

Analysis of a Strip Footing Using Constitutive Law

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Abstract In this paper an attempt has been made to obtain the bearing capacity and settlement of a strip footing using non-linear constitutive law of soil. Soil used in the study has been taken saturated clay. This approach gives directly the pressure-settlement characteristics of an actual footing, and therefore no interpolation and other formulae are needed for its analysis and design. The study includes the effect of parameters needed for describing a constitutive law and width of footing. Salient conclusions have been drawn based on the study.

Keywords Constitutive law of soil, Bearing capacity, Strip footing, Settlement curve

1. Introduction

Strip footings are commonly used as the foundations of buildings specifically upto four storeys. Bearing capacity and settlement at working pressure intensity are needed for its analysis and design. In the past, it was usual to design the footing using bearing capacity theories [14]; [9] and one dimensional consolidation theory proposed by [14]. Data obtained from static cone penetration tests have also been used for proportioning footing using related correlations [10]; [11]. Few investigators [2]; [3]; [4]; [8] have analysed problem using finite element and finite difference techniques. Method of characteristics have also been used by some researchers [7]; [9]; [13]; [17].

In this paper, work of [12] has been extended studying the effect of Kondner's hyperbola constants and width of footing. The analysis facilitates to obtain the pressure-settlement characteristics of an actual footing.

2. Analysis

The analysis is based on the following assumptions:

- 1) The soil mass has been assumed as semi-infinite and isotropic medium.
- 2) The whole soil mass supporting a footing has been divided into a large number of thin horizontal strips (Fig.1) in which stresses and strains have been obtained along any vertical section.
- 3) a) The stresses in each layer have been computed using Boussinesq's theory as the stress equations for various types of loads are available.

- b) The strains have been computed from the known stress condition using constitutive law of soil.

Following steps have been carried out for obtaining the pressure-settlement characteristics of the footing.

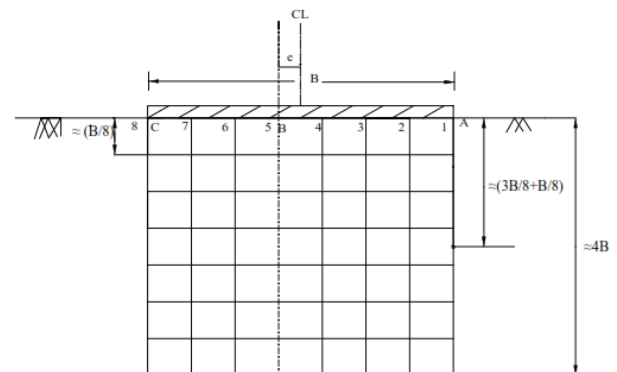


Figure 1. Soil strata divided into n-horizontal strips

3. Procedure

Procedure of the analysis is described in following steps:

- i) Divide the soil strata in thin layers upto significant depth ($\approx 4.0B$). Thickness of each layer may be taken equal to $B/8$, B bearing the width of footing.
- ii) Select about eight points on the base of footing including points A, B, and C. Further procedure is to determine the values of settlements of these points. For this, consider a vertical section passing through the any of the selected points.
- iii) For determining the settlement of the selected point on the base of footing, determine the settlement of each layer at that point. For example, consider fourth layer and vertical section passing through point 'A'. Depth of the centre of this layer below base of footing

will be $\left(\frac{3B}{8} + \frac{B}{16} \right)$.

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Determine the stresses (σ_z , σ_x and τ_{xz}) at this point due to applied stress using theory of elasticity. Using these values of stresses, obtain the values of principal stresses (σ_1 and σ_3), and their directions (θ_1 and θ_3) with vertical in usual way.

- iv) In case of clays value of principal strain (ε_1) in the direction major principal stress may be obtained as per Eq.(1)

$$\varepsilon_1 = \frac{a(\sigma_1 - \sigma_3)}{1 - b(\sigma_1 - \sigma_3)} \quad (1)$$

Where a and b are Kondner's hyperbola constants. Their values may be obtained by performing drained triaxial tests. These are independent to confining pressure σ_3 .

$$\varepsilon_3 = -\mu_2 \cdot \varepsilon_1 \quad (2a)$$

$$\mu_2 = \frac{-\sigma_3 + \mu_1 \sigma_1}{\sigma_1 - \mu_1 \sigma_3} \quad (2b)$$

$$\mu_1 = \frac{\mu}{1 - \mu} \quad (2c)$$

μ is Poisson's ratio

- v) Vertical strain at the selected point will then be:

$$\varepsilon_z = \varepsilon_1 \cos^2 \theta_1 + \varepsilon_3 \cos^2 \theta_3 \quad (3)$$

- vi) On multiplying with the thickness of the strip, settlement of the strip at the point under consideration is obtained. The evaluation of the total settlement along any vertical section is done by numerically integrating the quantity i.e.

Where

$$S_t = \int_0^n \varepsilon_z dz \quad (4)$$

S_t = total settlement along i^{th} vertical section,

ε_z = vertical strain at depth z below the base of footing.

dz = thickness of strips at depth z, and

n = number of strips in which the soil strata upto significant depth is divided. In this way values of total settlements along other vertical sections passing through the base of footing are obtained.

4. Results and Interpretation

Using the above analysis, pressure settlement characteristics of a strip footing have been obtained for the following parameters:

Width of footing, B: 0.5m, 1.0m, 1.5m, 2.0m

Hyperbolic Constitutive laws parameters

1/a (kN/m ²)	5000	7000	9091	12000	15000
1/b (kN/m ²)	35	50	64	80	100

Pressure Intensity, q (kN/m²): 5 to 120 (Depending on the values of 1/a and 1/b).

For illustration, for 1/a =15000; 1/b =100 and B=1.0m, settlements of equally spaced nine points of the base were obtained for different pressure intensities. Typical plots showing the settlement patterns for pressure intensities 25 and 50 are shown in Figs 2 and 3 respectively. It is evident that these settlement patterns are parabolic in nature having maximum settlement at the centre of the footing. It was concluded by earlier investigations [12]; [1] that the settlement of the equivalent rigid footing i.e. of the same width is almost equal to the average settlement of a flexible footing. It is obtained by dividing the area of the settlement diagram (i.e. as shown in Figs 2 and 3) by the width of footing. Plots of pressure versus settlement curves have been prepared using the average settlement i.e. considering the footing base as rigid.

Typical bearing pressure versus settlement curves for different combinations of 1/a and 1/b values are given in Figs 4, 5, 6, 7 and 8 considering B=1.0m. It is evident from these figures that pressure-settlement characteristics improves with the increase in 1/a and 1/b values. It is due to the fact that higher values of 1/a and 1/b indicate better soil.

This point has been highlighted in Figs.9 and 10, i.e. for a given value of 1/b, settlement of a footing same pressure intensity decreases with the increase in 1/a. Similar trend has been observed for a given value of 1/a, variation of settlement with respect to 1/b.

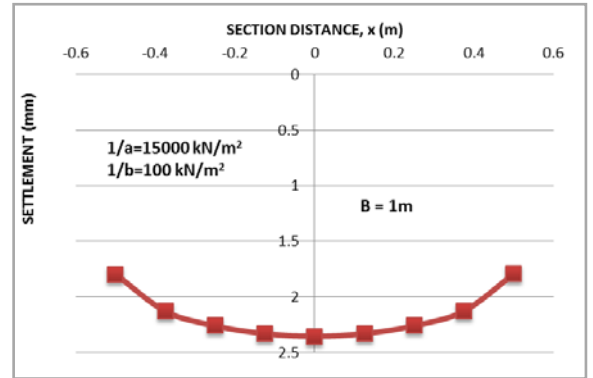


Figure 2. Settlement pattern for q = 25 kN/m²

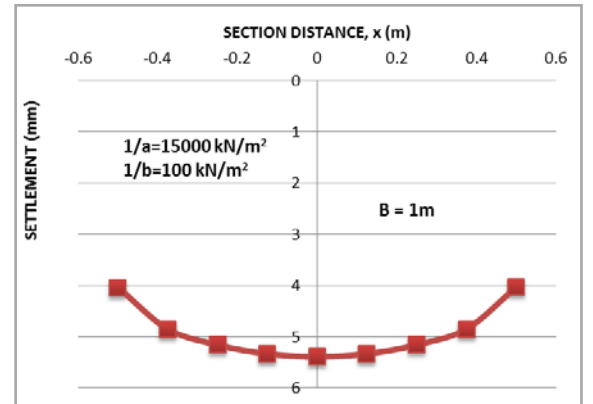


Figure 3. Settlement pattern for q = 50 kN/m²

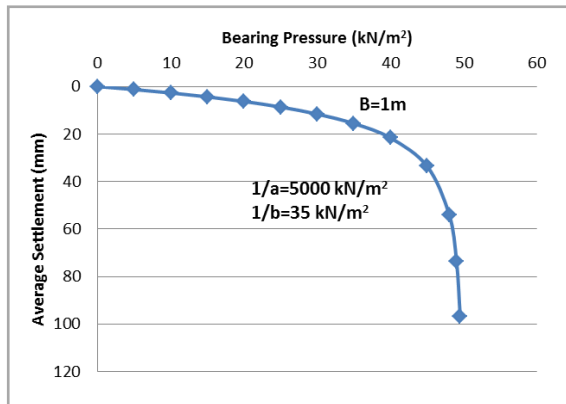


Figure 4. Pressure-average settlement curve

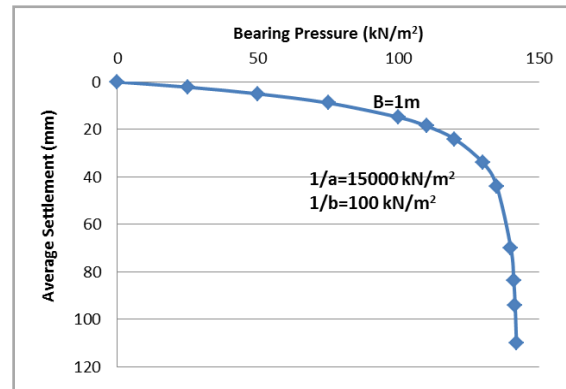


Figure 8. Pressure-average settlement curve

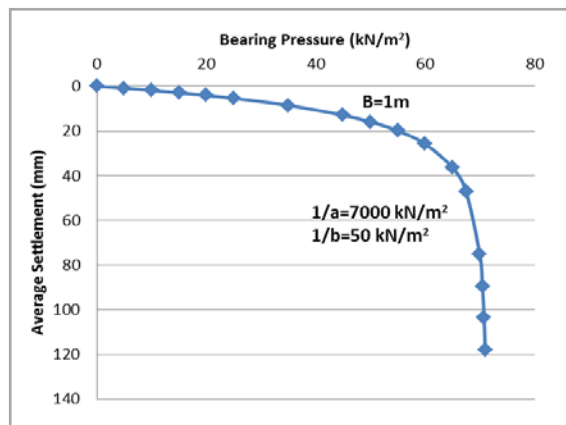


Figure 5. Pressure-average settlement curve

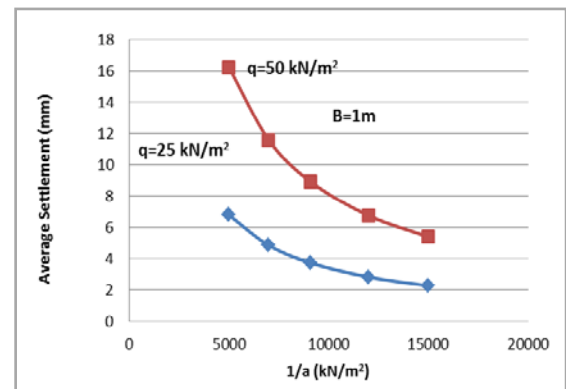
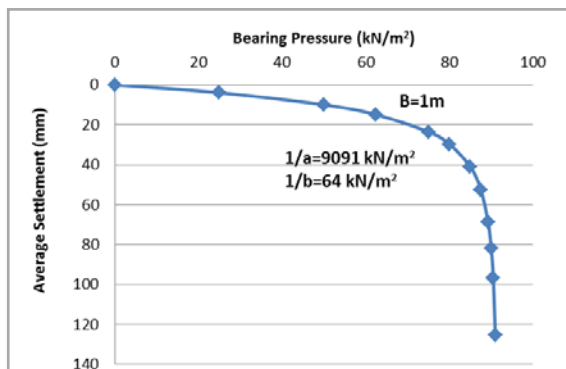
Figure 9. Average settlement - $1/a$ curve

Figure 6. Pressure-average settlement curve

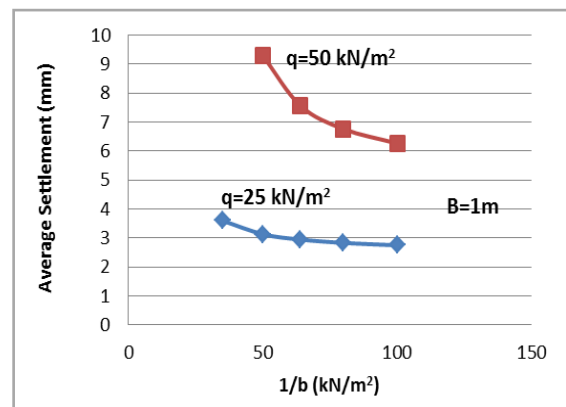
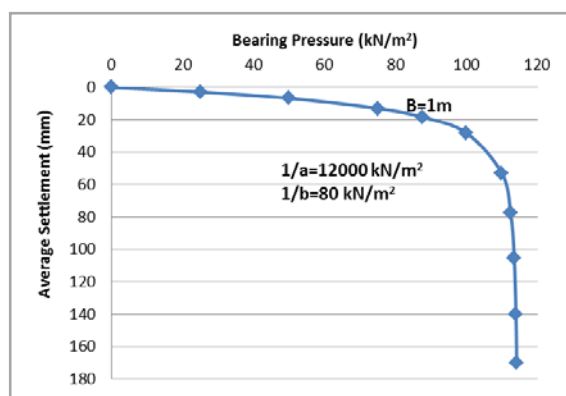
Figure 10. Average settlement - $1/b$ curve

Figure 7. Pressure-average settlement curve

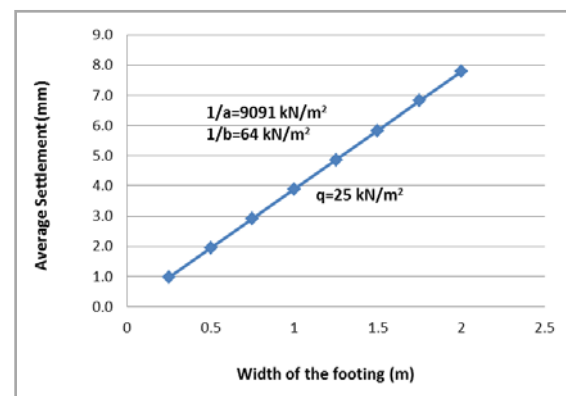


Figure 11. Pressure-average settlement curve

Settlement of a footing for given values of $1/a$ and $1/b$ and pressure intensity increases with the width of the footing linearly (Figs.11). These have been observed earlier also by many investigators including [15].

5. Conclusions

- 1) Evaluation of pressure-settlement characteristics of an actual footing using constitutive law of soil enable its complete proportioning. Bearing capacity can be obtained using intersection tangent method. Settlement of the footing for the design pressure can be read confidently from it.
- 2) Pressure-settlement characteristic of a footing improves with the increase in the values of Kondner's hyperbola parameters.

Further it may be noted from the pressure-settlement characteristics that the bearing capacity of the footing may conveniently obtained using intersection tangent method. Thus from Figs. 4 to 8, bearing capacities are obtained as below:

$1/a$ (kN/m ²)	$1/b$ (kN/m ²)	q_u (kN/m ²)	Fig. No
5000	35	45	4
7000	50	62	5
9000	60	83	6
12000	80	110	7
15000	100	130	8

Illustrative Example

Design a wall footing which carries a load of 40 kN/m. The properties of underground soil which is saturated clay has been obtained by performing triaxial test. It gives $1/a=15000$ kN/m²; $1/b=100$ kN/m²;

Assume permissible settlement as 20mm as Solution:

- a) Assume the width of the footing as 1.0 m. Obtain Pressure-settlement characteristics of the footing using the methodology described in the paper. It will work out as shown in Fig.8. This curve gives ultimate bearing capacity as 130 kN/m². Therefore, the footing wall will be able to carry a load equal to $(140/3) \times 1 = 43$ kN/m, hence safer it is more than 12 kN/m.
- b) Pressure on the footing = $(40/1) = 40$ kN/m². For pressure intensity of 40 kN/m², settlement of the footing works out to be about 5 mm which is much less than the permissible settlement 20mm.

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REFERENCES

- [1] Agarwal, R.K. (1986), "Behaviour of Shallow Foundations Subjected To Eccentric-Inclined Loads" Ph.D Thesis, University of Roorkee, Roorkee.
- [2] Chakraborty, Debarghya. "Bearing capacity of strip footings by incorporating a non-associated flow rule in lower bound limit analysis." *International Journal of Geotechnical Engineering* 10, no. 3 (2016): 311-315.
- [3] Dai, Zi Hang, and Xiang Xu. "Comparison of analytical solutions with finite element solutions for ultimate bearing capacity of strip footings." In *Applied Mechanics and Materials*, vol. 353, pp. 3294-3303. Trans Tech Publications, 2013.
- [4] Dai, Zi Hang, and Xiang Xu. "Comparison of analytical solutions with finite element solutions for ultimate bearing capacity of strip footings." In *Applied Mechanics and Materials*, vol. 353, pp. 3294-3303. Trans Tech Publications, 2013.
- [5] Han, Dongdong, Xinyu Xie, Lingwei Zheng, and Li Huang. "The bearing capacity factor N_γ of strip footings on $c-\phi-\gamma$ soil using the method of characteristics." *SpringerPlus* 5, no. 1 (2016): 1482.
- [6] Kumar, Jyant. " N_γ for rough strip footing using the method of characteristics." *Canadian Geotechnical Journal* 40, no. 3 (2003): 669-674.
- [7] Kumar, Jyant. "The variation of N_γ with footing roughness using the method of characteristics." *International Journal for Numerical and Analytical Methods in Geomechanics* 33, no. 2 (2009): 275-284.
- [8] Loukidis, D., and R. Salgado. "Bearing capacity of strip and circular footings in sand using finite elements." *Computers and Geotechnics* 36, no. 5 (2009): 871-879.
- [9] Meyerhof, G.G. (1951.), "Ultimate Bearing Capacity of Foundation", *Geotechnique*, Vol.2, No.4, pp.301-331.
- [10] Peck, R.B, Hanson, W.E and T.H. Thornburn (1974), "Foundation Engineering", 2nd Edition, John Wiley and sons, N.Y.
- [11] Saran, S., (2010), "Analysis and Design of Substructures", Oxford and IBH Publishing Co. Pvt. Ltd., New Delhi.
- [12] Sharan, U.N. (1977), "Pressure Settlement Characteristics of Surface Footings Using Constitutive Laws", Ph.D Thesis, University of Roorkee, Roorkee
- [13] Sun, Jian-Ping, Zhi-Ye Zhao, and Yi-Pik Cheng. "Bearing capacity analysis using the method of characteristics." *Acta Mechanica Sinica* 29, no. 2 (2013): 179-188.
- [14] Terzaghi, K. (1943), "Theoretical Soil Mechanics, John Wiley and Sons", Inc. N.Y.1943.
- [15] Terzaghi, K. and Peck, R B. (1967), "Soil Mechanics in Engineering Practice", John Wiley and Sons Inc. New York, 1967.