to\ the\ electric\ field\ E_2$

$P_1: Charge\ polarization\ density\ due\ to\ the\ electric\ field\ E_1$

Now assume that E_1 and E_2 are propagating together in the same nonlinear dispersive medium, in this case the pair of equations that represent the total electric field propagation is given as

$$\nabla^2 (E_1 + E_2) - \mu_0 \varepsilon_\infty \frac{\partial^2 (E_1 + E_2)}{\partial t^2} = \mu_0 \sigma \frac{\partial (E_1 + E_2)}{\partial t} + \mu_0 \frac{\partial^2 (P_1 + P_2)}{\partial t^2}. \tag{8a}$$

$$\frac{\partial^2 (P_1 + P_2)}{\partial t^2} + \gamma \frac{\partial (P_1 + P_2)}{\partial t} + \omega_0{}^2 (P_1 + P_2) - \frac{\omega_0{}^2}{Ned} (P_1 + P_2)^2 - \frac{\omega_0{}^2}{N^2 e^2 d^2} (P_1 + P_2)^3 = \frac{Ne^2}{m} (E_1 + E_2). \tag{8b}$$

We want to determine the propagation of the low amplitude wave E_1 in the presence of the high amplitude wave E_2, in other words we want to determine the propagation of E_1 when there is an energy coupling from E_2 as a result of the nonlinear interaction. In order to do that we subtract equations (7a,7b) from equations (8a,8b) respectively, which gives us

$$\nabla^2 (E_1) - \mu_0 \varepsilon_\infty \frac{\partial^2 (E_1)}{\partial t^2} = \mu_0 \sigma \frac{\partial (E_1)}{\partial t} + \mu_0 \frac{\partial^2 (P_1)}{\partial t^2}. \tag{9a}$$

$$\frac{\partial^2 (P_1)}{\partial t^2} + \gamma \frac{\partial (P_1)}{\partial t} + \omega_0{}^2 (P_1) - \frac{\omega_0{}^2}{Ned} \{P_1{}^2 + 2P_1 P_2\} - \frac{\omega_0{}^2}{N^2 e^2 d^2} \{P_1{}^3 + 3P_1{}^2 P_2 + 3P_1 P_2{}^2\} = \frac{Ne^2}{m} (E_1) \tag{9b}$$

From equations {(7a,7b),(9a,9b)} we can see that E_2 and E_1 are coupled to each other. Based on equations (9a,9b), we will investigate whether it is possible to amplify the low power electric field E_1, by drawing energy from a low loss cavity that is energized by the high power electric field E_2.

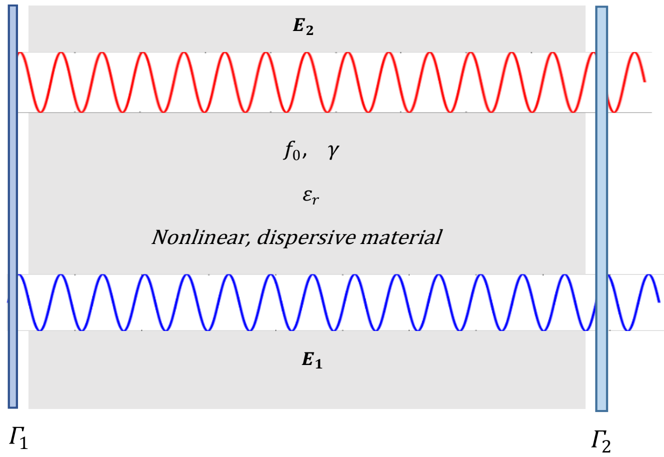

Figure 3. Two waves are propagating through a nonlinear dipersive medium placed in a cavity

3.1. Finite Difference Time Domain Formulation of Energy Coupling in a Nonlinear Dispersive Medium

As introduced in the previous section, E_2 is the high power wave that initiates the nonlinear energy coupling process and E_1 is the low power wave that will absorb energy from E_2. The finite difference time domain formulation of this process in a low-loss dispersive cavity requires the discretization of equations (7a,7b) and (9a,9b). Here we assume a one dimensional space along the x direction so that

$$\nabla^2(E) = \frac{\partial^2(E)}{\partial x^2} \tag{10}$$

As in every computational problem that has an open (or reflectionless) boundary, we have to define a perfectly matched layer (PML) that must effectively surround and limit the computational region due to memory restrictions [17,18]. Since we are carrying out a one dimensional simulation, there is no issue of precise surrounding of the computational region by the PML. A PML is an artificial region and it surrounds the entire computational domain in order to absorb reflections [20,21]. By minimizing reflections, a PML allows us to obtain realistic results in a limited computational domain of a given problem. One method of realizing a PML is to define a conductivity gradient, by which we assign a zero conductivity at the computational domain interface of the PML, and we gradually increase the conductivity towards the outer PML boundary. Based on Fig. 4 the conductivity gradient that represents the PML region is defined as follows:

$$\sigma(x) = \left\{ \begin{matrix} \sigma_0(\Delta - x)\,, 0 < x \leq \Delta \\ (x-(L-\Delta))\sigma_0\,, (L-\Delta) \leq\ x < L \end{matrix} \right\} \tag{11a}$$

$$\sigma(x=0) \gg \sigma_0\Delta\,, \sigma(x=L) \gg \sigma_0\Delta\ (Perfect\ electric\ conductors) \tag{11b}$$

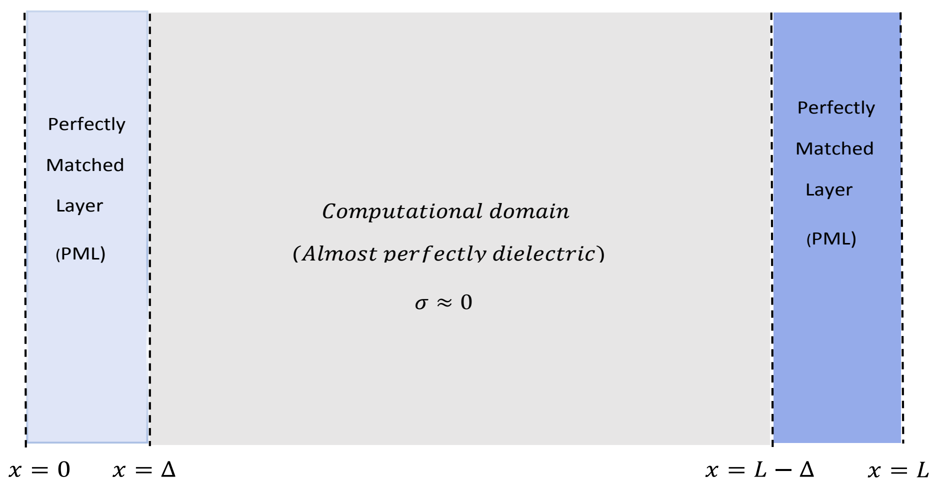

Figure 4. The domain of computation and the domain of termination (PML region)

In order to match the intrinsic impedance of the PML region, to the intrinsic impedance of the computational region, we equate their permittivities and permeabilities at the junctions

$$\varepsilon_\infty = \varepsilon_{Computational} = \varepsilon_{PML} \tag{12a}$$

$$\mu_0 = \mu_{Computational} = \mu_{PML} \tag{12b}$$

$$\lim_{x\to(L-\Delta)^-} \varepsilon_{Computational} = \lim_{x\to(L-\Delta)^+} \varepsilon_{PML} \tag{13a}$$

$$\lim_{x\to(\Delta)^-} \varepsilon_{PML} = \lim_{x\to(\Delta)^+} \varepsilon_{Computational} \tag{13b}$$

$$\lim_{x\to(L-\Delta)^-} \mu_{Computational} = \lim_{x\to(L-\Delta)^+} \mu_{PML} \tag{13c}$$

$$\lim_{x\to(\Delta)^-} \mu_{PML} = \lim_{x\to(\Delta)^+} \mu_{Computational} \tag{13d}$$

Having the PML regions of our problem defined based on equations {(11a,11b),(12a,12b),(13a-13d)}, we can now discretize equations (7a,7b) and (9a,9b) using the finite difference time domain method. Let us first discretize equations (7a,7b) as follows:

$$\begin{aligned}&\frac{E_2(i+1,j)-2E_2(i,j)+E_2(i-1,j)}{\Delta x^2}-\mu_0\varepsilon_\infty(i,j)\frac{E_2(i,j+1)-2E_2(i,j)+E_2(i,j-1)}{\Delta t^2}\\&=\mu_0\sigma(i,j)\frac{E_2(i,j)-E_2(i,j-1)}{\Delta t}+\mu_0\frac{P_2(i,j+1)-2P_2(i,j)+P_2(i,j-1)}{\Delta t^2}.\end{aligned} \tag{14}$$

$$\begin{aligned}&\frac{P_2(i,j+1)-2P_2(i,j)+P_2(i,j-1)}{\Delta t^2}+\gamma\frac{P_2(i,j)-P_2(i,j-1)}{\Delta t}+{\omega_0}^2\big(P_2(i,j)\big)\\&-\frac{{\omega_0}^2}{Ned}\big(P_2(i,j)\big)^2-\frac{{\omega_0}^2}{N^2e^2d^2}\big(P_2(i,j)\big)^3=\frac{Ne^2}{m}\big(E_2(i,j)\big).\end{aligned} \tag{15}$$

Our aim is to solve for $E_2(i,j+1)$ i.e. the value of E_2 at a given point at the next time step. Since E_2 is coupled to P_2, we first solve for $P_2(i,j+1)$ and then substitute it into the equation for $E_2(i,j+1)$. We keep on solving these two equations iteratively for all time steps and for all points in the spatial domain of a given one dimensional problem. For a higher accuracy of the resulting solution, we choose Δt and Δx as small as possible [19,20]. Now we discretize equations (9a,9b) and substitute the value of $P_2(i,j)$ obtained from equations (7a,7b)

$$\begin{aligned}&\frac{E_1(i+1,j)-2E_1(i,j)+E_1(i-1,j)}{\Delta x^2}-\mu_0\varepsilon_\infty(i,j)\frac{E_1(i,j+1)-2E_1(i,j)+E_1(i,j-1)}{\Delta t^2}\\&=\mu_0\sigma(i,j)\frac{E_1(i,j)-E_1(i,j-1)}{\Delta t}+\mu_0\frac{P_1(i,j+1)-2P_1(i,j)+P_1(i,j-1)}{\Delta t^2}.\end{aligned} \tag{16a}$$

$$\begin{aligned}&\frac{P_1(i,j+1)-2P_1(i,j)+P_1(i,j-1)}{\Delta t^2}+\gamma\frac{P_1(i,j)-P_1(i,j-1)}{\Delta t}+{\omega_0}^2\big(P_1(i,j)\big)-\frac{{\omega_0}^2}{Ned}\Big\{\big(P_1(i,j)\big)^2+2P_1(i,j)P_2(i,j)\Big\}\\&-\frac{{\omega_0}^2}{N^2e^2d^2}\Big\{\big(P_1(i,j)\big)^3+3\big(P_1(i,j)\big)^2P_2(i,j)+3P_1(i,j)\big(P_2(i,j)\big)^2\Big\}=\frac{Ne^2}{m}\big(E_1(i,j)\big).\end{aligned} \tag{16b}$$

By solving these four equations simultaneously, along with the initial condition and the boundary conditions of a given problem, we can get the time variations of E_1 and E_2 at any point in one dimensional space. In order to test our computational model, we have compared our computational results with the theoretical results in the well established context of nonlinear sum frequency generation in the appendix section. Now we move on to the simulation results.

4. Simulation Results

Simulation 1 - Part 1: Sweeping the pump wave frequency to maximize the intracavity energy density

Assume that a 250THz infrared stimulus wave E_{st} and a high power pump wave E_{hp} (frequency to be determined) are propagating inside a low-loss (high Q) cavity that has two reflecting walls. The reflecting wall on the left side can be thought as an optical isolator and has a reflection coefficient of $\Gamma_1 \approx 1$, the one on the right side represents a switch controlled optical band-pass filter with a frequency dependent reflection coefficient $\Gamma(f)$. Both waves are generated at x=0μm and at the time instant t=0. The waves and the parameters of the gain medium are as given below:

$E_{hp}(x=0\mu m,t)=2.25\times10^8\times\sin\big(2\pi\big(f_{pump}\big)t\big)\ V/m, for\ 0\le t\le 1ps\ (Ultrashort\ pulse)$

$E_{st}(x=0\mu m,t)=1\times\sin(2\pi(2.5\times10^{14})t)\ V/m, for\ 0\le t\le 30ps$

$Dielectric\ constant\ of\ the\ gain\ medium\ (\varepsilon_\infty)=8\ (\mu_r=1)$

$Resonance\ frequency\ of\ the\ gain\ medium = f_0 = 400THz$

$Damping\ coefficient\ of\ the\ gain\ medium: \gamma=1\times10^9 Hz$

$Time\ interval\ and\ duration\ of\ simulation: 0\le t\le 30ps$

$Spatial\ range\ of\ the\ gain\ medium: 0\mu m < x < 10\mu m$
$Right\ cavity\ wall\ location: x = 10\mu m;\ Left\ cavity\ wall\ location: x = 0\mu m$
$Electron\ density\ of\ the\ gain\ medium: N = 3.5 \times 10^{28}/m^3;\ Atomic\ diameter: d = 0.3\ nm$

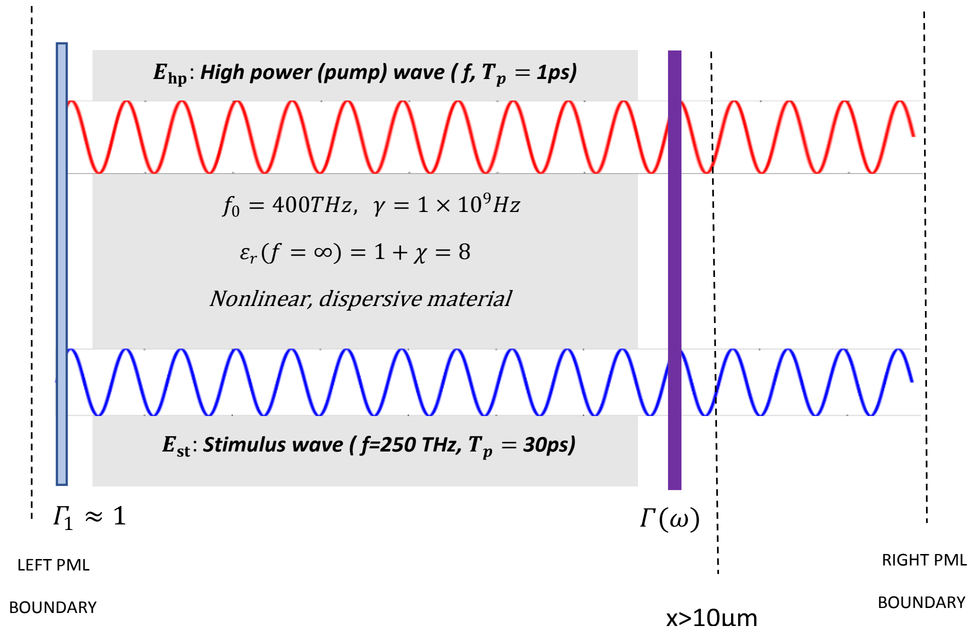

Figure 5. Configuration of the cavity and the dielectric material specifications for simulation1-part1

During the whole simulation time, $\Gamma(f) = 1$ for all f. The filter is used for post-processing of the results.
Our problem: Find the optimum pump wave frequency f_{pump} that maximizes $|E_{st}|$ in the cavity, for 10THz < $f_{pump} < 1000THz$ (THz to UV), for $0\mu m < x < 10\mu m, 0 \le t \le 30ps$, such that

$$\nabla^2(E_{hp}) - \mu_0\varepsilon_\infty \frac{\partial^2(E_{hp})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{hp})}{\partial t} + \mu_0\frac{\partial^2 P_{hp}}{\partial t^2}. \tag{7a}$$

$$\frac{\partial^2 P_{hp}}{\partial t^2} + \gamma\frac{\partial P_{hp}}{\partial t} + \omega_0{}^2(P_{hp}) - \frac{\omega_0{}^2}{Ned}(P_{hp})^2 - \frac{\omega_0{}^2}{N^2e^2d^2}(P_{hp})^3 = \frac{Ne^2}{m}(E_{hp}). \tag{7b}$$

$$\nabla^2(E_{st}) - \mu_0\varepsilon_\infty \frac{\partial^2(E_{st})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{st})}{\partial t} + \mu_0\frac{\partial^2(P_{st})}{\partial t^2}. \tag{9a}$$

$$\frac{\partial^2(P_{st})}{\partial t^2} + \gamma\frac{\partial(P_{st})}{\partial t} + \omega_0{}^2(P_{st}) - \frac{\omega_0{}^2}{Ned}\{P_{st}{}^2 + 2P_{st}P_{hp}\}$$
$$-\frac{\omega_0{}^2}{N^2e^2d^2}\{P_{st}{}^3 + 3P_{st}{}^2P_{hp} + 3P_{st}P_{hp}{}^2\} = \frac{Ne^2(E_{st})}{m} \tag{9b}$$

Initial conditions:

$$P_{hp}(x,0) = P_{hp}{}'(x,0) = E_{hp}(x,0) = E_{hp}{}'(x,0) = P_{st}(x,0) = P_{st}{}'(x,0) = E_{st}(x,0) = E_{st}{}'(x,0) = 0$$

Boundary and excitation conditions:

$E_{hp}(x = 0\mu m, t) = 2.25 \times 10^8 \times \sin(2\pi(f_{pump})t)\ V/m, for\ 0 \le t \le 1ps\ (Ultrashort\ pulse)$

$E_{st}(x = 0\mu m, t) = 1 \times \sin(2\pi(2.5 \times 10^{14})t)\ V/m, for\ 0 \le t \le 30ps$

$E_{hp}(x = 15\mu m, t) = E_{st}(x = 15\mu m, t) = 0\ for\ 0 < t < 30ps$

Absorbing boundary condition (perfectly matched layer):

$$\sigma(x) = \{\,(x - (L - \Delta))\sigma_0, (L - \Delta) \le\ x < L\,\}, for\ L = 15\mu m, \Delta = 2.5\mu m,\ \sigma_0 = 4.5 \times 10^8 S/m$$

Optical isolator condition: Full reflection at $x = 0\mu m$

$$\Gamma(x = 0\mu m, t) = 1\ (Reflection\ coefficient\ is\ equal\ to\ 1)$$

Switch controlled optical bandpass filter condition: Full reflection at $x = 10\mu m$ for t≤30 picoseconds, frequency dependent reflection at $x = 10\mu m$ after t=30 picoseconds;

$$|\Gamma(f')| = \begin{cases} 1\ for\ all\ f'\ ,for\ x = 10\mu m, 0 \le t \le 30ps \\ 1 - e^{-(\frac{(f'-f)}{\sqrt{2}THz})^2}\ ,for\ x = 10\mu m, t > 30ps \end{cases}$$

If amplified, the stimulus wave at t=30 picoseconds will not be monochromatic anymore, this will be due to the spectral broadening inside the cavity [11-13]. However, by adjusting the center frequency of the band-pass filter to be 250THz, we will get an amplified quasi-monochromatic output. The pump wave frequency will be varied from 10THz to 1000THz in 10THz increments and a pump wave frequency that yields a strong amplification of the stimulus wave will be chosen. The electric energy density W_e and the charge polarization density P_{hp} created by the pump wave, are plotted with respect to the pump wave frequency in Fig. 6 and Fig. 7 respectively. Stimulus wave amplitude gain versus pump wave frequency plot is shown in Fig. 8.

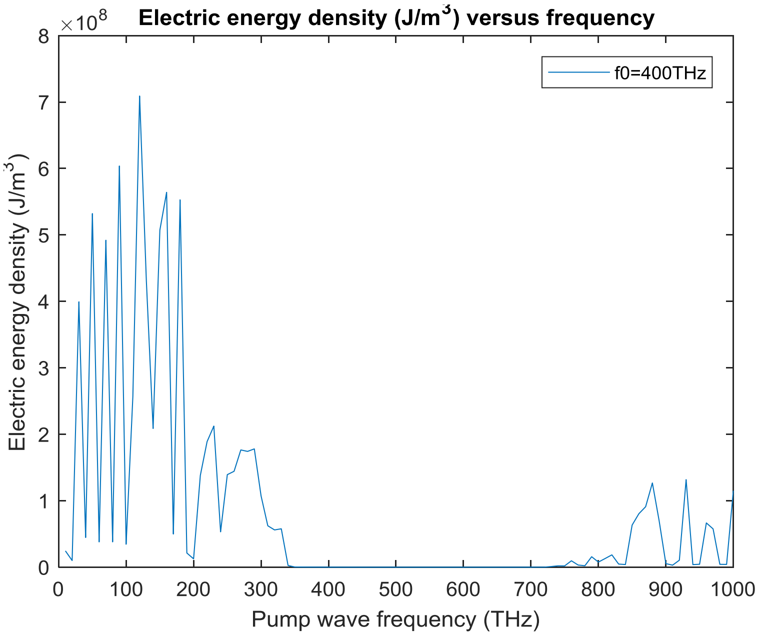

Figure 6. Maximum electric energy density created by the pump wave (for 0<t<30ps), as measured inside the cavity at x=5.73μm, versus the frequency of the pump wave

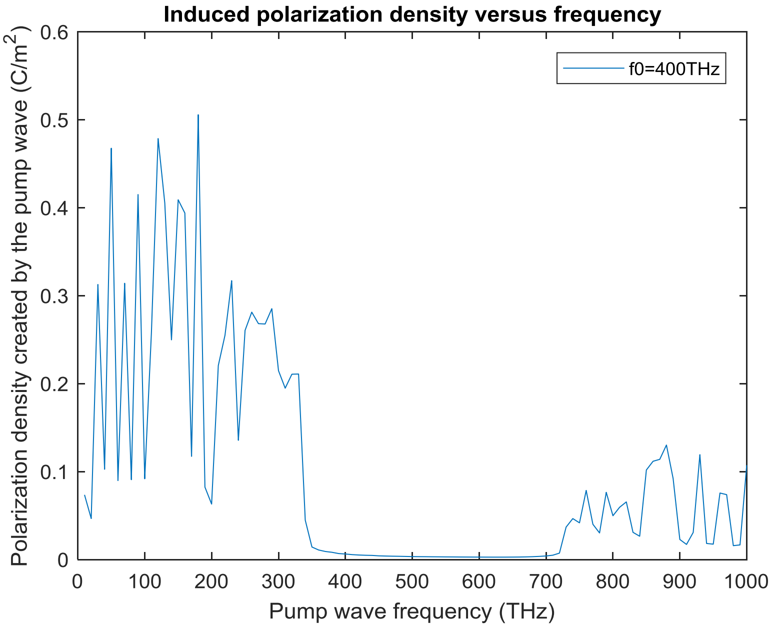

Figure 7. Maximum charge polarization density created by the pump wave (for 0<t<30ps), as measured inside the cavity at x=5.73μm, versus the frequency of the pump wave

The maximum amplitude of the stimulus wave, versus pump wave frequency plot is shown in Fig. 8. Let us investigate the major amplification peak of the stimulus wave at $f = 120THz$. The peak polarization density P_{hp} created by the pump wave (which acts as the coupling coefficient), is high at this frequency. The peak electric energy density W_e created by the pump wave is also high at $f = 120THz$. Therefore we have an amplification peak at $f = 120THz$. The same is true for all other peaks. If we have a look at Table 1, wherever the electric energy density and the polarization density created by the pump wave, are high, there is a stronger stimulus wave amplification.

Since the pump wave frequency of $f = 120THz$ maximizes the intra-cavity energy and amplifies the stimulus wave, we choose this frequency as the frequency of pump wave excitation in order to compute the amplification (gain) spectrum of the stimulus wave. However, as already mentioned, the amplified stimulus wave is not monochromatic anymore due to the spectral broadening inside the cavity. Therefore, we must make a seperate analysis to obtain the gain spectrum of the stimulus wave, under a 120THz pump wave excitation.

W_e: $Maximum\ electric\ energy\ density\ created\ by\ the\ pump\ wave\ in\ the\ cavity$

P_{hp}: $Maximum\ charge\ polarization\ density\ created\ by\ the\ pump\ wave\ in\ the\ cavity$

$A_{st,max}$: $Maximum\ stimulus\ wave\ amplitude\ in\ the\ cavity$

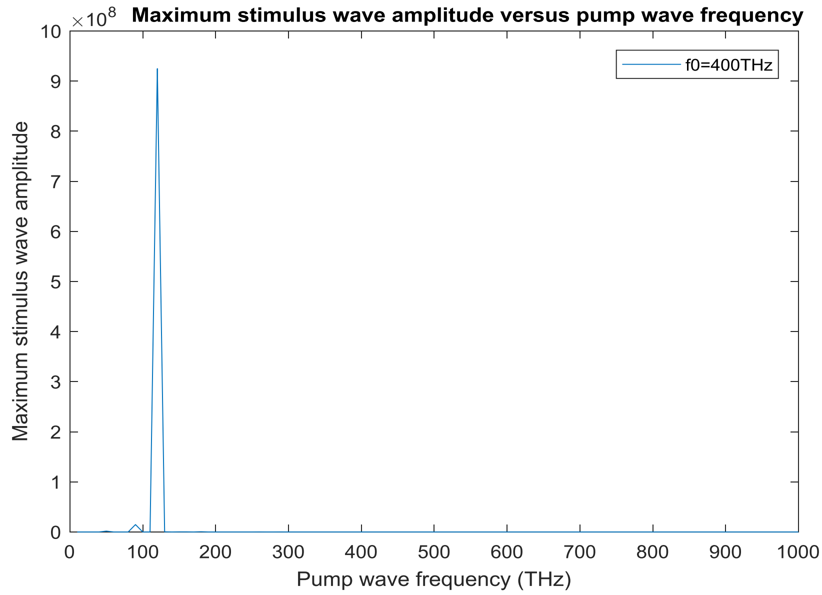

Figure 8. Maximum stimulus wave amplitude between 0<t<30ps, as measured inside the cavity at x=5.73μm, versus the frequency of the pump wave

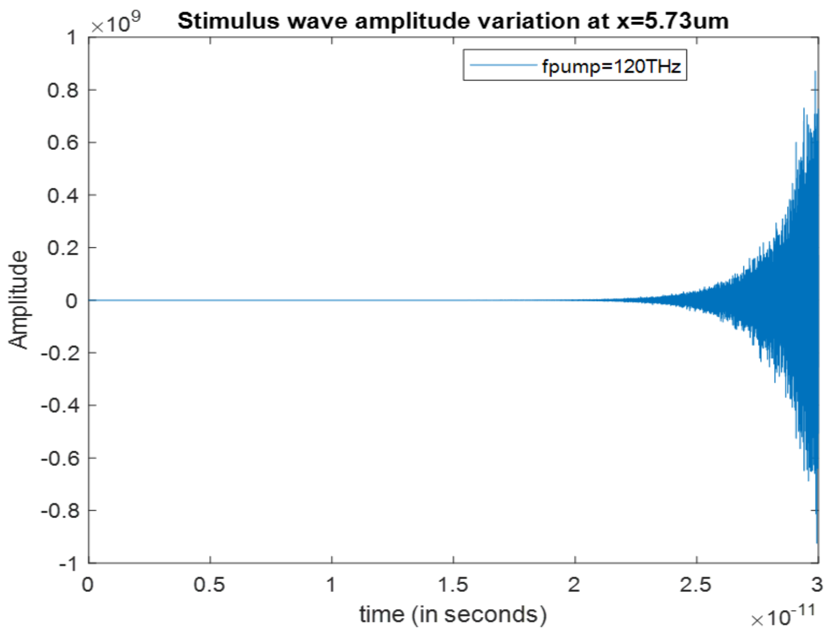

Figure 9. Stimulus wave amplitude variation at x=5.73μm for a pump wave frequency of 120THz

Table 1. Maximum stimulus wave amplitude (gain), maximum intracavity energy density created by the pump wave, maximum intracavity charge polarization density created by the pump wave, versus frequency of the pump wave. Gain maximizing pump wave frequency is indicated in bold

f(THz)	W_e	P_{hp}	$A_{st,max}$
10	24587858	0.073549	29.68782
20	10038549	0.046622	21.16144
30	3.99E+08	0.312706	37.48459
40	44698541	0.102643	8.542033
50	5.32E+08	0.467588	1677835
60	38094201	0.089774	53.50252
70	4.92E+08	0.314052	82.48303
80	38134542	0.090776	99.3708
90	6.04E+08	0.414954	14730996
100	34642254	0.091809	7.464663
110	2.57E+08	0.257834	3414.097
120	**7.09E+08**	**0.478612**	**9.25E+08**
130	4.31E+08	0.405224	260720.9
140	2.09E+08	0.249733	3352.826
150	5.07E+08	0.408981	167951.1
160	5.64E+08	0.393902	158069.7
170	49943526	0.117305	7.338003
180	5.53E+08	0.505682	436140.8
190	21394526	0.082377	83.99106
200	12609984	0.063049	20.24936
210	1.38E+08	0.220572	62.19482
220	1.89E+08	0.255036	279.9376
230	2.12E+08	0.317012	880.1543
240	53238963	0.13555	25.35636
250	1.39E+08	0.260475	1114.391
260	1.44E+08	0.281235	65709.25
270	1.76E+08	0.268166	3008.924
280	1.74E+08	0.267763	9804.008
290	1.78E+08	0.285167	61348.81
300	1.07E+08	0.214681	4015.914
310	62371087	0.194802	22.12135
320	56119926	0.210692	33.94152
330	57819494	0.210913	15.23713
340	2596324	0.044759	17.47628
350	348233.6	0.014213	11.1293
360	239292.5	0.010843	10.84791
370	179574.4	0.00915	10.6363
380	151083.5	0.008218	11.35786
390	111890.1	0.00678	10.74123
400	105920.5	0.006284	10.80236
410	107563.3	0.005577	10.65824
420	70488.12	0.005131	10.63149
430	85887.48	0.004835	10.64494
440	75463.67	0.004673	10.58697
450	59944.63	0.004191	10.55064
460	50390.5	0.003963	10.74716
470	48923.47	0.003791	10.61051
480	45454.85	0.003668	10.58697
490	43511.6	0.003503	10.68148
500	47541.63	0.003327	10.80808

f(THz)	W_e	P_{hp}	$A_{st,max}$
510	39410.43	0.00326	10.73229
520	37676.33	0.003208	10.72071
530	37889.28	0.003121	10.76524
540	35060.39	0.003065	10.70476
550	36954.81	0.002982	10.92333
560	36226.29	0.002944	10.87302
570	38002.19	0.00287	10.93508
580	35558.58	0.002833	10.90827
590	37412.51	0.002794	10.87437
600	36235.02	0.002764	10.92448
610	35652.4	0.00273	11.09937
620	36590.42	0.002712	11.07885
630	37191.04	0.002706	11.1156
640	38061.72	0.002724	11.1168
650	39609.29	0.002772	11.23375
660	41287.29	0.002839	11.31049
670	44207.1	0.002994	11.43198
680	48415.29	0.003224	11.7344
690	54558.22	0.00357	11.83012
700	62270.26	0.00406	12.03565
710	75584.37	0.004891	12.47808
720	107466.8	0.007278	13.18134
730	1009334	0.036879	21.99337
740	2078858	0.046548	33.81698
750	2006865	0.041713	30.761
760	9673896	0.078567	46.73325
770	3255280	0.040102	243.0622
780	2256265	0.030198	35.71366
790	15803535	0.076385	100.6912
800	7697947	0.049781	48.22146
810	13096084	0.059426	159.2424
820	18594988	0.06552	73.16598
830	4725648	0.030918	18.66814
840	4019403	0.026476	19.01043
850	63409966	0.101802	55.50794
860	80506507	0.111711	43.26424
870	91022624	0.113907	67.34349
880	1.27E+08	0.130152	63.39702
890	69961073	0.092079	306.5119
900	5394961	0.022793	27.93713
910	3089991	0.017161	278.0156
920	10570232	0.030793	192.1754
930	1.32E+08	0.119193	1344.705
940	3920668	0.01826	36.3059
950	4451290	0.017481	19.01912
960	66658661	0.075618	407.8365
970	57676989	0.073767	1606.28
980	4332888	0.015759	19.9839
990	4195312	0.016605	31.25524
1000	1.15E+08	0.1073	6115.038

Part 2: Investigating the gain spectrum of the stimulus wave for a pump wave frequency of 120THz

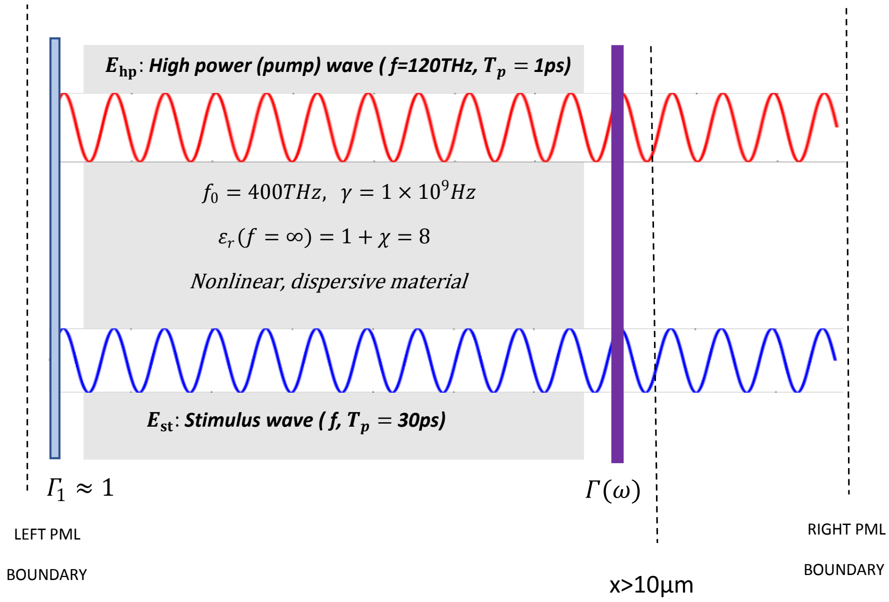

Figure 10. The configuration for computing the gain spectrum of the stimulus wave for $f_{pump} = 120THz$

Our problem: Under an energy maximizing 120THz pump wave excitation, find the maximum stimulus wave amplitude (gain) $|E_{st,max}|$ in the cavity for each stimulus wave frequency $f_{stimulus}$, in the range 10THz < $f_{stimulus} < 1000THz$ (THz to UV), for $\{\, 0\mu m < x < 10\mu m, 0 \le t \le 30ps\}$, such that

$$\nabla^2(E_{hp}) - \mu_0\varepsilon_\infty \frac{\partial^2(E_{hp})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{hp})}{\partial t} + \mu_0\frac{\partial^2 P_{hp}}{\partial t^2}. \tag{7a}$$

$$\frac{\partial^2 P_{hp}}{\partial t^2} + \gamma\frac{\partial P_{hp}}{\partial t} + {\omega_0}^2(P_{hp}) - \frac{{\omega_0}^2}{Ned}(P_{hp})^2 - \frac{{\omega_0}^2}{N^2e^2d^2}(P_{hp})^3 = \frac{Ne^2}{m}(E_{hp}). \tag{7b}$$

$$\nabla^2(E_{st}) - \mu_0\varepsilon_\infty\frac{\partial^2(E_{st})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{st})}{\partial t} + \mu_0\frac{\partial^2(P_{st})}{\partial t^2}. \tag{9a}$$

$$\frac{\partial^2(P_{st})}{\partial t^2} + \gamma\frac{\partial(P_{st})}{\partial t} + {\omega_0}^2(P_{st}) - \frac{{\omega_0}^2}{Ned}\{{P_{st}}^2 + 2P_{st}P_{hp}\}$$
$$-\frac{{\omega_0}^2}{N^2e^2d^2}\{{P_{st}}^3 + 3{P_{st}}^2P_{hp} + 3P_{st}{P_{hp}}^2\} = \frac{Ne^2(E_{st})}{m} \tag{9b}$$

Initial conditions:

$$P_{hp}(x,0) = {P_{hp}}'(x,0) = E_{hp}(x,0) = {E_{hp}}'(x,0) = P_{st}(x,0) = {P_{st}}'(x,0) = E_{st}(x,0) = {E_{st}}'(x,0) = 0$$

Boundary conditions:

$E_{hp}(x = 0\mu m, t) = 2.25 \times 10^8 \times \sin(2\pi(1.2 \times 10^{14})t)\ V/m, for\ 0 \le t \le 1ps, (120THz\,)$

$E_{st}(x = 0\mu m, t) = 1 \times \sin(2\pi(\,f_{stimulus}\,)t)\ V/m, for\ 0 \le t \le 30ps$

$E_{hp}(x = 15\mu m, t) = E_{st}(x = 15\mu m, t) = 0\ for\ 0 < t < 30ps$

Absorbing boundary condition (perfectly matched layer):

$$\sigma(x) = \{\,(x - (L - \Delta))\sigma_0\,, (L - \Delta) \le\ x < L\,\}, for\ L = 15\mu m, \Delta = 2.5\mu m,\ \sigma_0 = 4.5 \times 10^8 S/m$$

Optical isolator condition: Full reflection at x = 0µm

$$\Gamma(x = 0\mu m, t) = 1\ (Reflection\ coefficient\ is\ equal\ to\ 1)$$

Switch controlled optical bandpass filter condition: Full reflection at x = 10µm for t≤30 picoseconds, frequency dependent reflection at x = 10µm after t=30 picoseconds. For a given stimulus wave frequency (f), the magnitude frequency response of the filter is chosen to be

$$|\Gamma(f')| = \begin{cases} 1\ for\ all\ f'\ , for\ x = 10\mu m, 0 \le t \le 30ps \\ \\ 1 - e^{-\left(\frac{(f' - f)}{\sqrt{2}THz}\right)^2}\ , for\ x = 10\mu m, t > 30ps \end{cases}$$

The amplification spectrum of the stimulus wave for $f_{pump} = 120THz$ is tabulated below in Table 2. The stimulus wave is supplied to the cavity at t=0 as a quasi-monochromatic wave. For a given initial (at t=0) stimulus wave frequency, the center frequency of the band-pass filter is adjusted to be at the same frequency with the initial stimulus wave frequency. By doing so, we can observe how much gain can be obtained from the cavity for each initial stimulus wave frequency. We choose the pump wave frequency as $f = 120THz$ as it maximizes the amplification, and we sweep the stimulus wave frequency from 10THz to 1000THz in 10THz increments.

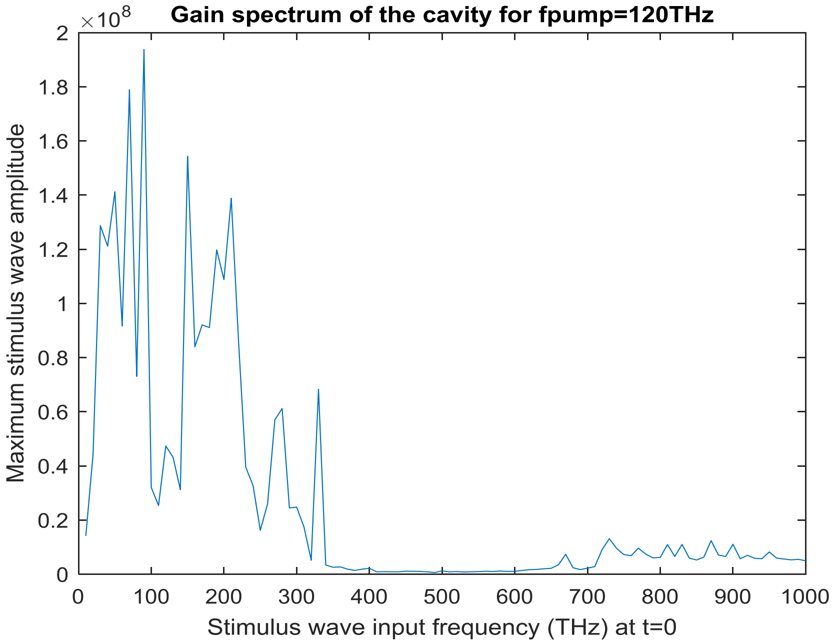

Figure 11. Gain spectrum of the stimulus wave for $f_{pump} = 120THz$ and $f_0 = 400THz$

Table 2. Gain spectrum of the stimulus wave for $f_{pump} = 120THz$ and $f_0 = 400THz$

f(THz)	Gain	f(THz)	Gain	f(THz)	Gain	f(THz)	Gain	f(THz)	Gain
10	14124439	210	1.39E+08	410	875477.2	610	1393617	810	10940196
20	44346785	220	86250208	420	932991.1	620	1670565	820	6544280
30	1.29E+08	230	39455150	430	906554.7	630	1767159	830	10986241
40	1.21E+08	240	32766548	440	886455.3	640	2001623	840	5927739
50	1.41E+08	250	16232796	450	1104075	650	2180135	850	5260996
60	91652199	260	25982193	460	1085080	660	3459366	860	6291252
70	1.79E+08	270	57040620	470	1025036	670	7338895	870	12397758
80	73052298	280	61175896	480	875256.9	680	2399296	880	7090064
90	1.94E+08	290	24477518	490	624327.7	690	1685534	890	6536399
100	32041383	300	24763425	500	1231953	700	2243919	900	11063978
110	25362086	310	17486470	510	863688.6	710	2802797	910	5716487
120	47337188	320	5162716	520	974822.8	720	9071625	920	6974320
130	43165734	330	68281614	530	814416.5	730	13108154	930	5842117
140	31231630	340	3418249	540	887631.1	740	9529488	940	5736888
150	1.54E+08	350	2548473	550	971105.8	750	7241604	950	8216205
160	83973170	360	2697356	560	1090509	760	6847708	960	5951542
170	92068095	370	1881945	570	994421.7	770	9602161	970	5608217
180	91082666	380	1398025	580	1164989	780	7425494	980	5305001
190	1.2E+08	390	1865132	590	1031798	790	6026434	990	5486357
200	1.09E+08	400	2195218	600	1063269	800	6214589	1000	4972009

Simulation 2:

Part 1: Sweeping the pump wave frequency to maximize the intracavity energy density

Assume that a 300THz infrared stimulus wave E_{st} and a high power pump wave E_{hp} (frequency to be determined) are propagating inside a low-loss (high Q) cavity that has two reflecting walls. The reflecting wall on the left side can be thought as an optical isolator and has a reflection coefficient of $\Gamma_1 \approx 1$, the one on the right side represents a switch controlled optical band-pass filter with a frequency dependent reflection coefficient $\Gamma(f)$. Both waves are generated at x=0μm and at the time instant t=0. The waves and the parameters of the gain medium are as given below:

$E_{hp}(x = 0\mu m, t) = 5 \times 10^8 \times \sin\big(2\pi\big(f_{pump}\big)t\big)\ V/m, for\ 0 \le t \le 1ps\ (Ultrashort\ pulse)$

$E_{st}(x = 0\mu m, t) = 1 \times \sin(2\pi(3 \times 10^{14})t)\ V/m, for\ 0 \le t \le 10ps$

$Dielectric\ constant\ of\ the\ gain\ medium\ (\varepsilon_\infty) = 10\ (\mu_r = 1)$

$Resonance\ frequency\ of\ the\ gain\ medium =\ f_0 = 800THz$

$Damping\ coefficient\ of\ the\ gain\ medium: \gamma = 1 \times 10^7 Hz$

$Time\ interval\ and\ duration\ of\ simulation: 0 \le t \le 10ps$

$Spatial\ range\ of\ the\ gain\ medium: 0\mu m < x < 10\mu m$

$Right\ cavity\ wall\ location: x = 10\mu m;\ Left\ cavity\ wall\ location: x = 0\mu m$

$Electron\ density\ of\ the\ gain\ medium: N = 3.5 \times 10^{28}/m^3;\ Atomic\ diameter: d = 0.3\ nm$

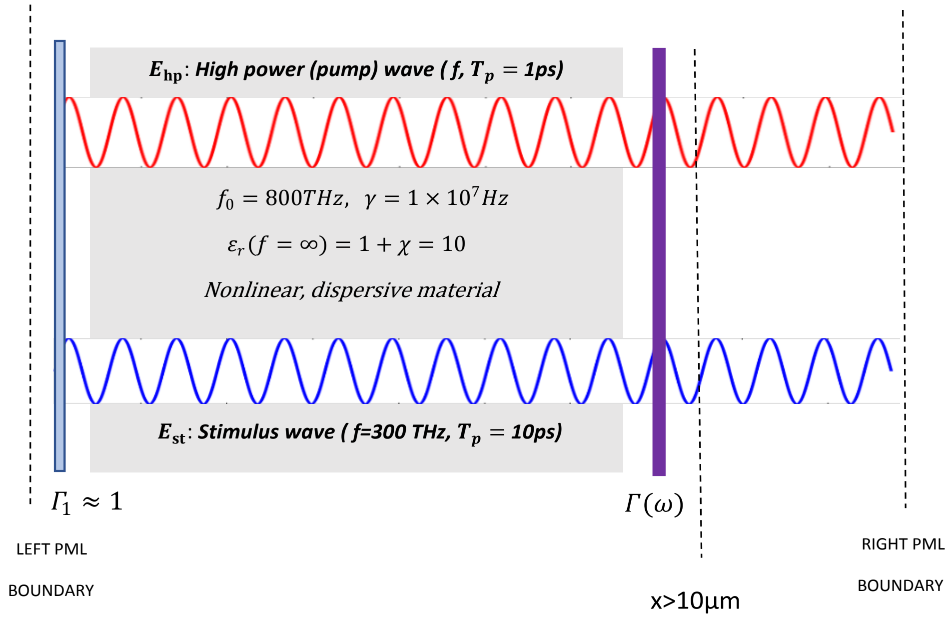

Figure 12. Configuration of the cavity and the dielectric material specifications for simulation2-part1

During the whole simulation time, $\Gamma(f) = 1$ for all f. The filter is used for post-processing of the results.
Our problem: Find the optimum pump wave frequency f_{pump} that maximizes $|E_{st}|$ in the cavity, for 10THz < f_{pump} $< 1000THz$ (THz to UV), for $0\mu m < x < 10\mu m, 0 \le t \le 10ps$, such that

$$\nabla^2(E_{hp}) - \mu_0\varepsilon_\infty \frac{\partial^2(E_{hp})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{hp})}{\partial t} + \mu_0\frac{\partial^2 P_{hp}}{\partial t^2}. \quad (7a)$$

$$\frac{\partial^2 P_{hp}}{\partial t^2} + \gamma\frac{\partial P_{hp}}{\partial t} + {\omega_0}^2(P_{hp}) - \frac{{\omega_0}^2}{Ned}(P_{hp})^2 - \frac{{\omega_0}^2}{N^2e^2d^2}(P_{hp})^3 = \frac{Ne^2}{m}(E_{hp}). \quad (7b)$$

$$\nabla^2(E_{st}) - \mu_0\varepsilon_\infty \frac{\partial^2(E_{st})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{st})}{\partial t} + \mu_0\frac{\partial^2(P_{st})}{\partial t^2}. \quad (9a)$$

$$\frac{\partial^2(P_{st})}{\partial t^2} + \gamma\frac{\partial(P_{st})}{\partial t} + {\omega_0}^2(P_{st}) - \frac{{\omega_0}^2}{Ned}\{{P_{st}}^2 + 2P_{st}P_{hp}\}$$
$$-\frac{{\omega_0}^2}{N^2e^2d^2}\{{P_{st}}^3 + 3{P_{st}}^2P_{hp} + 3P_{st}{P_{hp}}^2\} = \frac{Ne^2(E_{st})}{m} \quad (9b)$$

Initial conditions:

$$P_{hp}(x,0) = P_{hp}{'}(x,0) = E_{hp}(x,0) = E_{hp}{'}(x,0) = P_{st}(x,0) = P_{st}{'}(x,0) = E_{st}(x,0) = E_{st}{'}(x,0) = 0$$

Boundary and excitation conditions:

$$E_{hp}(x = 0\mu m, t) = 5 \times 10^8 \times \sin\left(2\pi\left(f_{pump}\right)t\right)\ V/m, for\ 0 \le t \le 1ps\ (Ultrashort\ pulse)$$

$$E_{st}(x = 0\mu m, t) = 1 \times \sin(2\pi(3 \times 10^{14})t)\ V/m, for\ 0 \le t \le 10ps$$

$$E_{hp}(x = 15\mu m, t) = E_{st}(x = 15\mu m, t) = 0\ for\ 0 < t < 10ps$$

Absorbing boundary condition (perfectly matched layer):

$$\sigma(x) = \{\, (x - (L - \Delta))\sigma_0\ , (L - \Delta) \le\ x < L \,\}, for\ L = 15\mu m, \Delta = 2.5\mu m,\ \sigma_0 = 4.5 \times 10^8 S/m$$

Optical isolator condition: Full reflection at $x = 0\mu m$

$$\Gamma(x = 0\mu m, t) = 1\ (Reflection\ coefficient\ is\ equal\ to\ 1)$$

Switch controlled optical bandpass filter condition: Full reflection at $x = 10\mu m$ for t≤10 picoseconds, frequency dependent reflection at $x = 10\mu m$ after t=10 picoseconds;

$$|\Gamma(f')| = \begin{cases} 1\ for\ all\ f'\ , for\ x = 10\mu m, 0 \le t \le 10ps \\ 1 - e^{-\left(\frac{(f'-f)}{\sqrt{2}THz}\right)^2}\ , for\ x = 10\mu m, t > 10ps \end{cases}$$

If amplified, the stimulus wave at t=10 picoseconds will not be monochromatic anymore, this will be due to the spectral broadening inside the cavity. However, by adjusting the center frequency of the band-pass filter to be 300THz, we will get an amplified quasi-monochromatic output. The pump wave frequency will be varied from 10THz to 1000THz in 10THz increments and a pump wave frequency that yields a strong amplification of the stimulus wave will be chosen. The electric energy density W_e and the charge polarization density P_{hp} created by the pump wave, are plotted with respect to the pump wave frequency in Fig. 13 and Fig. 14 respectively. Stimulus wave amplitude gain versus pump wave frequency plot is shown in Fig. 15.

The maximum amplitude of the stimulus wave, versus pump wave frequency plot is shown in Fig. 15. Let us investigate the major amplification peak of the stimulus wave at $f = 350THz$. The peak polarization density P_{hp} created by the pump wave (which acts as the coupling coefficient), is high at this frequency. The peak electric energy density W_e created by the pump wave is also high at $f = 350THz$. Therefore we have an amplification peak at $f = 350THz$. The same is true for all other peaks. If we have a look at Table 3, wherever the electric energy density and the polarization density created by the pump wave, are high, there is a stronger stimulus wave amplification.

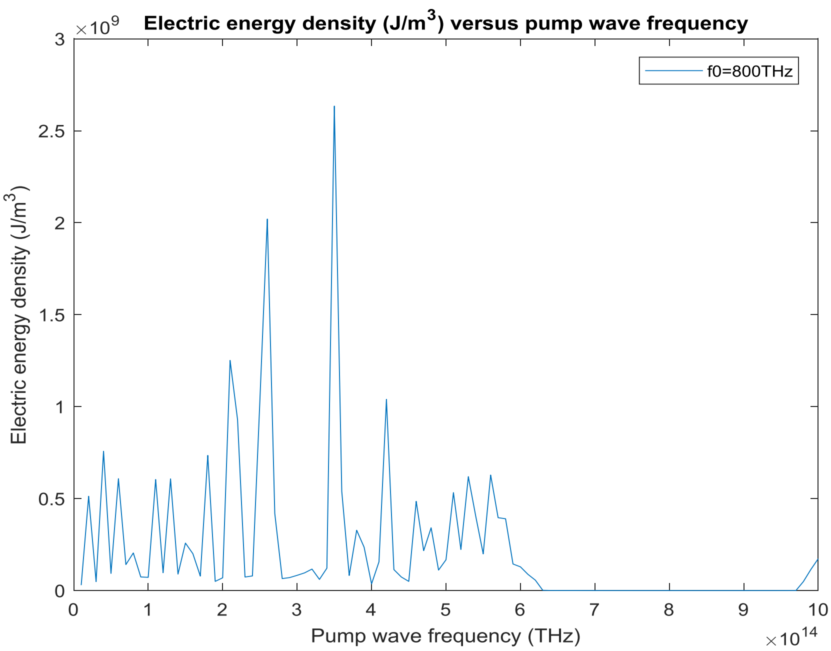

Figure 13. Maximum electric energy density created by the pump wave (for 0<t<10ps), as measured inside the cavity at x=5.73µm, versus the frequency of the pump wave

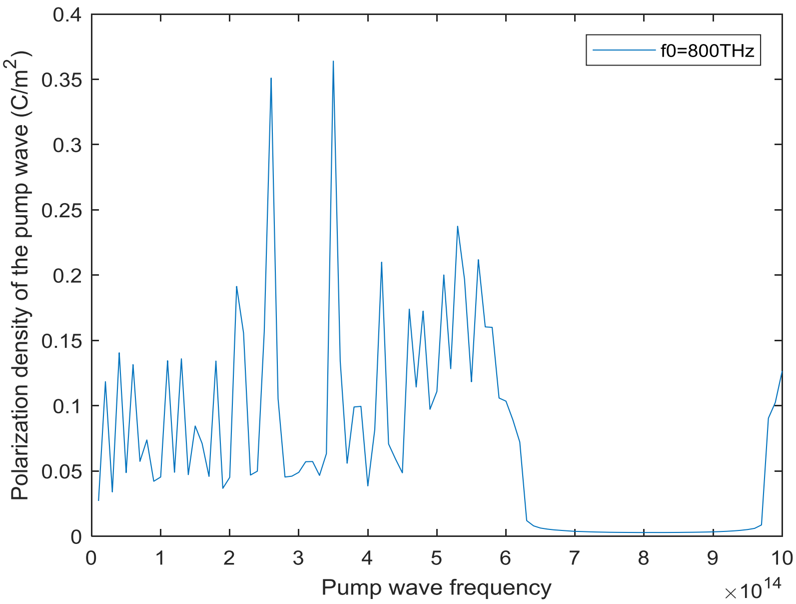

Figure 14. Maximum charge polarization density created by the pump wave (for 0<t<10ps), as measured inside the cavity at x=5.73μm, versus the frequency of the pump wave

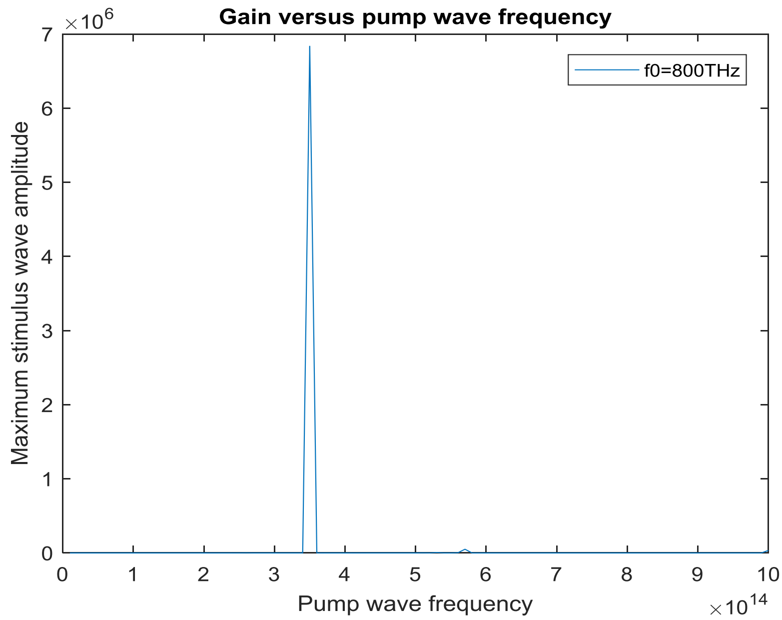

Figure 15. Maximum stimulus wave amplitude between 0<t<10ps, as measured inside the cavity at x=5.73μm, versus the frequency of the pump wave

Since the pump wave frequency of $f = 350THz$ maximizes the intra-cavity energy and amplifies the stimulus wave, we choose this frequency as the frequency of pump wave excitation in order to compute the amplification (gain) spectrum of the stimulus wave. However, as already mentioned, the amplified stimulus wave is not monochromatic anymore due to the spectral broadening inside the cavity. Therefore, we must make a seperate analysis to obtain the gain spectrum of the stimulus wave, under a 350THz pump wave excitation.

$W_e: Maximum\ electric\ energy\ density\ created\ by\ the\ pump\ wave\ in\ the\ cavity$

$P_{hp}: Maximum\ charge\ polarization\ density\ created\ by\ the\ pump\ wave\ in\ the\ cavity$

$A_{st,max}: Maximum\ stimulus\ wave\ amplitude\ in\ the\ cavity$

Table 3. Maximum stimulus wave amplitude (gain), maximum intracavity energy density created by the pump wave, maximum intracavity charge polarization density created by the pump wave, versus frequency of the pump wave. Gain maximizing pump wave frequency is indicated in bold

f(THz)	$A_{st,max}$	W_e	P_{hp}
10	5.883986	29699657	0.027091
20	13.42542	5.13E+08	0.118397
30	12.99005	48849682	0.033976
40	9.096766	7.58E+08	0.14055
50	17.10611	93371276	0.048789
60	4.474521	6.09E+08	0.131476
70	5.069646	1.42E+08	0.057396
80	3.561172	2.05E+08	0.073801
90	2.641755	74279614	0.042153
100	4.077828	72125541	0.045394
110	8.999627	6.04E+08	0.13448
120	2.613823	97309728	0.049051
130	3.933646	6.07E+08	0.135959
140	2.685399	89828810	0.047224
150	2.446767	2.58E+08	0.08447
160	2.277283	2.01E+08	0.071119
170	2.568835	78977510	0.045889
180	3.640279	7.35E+08	0.134221
190	3.496944	50155368	0.036736
200	5.209949	69704490	0.045108
210	39.38512	1.25E+09	0.19136
220	17.4777	9.3E+08	0.155742
230	3.781144	73950854	0.046904
240	6.847807	79718064	0.049873
250	66.4612	1.03E+09	0.156807
260	1354.073	3.03E+09	0.350968
270	14.35446	4.2E+08	0.105385
280	13.4472	65372520	0.045401
290	6.311408	70860412	0.046055
300	2.760757	83057212	0.048992
310	2.27222	96378435	0.057127
320	2.685882	1.17E+08	0.057266
330	2.403513	60937048	0.046658
340	1.907491	1.22E+08	0.06329
350	**6835947**	**2.2E+09**	**0.36398**
360	51.09379	5.4E+08	0.134201
370	2.244881	82294755	0.055986
380	4.045674	3.28E+08	0.099051
390	2.385328	2.36E+08	0.099527
400	1.914928	36407088	0.038589
410	2.245768	1.57E+08	0.081641
420	60.8144	1.04E+09	0.209959
430	2.316941	1.14E+08	0.070736
440	2.424293	74048560	0.059251
450	1.903607	50132227	0.048627
460	22.94166	4.85E+08	0.173972
470	2.353593	2.17E+08	0.114367
480	11.76979	3.42E+08	0.172466
490	7.203818	1.12E+08	0.097311
500	7.889092	1.68E+08	0.110958

f(THz)	Gain	We	Php
510	1211.665	5.33E+08	0.200157
520	12.03636	2.24E+08	0.128374
530	6380.668	6.2E+08	0.237481
540	197.2572	4.04E+08	0.196768
550	2.228608	2E+08	0.118366
560	3341.632	6.28E+08	0.211868
570	50526.02	3.97E+08	0.160384
580	1.952741	3.91E+08	0.160039
590	1.836473	1.45E+08	0.105914
600	2.067759	1.29E+08	0.103506
610	2.159113	88756343	0.089169
620	7.876075	56936698	0.072166
630	2.014397	1719389	0.012014
640	2.002486	867505.3	0.008003
650	1.99723	594922.3	0.006319
660	1.992377	454115.5	0.005502
670	1.992079	380584.9	0.004912
680	1.99609	327444.1	0.004525
690	1.992056	281248.6	0.004147
700	1.994624	254086.5	0.003786
710	2.003631	239090.8	0.003631
720	2.008237	224470.1	0.003453
730	2.014686	212223.1	0.003299
740	2.08819	194997.7	0.00319
750	2.020707	189412.7	0.003079
760	2.026132	182209.2	0.003019
770	2.031172	171364.8	0.002977
780	2.106717	167320.3	0.002944
790	2.062642	160819.4	0.002907
800	2.123964	154567	0.00289
810	2.074114	150234.4	0.002923
820	2.043318	146722.7	0.002912
830	2.033978	142738.8	0.00291
840	2.02517	137337.5	0.002947
850	2.07065	135352.4	0.002973
860	2.042151	133736	0.003041
870	2.013683	130354.1	0.003091
880	2.048405	129162.4	0.003179
890	1.986717	133554.1	0.003284
900	2.000912	135280.2	0.003413
910	1.981226	141468.3	0.003604
920	2.02185	150727.1	0.003847
930	2.017607	161806.4	0.004165
940	2.009308	175362.1	0.004584
950	2.004022	198713	0.005175
960	2.00816	238825.9	0.006067
970	2.008864	307733	0.008694
980	122.4469	48819069	0.090452
990	404.8945	1.14E+08	0.10242
1000	34391.77	1.74E+08	0.126597

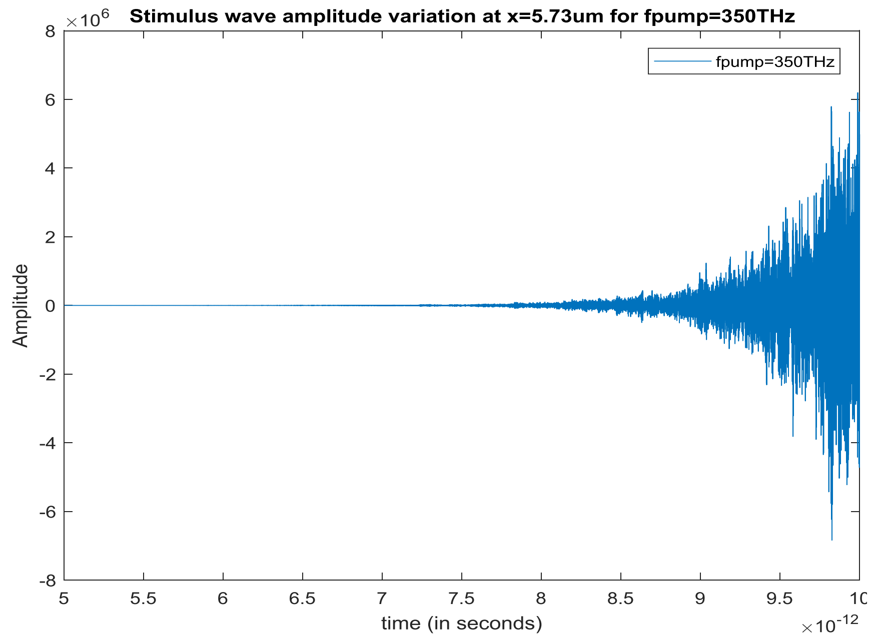

Figure 16. Stimulus wave amplitude variation at x=5.73μm for a pump wave frequency of 350THz

Part 2: Investigating the gain spectrum of the stimulus wave for a pump wave frequency of 350THz

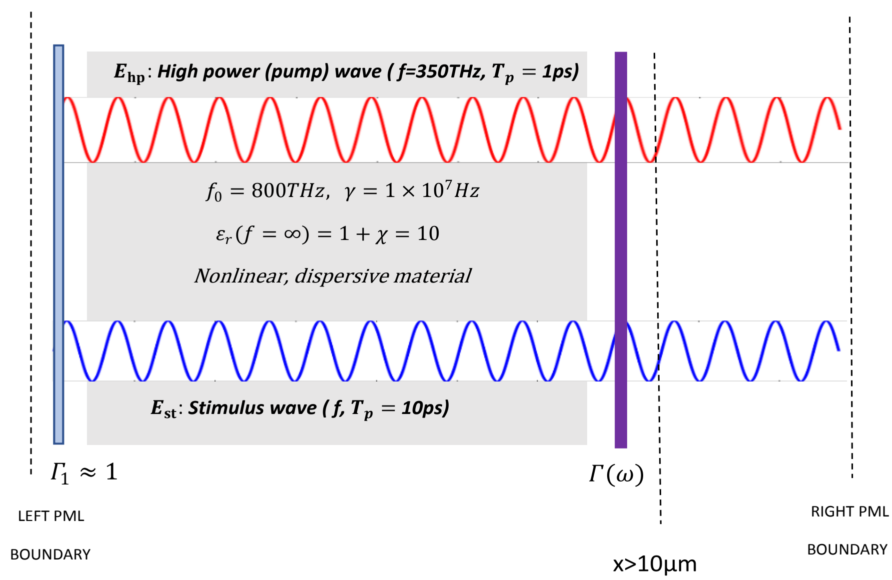

Figure 17. The configuration for computing the gain spectrum of the stimulus wave for $f_{pump} = 350THz$

Our problem: Under an energy maximizing 350THz pump wave excitation, find the maximum stimulus wave amplitude (gain) $|E_{st,max}|$ in the cavity for each stimulus wave frequency $f_{stimulus}$, in the range 10THz < $f_{stimulus} < 1000THz$ (THz to UV), for $\{ 0\mu m < x < 10\mu m, 0 \le t \le 10ps\}$, such that

$$\nabla^2(E_{hp}) - \mu_0\varepsilon_\infty\frac{\partial^2(E_{hp})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{hp})}{\partial t} + \mu_0\frac{\partial^2 P_{hp}}{\partial t^2}. \tag{7a}$$

$$\frac{\partial^2 P_{hp}}{\partial t^2} + \gamma\frac{\partial P_{hp}}{\partial t} + \omega_0{}^2(P_{hp}) - \frac{\omega_0{}^2}{Ned}(P_{hp})^2 - \frac{\omega_0{}^2}{N^2e^2d^2}(P_{hp})^3 = \frac{Ne^2}{m}(E_{hp}). \tag{7b}$$

$$\nabla^2(E_{st}) - \mu_0\varepsilon_\infty\frac{\partial^2(E_{st})}{\partial t^2} = \mu_0\sigma\frac{\partial(E_{st})}{\partial t} + \mu_0\frac{\partial^2(P_{st})}{\partial t^2}. \tag{9a}$$

$$\frac{\partial^2(P_{st})}{\partial t^2} + \gamma\frac{\partial(P_{st})}{\partial t} + \omega_0{}^2(P_{st}) - \frac{\omega_0{}^2}{Ned}\{P_{st}{}^2 + 2P_{st}P_{hp}\}$$
$$-\frac{\omega_0{}^2}{N^2e^2d^2}\{P_{st}{}^3 + 3P_{st}{}^2P_{hp} + 3P_{st}P_{hp}{}^2\} = \frac{Ne^2(E_{st})}{m} \tag{9b}$$

Initial conditions:

$$P_{hp}(x,0) = P_{hp}{}'(x,0) = E_{hp}(x,0) = E_{hp}{}'(x,0) = P_{st}(x,0) = P_{st}{}'(x,0) = E_{st}(x,0) = E_{st}{}'(x,0) = 0$$

Boundary and excitation conditions:

$E_{hp}(x = 0\mu m, t) = 5 \times 10^8 \times \sin(2\pi(3.5 \times 10^{14})t)\ V/m, for\ 0 \le t \le 1ps, (350THz\ )$

$E_{st}(x = 0\mu m, t) = 1 \times \sin(2\pi(\ f_{stimulus}\ )t)\ V/m, for\ 0 \le t \le 10ps$

$E_{hp}(x = 15\mu m, t) = E_{st}(x = 15\mu m, t) = 0\ for\ 0 < t < 10ps$

Absorbing boundary condition (perfectly matched layer):

$$\sigma(x) = \{\ (x - (L - \Delta))\sigma_0\ , (L - \Delta) \le\ x < L\ \}, for\ L = 15\mu m, \Delta = 2.5\mu m,\ \sigma_0 = 4.5 \times 10^8 S/m$$

Optical isolator condition: Full reflection at $x = 0\mu m$

$$\Gamma(x = 0\mu m, t) = 1\ (Reflection\ coefficient\ is\ equal\ to\ 1)$$

Switch controlled optical bandpass filter condition: Full reflection at $x = 10\mu m$ for t≤10 picoseconds, frequency dependent reflection at $x = 10\mu m$ after t=10 picoseconds. For a given stimulus wave frequency (f), the magnitude frequency response of the filter is chosen to be

$$|\Gamma(f')| = \begin{cases} 1\ for\ all\ f'\ , for\ x = 10\mu m, 0 \le t \le 10ps \\ 1 - e^{-(\frac{(f'-f)}{\sqrt{2}THz})^2}\ , for\ x = 10\mu m, t > 10ps \end{cases}$$

Table 4. Gain spectrum of the stimulus wave for $f_{pump} = 350THz$ and $f_0 = 800THz$

f_{st} (THz)	Gain	f_{st} (THz)	Gain	f_{st} (THz)	Gain	f_{st} (THz)	Gain
10	155844.9	260	1501566	510	520598.5	760	39215.57
20	798352.4	270	269297.9	520	220764.7	770	15594.62
30	31826.07	280	39901.23	530	452107.7	780	38361.46
40	34628	290	77451.99	540	833534.4	790	16695.09
50	142339.1	300	167516	550	2134640	800	5713.115
60	94490.95	310	72228.5	560	1208588	810	7659.54
70	318353.9	320	288013	570	32903344	820	5866.713
80	282794.1	330	1451155	580	9814840	830	8579.433
90	57909.77	340	3397976	590	10268905	840	4303.018
100	224267.3	350	5710834	600	863060.1	850	6155.742
110	2894406	360	169142.8	610	636619.8	860	9586.705
120	150805.7	370	168433	620	226696.8	870	6767.037
130	117805.1	380	154531.2	630	59811.9	880	5462.442
140	60393.88	390	236854.6	640	13603.53	890	41621.51
150	35616.27	400	511870.7	650	61635.65	900	49452.8
160	21939.82	410	425733.8	660	27506.4	910	25727.11
170	17310.18	420	16540260	670	21122.59	920	269359.6
180	67314.37	430	18839603	680	33171.2	930	1753751
190	34687.58	440	9002002	690	14305.75	940	77485.81
200	28112.21	450	515299.4	700	6754.221	950	33980.92
210	153257.3	460	1223315	710	14424.94	960	60528.96
220	308297.8	470	293196.6	720	21193.08	970	83267.03
230	50972.36	480	72246.41	730	12675.06	980	393314.5
240	259557.2	490	40842.28	740	10614.65	990	590517.8
250	41309.18	500	143052.1	750	5628.318	1000	1808326

The amplification spectrum of the stimulus wave for $f_{pump} = 350THz$ is tabulated below in Table 4. The stimulus wave is supplied to the cavity at t=0 as a quasi-monochromatic wave. For a given initial (at t=0) stimulus wave frequency, the center frequency of the band-pass filter is adjusted to be at the same frequency with the initial stimulus wave frequency. By doing so, we can observe how much gain can be obtained from the cavity for each stimulus wave frequency. We choose the pump wave frequency as $f = 350THz$ as it maximizes the amplification, and we sweep the stimulus wave frequency from 10THz to 1000THz in 10THz increments.

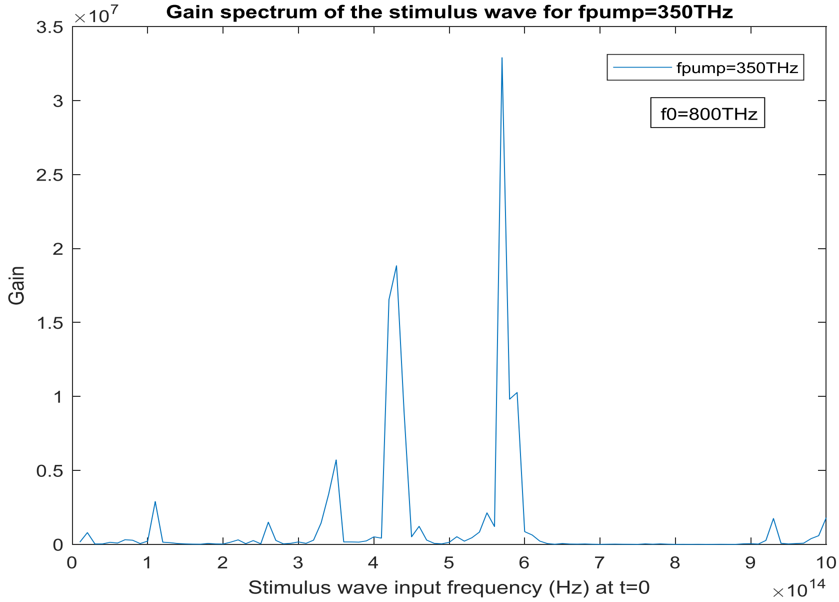

Figure 18. Gain spectrum of the stimulus wave for $f_{pump} = 350THz$ and $f_0 = 800THz$

5. Conclusions

In order to amplify a low power stimulus wave via nonlinear wave mixing in a low-loss optical microcavity, a high intracavity energy is required. As a summary for intracavity energy maximization and efficient wave amplification, the following requirements should be met:

- The cavity should have a high quality (Q) factor.
- For a given resonance frequency (f_0) of the material, the high power pump wave frequency f_{pump} must be adjusted to maximize the electric energy density inside the cavity.

Once these requirements are satisfied, it is possible to amplify a low power input wave, with a very large gain coefficient, by mixing it with an intense pump wave of very short duration, in a wide range of frequencies, inside a low-loss optical microcavity.

6. Appendix

6.1. Validation of the Computational Model and Results

In this section, we will compare our finite difference time domain based computational model with the theoretical formula in the well-established context of *sum frequency generation via nonlinear wave mixing*, in the following example.

Example: Nonlinear sum frequency generation (frequency upconversion)

This example is about the generation of a higher frequency component (ω_3), *by mixing of two monochromatic waves with frequencies* ω_1 *and* ω_2, *such that* $\omega_3 = \omega_2 + \omega_1$. *In order to achieve this, at least one of the waves must have a high intensity, so that nonlinearity arises, and wave mixing occurs.*

The high amplitude pump wave $\boldsymbol{E_2}$ is generated at x=2.5μm. It has an amplitude of A_2V/m and a frequency of 180THz.

$\boldsymbol{E_2}(x = 2.5\mu m, t) = A_2 \times \sin(2\pi(1.8 \times 10^{14})t + \varphi_2)\ V/m$

The input wave $\boldsymbol{E_1}$ is generated at x=2.5μm. It has an amplitude of A_1V/m and a frequency of 120THz.

$\boldsymbol{E_1}(x = 2.5\mu m, t) = A_1 \times \sin(2\pi(1.2 \times 10^{14})t + \varphi_1)\ V/m$

$For\ simplicity, assume\ that\ \varphi_1 = 0, \varphi_2 = 0$

Range of independent simulation variables: $0 \leq x \leq 10\mu m, 0 \leq t \leq 60ps$

$Resonance\ frequency\ of\ the\ interaction\ medium\!: f_0 = 1.1 \times 10^{15} Hz$

$Damping\ coefficient\ of\ the\ interaction\ medium: \gamma = 1 \times 10^{12} Hz$
$Dielectric\ constant\ of\ the\ interaction\ medium\ (\varepsilon_\infty) = 1 + \chi = 10\ (\mu_r = 1)$
$Left\ perfectly\ matched\ layer\ (left\ absorption\ layer)\ is\ from\ x = 0\ to\ x = 2.25\mu m$
$Right\ perfectly\ matched\ layer\ (right\ absorption\ layer)\ is\ from\ x = 7.75\mu m\ to\ x = 10\mu m$

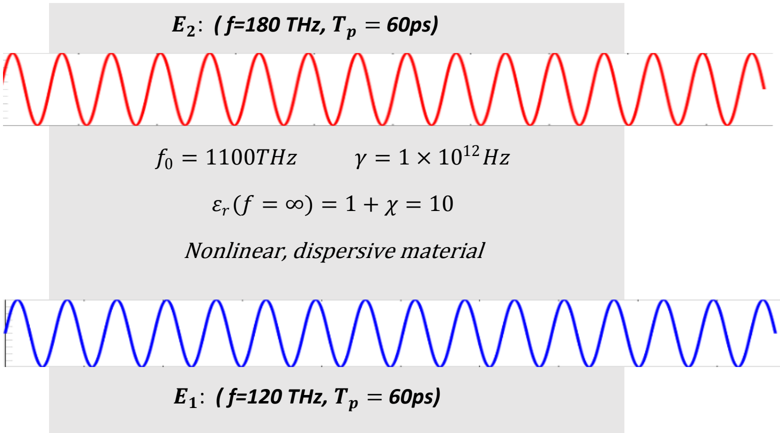

Figure 19. Configuration for frequency upconversion

The theoretical formula for frequency upconversion efficiency, which is derived from the solution of nonlinear wave equation that is based on material nonlinearity coefficient, is given as [1,4]

$$\eta_{theoretical} = \frac{\omega_3}{\omega_2}\left(\sin\sqrt{2d^2 n^3 \omega_3{}^2 (cn\varepsilon_0 A_2{}^2) L^2}\ \right)^2 = \frac{\omega_3}{\omega_2}\left(\sin\sqrt{2d^2 n^4 \omega_3{}^2 c\varepsilon_0 A_2{}^2 L^2}\ \right)^2 \quad (7)$$

$\omega_2 = Frequency\ of\ the\ pump\ wave, \omega_1 = Frequency\ of\ the\ input\ wave$

d= Material nonlinearity coefficient, n= Refractive index

A_2= Pump wave amplitude, A_1= Input wave amplitude, L= Length of the nonlinear media

$\omega_3 = \omega_1 + \omega_2 = Frequency\ of\ the\ upconverted\ wave$

Our computational model is based on the finite difference time domain discretization of the nonlinear electron motion equation that involves the resonance frequency and the damping coefficient of the interaction medium. Coupled with the wave equation, the total wave $E = E_1 + E_2$ can be evaluated from:

$$\frac{E(i+1,j)-2E(i,j)+E(i-1,j)}{\Delta x^2} - \mu_0\varepsilon_\infty(i,j)\frac{E(i,j+1)-2E(i,j)+E(i,j-1)}{\Delta t^2} = \mu_0\sigma(i,j)\frac{E(i,j)-E(i,j-1)}{\Delta t} + \mu_0\frac{P(i,j+1)-2P(i,j)+P(i,j-1)}{\Delta t^2}. \quad (8a)$$

$$\frac{P(i,j+1)-2P(i,j)+P(i,j-1)}{\Delta t^2} + \gamma\frac{P(i,j)-P(i,j-1)}{\Delta t} + \omega_0{}^2\big(P(i,j)\big) - \frac{\omega_0{}^2}{Ned}\big(P(i,j)\big)^2 - \frac{\omega_0{}^2}{N^2e^2d^2}\big(P(i,j)\big)^3 = \frac{Ne^2}{m}\big(E(i,j)\big). \quad (8b)$$

For a time interval of $0 \le t \le t_{max}$, the computational formula for *frequency upconversion efficiency* is

$$\eta_{computational} = \frac{\text{Intensity of the } \omega_3 \text{ frequency component of the total wave at } t{=}t_{max}}{\text{Intensity of the } \omega_2 \text{ frequency component of the total wave at } t{=}0} \quad (9)$$

In this example, we have used the following values for each efficiency formula

$\omega_2 = Frequency\ of\ the\ pump\ wave = (2\pi \times 180)THz\ \omega_1 = Frequency\ of\ the\ input\ wave = (2\pi \times 120)THz$

L =Length of the nonlinear dispersive media=3.33 micrometers (from x=3.33µm to 6.66µm)

$\omega_3 = Frequency\ of\ the\ upconverted\ wave = 2\pi \times 300THz$, n= $Refractive\ index = \sqrt{10}$

d = *Material nonlinearity coefficient* = 6.3×10^{-22} (The theoretical and the computational results agree for this value of d for a sample pump wave amplitude of $A_2 = 10^9 V/m$. Our aim is to see if the results also agree for all the other pump wave amplitudes for this value of d)

A_2=Pump wave amplitude (varied from $5 \times 10^7 V/m$ to $2 \times 10^9 V/m$)

A_1=Input wave amplitude = $A_2/100$ $(A_1 \ll A_2)$

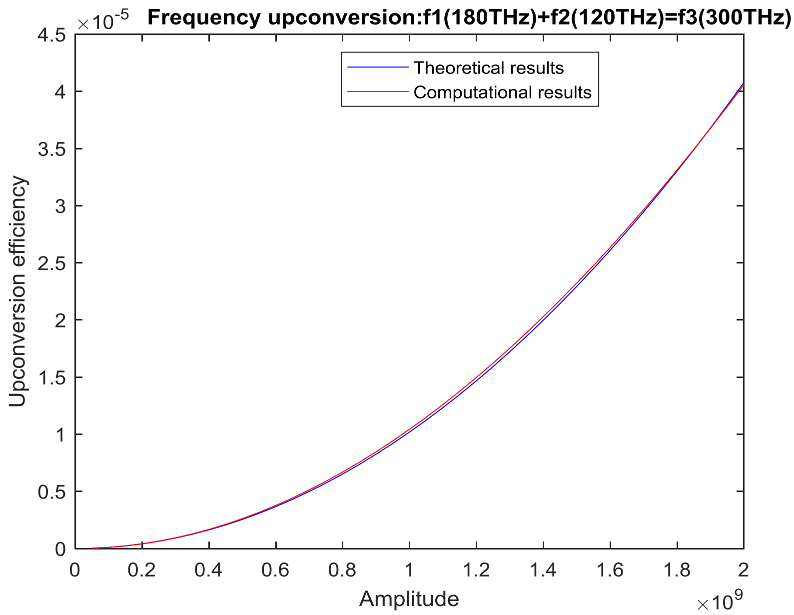

Figure 20. Comparison of the frequency upconversion efficiencies for f_3=300THz and d= 6.3×10^{-22}, versus the pump wave amplitude

REFERENCES

[1] Boyd Robert. W., Nonlinear Optics, Academic Press, New York, 2008.

[2] Fox Mark, Optical properties of solids, Oxford University Press, New York, 2002.

[3] Balanis Constantine. A., Advanced Engineering Electromagnetics, John Wiley & Sons, New York, 1989.

[4] Bahaa E. A. Saleh, Malvin Carl Teich, Fundamentals of Photonics, Wiley-Interscience, New York, 2007.

[5] Silfvast William.T., Laser Fundamentals, Cambridge University Press, New York, 2004.

[6] Junkichi Satsuma, Nobuo Yajima, "Initial Value Problems of One-Dimensional Self-Modulation of Nonlinear Waves in Dispersive Media", *Progress of Theoretical Physics Supplement*, Volume 55, January 1974.

[7] Taflove Allen, Hagness Susan.C., Computational Electrodynamics: The Finite-Difference Time-Domain Method, Artech House, Boston, 2005.

[8] Amit S. Nagra, Robert A. York, "FDTD Analysis of Wave Propagation in Nonlinear Absorbing and Gain Media", IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 46, NO. 3, MARCH 1998.

[9] Murray K. Reed, Michael K. Steiner-Shepard, Michael S. Armas, Daniel K. Negus, "Microjoule-energy ultrafast optical parametric amplifiers", Journal of the Optical Society of America B, Volume 12, Issue 11, 1995.

[10] Anna G. Ciriolo, Matteo Negro, Michele Devetta, Eugenio Cinquanta, Davide Faccialà, Aditya Pusala, Sandro De Silvestri, Salvatore Stagira, and Caterina Vozzi, "Optical Parametric Amplification Techniques for the Generation of High-Energy Few-Optical-Cycles IR Pulses for Strong Field Applications", MDPI APPLIED SCIENCES, MARCH8 2017.

[11] E. A. Migal, F. V. Potemkin, and V. M. Gordienko, "Highly efficient optical parametric amplifier tunable from near-to mid-IR for driving extreme nonlinear optics in solids", Optics Letters, Vol. 42, Issue 24, pp. 5218-5221, 2017.

[12] Chaohua Wu, Jingtao Fan, Gang Chen, and Suotang Jia, "Symmetry-breaking-induced dynamics in a nonlinear microresonator", Opt. Express 27(20), 28133-28142 (2019).

[13] Sung Bo Lee, Hyeon Sang Bark, and Tae-In Jeon, "Enhancement of THz resonance using a multilayer slab waveguide for a guided-mode resonance filter", Opt. Express 27(20), 29357-29366 (2019).

[14] Houssein El Dirani, Laurene Youssef, Camille Petit-Etienne, Sebastien Kerdiles, Philippe Grosse, Christelle Monat, Erwine Pargon, and Corrado Sciancalepore, "Ultralow-loss tightly confining Si3N4 waveguides and high-Q microresonators", Opt. Express, Vol. 27, Issue 21, pp. 30726-30740 (2019).

[15] Ivan S. Maksymov; Andrey A. Sukhorukov; Andrei V. Lavrinenko; Yuri S. Kivshar, "Comparative Study of FDTD-Adopted Numerical Algorithms for Kerr Nonlinearities", IEEE Antennas and Wireless Propagation Letters, Year: 2011 | Volume: 10 | Journal Article | Publisher: IEEE.

[16] E. Valentinuzzi, "Dispersive properties of Kerr-like nonlinear optical structures", Journal of Lightwave Technology, Year: 1998 | Volume: 16, Issue: 1 | Journal Article | Publisher: IEEE.

[17] Ozlem Ozgun, Mustafa Kuzuoglu, Metamaterials and Numerical Models, Nova Novinka; UK ed. Edition, 2011.

[18] Ozlem Ozgun, Mustafa Kuzuoglu, MATLAB-based Finite Element Programming in Electromagnetic Modeling, CRC Press, 1st edition, 2018.

[19] Mohammad Amin Izadi, Rahman Nouroozi, "Adjustable Propagation Length Enhancement of the Surface Plasmon Polariton Wave via Phase Sensitive Optical Parametric Amplification", Scientific Reports volume 8, Article number: 15495 (2018).

[20] Ozlem Ozgun, Mustafa Kuzuoglu, "Non-Maxwellian Locally-Conformal PML Absorbers for Finite Element Mesh Truncation", IEEE Transactions on Antennas and Propagation 55(3): 931 - 937 • April 2007.

[21] Ozlem Ozgun, Mustafa Kuzuoglu, "Near-field performance analysis of locally-conformal perfectly matched absorbers via Monte Carlo simulations", Journal of Computational Physics, Volume 227 Issue 2, December, 2007, Pages 1225-1245.