

A Novel Mathematical Model for Deterministic Time-cost Trade-off Based on Path Constraint

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Abstract In this paper a new deterministic mathematical model for time-cost trade-off is introduced. The proposed method is based on path constraints in a network while other similar methods are modelled based on activities. Through this paper, the proposed model is constructed and then validated by testing in a case study. The achieved results from model are compared with manual solution. The results of both proposed model and manual solution are exactly same which proves the accuracy of the model. Moreover, using network paths for constructing the constraints has the benefit of a lower number of constraints and leading to a faster and more straightforward solution. Lower number of constraints might lead to less computing effort. This can provide us an opportunity to solve some networks which are NP-hard by other similar approaches. Finally can summarize that the main advantages of the proposed model are simplicity in modelling and the usage of a fewer equations.

Keywords Time-cost Trade off, Linear Programming, Mathematical Modeling

1. Introduction

Time is an extremely important factor in construction projects. Commonly penalties are imposed on projects that run over time. Furthermore indirect costs are highly dependent to the duration of the project thus a faster project is often a less expensive project. Finding the optimum trade-off between cost and time is one of the most challenging aspects of project management.

Reducing the duration of a project is accomplished by reducing the duration of (some of) its constituent activities. This typically requires an increasing in resources costs such as technology, level of competency of crews, equipment and working hours assigned to those activities. On one hand, increasing resources will increase the direct cost of the activities, but on the other hand, by saving project time, the indirect cost of the project will be reduced. Balancing between the increase of direct costs and the decrease of indirect costs is the issue that time-cost trade-off techniques deal with[1]. If a project is running behind schedule or is required to be finished earlier, then the plan needs to be revised, the manager can compress some critical activities to save time[2]. This process is called time-cost trade-off (TCT) analysis. (Fig 1) shows a time-cost trade-off curve, which

presents the relationship between project duration and project cost. Indirect costs decline linearly with project duration. The normal duration is the time that would result from selecting resources which minimize the direct cost of each activity. Thus reducing the duration increases direct cost. Furthermore the increase in direct cost will be different for different activities. TCT seeks to find a way to minimize the cost increase while project duration is cut.

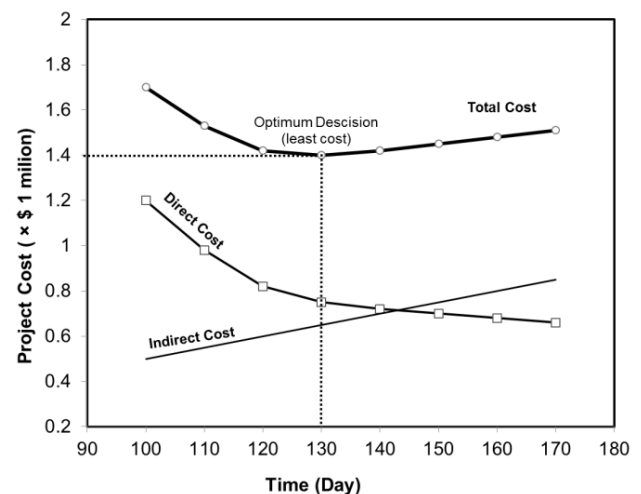


Figure 1. Project time-cost relationship

In typical projects, finding the optimal solution for project compression would be difficult, due to the large number of activities[3]. Only critical activities should be compressed. As the project gets shorter the number of critical paths may

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increase when the number of critical paths is increased. These must be considered by the TCT process as well. However, the execution time required by this process might be unfeasible even with very fast computers, because the problem becomes a classical NP-hard problem and finding a solution for it is computationally intractable.

2. Existing Time–cost Trade-off Techniques

The related research in TCT can be classified into two sub-areas; mathematical programming models and heuristic methods. The details of these efforts since the early 1960s are compared in literature such as [1],[4-6]. The main drawbacks of mathematical programming models which were accounted are complexity of formulation, local minimum solutions and inability to deal with large projects ([1],[6-9]). On the other hand, the main criticism of heuristic models is inability to guarantee optimum solutions.

In two past decades, the significant numbers of studies on TCT have been allocated to hire the Artificial Intelligence (AI) methods for solving TCT problems. According to the fast growth in computer science these methods were highlighted, especially genetic algorithms (GAs) ([8-9], [10-23]). GAs uses random procedure for finding optimum solution and may it is required to pass the large computational process to find a solution. Despite of uncomplicated modeling process of solving TCT by GAs, the main inadequacy of these methods is difficulties to deal with more than one objective [8-9]. Xiong and Kuang [14] used ant colony optimization (ACO) with modified adaptive weight approach to deal with TCT problem. Geem [15] used similar approach by hiring harmony search. Eshtehardian, et al. [16] proposed a fuzzy based approach to deal with uncertainties in TCT and employed a multi-objective GA for acquiring a near optimum solution. Ammar [17] considered TCT when discounted cash flow is taken into account and solved this integrated problem heuristically. Aghassi, et al. [18] implemented a new multi attribute fitness function in order to solve TCT problem by GA and recently Ghoddousi, et al. [19] considered TCT with resource leveling simultaneously in a framework and used non-domination based GA to solve it. These models cannot guarantee the optimum solution, because for solving problems in these heuristic approaches, random search methods and evolutionary assessment are used and they do not guarantee to find the best feasible solution for every TCT problem.

In summary, as discussed above, the modeling process with mathematical approaches is problematical. It means that large TCT problems cannot be solved optimally. In order to deal with this issue the application of heuristic methods has been highlighted and widely used in literature, but there is no proof that how much these techniques are accurate. This paper firstly, presents a new mathematical modeling process for TCT analysis that in comparison to previously introduced mathematical models has some advantages; secondly the

proposed model is tested in a case study. Finally the manual results are compared with computer solution results. The main objective of this paper is proposing a method for cutting the size of constraints which can lead to providing an opportunity for solving larger sizes of TCT problems optimally.

3. Methodology

In this paper, two bellow approaches for time-cost trade-off analysis are discussed,

- Specific time for compression.
- Unspecific time for compression. (optimal decision)

In the deterministic solution of time-cost trade-off problems few points have to be observed. They are,

- Set of activities on critical paths consist of less costs are chosen.
- In the process of compression the critical paths should stay in critical condition.
- The number of critical paths should not reduce.

Due to the above constraints the computation of each process must be performed on the network. The computation is not only time-consuming but also may probably produce errors. One of most difficulties in the previous models of time-cost trade-off is huge number of parameters and equations. Because the equations must be set up between each activity in network linked together. So, in large TCT problems, a huge number of equations should be defined. In terms of computing process may normal PC are able to handle this job, however, the human errors in modeling process still remains as one the challenging points for this techniques. Therefore, the reduction in the number of equations in deterministic TCT problems, can lead to decrease the computing time and risk of human errors in modeling process. The main idea of our model is to decrease the number of equations in the networks especially when the number of paths is less than number of activities.

3.1. Model Formulation

The proposed model consists of low parameters and equation in compare to the previous mathematical time-cost trade-off models, because the previous models based on activities and this model based on the paths. In the normal cases that the number of paths is less than the number of activities, the number of constraints in the modeling process will significantly reduce

For each current path within the network an expression representing the length of the path is introduced,

$$TT(m) = \sum_{i=1}^n t_{n_i} \quad (1)$$

Where,

T_{n_i} = normal time correspond to activity i

For example, if the path number 1 consist of activities number A, C, F, H then,

$$TT(1) = T_{n_A} + T_{n_C} + T_{n_F} + T_{n_H} \quad (2)$$

For finding available paths in a network automatically, some techniques such as dynamic programming can be used, such as [20].

Determine the longest path length which is introduced by T.

$$T = \text{Max}(TT(i)) \quad (3)$$

For each activity determine X_i as the rate of compression.

For each activity determine C_i as the rate of changing cost for decreasing time of activity i .

For instance, it is assumed that cost and time are related together linearly, so,

C_C =normal cost

C_N = crash cost

T_N = Normal time

T_C = Crash time

$$C_i = \frac{C_C - C_N}{T_N - T_C} \quad (4)$$

In the other word, the expresses the additional cost which is required to complete activity one time unit earlier.

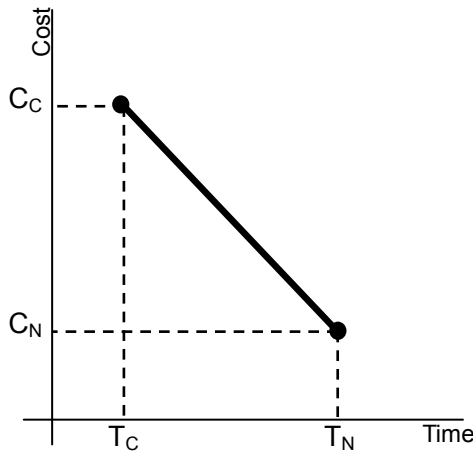


Figure 2. Relationship between time and cost of activity in this model

1-The rate of compression of activity i is defined by X_i

2-The rate of compression for path (α) is defined by $XT(\alpha)$ as bellow,

$$XT(\alpha) = \sum X_i \quad \forall i \in \alpha \quad (5)$$

3-The rate of compression for the whole network is defined as X by ,

$$X = \text{Max}(XT_i) \quad (6)$$

4-Take an arbitrary path (α) and set up an expression between the path and all other paths (β),

$$\sum X_i = \sum X_j + (TT(i) - TT(j)) \quad \forall i \in \alpha \text{ and } j \in \beta \quad (7)$$

i remains constant in the all of above equations while j is changing, hence with n independent path there are $n - 1$ equations in this step.

5- The following constraints are existing for all activities.

$$0 \leq X_i \leq (T_{n_i} - T_{C_i}) \quad (8)$$

6-The objective function is defined as follows,

$$Z = \sum X_i C_i + K(T - X) \quad (9)$$

Where,

K = indirect cost per time unit

Since the time constraints for each activity is replaced by the constraints on path, it is obviously observed that the number of equations are reduced when the model is compared with all known previous models. Therefore, it can be expected that TCT problems be solved easier and faster by introduced model. However, it is rarely possible and for very complex networks that the number of activities be less than the number of paths, so in that case, it is preferred to construct constraints on activities same as previous introduced models.

4. Case Study

Validation process consists of testing the proposed TCT model in a case study and then comparing the computing results achieved from mathematical model with manual solution results. The required case study should cover all possible situations in compressing process. By looking at literature [1, 2, 11] a CPM network has been designed for testing the model (Fig. 2) that includes the following features,

- Incompressible activity (E).
- After the forth step of compression a new path is added as critical path, so a simultaneous compression on this path must be performed and also after sixth step another path will be critical.

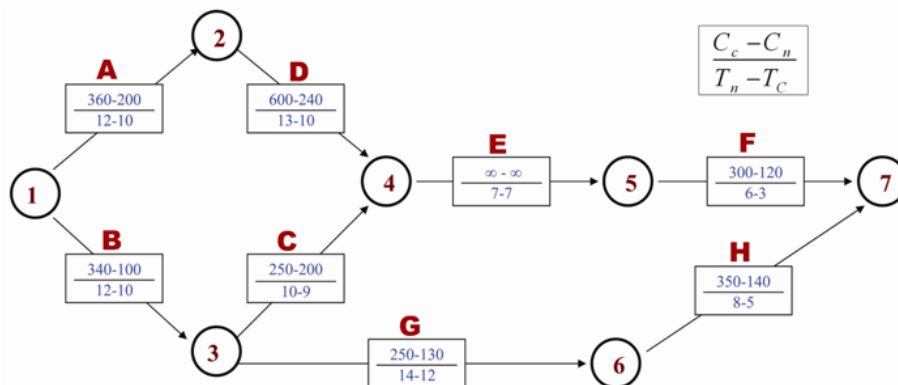


Figure 3. Case Study Network

Table 1. Case Study Data

Activity	Time		Cost		Cost to reduce per time unit
	Normal	Crash	Normal	Crash	
A	12	10	200	360	80
B	12	10	100	340	120
C	10	9	200	250	50
D	13	10	240	600	120
E	7	7	∞	∞	∞
F	6	3	120	300	60
G	14	12	130	250	60
H	8	5	140	350	70

Despite a CPM network with described characteristics is taken to control the model, however, since the model is based on logic of network the proposed model can be used in all other scheduling networks that uses critical path method such as PERT, GERT and PN as well.

5. Manual Solution

Table 2. Manual Solution

Phase	compression			Project Duration	Direct Cost	Indirect Cost	Total Cost
	Selected Activity	Unit	Cost				
Before compression	-	-	-	38	1130	2660	3790
1	F	1	60	37	1190	2590	3780
2	F	1	60	36	1250	2520	3770
3	F	1	60	35	1310	2450	3760
4	A	1	80	34	1390	2380	3770
5	A	1	120	33	1570	2310	3880
	G	1	60				
6	D	1	120	32	1750	2240	3990
	G	1	60				
7	D	1	120	31	1990	2170	4160
	E	1	50				
	H	1	70				

indirect cost per time
unit = 70

6. Comparison between Handy Solution and the Computed Result

In order to solve this model by computer, the Lingo optimization software was hired and the case study was modeled as bellow,

```
SETS,
ACTIVITY/1..8/,TN,TC,CN,CC,C,X;
ENDSETS
```

```
@for(activity(i),@gin(x(i)));
[OBJ]Min=total_cost;
@for(activity(i),X(i)>=0);
@FOR(ACTIVITY(I),C(I)=(Cc(I)-Cn(I))/(TN(I)-TC(I)
));
```

```
@FOR(ACTIVITY(I),X(I)<=(TN(I)-TC(I)));
```

```
T1=@SUM(ACTIVITY(I)|(I#EQ#1)#OR#(I#EQ#4)#
```

```
OR#(I#EQ#5)#OR#(I#EQ#6),X(I));
```

```
T2=@SUM(ACTIVITY(I)|(I#EQ#2)#OR#(I#EQ#3)#
OR#(I#EQ#5)#OR#(I#EQ#6),X(I));
```

```
T3=@SUM(ACTIVITY(I)|(I#EQ#2)#OR#(I#EQ#7)#
OR#(I#EQ#8),X(I));
```

```
X(1)+X(4)+X(5)+X(6)=A;
```

```
X(1)+X(4)+X(5)+X(6)=X(2)+X(3)+X(5)+X(6)+3;
```

```
X(1)+X(4)+X(5)+X(6)=X(2)+X(7)+X(8)+4;
```

```
DATA,
```

```
TN=12,12,10,13,7,1,6,14,8;
```

```
TC=10,10,9,10,7,3,12,5;
```

```
CC=360,340,250,600,1,01,300,250,350;
```

```
CN=200,100,200,240,1,120,130,140;
```

```
END data
```

```
total_cost=direct_cost+indirect_cost+Compressive_co
st;
```

```

direct_cost=@sum(activity(i),CC(i));
indirect_cost=(38-(x(1)+x(4)+x(6)))*70;
Compressive_cost=@SUM(ACTIVITY(i),C(I)*X(I));

```

```

X(5)      0.000000
X(6)      3.000000
X(7)      2.000000
X(8)      1.000000

```

The in above program controls the rate of compression and if it is not given the program will find the optimum decision in terms of least cost.

After run this model, the bellow result has been taken.

1- Unknown time for compression. (Optimal decision- least cost)

Variable	Value
TOTAL_COST	3760.000
DIRECT_COST	1990.000
INDIRECT_COST	2170.000
COMPRESSIVE_COST	860.0000

2- Specific time for compression.

In this case, it is supposed that project needs to be accomplished earlier, for example 2 time unit(A=2).

Variable	Value
TOTAL_COST	3770.010
DIRECT_COST	1250.010
INDIRECT_COST	2520.000
COMPRESSIVE_COST	120.0000
X(6)	2.000000

```

X(1)      1.000000
X(2)      0.000000
X(3)      1.000000
X(4)      3.000000

```

The achieved results for two scenarios are exactly same as results achieved from manual solution. It shows that model works precisely and the ability of the model in overcoming all possible situation during compressing process.

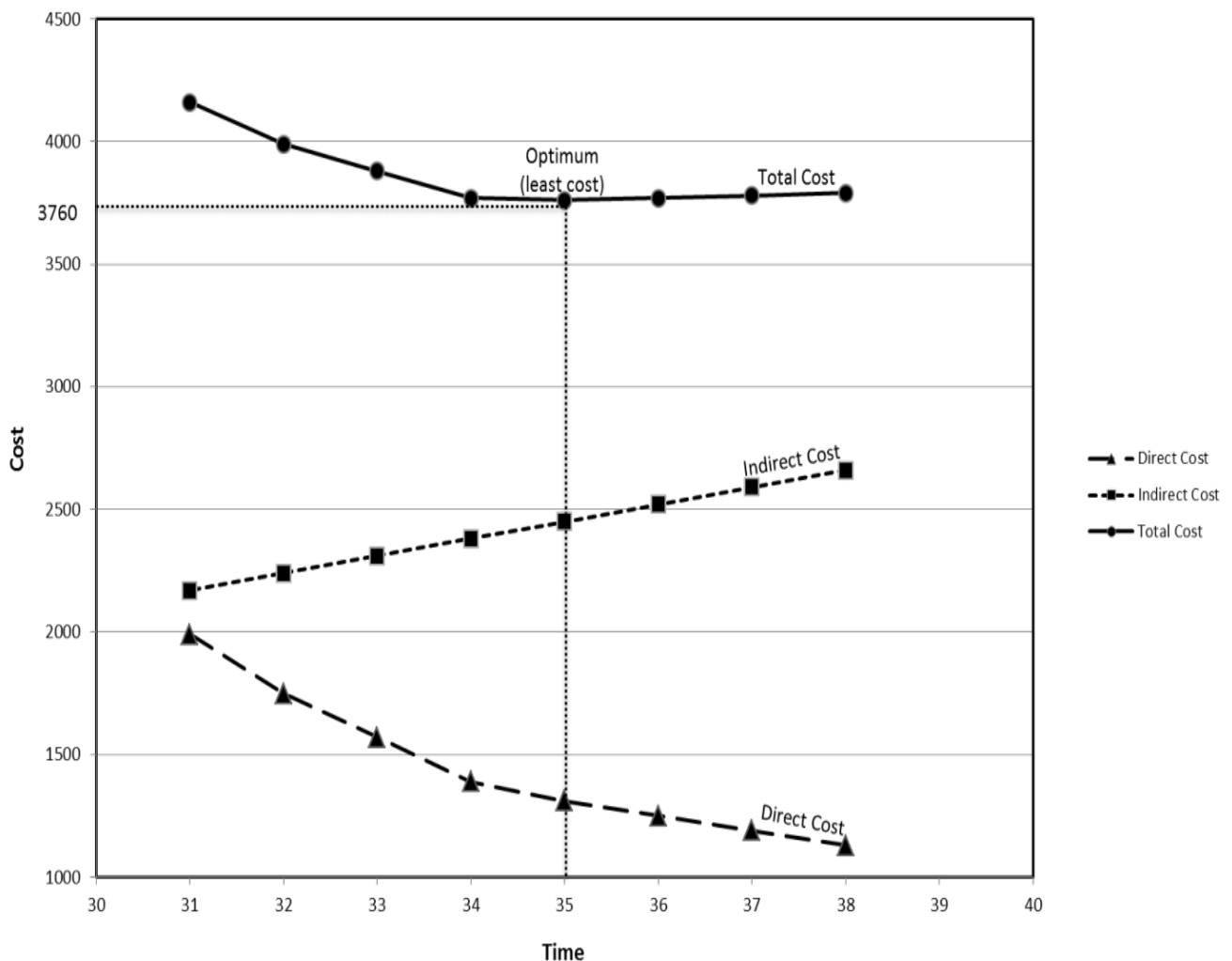


Figure 4. Result of relationship between time and cost of activity

7. Conclusions

A new mathematical method for modeling time-cost trade-off for analysis was introduced. The constraints of proposed model based on the path rather than activities, so, in the networks that the number of paths is lower than the number of activities, the number of equations would be less than the previous time-cost trade-off models. So, the time of solution and simplicity of model are main two advantages of this model.

The model was tested in a case study and computational results were validated by comparing with results of manual solution. The manual solution shows that the model is accurate and modeling process proves that a lesser number of constraints is needed in comparison to the activity based model.

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