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Conservation of a Resource-Consumer Systemby Imposing Taxation on Pollution

Jiwei He[1,2]

[1]School of International Trade and Economics, University of International Business and Economics, Beijing, 100029, China
[2]School of Mathematics and Physics, North China Electric Power University, Baoding 071003, China

Abstract This paper studies the effects of pollution taxation on population in a polluted environment. We improve the classical Gallopin resource-consumer model, and assume that pollution taxation is imposed on toxicant emitters if the emission exceeds the amount permitted. We give some sufficient conditions under which the species in Gallopin system persists or becomes extinct. The threshold between persistence and extinction in some cases is obtained. We also verify these results with the help of numerical simulation.

Keywords Resource-consumer system, Pollution taxation, Persistence

1. Introduction[1]

Uncontrolled contribution of toxicant to the environment has led many species to extinction and several others at the verge of extinction. It has been shown that toxicant can decrease the birth rate, increase the death rate and reduce the carrying capacity of the environment. One efficient way to control the environment pollution is imposing pollution taxation on toxicant emitters. In this paper, we introduce a mathematical model to analyze the effects of pollution taxation on Gallopin system in a polluted environment.

The study of deterministic dynamic population models with toxicant effects was established by Hallam and his colleagues in the 1980s [1, 2, 3]. This model has been improved by many scholars [4, 5, 6]. Srinivasu [7] improved the model of Hallam by considering the direct effects of the toxicant in the environment on the population, and determined aclean-up policy to be implemented at the source of pollutants in order to conserve a population. He et al [8] studied the survival of a single species in the polluted environment, considering the organism's uptake of toxicant from the environment and egestion of the internal toxicant to the environment. He and Ma [9] studied a Gallopin resource-consumer model, and obtained the threshold between the persistence and extinction of the consumer. Buonomo [10] et al studied the effects of variation of the population amount on the toxicant concentration in the organism and environment. He and Wang [11, 12] improved the ordinary differential equation system induced by Buonomo [10], and also applied to the Gallopin resource - consumer system. He [13] studied the effects of taxation on a single logistic species in the polluted environment.

This paper improves the classical Gallopin resource - consumer system in a polluted environment, on the assumption of the living organisms absorbing part of the toxicant into their bodies so that the dynamics of the population is affected by this (internal) toxicant. It is assumed that pollution taxation is imposed on toxicant emitters if their emission exceeds the amount permitted by the government. This paper only discusses the conditions of the emission beyond the limit. It also considers the negative effects of taxation evasion. The sufficient conditions for persistence or extinction of the population are obtained, and some numerical simulation is given.

2. The Model

The study is relied on the hypothesis of a complete spatial homogeneous environment without migration. Let $x(t)$ represent the population of the consumer species at time t, $a(t)$ denote the quantity of the resources at time t, $C_0(t)$ be the toxicant concentration in a body, $C_e(t)$ be the toxicant concentration of the environment at time t, and $I(t)$ be the pollution taxation imposed on the emitters. A single species resource-consumer model of Gallopin system [9, 14, 15] in a polluted environment is improved as follows (M_1):

$$\frac{dx(t)}{dt} = x(t)(r_0 - \alpha C_0(t) - b\omega e^{-\frac{na(t)}{x(t)}}), \quad (1)$$

* Corresponding author:
hejiwei313@163.com (Jiwei He)
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$$\frac{da(t)}{dt} = f(t) - \omega x(t)(1 - e^{-\frac{na(t)}{x(t)}}), \tag{2}$$

$$\frac{dC_0(t)}{dt} = kC_e(t) - (g+m)C_0(t), \tag{3}$$

$$\frac{\mathrm{dC_e(t)}}{\mathrm{dt}} = -k_1 C_e(t)x(t) + g_1 x(t)C_0(t)$$
$$-hC_e(t) + u(t) - \rho I(t), \tag{4}$$

$$\frac{dI(t)}{dt} = \theta(C_e(t) - A) - \theta_0 I(t). \tag{5}$$

In this model, $f(t)$ is the continuous growth rate of the resource in the absence of the consumer. $u(t)$, the exogenous toxicant input rate, defined on $[0,+\infty)$, is a bounded non-negativecontinuous function, and $\sup_{t\geq 0} u(t) := u_1 > 0$. $r_0, \alpha, b, \omega, n, k, g, m, k_1, g_1, h, \rho$ are positive constants. r_0 is the intrinsic growth rate of the species in the absence of the toxicant in the environment. α represents the species response to the toxicant present in the organism. k denotes an organism's net uptake rate of the toxicant from the environment. g and m represent the egestion and deputation rates of the toxicant in an organism. The constant k_1 is given by km_0/m_e, where m_0 and m_e represent the average mass of an individual in the population and the total mass of the medium in the environment, respectively. Similarly $g_1 = gm_0/m_e$. h denotes the loss rate of the toxicant in the environment due to natural degradation. ρ is the taxation repulsion coefficient. θ is the coefficient of the taxation imposed on the emitters. Taxation is imposed only if C_e exceeds the permitted limit A (For convenience sake, we assume C_e always exceeds A, the limit is up to which there is bare harm to the population.). θ_0 denotes some practical difficulties(such as tax evasion) on implementing the tax system. The initial values are $x(0) > 0$, $a(0) > 0$, $0 \leq C_0(0) < 1$, $0 \leq C_e(0) < 1$, $I(0) \geq 0$.

Some notations defined:

$$M_a = a(0) + \int_0^{+\infty} f(t)dt, \quad M_x = \frac{bn\omega}{b\omega - r_0} M_a,$$

$$\langle v(t)\rangle = \frac{1}{t}\int_0^t v(s)ds,$$

$$M_y = \begin{cases} e^{\frac{r_0 + n\omega - b\omega}{b\omega}}, & b\omega > r_0 + n\omega, \\ 1, & b\omega \leq r_0 + n\omega. \end{cases}$$

Definition 1. A population x is said to be weakly persistent if $\limsup_{t\to+\infty} x(t) = 0$; it goes to extinction if $\lim_{t\to+\infty} x(t) = 0$.

We have the following lemmas similar to lemma 1 and lemma 2 in Ref. 9.

Lemma 1. For model (M_1), the field $\mathrm{S} = \{(x(t), a(t), C_0(t), C_e(t), I(t)) : x(t) > 0, a(t) > 0, C_0(t) > 0, C_e(t) > 0, I(t) > 0\}$ is an invariant set.

Lemma 1 shows that if the initial values of model (M_1) are in set S, then the solutions are always in S.

Both $C_0(t)$ and $C_e(t)$ in model (M_1) are concentrations of the toxicant, so the inequalities

$$0 \leq C_0(t) \leq 1,\ 0 \leq C_e(t) \leq 1$$

must be satisfied to be realistic. These limitations should be reflected in some conditions among the coefficients of model (M_1). These conditions are shown in the following lemma.

Lemma 2. For model (M_1), if $g \leq k \leq g+m$, $u_1 < h$, then $0 \leq C_0(t) \leq 1, 0 \leq C_e(t) \leq 1$ for all $t > 0$.

If the continuous growth rate of the resourcesatisfies some conditions, then the amount of the resource, population and pollution taxation must be limited.

Lemma 3. For model (M_1), if $\int_0^{+\infty} f(t)dt < +\infty, b\omega - r_0 > 0$, then $a(t) \leq M_a$, $\limsup_{t\to+\infty} x(t) \leq M_x$ and $\limsup_{t\to+\infty} I(t) \leq \frac{\theta}{\theta_0}$.

Proof. Equation (2) means $\frac{da(t)}{dt} \leq f(t)$. Then

$$a(t) \leq a(0) + \int_0^t f(s)ds \leq a(0) + \int_0^{+\infty} f(t)dt = M_a.$$

From equation (1), we have

$$\frac{dx(t)}{dt} \leq x(t)\left(r_0 - b\omega\left(1 - \frac{na(t)}{x(t)}\right)\right)$$
$$= bn\omega a(t) - (b\omega - r_0)x(t)$$
$$\leq bn\omega M_a - (b\omega - r_0)x(t)$$

Based on the standard comparison theorem, we get

$$\limsup_{t\to+\infty} x(t) \leq \frac{bn\omega}{b\omega - r_0} M_a = M_x.$$

Analogously, from equation (5) we obtain

$$\limsup_{t\to+\infty} I(t) \leq \theta/\theta_0.$$

3. Main Results

The problem we are interested in is to find the conditions under which the population in model (M_1) is persistent or goes to extinction.

Theorem 1. For model (M_1), if $g \leq k \leq g+m, u_1 < h$ and $\liminf_{t\to+\infty} f(t) > 0$, $\liminf_{t\to+\infty}\langle u(t)\rangle < (h + \frac{\rho\theta}{\theta_0})\frac{r_0(g+m)}{k\alpha} - \frac{\rho\theta}{\theta_0}A$, then the population $x(t)$ will be weakly persistent.

Proof. If the conclusion of theorem is false, i.e. $\lim_{t\to+\infty} x(t) = 0$, there will be a contradiction.

Firstly, we will show that

$$\liminf_{t\to+\infty}\langle C_0(t)\rangle < \frac{r_0}{\alpha}.$$

Equations (3), (4) and (5) can be rewritten as:

$$\frac{\mathrm{C_0(t) - C_0(0)}}{\mathrm{t}} = \mathrm{k}\langle \mathrm{C_e(t)}\rangle - (\mathrm{g+m})\langle \mathrm{C_0(t)}\rangle, \tag{6}$$

$$\frac{\mathrm{C_e(t) - C_e(0)}}{\mathrm{t}} = -\mathrm{k_1}\langle \mathrm{C_e(t)x(t)}\rangle + \mathrm{g_1}\langle \mathrm{C_0(t)x(t)}\rangle$$
$$-h\langle C_e(t)\rangle + \langle u(t)\rangle - \rho\langle I(t)\rangle. \tag{7}$$

$$\frac{I(t) - I(0)}{t} = \theta\langle C_e(t)\rangle - \theta A - \theta_0\langle I(t)\rangle. \tag{8}$$

Since $C_0(t)$ and $C_e(t)$ are bounded, then

$$\lim_{t\to+\infty}\langle C_e(t)x(t)\rangle=\lim_{t\to+\infty}\langle C_0(t)x(t)\rangle=0.$$

From expression (6) (7) and (8), we can obtain that

$$\liminf_{t\to+\infty}\langle u(t)\rangle=\left(h+\frac{\rho\theta}{\theta_0}\right)\liminf_{t\to+\infty}\langle C_e(t)\rangle-\frac{\rho\theta}{\theta_0}A.$$
$$=(h+\frac{\rho\theta}{\theta_0})\frac{g+m}{k}\liminf_{t\to+\infty}\langle C_0(t)\rangle-\frac{\rho\theta}{\theta_0}A.$$

Then $\liminf_{t\to+\infty}\langle C_0(t)\rangle<\frac{r_0}{\alpha}$ follows immediately according to the conditions of the theorem.

Since $\liminf_{t\to+\infty} f(t)>0$, there exist $t_1>0$ and $0<\delta\le\omega na(t_1)$ such that $f(t)>\delta$ for $t\ge t_1$. Taylor's theorem enables us to write $e^{-\frac{na(t)}{x(t)}}\ge 1-\frac{na(t)}{x(t)}$. Then from equation (2) we have

$$\frac{da(t)}{dt}\ge\delta-\omega x(t)\left(1-\left(1-\frac{na(t)}{x(t)}\right)\right)$$
$$=\delta-\omega na(t),\quad t\ge t_1.$$

The solution of the differential equation $\frac{dv}{dt}=\delta-\omega n\cdot v$, starting from $(t_1,\ a(t_1))$, is

$$v(t)=\frac{\delta}{\omega n}+(a(t_1)-\frac{\delta}{\omega n})e^{-\omega n(t-t_1)}.$$

Then from a standard comparison theorem, we have $a(t)\ge v(t),\ t>t_1$. Since $0<\delta\le\omega na(t_1)$, then $v(t)\ge\frac{\delta}{\omega n}=:\bar a>0,\ t\ge t_1$. That is $a(t)\ge\bar a>0,\ t\ge t_1$. Then $\liminf_{t\to+\infty} a(t)=:m_a>0$.

For any $e^x>x$, then $e^{-x}<\frac{1}{x}$. So equation (1) implies that

$$\frac{dx(t)}{dt}\ge x(t)\left(r_0-\alpha C_0(t)-b\omega\cdot\frac{x(t)}{na(t)}\right)$$
$$\ge x(t)\left(r_0-\alpha C_0(t)-\frac{b\omega}{n\bar a}x(t)\right),\ \ t>t_1.$$

Dividing this inequality by $x(t)$ and then integrating it from t_1 to t, we can get that

$$\ln\frac{x(t)}{x(t_1)}\ge r_0(t-t_1)-\alpha\int_{t_1}^t C_0(s)ds-\frac{b\omega}{n\bar a}\int_{t_1}^t x(s)ds,$$
$$\frac{1}{t-t_1}\ln\frac{x(t)}{x(t_1)}+\frac{b\omega}{n\bar a}\cdot\frac{1}{t-t_1}\int_{t_1}^t x(s)ds$$
$$\ge r_0-\alpha\cdot\frac{1}{t-t_1}\int_{t_1}^t C_0(s)ds.$$

Then we take the upper limit on both sides of the last inequality,

$$\limsup_{t\to+\infty}\frac{1}{t-t_1}\ln\frac{x(t)}{x(t_1)}+\limsup_{t\to+\infty}\frac{b\omega}{n\bar a}\cdot\frac{1}{t-t_1}\int_{t_1}^t x(s)ds$$
$$\ge r_0-\alpha\cdot\liminf_{t\to+\infty}\frac{1}{t-t_1}\int_{t_1}^t C_0(s)ds. \tag{9}$$

The following expression is obvious to be obtained:

$$\langle C_0(t)\rangle=\frac{1}{t}\int_0^t C_0(s)ds$$
$$=\frac{1}{t}\int_0^{t_1} C_0(s)ds+\frac{t-t_1}{t}\cdot\frac{1}{t-t_1}\int_{t_1}^t C_0(s)ds.$$

Which leads to

$$\liminf_{t\to+\infty}\langle C_0(t)\rangle=\liminf_{t\to+\infty}\frac{1}{t-t_1}\int_{t_1}^t C_0(s)ds<\frac{r_0}{\alpha}.$$

The assumption $\lim_{t\to+\infty} x(t)=0$ gives that $\limsup_{t\to+\infty}\frac{1}{t-t_1}\int_{t_1}^t x(s)ds=0$,then by inequality (9), we have $\limsup_{t\to+\infty}\frac{1}{t-t_1}\ln\frac{x(t)}{x(t_1)}>0$. This is impossible. In fact, from the assumption $\lim_{t\to+\infty} x(t)=0$, we know that

$$\limsup_{t\to+\infty}\frac{1}{t-t_1}\ln\frac{x(t)}{x(t_1)}\le 0.$$

So the conclusion of theorem 1 is true.

Theorem 2. For model (M_1), if the conditions of lemmas 2 and 3 are satisfied and one of the following condition groups holds, then the population will go to local extinction.

(I) $\liminf_{t\to+\infty}\langle u(t)\rangle>\frac{(h+\rho\theta/\theta_0)r_0(g+m)}{k\alpha}-\frac{\rho\theta A}{\theta_0}$, and

$$\begin{cases}(h+\rho\theta/\theta_0)(g+m)>kg_1M_x,\\ k_1\alpha\le g_1r_0.\end{cases}$$

(II) $\liminf_{t\to+\infty}\langle u(t)\rangle>\frac{(h+\rho\theta/\theta_0)r_0(g+m)}{k\alpha}+\frac{k_1\alpha-r_0g_1}{\alpha}M_x-\frac{\rho\theta A}{\theta_0}$, and

$$\begin{cases}(h+\rho\theta/\theta_0)(g+m)>kg_1M_x,\\ k_1\alpha>g_1r_0.\end{cases}$$

(III) $\liminf_{t\to+\infty}\langle u(t)\rangle>\frac{(h+\rho\theta/\theta_0)r_0(g+m)}{k\alpha}-\frac{\rho\theta A}{\theta_0}+\frac{A}{k\alpha}$, and

$$\begin{cases}(h+\rho\theta/\theta_0)(g+m)\le kg_1M_x,\\ k_1\alpha>g_1r_0.\end{cases}$$

(IV) $\liminf_{t\to+\infty}\langle u(t)\rangle>\frac{hr_0(g+m)}{k\alpha}-\frac{\rho\theta A}{\theta_0}+\frac{B}{k\alpha}$, and $\begin{cases}h(g+m)\le kg_1M_x,\\ r_0g_1\le k_1\alpha\end{cases}$, where

$$B:=\max\{0,\ A\},$$
$$A:=k(k_1\alpha-r_0g_1)M_x+b\omega(kg_1M_x$$
$$-(h+\rho\theta/\theta_0)(g+m))M_y.$$

Proof. Assume that $\limsup_{t\to+\infty} x(t)>0$. From equation (1), we can get that

$$\frac{1}{t}\ln\frac{x(t)}{x(0)}=r_0-\alpha\langle C_0\rangle-b\omega\left\langle e^{-\frac{na}{x}}\right\rangle, \tag{10}$$
$$C_0x=\frac{1}{\alpha}(r_0x-b\omega xe^{-\frac{na}{x}}-\frac{dx}{dt}),$$
$$\langle C_0x\rangle=\frac{1}{\alpha}(r_0\langle x\rangle-b\omega\left\langle xe^{-\frac{na}{x}}\right\rangle-\frac{x(t)-x(0)}{t}). \tag{11}$$

From (8), we have

$$\langle I(t)\rangle=\frac{\theta}{\theta_0}\langle C_e(t)\rangle-\frac{\theta}{\theta_0}A-\frac{1}{\theta_0}\frac{I(t)-I(0)}{t} \quad (12)$$

Substitution of (11), (12) into (7) gives

$$\langle C_e\rangle=\frac{1}{h+\frac{\rho\theta}{\theta_0}}(-k_1\langle C_e x\rangle+\frac{g_1}{\alpha}(r_0\langle x\rangle-b\omega\left\langle xe^{-\frac{na}{x}}\right\rangle$$
$$-\frac{x(t)-x(0)}{t})+\langle u(t)\rangle+\frac{\rho\theta}{\theta_0}A+\frac{\rho}{\theta_0}\frac{I(t)-I(0)}{t}).$$

Equation (6) implies that

$$\langle C_0\rangle=\frac{1}{g+m}\left(k\langle C_e\rangle-\frac{C_0(t)-C_0(0)}{t}\right),$$
$$=\frac{k}{(g+m)\left(h+\frac{\rho\theta}{\theta_0}\right)\alpha}(-k_1\alpha\langle C_e x\rangle+g_1r_0\langle x\rangle$$
$$-\ g_1b\omega\left\langle xe^{-\frac{na}{x}}\right\rangle+\alpha\langle u(t)\rangle+\frac{\rho\theta}{\theta_0}A\alpha)+o(1). \quad (13)$$

Here $o(1)$ is an infinitesimal, that is to say its limitation is 0 as $t$tends to infinity. Substituting into (10), we get

$$\frac{(h+\rho\theta/\theta_0)(g+m)}{t}\ln\frac{x(t)}{x(0)}$$
$$=\left(h+\frac{\rho\theta}{\theta_0}\right)r_0(g+m)+kk_1\alpha\langle C_e x\rangle-kg_1r_0\langle x\rangle$$
$$+kg_1b\omega\left\langle xe^{-\frac{na}{x}}\right\rangle-k\alpha\langle u\rangle-k\alpha\rho\theta A/\theta_0$$
$$-b\omega(g+m)(h+\rho\theta/\theta_0)\left\langle e^{-\frac{na}{x}}\right\rangle+o(1).$$

By the use of $C_e(t)\le 1,\ t\in R_+$, the above expression can be rewritten as

$$k\alpha\langle u\rangle\le\left(h+\frac{\rho\theta}{\theta_0}\right)r_0(g+m)$$
$$-k\alpha\rho\theta A/\theta_0+\langle F(a,x)\rangle+o(1),$$

where

$$F(a,x)\coloneqq kk_1\alpha x-kg_1r_0x+kg_1b\omega xe^{-\frac{na}{x}}$$
$$-b\omega(g+m)(h+\rho\theta/\theta_0)e^{-\frac{na}{x}}.$$

Therefore

$$k\alpha\liminf_{t\to+\infty}\langle u\rangle\le\left(h+\frac{\rho\theta}{\theta_0}\right)r_0(g+m)$$
$$-k\alpha\rho\theta A/\theta_0+\limsup_{t\to+\infty}\langle F(a,x)\rangle$$
$$\le\left(h+\frac{\rho\theta}{\theta_0}\right)r_0(g+m)-k\alpha\rho\theta A/\theta_0+\max_D F(a,x), \quad (14)$$

where $D=\{(a,x): a\in[0,M_a], x\in[0,M_x]\}$. Let $y(t)=e^{-\frac{na(t)}{x(t)}}$. Define

$$G(x,y)\coloneqq F(a,x)=kk_1\alpha x-kg_1r_0x+kg_1b\omega xy$$
$$-b\omega(g+m)(h+\rho\theta/\theta_0)y.$$

Now we want to know the range of $y(t)$. Using equations (1) and (2),

$$\frac{d}{dx}\left(\frac{a}{x}\right)=\frac{1}{x^2}\left(x\frac{da}{dt}-a\frac{dx}{dt}\right),$$
$$\ge\frac{1}{x^2}\left(xf(t)-\omega x^2\left(1-e^{-\frac{na}{x}}\right)-ax\left(r_0-b\omega e^{-\frac{na}{x}}\right)\right)$$
$$\ge\frac{1}{x^2}\left(xf(t)-\omega x^2\left(1-e^{-\frac{na}{x}}\right)-ax\left(r_0-b\omega e^{-\frac{na}{x}}\right)\right)$$
$$\ge\frac{1}{x^2}\left(-\omega x^2\left(1-e^{-\frac{na}{x}}\right)-ax\left(r_0-b\omega e^{-\frac{na}{x}}\right)\right)$$
$$\ge\omega(1-\frac{na}{x}-1)-\frac{a}{x}(r_0-b\omega(1-\frac{na}{x}))$$
$$=(b\omega-r_0-n\omega)\frac{a}{x}-bn\omega(\frac{a}{x})^2.$$

If $b\omega-r_0-n\omega>0$, then the standard comparison theorem results in

$$\liminf_{t\to+\infty}\frac{a}{x}\ge\frac{b\omega-r_0-n\omega}{bn\omega}.$$

If $b\omega-r_0-n\omega\le 0$, then at least $\liminf_{t\to+\infty}\frac{a}{x}\ge 0$.

According to the meaning of M_y, we have $\limsup_{t\to+\infty}y(t)\le M_y$. Thus $\max_D F(a,x)\le\max_{D'}G(x,y)$, where

$$D'=\{(x,y): x\in[0,M_x],\ y\in[0,M_y]\}.$$

Let $\frac{\partial G}{\partial x}=\frac{\partial G}{\partial y}=0$. We get

$$p:=\begin{cases}x=\dfrac{(h+\rho\theta/\theta_0)(g+m)}{kg_1},\\ y=\dfrac{r_0g_1-k_1\alpha}{b\omega g_1}.\end{cases}$$

Since $x\ge 0, y\ge 0$, then $r_0g_1\ge k_1\alpha$. The value of $G(x,y)$ at point p gives

$$G(p)=k_1\alpha(h+\rho\theta/\theta_0)(g+m)/g_1-r_0(h+\rho\theta/\theta_0)(g+m)$$
$$+(h+\rho\theta/\theta_0)(g+m)(r_0g_1-k_1\alpha)/g_1$$
$$-(h+\rho\theta/\theta_0)(g+m)(r_0g_1-k_1\alpha)/g_1$$
$$=(h+\rho\theta/\theta_0)(g+m)(k_1\alpha-r_0g_1)/g_1\le 0$$

Since $G(0,0)=0$, we can assert that $\max_{D'}G(x,y)$ must be taken on the boundary of D'.

When $(h+\rho\theta/\theta_0)(g+m)>kg_1M_x$ and $k_1\alpha>r_0g_1$, then

$$G(0,y)=-\left(h+\frac{\rho\theta}{\theta_0}\right)b\omega(g+m)y\le 0;$$
$$G(M_x,y)=k(k_1\alpha-r_0g_1)M_x$$
$$+b\omega y\left(kg_1M_x-\left(h+\frac{\rho\theta}{\theta_0}\right)(g+m)\right)$$
$$\le k(k_1\alpha-r_0g_1)M_x;$$
$$G(x,0)=k(k_1\alpha-r_0g_1)x\le k(k_1\alpha-r_0g_1)M_x;$$
$$G(x,M_y)=kx(k_1\alpha-r_0g_1+b\omega g_1M_y)$$

$$-\left(h+\frac{\rho\theta}{\theta_0}\right)b\omega(g+m)M_y$$
$$\le kM_x\left(b\omega g_1M_y+k_1\alpha-r_0g_1\right)$$
$$-\left(h+\frac{\rho\theta}{\theta_0}\right)b\omega(g+m)M_y$$
$$=k(k_1\alpha-r_0g_1)M_x$$
$$+b\omega\left(kg_1M_x-\left(h+\frac{\rho\theta}{\theta_0}\right)(g+m)\right)M_y$$
$$\le k(k_1\alpha-r_0g_1)M_x.$$

So $\max_{D'} G(x,y)\le k(k_1\alpha-r_0g_1)M_x$, and since $G(M_x,0)=k(k_1\alpha-r_0g_1)M_x$, then $\max_{D'} G(x,y)=k(k_1\alpha-r_0g_1)M_x$.

Similarly, when

$$(h+\rho\theta/\theta_0)(g+m)>kg_1M_x, k_1\alpha\le g_1r_0,$$

$\max_{D'} G=0$;

when $(h+\rho\theta/\theta_0)(g+m)\le kg_1M_x, k_1\alpha>g_1r_0$,

$\max_{D'} G=A$;

when $(h+\rho\theta/\theta_0)(g+m)\le kg_1M_x, k_1\alpha\le g_1r_0$,

$\max_{D'} G=\max\{0,A\}$.

Using inequality (14), it contradicts with the conditions of theorem 2 accordingly.

4. Numerical Simulation

The relations obtained from the theorems are well consistent with the actual data observed. In application of MATLAB software package, graphs are plotted for different values obtained in order to confirm the conclusion we have acquired above. Set $r_0=2, \alpha=1, b=1, \omega=2, n=1, f(t)=1+e^{-t}, k=1, g=1, m=2, h=1, \theta_0=0.5, A=0.2$, and the initial values are $x(0)=0.5, a(0)=0.2$, $C_0(0)=0.1, C_e(0)=0.1, I(0)=0$. The following Figure 1 describes the behavior of the population for different exogenous toxicant input rate u in the absence of pollution taxation, Figure 2 shows the behavior of the population in the presence and absence of pollution taxation.

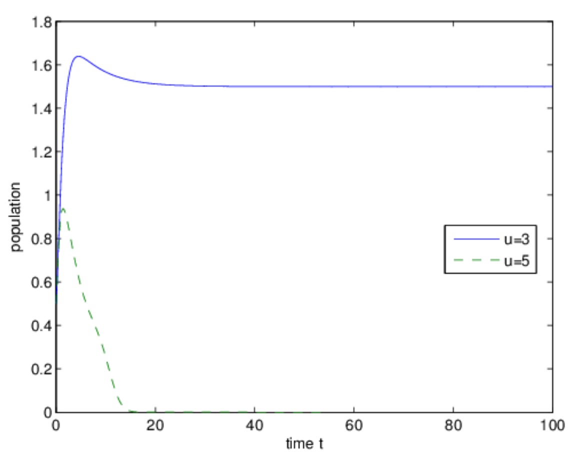

Figure 1. The long behavior of population when $\rho=0$

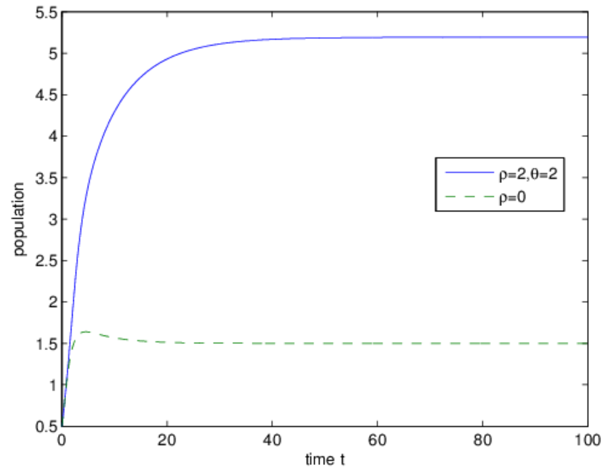

Figure 2. The long behavior of population when $u=3$

Figure 1 and Figure 2 show that the amount of the population decreases with the increase of the exogenous toxicant input rate, and even goes to extinction. However, when the pollution taxation is imposed on the toxicant emitters, the amount of the population increases to 5.25 from 1.5 rapidly.

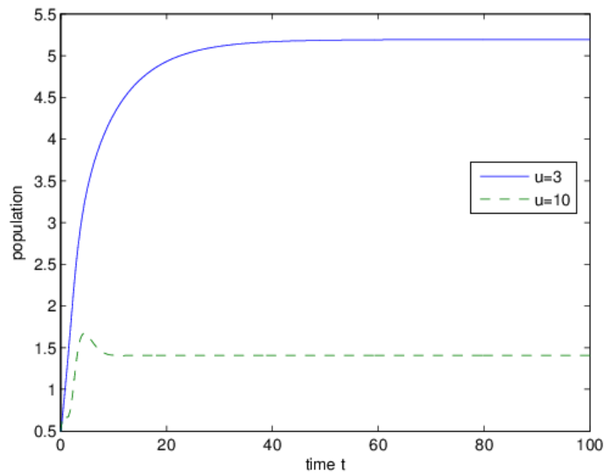

Figure 3. The long behavior of population when $\rho=2, \theta=3$

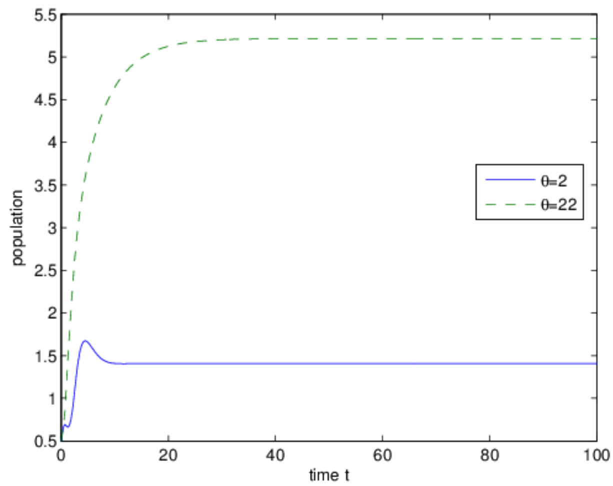

Figure 4. The long behavior of population when $\rho=2, u=10$

Figure 3 denotes the behavior of the population for different rate of emission of toxicant. We can observe that when u increases from 3 to 10, there will be a sharp decrease of the amount of the population. Nevertheless, when the heavy taxation $(\theta = 22)$ is imposed on the emitters, the balanced amount of the population will return to 5.25 from 1.4 quickly (Figure 4). The taxation coefficient θ is the key parameter which needs to be chosen very carefully in order to maintain the equilibrium level of population.

5. Conclusions

The main focus of this paper is to analyze the long behavior of Gallopin resource-consumer system, when the population is affected by toxicant emitted into the environment by external sources. It is further assumed that the cumulative rate of emission is reduced due to the levy of taxation. The model (M_1) reduces to the system in [9] if there is no pollution taxation. Conditions which guarantee the persistence or extinction of the system are also given. The threshold between persistence and extinction is $\liminf_{t\to+\infty}\langle u(t)\rangle = \left(h+\frac{\rho\theta}{\theta_0}\right)\frac{r_0(g+m)}{k\alpha} - \frac{\rho\theta}{\theta_0}A$, when the conditions of theorem 1 and (I) of theorem 2 are satisfied. In [9] this threshold is $\liminf_{t\to+\infty}\langle u(t)\rangle = \frac{hr_0(g+m)}{k\alpha}$. By numerical simulation, it is shown that the amount of population decreases as the cumulative rate of the emission of the toxicant from external sources increases. So we need to control the emission rate of toxicant from external sources. Moreover, we note that when taxation are imposed on emitters of toxicant, the amount of population increases and shift more near to the level when the ecosystem is in the absence of toxicant.

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