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Numerical Evaluation of Longitudinal Modulus of Elasticity of *Eucalyptus grandis* Timber Beams

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Abstract Currently, the standard NBR 7190:1997 (Design of Wood Structures) makes no reference to any test aimed to determine the stiffness and strength of structural-sized lumber components, restricting this analysis to small and clear specimens. Methodologies proposed by international standard do not include optimum criteria in their calculation models. This study presents an alternative methodology to determine the longitudinal modulus of elasticity in *Eucalyptus grandis* timber beams, based on a combination between the Finite Element and the Least Square Methods. Besides the use of numerical methods, the modulus of elasticity was also analytically obtained by the equation presented in NBR 7190:1997, concerning the static three-point bending test, adapted to a non-destructive testing condition. Results found for the elastic modulus for the *Eucalyptus grandis* species showed statistical equivalence between the methodologies, which implies the reliability of using Brazilian standard for the characterization of structural components subjected to bending. However, these results cannot be extended for other woods from the same species or different species, justifying the use of the numerical calculation approach discussed in this paper.

Keywords Modulus Of Elasticity, Lumber, Finite Element Method, Least Square Method, Bending Beams

1. Introduction

Wood has been widely used to build structures because it is a renewable material with an excellent strength-to-density ratio in compare to steel[1].

Design of timber structures, as well as of other materials, requires the knowledge of some variables, including the modulus of elasticity, obtained through experimental tests recommended by standard documents, which can be destructive or not.

Being wood orthotropic and heterogeneous, experiments aimed at characterizing this material, by evaluating its bending behaviour, must be held on structural-sized components in order to improve their reliability. In this context, only international standards can be mentioned, since the Brazilian standard[2], which deals with wood characterization, only includes destructive testing conditions using small and clear specimens.

American standard[3] recommends the static four-point bending test (Figure 1) to determine the bending modulus of elasticity (E_m), expressed by the Equation 1, being F the applied force, provided that the limit of proportionality is not exceeded, L the span between supports, a the distance between the applied forces, b the specimen width, h the specimen thickness and δ the deflection at the middle of the span.

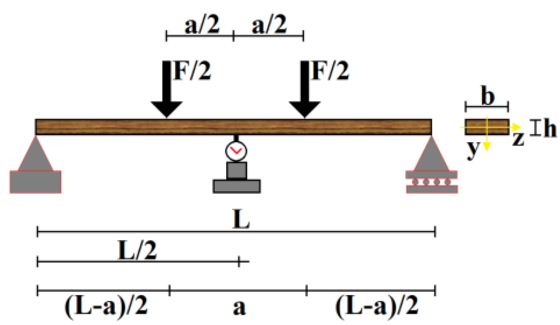

Figure 1. Static four-points bending tests[3]

$$E_m = \frac{F \cdot a \cdot (3 \cdot L^2 - 4 \cdot a^2)}{4 \cdot b \cdot h^3 \cdot \delta} \quad (1)$$

American standard[4] uses the static four-point bending structural model and indicates the flatwise position to determine the strength and stiffness properties of wood, as

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depicted in Figure 2.

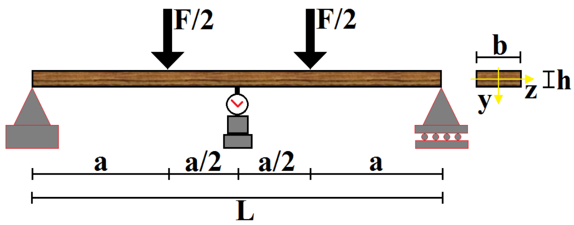

Figure 2. Four-point static bending[4]

The apparent modulus of elasticity (E_a), calculated by standard[4], is obtained by the Equation 2.

$$E_a = \frac{a\cdot\left(3\cdot L^2 - 4\cdot a^2\right)}{2\cdot b\cdot h^3}\cdot\left(\frac{F_{50\%} - F_{10\%}}{\delta_{50\%} - \delta_{10\%}}\right) \quad (2)$$

In the Equation 2, $F_{50\%}$ and $F_{10\%}$ are the forces (N) corresponding to 10% and 50% of the maximum load applied to the specimen, $\delta_{50\%}$ and $\delta_{10\%}$ are the displacements (mm) corresponding to 10% and 50 % of the maximum load, *b* and *h* correspond respectively to the width and height (mm) of the specimen cross-section, *a* is the distance between the support and the point of load application and *L* the span between supports.

American standard[5] uses the static three-point bending test to determine the apparent elastic modulus (Equation 3) of boards in the edgewise position (Figure 3), where I_z is the moment of inertia about the axis *z*.

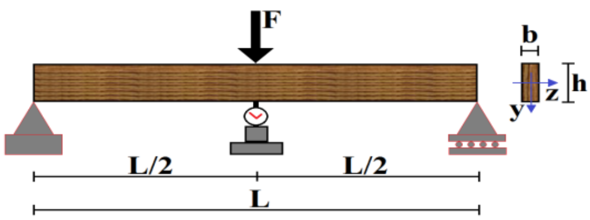

Figure 3. Static three-point bending tests[5]

$$E_a = \frac{L^3}{48\cdot I_z}\cdot\left(\frac{F_{50\%} - F_{10\%}}{\delta_{50\%} - \delta_{10\%}}\right) \quad (3)$$

The two calculation methods proposed by standards[4] and[5] can produce different results for the elastic modulus, justified by the intensity and amplitude of the shear force, leading to apparent values for the flexural modulus of elasticity, since the shear contribution is not taken into account in the beam model used to develop those equations[6].

However, Bodig and Jaine[7] state that the higher the ratio of the beam length (L) to the cross-section height (h), the lower the shear contribution is to the deflection calculus. For *L/h* ratios equal to or greater than 21 this contribution is virtually negligible[8].

The ASTM D 4761-96[5], which adopts the three-point bending test, suggests that the *L/h* ratio when testing boards in the edgewise position should be between 17 and 21. In these cases, the apparent elastic modulus (E_a) approaches the bending modulus of elasticity (E_m).

The European standard (EN 789)[9] (Portuguese version) recommends the static four-point bending test is in accordance to the curvature method, recording the vertical displacement at mid-span, however another point might be situated between the load set-points, preferably one as far as possible from the mid-span. Interpretation of results is done from the record of the curvature variation due to the applied loads. In this case, the bending modulus of elasticity is based exclusively on deformation under bending moment. Equation 4 expresses the flexural modulus of elasticity.

$$E_m = \left(\frac{F_{40\%} - F_{10\%}}{\delta_{40\%} - \delta_{10\%}}\right)\cdot\frac{L_1^{\,2}\cdot L_2}{16\cdot I_z} \quad (4)$$

In the Equation 4, $F_{40\%}$ and $F_{10\%}$ are the forces (*kgf*) corresponding to 10% and 40% of the maximum load applied to the specimen; $\delta_{40\%}$ and $\delta_{10\%}$ are the displacements (*cm*) corresponding to 10% and 40% of the maximum load; L_1 the reference distance for measuring the displacement of the center span (*cm*) and L_2 half the length of the shear zone (*cm*).

Structural wood composites, such as Laminated Veneer Lumber (LVL), Laminated Strand Lumber (LSL), Oriented Strand Lumber (OSL) and Parallel Strand Lumber (PSL) are usually characterized in bending based on the calculus premises of American standard (ASTM D 5456)[10], which uses the static three-point bending test to determine the modulus of elasticity and bending stiffness of veneers both in the flatwise and edgewise positions, according to the veneer disposition in the element[11, 12].

In Brazil, investigations[13-15] on the characterization of wood components through destructive tests follow the premises and calculation methods contained in international standard.

As previously dicussed, characterization of structural timber components can also be done by means of non-destructive tests, aimed at determining the physical and mechanical properties of a structural element without changing its usabilities[16]. Non-destructive testing has the advantage of not needing specimen extraction, thus enabling the study of its structural integrity[17-24], which is normally done by transverse vibration and ultrasound techniques.

As explained, the calculation models proposed by standards to determine the flexural modulus of elasticity of structural timber components (via destructive testing) do not include optimization methodologies. As for the conventional non-destructive tests, the need to acquire specialized equipment for determining the modulus of elasticity must be emphasized.

This paper suggests an alternative methodology to calculate the longitudinal modulus of elasticity of *Eucalyptus grandis* structural-sized timber beams, based on the Finite Element and the Least Square methods, together with the equation used to calculate the modulus of elasticity proposed by the Brazilian standard[2] concerning the three-point bending test under a non-destructive testing condition.

2. Materials and Methods

To determine the modulus of elasticity, 20 *Eucalyptus Grandis* elements with dimensions of $6cm \times 16cm \times 200cm$ were used, respecting the $L \geq 21 \cdot h$ ratio and neglecting the influence of shear force on the deflection calculus[8].

Experiments held to determine the modulus of elasticity are considered as non-destructive tests, because the highest displacement value found is limited to the *L/200* ratio, where *L* is the span between supports, expressed in centimeters. This is an average of small displacements defined by standard (NBR 7190)[2], which implies physical and geometrical linearity for the timber beams.

In this paper, the modulus of elasticity is evaluated according to two distinct mathematical calculation models, both making use of the static three-point bending structural scheme. In the first, adapted from[2], the modulus of elasticity (E_{eq}) is determined by the Equation 5, where F^* is the force responsible for causing a displacement equal to *L/200*, and *b* and *h* are the basis and height measurements of the pieces cross-section. In the second, as an alternative way of calculation, it is proposed that the value of the effective modulus of elasticity (E_{ef}) be determined according to the structural testing scheme shown in Figure 4.

$$E_{eq} = \frac{50 \cdot F^* \cdot L^2}{b \cdot h^3} \tag{5}$$

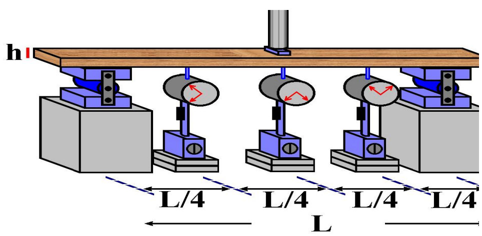

Figure 4. Alternative test to determine the effective modulus of elasticity

For each one of the structural tests held, three dial gauges were placed along the piece, distant from *L/4* from each other. Displacement readings on the gauges equidistant from the supports are done when the magnitude of displacement at mid-span approaches the *L/200* ratio.

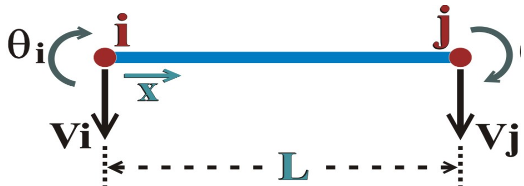

Figure 5. Finite-element degrees of freedom

To calculate the effective modulus of elasticity, a computer software (E_{ef}) that makes use of *Mathcad 2000®* programming language was developed, in accordance to the fundamentals of the Finite Element Method (FEM) applied to the Virtual Work Principle (VWP), including the Bernoulli's kinematic model of beam deformation and ignoring the forces per unit of volume and surface area in those calculations. The finite element has two degrees of freedom per node, two translations and two rotations (Figure 5), developed with the use of third-degree polynomial functions as displacement field approximation.

Displacement values read on each indicator dial are grouped together in vector form ($\boldsymbol{U}^{(exp)}$) and entered into the *Eotm* software, in order to calculate the optimal value of the modulus of elasticity of the components.

Based on the fundamentals of FEM, *Eotm* software determines a numerical displacement vector ($\boldsymbol{U}^{(num)}$), where the dependent variable is the modulus of elasticity of the structural element.

Having the displacement vector determined by the software and the experimental displacement vector, a function is formulated, based on the Least Square Method, which gives the value of the modulus of elasticity so that the amount of waste generated both by the numerical and experimental solutions can be kept to a minimum (Equation 6), obtained through the Newton-Raphson Method.

$$f(E) = \frac{1}{2}\sum_{i=1}^{n}\left(U_i^{(exp)} - U_i^{(num)}\right)^2 \tag{6}$$

Statistical equivalence between the values of E_{eq} (equivalent longitudinal modulus of elasticity) and E_{ef} (effective modulus of elasticity) is evaluated by using a confidence interval for the medians, which is expressed by the Equation 7, being μ the difference between population means, $\overline{x}_m$ the difference between sample means, *n* the sample size, S_m the difference between sample standard deviations, and $t_{\alpha/2,n-1}$ the tabulated value from t Student's distribution, with *n-1* degrees of freedom and significance level α.

$$\overline{x}_m - t_{\alpha/2,n-1} \cdot S_m / \sqrt{n} \leq \mu \leq \overline{x}_m + t_{\alpha/2,n-1} \cdot S_m / \sqrt{n} \tag{7}$$

Anderson-Darling test was used to confirm that the sets of values for the modulus of were normally distributed, validating the confidence interval studied herein.

3. Results and Conclusions

The Elasticity values E_{eq} and E_{ef} found for the *Eucalyptus grandis* components are shown in Table 1.

Table 1. E_{eq} and E_{ef} values

	E_{eq} (MPa)	E_{ef} (MPa)		E_{eq} (MPa)	E_{ef} (MPa)
1	14133	13760	11	14636	15123
2	15175	15326	12	17599	17372
3	14261	14844	13	17166	17420
4	16347	16151	14	17481	17945
5	15384	14827	15	16956	17638
6	16823	17423	16	15789	15376
7	18581	18224	17	18326	19482
8	16552	16813	18	16854	16911
9	15354	15836	19	15980	15466
10	17237	16867	20	17022	16542

P-values found for the equivalent and effective elasticity modulus are respectively *0.667* and *0.794*, both greater than 0.05, confirming the normality test[25]. Figures 5 and 6 show the normal probability test for the moduli of elasticity E_{eq} and E_{ef}, respectively.

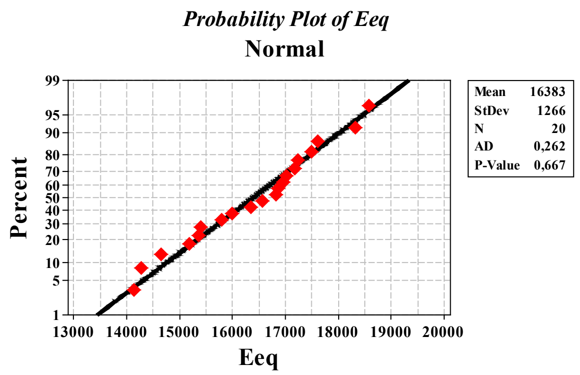

Figure 5. Normal probability plot for the modulus of elasticity E_{eq}

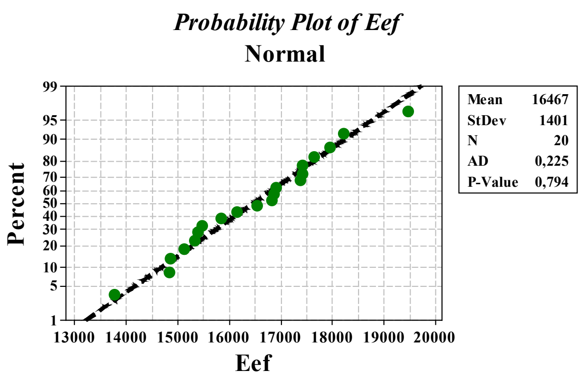

Figure 6. Normal probability plot for the modulus of elasticity E_{ef}

The confidence interval between the E_{eq} and E_{ef} values is $-940.18 \le \mu \le 771.18$ and, since zero belongs to the interval, they are proven to be statistically equivalent. The confidence interval of the modulus of elasticity responses is shown in Figure 7.

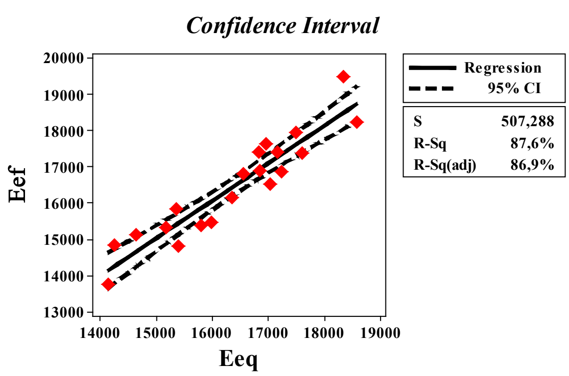

Figure 7. Confidence interval plot for E_{eq} and E_{ef} responses

The graph of linear regression between the elasticity values for both calculation methodologies is shown in Figure 8, whose the regress equation and correlation coefficient (R^2) found are respectively $E_{ef} = -500+1.036 \cdot E_{eq}$ and *0.88*.

Since the numerical methodology proposed in this paper is based on optimization concepts, it allows a more reliable calculation of the modulus of elasticity in structural-sized timber components when compared to other methodologies.

The loading restriction at *L/200*[2] provides a non-destructive testing, which allow the reuse of the structural timber sample.

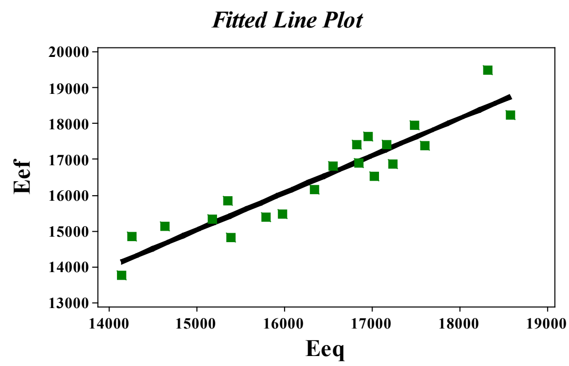

Figure 8. Linear regression between E_{eq} and E_{ef}

The ratio $L \ge 21 \cdot h$ was able to use the Bernoulli Beam Theory in both methodologies which considered no shear effects in the calculus of displacements[8].

Statistical equivalence between the elasticity values E_{eq} and E_{ef} point to the fact that the methodology recommended by Brazilian standard[2] was also proven to be efficient for *Eucalyptus grandis* structural-sized timber components. However, these results may not apply to wood components and/or species other than the ones herein evaluated. It is worthy of note that the presented methodology can also be used to characterize specimens, and not only lumber, but also glued-laminated timber beams, wood panels and others.

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