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Aseismic Ground Deformation in a Viscoelastic Layer over Lying a Viscoelastic Half-Space Model of the lithosphere-Asthenosphere System

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Abstract The process of stress –accumulation near earthquake faults during the aseismic period in between two major seismic events in seismically active regions has become a subject of research during the last few decades. A long strike-slip fault in a viscoelastic layer over a viscoelastic half space representing the lithosphere-asthenosphere system has been considered here. Stresses accumulate in the region due to various tectonic processes, such as mantle convection and plate movements etc, which ultimately leads to movements across the fault. In the present paper, a two-dimensional model of the system is considered and expressions for displacements, stresses and strains in the model have been obtained using suitable mathematical techniques developed for this purpose. A detailed study of these expressions may give some ideas about the nature of stress accumulation in the system, which in turn will be helpful in formulating an effective earthquake prediction programme.

Keywords Viscoelastic Layered Model, Aseismic Period, Stress Accumulation, Mantle Convection, Plate Movements, Tectonic Process, And Earthquake Prediction

1. Introduction

Modeling of dynamic processes leading to an earthquake is one of the main concerns of seismologist. Two consecutive seismic events in a seismically active region are usually separated by a long aseismic period during which slow and continuous aseismic surface movements are observed with the help of sophisticated measuring instruments. Such aseismic surface movements indicate that slow aseismic change of stress and strain are occurring in the region which may eventually lead to sudden or creeping movements across the seismic faults situated in the region.

It is therefore seems to be an essential feature to identify the nature of the stress and strain accumulation in the vicinity of seismic faults situated in the region by studying the observed ground deformations during the aseismic period. A proper understanding of the mechanism of such aseismic quasi static deformation may give us some precursory information regarding the impending earthquakes.

A pioneering work involving static ground deformation in elastic media were initiated by ([8], [9]), ([21]), ([2]), ([3]),([6]), ([23]). Reference ([22]) has discussed various aspects of fault movement in his book. Reference ([5]) has discussed stress accumulation near buried fault in lithosphere-asthenosphere system.

In most of these studies the medium were taken to be elastic and /or viscoelastic, layered or otherwise. We now focus on some of the reasons of consideration of viscoelastic layer over viscoelastic half space model.

The laboratory experiments on rocks at high temperature and pressure indicates the imperfect elastic behavior of the rocks situated in the lower lithosphere and asthenosphere.

Investigations on the post-glacial uplift of Fennoscandia and parts of Canada indicate that at the termination of the last ice age, which happened about 10 millennia ago a 3km. ice cover melted gradually leading to upliftment of the regions .Evidence of this upliftment has been discussed in Fairbridge (1961), Schofield (1964), Chathles (1975), if the Earth were perfectly elastic, this deformation would be managed after the removal of the load, but it did not happened, indicating that the Earth crust and upper mantle is not perfect elastic but rather viscoelastic in nature.

Therefore in the present case we consider a long strike-slip fault situated in a viscoelastic layer over a viscoelastic half -space which is surface breaking in nature. The medium is under the influence of tectonic forces due to mantle convection or some related phenomena. The fault undergoes

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a creeping movement when the stresses in the region exceed certain threshold values.

2. Formulation

We consider a long strike-slip fault F width D situated in a viscoelastic layer over a viscoelastic half space of linear Maxwell type.

We choose our rectangular Cartesian coordinates (y_1, y_2, y_3) such that, the free surface is the plane $y_3 = 0$ the fault F_1 is in the plane $y_2 = 0$, y_1-axis is perpendicular to plane of the fault and y_3 axis pointing downwards as shown in the Fig: 1.Since our model is a viscoelastic layer of finite depth say H, over a viscoelastic half space.

∴ The interface is the plane $y_3 = H$.

We consider the fault F_1 in the region $y_2 = 0$, $0 \le y_3 \le D_1 \le H$

The half space is $y_3 \ge H$.

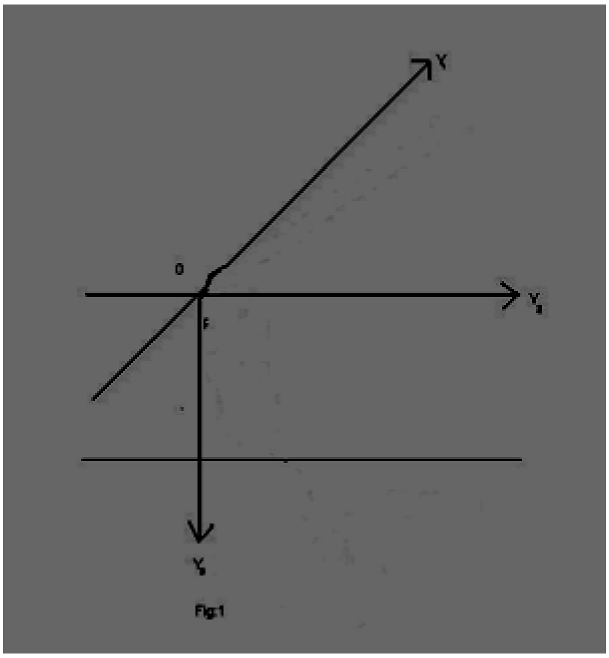

Figure 1. Section of the model by the plane y_1=0

Since we are concentrating 2D problem in which the length of the fault is large compared to the depth of it.

We take displacement, stress, strain to be in dependent of y_1 and dependent on y_2, y_3, t where't' is the time measured after the establishment of aseismic state.

Let the components of displacement, stress and strain in the layer M_1 be u_1, τ^1_{12}, τ^1_{13}, e^1_{12}, e^1_{13} respectively and in the half space M_2 be u_2, τ^2_{12}, τ^2_{13}, e^2_{12}, e^2_{13}.

Since we are considering strike-slip movement only. we are not considering the other components of displacements, stresses, strains (u_2, u_3, τ_{22}, τ_{23}, etc.)

In our model M_1 is isotropic-homogeneous layer of rigidly modus μ_1, effective viscosity η_1 (say).M_2 is another isotropic homogeneous half space of rigidity modulus μ_2 and effective viscosity η_2 (say).

2.1. Constitutive Equations

For perfect elastic body we know as soon as the stress is removed. The strain also disappears but for viscoelastic body, the strain does not at once remove.

To incorporate the viscoelastic effects stress-strain relations of a slightly different form have been suggested.

We have the constitutive equations (i.e., stress -strain relation) as, for the layer M_1,

$$\left(\frac{1}{\eta_1} + \frac{1}{\mu_1}\frac{\partial}{\partial t}\right)\tau'_{12} = \frac{\partial}{\partial t}(e'_{12}) = \frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial y_2}\right) \quad (1.1)$$

$$\left(\frac{1}{\eta_1} + \frac{1}{\mu_1}\frac{\partial}{\partial t}\right)\tau'_{13} = \frac{\partial}{\partial t}(e'_{13}) = \frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial y_3}\right) \quad (1.2)$$

$$(-\infty < y_2 < \infty,\ 0 \le y_3 \le H,\ t \ge 0)$$

For the half space M_2

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{12} = \frac{\partial}{\partial t}(e^2_{12}) = \frac{\partial}{\partial t}\left(\frac{\partial u_2}{\partial y_2}\right) \quad (1.3)$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{13} = \frac{\partial}{\partial t}(e^2_{13}) = \frac{\partial}{\partial t}\left(\frac{\partial u_2}{\partial y_3}\right) \quad (1.4)$$

$$(-\infty < y_2 < \infty,\ y_3 \ge H,\ t \ge 0)$$

2.2. Stress Equation of Motion

We have the stress equation of motion as

$$\frac{\partial}{\partial y_1}(\tau^1_{11}) + \frac{\partial}{\partial y_2}(\tau^1_{12}) + \frac{\partial}{\partial y_3}(\tau^1_{13}) = \rho\frac{\partial^2 u_1}{\partial t^2}$$

Since displacement, stress and strain are independent of y_1 therefore the stress equation becomes,

$$\frac{\partial}{\partial y_2}(\tau^1_{12}) + \frac{\partial}{\partial y_3}(\tau^1_{13}) = \rho\frac{\partial^2 u_1}{\partial t^2},$$

ρ= average density of M_1

Since our observation is in the aseismic period while the order of displacement is 10^{-4} c.g.s. unit or less.

∴ The inertial term $\frac{\partial^2 u_1}{\partial t^2}$ is extremely small. Therefore, we neglect it.

Stress equation of motion for the layer M_1 is

$$\frac{\partial}{\partial y_2}(\tau^1_{12}) + \frac{\partial}{\partial y_3}(\tau^1_{13}) = 0$$

$$\text{for}(-\infty < y_2 < \infty, 0 \le y_3 \le H, t \ge 0) \quad (1.5)$$

Similarly for the half space M_2

$$\frac{\partial}{\partial y_2}(\tau^2_{12}) + \frac{\partial}{\partial y_3}(\tau^2_{13}) = 0$$

$$\text{for}(-\infty < y_2 < \infty, y_3 \ge H, t \ge 0) \quad (1.6)$$

Now differentiating partially (1) With respect to y_2, (2) with respect to y_3 and adding and using (5) we get,$\nabla^2 u_1 = 0$ for

$$(-\infty < y_1 < \infty,\ 0 \le y_3, \le H,\ t \ge 0) \quad (1.7)$$

[by (1.3), (1.4), (1.6)]

Similarly, $\nabla^2 u_2 = 0$ for $(-\infty < y_2 < \infty, y_3 \geq H, t \geq 0)$ (1.8)

We have the following boundary conditions:

Boundary Conditions on Stress

Continuity in stress condition:

$$\tau^1_{13} = \tau^2_{13} = \tau_H \text{ (say) on } y_3 = H \ (-\infty<y_2<\infty, t\geq 0) \quad (1.9)$$

$$\tau^1_{12} = \tau_\infty(t)(|y_2| \to \infty, t \geq 0) \quad (1.10),$$

$$\tau^2_{13} \to 0 \text{ as } y_3 \to \infty, (-\infty < y_2 < \infty, t \geq 0) \quad (1.11)$$

Where, $\tau_\infty(t)$ is the tectonic force which arise due to convection current acting at a far distance from the fault and always keeps the layer in a stressed state.

Boundary Conditions on displacement

As we have assumed the layer and half space are in welded contact.

The displacement is continuous.

$$\therefore \ (u_1)_{y_3=H} = (u_2)_{y_3=H} \quad (1.11.1)$$

Initial Conditions:

$$(\tau^1_{12})_{t=0} = \tau_\infty(0)(-\infty<y_2<\infty, 0\leq y_3 \leq H) \quad (1.12)$$

Taking Laplace transformation from (7) to (11a)

$$\nabla^2 \bar{u}_1 = 0 \quad (1.13)$$

$$\bar{\tau}^1_{13} = \bar{\tau}^2_{13} = \tau_H / P \text{ (say) on } y_3 = H,$$

$$\bar{\tau}^1_{12} = \bar{\tau}_\infty(P) \text{ as } |y_2| \to \infty \quad (1.14)$$

$$(\bar{u}_1)_{y_3=H} = (\bar{u}_2)_{y_3=H} \quad (1.15)$$

We are to solve the boundary value problem (1.12) to (1.15)

3. Solutions in Absence of Fault Creep ([24], [25])

Solving the boundary value problem, (1.12)-(1.15)

We get,

$$u_1(y_2, y_3, t) = (u_1)_0 + (y_2/\mu_1)\left[(\tau_\infty(t) - \tau_\infty(0)) + \left(\frac{\mu_1}{\eta_1}\right)\int_0^t \tau_\infty(\tau)d\tau\right] + y_3\tau_H t/\eta_1 \quad (2.1)$$

$$\tau^1_{12} = \tau_\infty(t) - \left(\tau_\infty(0) - (\tau^1_{12})_0\right)e^{-\frac{\mu_1}{\eta_1}t} \quad (2.1.1)$$

$$\tau^1_{13} = \tau_H\left(1 - e^{-\frac{\mu_1}{\eta_1}t}\right) + (\tau^1_{13})_0 e^{-\frac{\mu_1}{\eta_1}t} \quad (2.1.2)$$

If we take $\tau_\infty(t)$ = constant = τ_∞ (say)

Then,

$$u_1(y_1, y_3, t) = (u_1)_0 + \frac{y_2}{\eta_1}\tau_\infty t + y_3 . \frac{\tau_H}{\eta_1} t \quad (2.2)$$

$$\therefore \ e^1_{12} = \frac{\partial u_1}{\partial y_2} = \frac{\partial}{\partial y_2}(u_1)_0 + \frac{\tau_\infty}{\eta_1} t \quad (2.3)$$

$$\therefore \ \frac{d}{dt}(e^1_{12}) = \frac{\tau_\infty}{\eta_1} \quad (2.4)$$

If we take τ_∞ = 200 bars = 200 x 10^6 dynes/cm^2 and η_1 = 10^{21} poise then, $\frac{\tau_\infty}{\eta_1}$ = 2x10^{-13} /sec = 6 x 10^{-6} / year which tally with the observational value which is order 10^{-6} to 10^{-7}.

4. Displacements and Stresses after the Commencement of Fault Creep across the Fault F_1:([24],[25])

In our paper we are considering 1 creeping fault in the layer M_1.

If the creep commences across F at time $t = T_1$, then the relations (1.1) to (1.10) are satisfied and we have the following creeping conditions.

$$[u_1]_{F_1} = u_1(t_1) f_1(y_3) H(t_1) \quad (3.1)$$

Where F is the fault located in the region $y_2 = 0$, and $0 \leq y_3 \leq D_1$ and $t_1 = t - T_1$,

$[u_1]_{F_1}$ is the discontinuity in the displacement u_1 across the fault F_1.

$$(u_1)_{F_1} = \lim_{y_2 \to 0+} u_1 - \lim_{y_2 \to 0-} u_1 \quad (0 \leq y_3 \leq D_1) \quad (3.1.2)$$

$\therefore$ $[u_1]_{F_1}$ = 0 for $t_1 \leq 0$ and $H(t_1)$ Heaviside unit step function.

$\therefore$ The velocity of the creep is

$$\frac{\partial}{\partial t}(u_1)_{F_1} = V_1(t_1) f_1(y_3) H(t_1)$$

where

$$V_1(t_1) = \frac{\partial}{\partial t}u_1(t_1) = \frac{\partial}{\partial t_1}u_1(t_1)$$

Taking Laplace transformation of (3.1)

$$[\bar{u}_1] = \bar{u}_1(p_1) f(y_3) \text{ for, } (y_2 = 0, d_1 \leq y_3 \leq D_1) \quad (3.2)$$

where $\bar{u}_1(p_1)$ is the Laplace transformation of $u_1(t_1)$ with respect to t_1 and is given by

$$\bar{u}_1(p_1) = \int_0^\infty e^{-P_1 t_1} u_1(t_1) dt_1$$

Now we try to find u_1, $\tau_{12}{}^1$, $\tau_{13}{}^1$ in the form,

$$u_1 = (u_1)_1 + (u_1)_2,\ \tau_{12}{}^1 = (\tau^1_{12})_1 + (\tau^1_{12})_2,\ \tau_{13}{}^1 = (\tau^1_{13})_1 + (\tau^1_{13})_2 \quad (3.3)$$

where $(u_1)_1$, $(\tau_{12}{}^1)_1$, $(\tau_{13}{}^1)_1$ are continuous every where and are therefore given by (2.1), (2.1.1), (2.1.27).

We are only to find $(u_1)_2$, $(\tau_{12}{}^1)_2$, $(\tau_{13}{}^1)_2$ which depend on the fault creep across F_1. The values of $(u_1)_2$, $(\tau_{12}{}^1)_2$, $(\tau_{13}{}^1)_2$ are assumed to be zero for $t \leq T_1$.

We have the new constitutive equations

$$\left(\frac{1}{\eta_1} + \frac{1}{\mu_1}\frac{\partial}{\partial t_1}\right)(\tau^1_{12})_2 = \frac{\partial}{\partial t}(e^1_{12})_2 = \frac{\partial}{\partial t_1}\frac{\partial}{\partial y_2}(u_1)_2 \quad (3.4)$$

$$\left(\frac{1}{\eta_1} + \frac{1}{\mu_1}\frac{\partial}{\partial t_1}\right)(\tau^1_{13})_2 = \frac{\partial}{\partial t_1}(e^1_{13})_2 = \frac{\partial}{\partial t_1}\frac{\partial}{\partial y_3}(u_1)_2 \quad (3.5)$$

Now stress equation of motion

$$\frac{\partial}{\partial y_2}(\tau_{12}^1)_2 + \frac{\partial}{\partial y_2}(\tau_{13}^1)_2 = 0 \quad (3.6)$$

Proceeding as before, we get,

$$\nabla^2 (u_1)_2 = 0 \quad (3.7)$$

New set of boundary conditions

$$(\tau_{13}{}^1)_2 = 0,\ y_3 = H\ (-\infty \le y_2 < \infty,\ t_1 \ge 0) \quad (3.8)$$

$$(\tau_{12}{}^1)_2 = 0,\ |y_2| \to \infty (y_3 \ge 0,\ t_1 \ge 0) \quad (3.9)$$

We are to solve (3.7) – (3.9)

We take Laplace Transformation of (3.7) – (3.9)

$$\nabla^2 (\bar{u}_1)_2 = 0 \quad (3.10),$$

$$(\bar{\tau}_{13}{}^1)_2 = 0 \text{ on } y_3 = H(-\infty < y_2 < \infty,\ t_1 \ge 0),$$

$$(\bar{\tau}_{12}{}^1)_2 \to 0 \text{ on } (y_2) \to \infty \quad (3.11)$$

We shall solve (3.7) to (3.11) with the creep condition (3.1), (3.2) by modified Green's function technique by ([9]),([21]), to get $(\bar{u}_1)_2$ and then using

$$(\bar{\tau}_{12}^1)_2 = \bar{\mu}_1 \frac{\partial(\bar{u}_1)_2}{\partial y_2} \quad (3.12)$$

$$(\bar{\tau}_{13}^1)_2 = \bar{\mu}_1 \frac{\partial(\bar{u}_1)_2}{\partial y_3} \quad (3.13)$$

We shall get $(\bar{\tau}_{12}{}^1)_2$, $(\bar{\tau}_{13}{}^1)_2$, finally, inverse Laplace Transform of which will give $(u_1)_2\ (\tau_{12}{}^1)_2, (\tau_{13}{}^1)_2$.

Now let $Q\ (y_1, y_2, y_3)$ be any point in the M_1 and $P\ (x, x_2, x_3)$ be any point on F_1. The by ([9]),

$$(\bar{u}_1)_2(Q) = \int_{F_1} [(\bar{u}_1)_2(P)] G(P,Q) dx_3 \quad (3.14)$$

where $G(P, Q)$ is the G.F. and

$(\bar{u}_1)_2\ (P)$ is the discontinuity in

$[(\bar{u}_1)_2]$ across F_1 and is given by

$[(\bar{u}_1)_2\ (P)] = \bar{u}_1\ (P) f(x_3)$

where P= Laplace transformation variable.

Therefore, from we get,

$$(\bar{u}_1)_2(Q) = \int_{d_1}^{D_1} \bar{u}_1(P) f_1(x_3) G(P,Q) dx_3 \quad (3.15)$$

where the Green's function,

$$G(P,Q) = \bar{\mu}_1\ (\partial/\partial x_2)\ G_1(P,Q)$$

where, $G_1(P,Q)=(1/4\pi\ \bar{\mu}_1)\ \{\log[(x_2-y_2)^2+(x_3-y_3)^2]+\log[(x_2-y_2)^2+(x_3+y_3)^2]\}-1/4\pi\bar{\mu}_1 \sum_1^\infty [(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\{\log[(x_2-y_2)^2+(x_3-2mH-y_3)^2]+\log[(x_2-y_2)^2+(x_3-2mH+y_3)^2]+\log[(x_2-y_2)^2+(x_3+2mH+y_3)^2]+\log[(x_2-y_2)^2+(x_3+2mH-y_3)^2]\}$, where, $0<=y_3<=H$. (3.16)

$$G(P,Q) = -(1/2\times 3.142)\left[\frac{(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3-x_3)^2\right]} + \frac{(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3+x_3)^2\right]}\right] + (1/2\pi)\sum_1^\infty [(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\{(x_2-y_2)/[(x_2-y_2)^2+(x_3-2mH-y_3)^2]+\{(x_2-y_2)/[(x_2-y_2)^2+(x_3-2mH+y_3)^2]+\{(x_2-y_2)/[(x_2-y_2)^2+(x_3+2mH+y_3)^2]+\{(x_2-y_2)/[(x_2-y_2)^2+(x_3+2mH-y_3)^2]\quad (3.17)$$

Thus we have,

$$(\bar{u}_1)_2(Q) = \int_F u_1(P)f(x_3)\left[\frac{(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3-x_3)^2\right]} + \frac{(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3+x_3)^2\right]}\right]dx_3 + (1/2\pi)\int_F \sum_1^\infty [(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\{(x_2-y_2)/[(x_2-y_2)^2+(x_3-2mH-y_3)^2]+\{(x_2-y_2)/[(x_2-y_2)^2+(x_3-2mH+y_3)^2]+\{(x_2-y_2)/[(x_2-y_2)^2+(x_3+2mH+y_3)^2]+\{(x_2-y_2)/[(x_2-y_2)^2+(x_3+2mH-y_3)^2]dx_3 \quad (3.18)$$

where $f(x_3)$ = creep function.

On the fault $F_1, x_2 = 0$

Let,

$$\varphi_1(y_2,y_3)= \int_0^D f(x_3')\left[\frac{(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3-x_3)^2\right]} + \frac{(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3+x_3)^2\right]}\right]dx_3 + (1/2\pi)\int_F \sum_1^\infty [(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\{y_2/[(x_2-y_2)^2+(x_3-2mH-y_3)^2]+\{y_2/[(x_2-y_2)^2+(x_3-2mH+y_3)^2]+\{y_2/[(x_2-y_2)^2+(x_3+2mH+y_3)^2]+\{y_2/[(x_2-y_2)^2+(x_3+2mH-y_3)^2]dx_3$$

$$\therefore\ (\bar{u}_1)_2(Q) = \frac{1}{2\pi}\bar{u}_1(P)\varphi_1(y_2,y_3) \quad (3.19)$$

Therefore we get,

$$(\bar{\tau}_{12}^1) = \frac{\mu_1 P}{P+\frac{\mu_1}{\eta_1}}\frac{\bar{u}_1(P)}{2\pi}\psi_1(y_2,y_3)$$

Where, $\Psi_1(y_2,y_3)= \int_F f(x_3)\left[\frac{(y_3-x_3)^2-(y_2-x_2)^2}{\left[(y_2-x_2)^2+(y_3-x_3)^2\right]} + \frac{(y_3+x_3)^2-(y_2-x_2)^2}{\left[(y_2-x_2)^2+(y_3+x_3)^2\right]}\right]dx_3 + [(1/2\pi)\int_F \sum_1^\infty [(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\ \{(x_2^2+(x_3-2Mh-y_3)^2-4x_2y_2+3y_2^2)\ /[(x_2-y_2)^2+(x_3-2mH-y_3)^2]^2+((x_2^2+(x_3-2Mh+y_3)^2-4x_2y_2+3y_2^2)/[(x_2-y_2)^2+(x_3-2mH+y_3)^2]^2+((x_2^2+(x_3+2Mh+y_3)^2-4x_2y_2+3y_2^2)/[(x_2-y_2)^2+(x_3+2mH+y_3)^2]^2+((x_2^2+(x_3+2Mh-y_3)^2-4x_2y_2+3y_2^2)/[(x_2-y_2)^2+(x_3+2mH-y_3)^2]^2dx_3]\}$

On the fault $F_1, x_2 = 0$

$$\Psi_1(y_2,y_3)= \int_F f(x_3)\left[\frac{(y_3-x_3)^2-(y_2)^2}{\left[(y_2)^2+(y_3-x_3)^2\right]} + \frac{(y_3+x_3)^2-(y_2)^2}{\left[(y_2)^2+(y_3+x_3)^2\right]}\right] + [(1/2\pi)\int_F \sum_1^\infty [(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\ \{((x_3-2Mh-y_3)^2+3y_2^2)\ /[(y_2)^2+(x_3-2mH-y_3)^2]^2+((x_3-2Mh+y_3)2+3y22)\ /[(y_2)^2+(x_3-2mH+y_3)^2]^2+((x_3+2Mh+y_3)^2+3y_2^2)\ /[(y_2)^2+(x_3+2mH+y_3)^2]^2+((x_3+2Mh-y_3)^2+3y_2^2)\ /[(y_2)^2+(x_3+2mH-y_3)^2]^2dx_3]\} \quad (3.21)$$

Similarly we get,

$$\therefore\ (\bar{\tau}_{13}^1)_2 = \frac{\mu_1}{2\pi}\psi_2(y_2,y_3)\frac{P}{P+\frac{\mu_1}{\eta_1}}\bar{u}_1(P) \quad (3.22)$$

Where, $\Psi_2(y_2,y_3)= \int_F f(x_3)\ [\frac{(y_3-x_3)(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3-x_3)^2\right]^2}+\frac{(y_3+x_3)(y_2-x_2)}{\left[(y_2-x_2)^2+(y_3+x_3)^2\right]^2}]dx_3+\partial/\partial y_3[(1/2\pi)\int_{d1}\sum_1^{\infty}$

$[(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\{y_2/[(x_2-y_2)^2+(x_3-2mH-y_3)^2]+\{y_2/[(x_2-y_2)^2+(x_3-2mH+y_3)^2]+\{y_2/[(x_2-y_2)^2+(x_3+2mH+y_3)^2]+\{y_2/[(x_2-y_2)^2+(x_3+2mH-y_3)^2\ ]dx_3\ ]$

On the fault ,$x_2=0$

$$\Psi_2(y_2,y_3)= \int_F f(x_3)\ [\frac{(y_3-x_3)y_2}{\left[(y_2)^2+(y_3-x_3)^2\right]^2}+\frac{(y_3+x_3)y_2}{\left[(y_2)^2+(y_3+x_3)^2\right]^2}]dx_3+\partial/\partial y_3[(1/2\pi)\int_{d1}\sum_1^{\infty}[(\bar{\mu}_1-\bar{\mu}_2)/(\bar{\mu}_1+\bar{\mu}_2)]^m\{y_2/[(y_2)^2+(x_3-2mH-y_3)^2]+\{y_2/[(y_2)^2+(x_3-2mH+y_3)^2]+\{y_2/[(y_2)^2+(x_3+2mH+y_3)^2]+\{y_2/[(y_2)^2+(x_3+2mH-y_3)^2\ ]dx_3\ ] \quad (3.23)$$

Taking Laplace transformation we get,

$$u_1(y_1,y_3,t)=(u_1)_0+\frac{y_2}{\eta_1}\left\{\tau_\infty(t)-\tau_\infty(0)+\frac{\mu_1}{\eta_1}\int_0^t\tau_\infty(\tau)d\tau\right\}+y_3.\frac{\tau_H}{\eta_1}t+[H(t-T_1)/2\pi][\ \{(y_2)/[(y_2)^2+(x_3-y_3)^2]+(y_2)/[(y_2)^2+(x_3+y_3)^2]\}-\sum_1^{\infty}(a/b)^m\{1+\sum_{r=1}^{m}\binom{m}{r}[b_1^r/(r-1)!]t^{(r-1)}e^{(-a_1t)}[(y_2)/[(y_2)^2+(x_3-2mH-y_3)^2]+(y_2)/[(y_2)^2+(x_3+2mH-y_3)^2+(y_2)/[(y_2)^2+(x_3-2mH+y_3)^2]+(y_2)/[(y_2)^2+(x_3+2mH+y_3)^2\ ]]dx_3 \quad (3.24)$$

$$\tau^1_{12}(y_2,y_3,t)=\tau_\infty(t)-[\tau_\infty(0)-(\tau^1_{12})_0]e^{(-\mu_1/\eta_1)t}+(H(t-T_1)/2\pi)\int_F f(x_3)[\{(x_3-y_3)^2-y_2^2\}/\{(x_3-y_3)^2+y_2^2\}^2+\{(x_3+y_3)^2-y_2^2\}/\{(x_3+y_3)^2+y_2^2\}^2]-a_1[(H(t-T_1)/2\pi)\int_F\sum_1^{\infty}(a/b)^m\{1+\sum_{r=1}^{m}\binom{m}{r}[b_1^r/(r-1)!]t^{(r-1)}e^{(-a_1t)}\{((x_3-2mH-y_3)^2+3y_2^2)/[(y_2)^2+(x_3-2mH-y_3)^2]^2+((x_3-2mH+y_3)^2+3y_2^2)/[(y_2)^2+(x_3-2mH+y_3)^2]^2+((x_3+2mH+y_3)^2+3y_2^2)/[(y_2)^2+(x_3+2mH+y_3)^2]^2+((x_3+2mH-y_3)^2+3y_2^2)/[(y_2)^2+(x_3+2mH-y_3)^2\ ]^2dx_3\ ]\} \quad (3.25)$$

$$e_{12}^1(y_2,y_3,t)=(e_{12}^1)_0+(1/\mu_1)(\tau_\infty(t)-\tau_\infty(0)+\mu_1/\eta_1\int_0^t\tau_\infty(\tau)d\lambda)+(\tau_H/\eta_1)y_3t+[(H(t-T)/2\pi)\int_F\sum_1^{\infty}[(a/b)^m\{1+\sum_{r=1}^{m}\binom{m}{r}[b_1^r/(r-1)!]t^{(r-1)}e^{(-a_1t)}\ ]\{((x_3-2mH-y_3)^2-3y_2^2)/[(y_2)^2+(x_3-2mH-y_3)^2]^2+((x_3-2mH+y_3)^2+3y_2^2)/[(y_2)^2+(x_3-2mH+y_3)^2]^2+((x_3+2mH+y_3)^2-3y_2^2)/[(y_2)^2+(x_3+2mH+y_3)^2]^2+((x_3+2mH-y_3)^2-3y_2^2)/[(y_2)^2+(x_3+2mH-y_3)^2]^2dx_3]\} \quad (3.26)$$

where, $a=(\mu_1/\mu_2)-1, b=(\mu_1/\mu_2)+1, a_1=(\mu_1/b)[(1/\mu_1)+(1/\mu_2)]$

$b_1=(\mu_1\mu_2/\mu_1^2-\mu_2^2)[(\mu_2/\eta_2)-(\mu_1/\eta_1)], a_3=a_1-(\mu_1/\eta_1)$

$T_r(t)=e^{-(\mu_1/\eta_1)t}[1-e^{a_3t}e_{r-1}(a_3t)]$,

where,

$e_{r-1}(a_3t)=1+(a_3t)+(a_3t)^2/2+\ldots\ldots\ldots\ldots\ldots\ldots+(a_3t)^{r-1}/(r-1)!$

$r-1>=0, e_0(a_3t)=1.$ (A)

5. Numerical Computations

Following[1] and recent studies on rheological behavior of crust and upper mantle by ([4],[7]) the values of the model parameters are taken as :

$\mu_1=3.10^{11}$dynes/cm^2,$\mu_2=2.10^{11}$ dynes/cm^2 ,$\eta_1=2.10^{21}$ poise and $\eta_2=3.\ 10^{21}$ poise .

D_1 = Depth of the fault = 40 km.,[nothing that the depth of all major earthquake faults is in between 10-35 km].

$\tau_\infty(t)$ =constant= $\tau_\infty=2\times10^8$ dynes/cm^2 (200 bars),[post seismic observations reveal that in most of the cases, stress released in major earthquake are of the order of 200 bars or less, in extreme cases, it may be 400 bars.]

$\tau^1_{12}(y_2,y_3,0)=5\times10^7$dyne/cm^2 (50 bars),and $\tau_\infty(0)=0$

We take the creep function,

$f_1(x_3)=1-\frac{3}{D_1^2}x_3^2+\frac{2}{D_1^3}x_3^3$,with

U=1cm/year, satisfying the conditions stated in $(C_1)-(C_2)$.

The rate of change of shear strain is:

$$\partial/\partial t(e_{12}^1)=1/\eta_1+(\tau_H/\eta_1)y_3+[(H(t-T)/2\pi)]\int_F\sum_1^{\infty}[(a/b)^m\{1+\sum_{r=1}^{m}\binom{m}{r}[b_1^r/(r-1)!][-a_1t^{(r-1)}e^{(-a_1t)}+(r-1)t^{r-2}e^{(-a_1t)}]\{((x_3-2mH-y_3)^2-3y_2^2)/[(y_2)^2+(x_3-2mH-y_3)^2]^2+((x_3-2mH+y_3)^2+3y_2^2)/[(y_2)^2+(x_3-2mH+y_3)^2]^2+((x_3+2mH+y_3)^2-3y_2^2)/[(y_2)^2+(x_3+2mH+y_3)^2]^2+((x_3+2mH-y_3)^2-3y_2^2)/[(y_2)^2+(x_3+2mH-y_3)^2]^2dx_3]\} \quad (3.27)$$

The rate of shear stress 12 is:

$$[\partial\tau^1_{12}(y_2,y_3,t)/\partial t]=^{-(\mu_1/\eta_1)}[\tau_\infty(0)-(\tau^1_{12})_0]e^{-(\mu_1/\eta_1)t}+[(H(t-T_1)/2\pi)\int_F\sum_1^{\infty}(a/b)^m\{1+\sum_{r=1}^{m}\binom{m}{r}[b_1^r/(r-1)!][-a_1t^{(r-1)}e^{(-a_1t)}+(r-1)t^{r-2}e^{(-a_1t)}]\{((x_3-2mH-y_3)^2+3y_2^2)/[(y_2)^2+(x_3-2mH-y_3)^2]^2+((x_3-2mH+y_3)^2+3y_2^2)/[(y_2)^2+(x_3-2mH+y_3)^2]^2+((x_3+2mH+y_3)^2+3y_2^2)/[(y_2)^2+(x_3+2mH+y_3)^2]^2+((x_3+2mH-y_3)^2+3y_2^2)/[(y_2)^2+(x_3+2mH-y_3)^2]^2dx_3]\} \quad (3.28)$$

6. Discussions and Conclusions

(A) Variation of displacement due to the creep movement across the fault after t_1=1year.

Equation (3.24) gives the displacement U1 at any point (y_2,y_3) at any time t, due to the fault movement across the fault F. Fig 2 shows the variation of displacement U1 with depth y_3 for y_2=8km. t=1year.It is observed that maximum displacement occurred at y_3=0 and it's magnitude is 2.25 cm/year and it gradually decreases to zero at a depth about 100km from the free surface.

We observe that the displacement has a discontinuity at y_2 =0km and is antisymmetric about the fault plane.

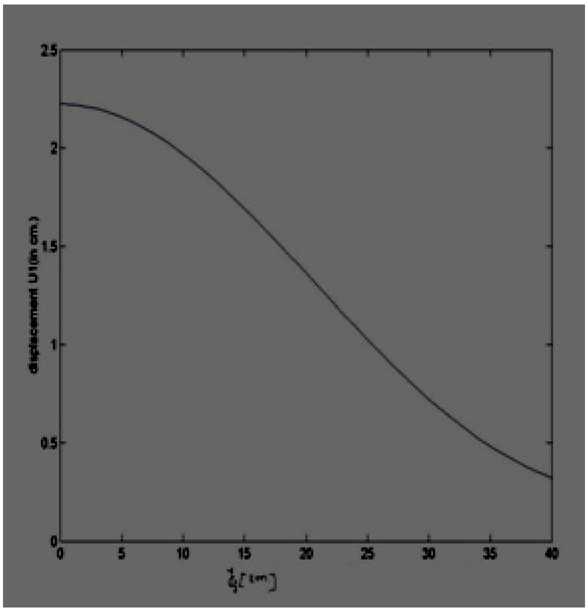

Figure 2. shows the variation of displacement U1 with depth y_3 for y_2=8km. t_1=1year due to fault movement

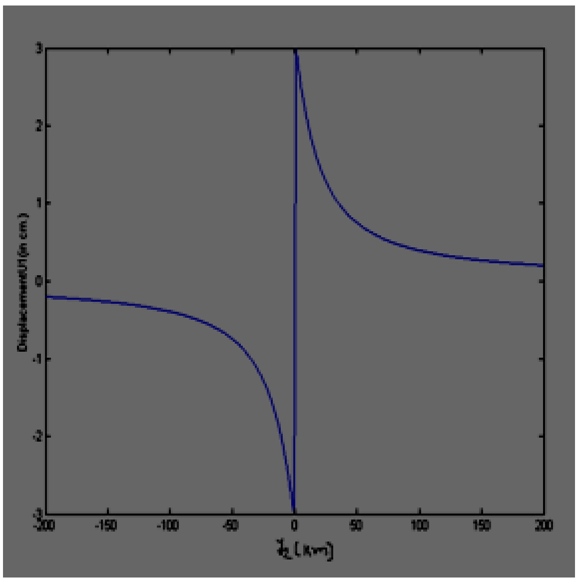

Figure 3. shows the variation of surface displacement U1 for y_3=0km.and t_1=1year with y_2 due to the fault movement across the fault F

(B) Rate of change of surface displacement before fault movement.

Equation (3.24) gives the displacement u_1 at any point (y_2, y3) at any time t, before the fault movement across the fault F. We have $\partial u_1/\partial t = y_2 \tau_\infty(t)/\eta_1$ which represents a straight line through the origin with slope <<1.

(C) Variation of stress 12(t_{12} , which is the main driving force) due to the creep movement across the fault after t_1=1year.

Equation (3.25) gives the stress component 12 at any point (y_2,y_3) at any time t, due to the fault movement across the fault F.

Fig 4 shows the variation of stress t_{12} with y_3 for y_2=5km.t=1year due to the fault movement across the fault F. We observe that stress 12 releases up to a depth about 20km. and then begin to accumulate. The maximum accumulation occurred at a depth about 40km with a magnitude of about 0.39 bar per year but the rate of accumulation gradually decreases to zero at a depth of about 150km.

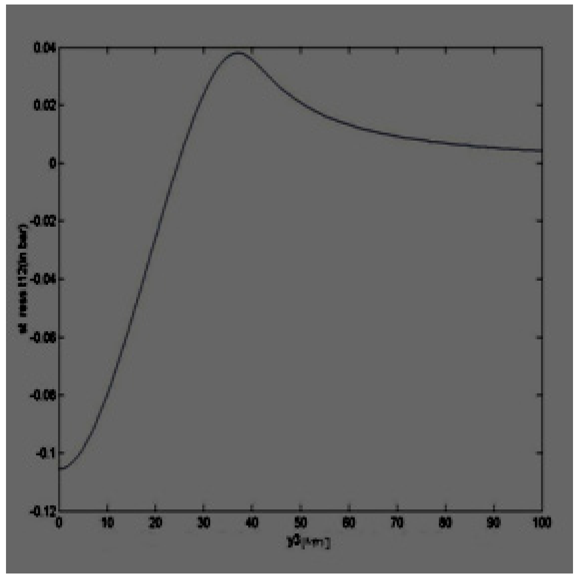

Figure 4. shows the variation of stress t_{12} with y_3 for y_2=5km.t_1=1year due to the fault movement across the fault F

(D) Variation of shear stress t_{12} with time ‘t’.

Equation (3.28) gives the rate of change of shear stress t_{12} . Fig 5 shows the variation of stress t_{12} with time ‘t’. It is observed that the graph is a straight line parallel to time axis, i.e. the rate of change of shear stress is constant.

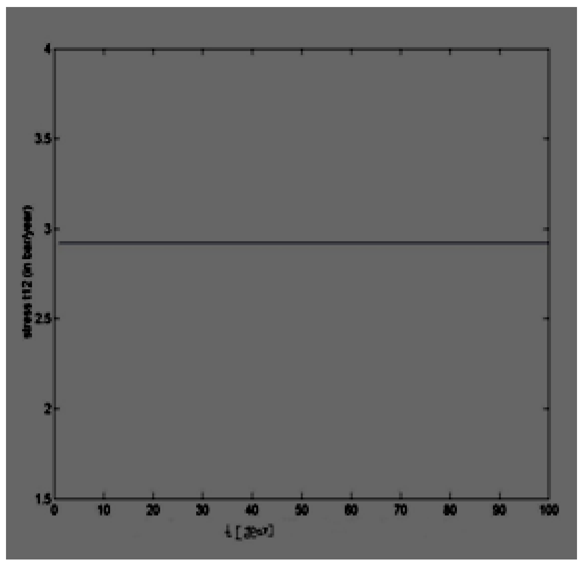

Figure 5. variation of shear stress t_{12} with time ‘t’ for y_2=5km. y_3=10km

(E) Variation of surface shear strain E_{12} (say) for t_1=1 year for y_3=0km. with y_2 due to the fault movement.

Equation (3.26) gives the variation of surface shear strain E_{12} .Fig 6 shows the variation of surface shear strain E_{12} with y_2 for y_3=0km and t=1 year due to the fault movement. It is observed that the surface shear strain is maximum near y_2=0, it’s magnitude is $2*10^{-6}$ and gradually decreases as we go away from the fault.

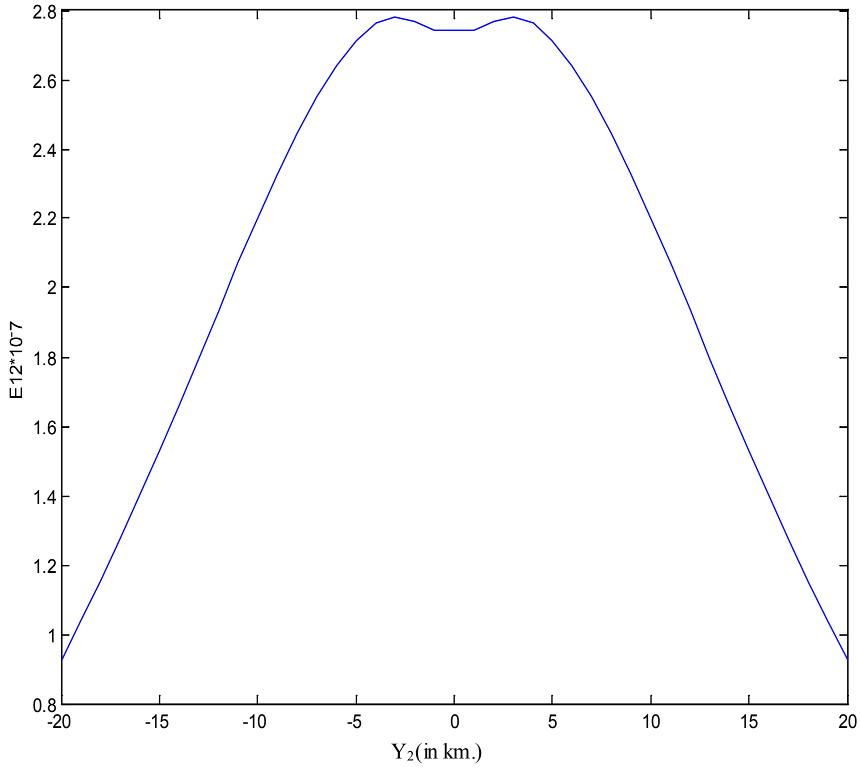

Figure 6. variation of surface shear strain E_{12} for t_1=1 year, with y_2 for y_3=0km.due to the fault movement

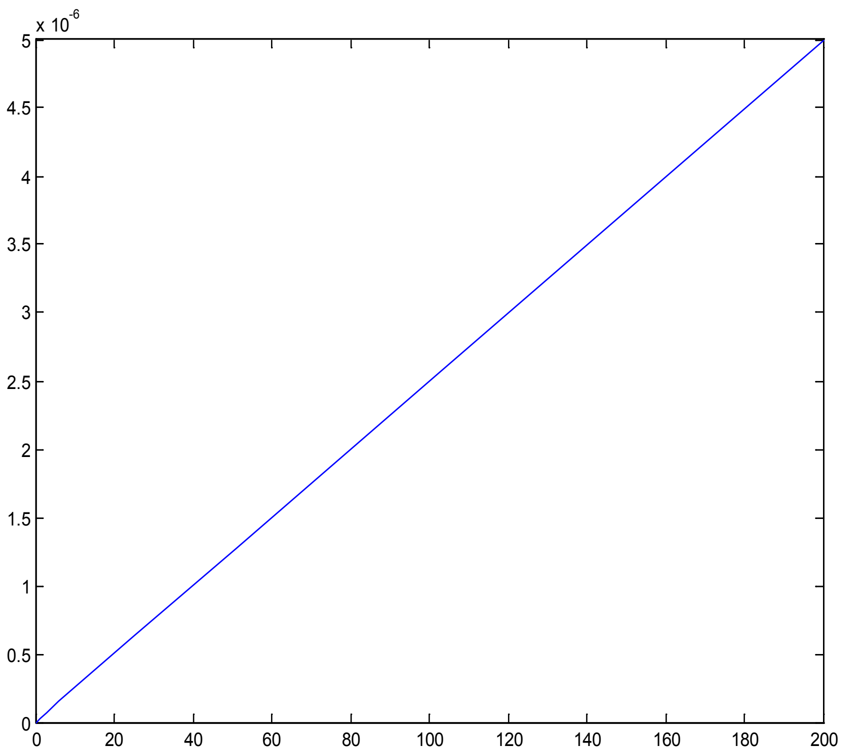

Figure 7. Rate of change of surface shear strain e_{12} for y_3=0,y_2=5km

(F)Rate of change of surface shear strain:

Equation (3.27) gives the rate of change of shear strain e_{12}. Fig 7 shows the variation of stress e_{12} with time'
t'. It is observed that the graph is a straight line through the origin and it's magnitude is of order 10^{-6} which well matched with the observational data.

[U1 is the component of displacement, t_{12} is stress component and E_{12} is the component of shear strain due to the fault movement]

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REFERENCES

[1] Aki, K., Richards, P.G. Quantitative Seismology (2nd Ed), University Science Books.1980.

[2] Cohen, S.C. Post-seismic viscoelastic deformation and stress 2, stress theory and computation : Dependence of displacement, strain and stress on fault-parameters. J. Geophys. Res., vol.85, No. B6, 3151-3158. 1980.

[3] Cohen, S.C. Post-seismic viscoelastic surface deformations and stress 1, theoretical considerations: Displacement and strain calculations. J. Geophys. Res., 85, No. B6, 3131-3150. 1980.

[4] Chift. P, Lin, J. Barcktiausen, U. Marine and Petroleum Geology 19,951-970. 2002.

[5] Ghosh, U. and Sen, S.: Stress accumulation near buried fault in lithosphere-asthenosphere system; International Journal of Computing, vol.I,Issue4, pp.786-795. Oct,2011.

[6] Ghosh, U. Mukhopadhyay, A. and Sen, S:On two interacting creeping vertical surface breaking strike-slip faults in a two layered model of the lithosphere. Physics of the Earth and planetary interior.70,119-129.1992

[7] Karato Shun-Ichiro: Rheology of the Earth's mantle: A historical review Gondwana Research-Vol.18-issue-1,pp. 17-45. 2010

[8] Maruyama, T. Statical elastic dislocations in an infinite and semi-infinite medium. Bull. Earthquake Res. Inst. Tokyo Univ., 42, 289-368. 1964.

[9] Maruyama, T. On two dimensional dislocations in an infinite and semi-infinite medium. Bull. Earthquake Res. Inst. Tokyo Univ.; 44 (Part 3), 811-871. 1966.

[10] Mukhopadhyay, A. and Mukherji, P. On stress accumulation and fault slip in the lithosphere. Indian Journal of Meteorology, Hydrology and Geophysics (Mausam), vol.30, pp. 353-358. 1979.

[11] Mukhopadhyay, A. *et al.* On stress accumulation in a visco-elastic lithosphere; proceedings of the sixth international symposium on earthquake engineering, Roorke, vol.1, pp. 71-76.1978. (with P. Mukherji).

[12] Mukhopadhyay, A. *et al.* On the interaction between two locked strike-slip faults : proceedings of the sixth international symposium on earthquake engineering, Roorke, vol.1, pp. 77-82. 1978. (with S. Sen and M. Maji).

[13] Mukhopadhyay, A. *et al.* On stress accumulation near finite rectangular fault. Indian Journal of Meteorology, hydrology and geophysics (Mausam), vol.30, pp. 347-352 .1979. (with B.P. Paul and S. Sen).

[14] Mukhopadhyay, A. *Et. al.* On stress accumulation in the lithosphere and interaction between two-strike-slip faults. Indian Journal of Meteorology, Hydrology and Geophysics (Mausam), vol.30, pp. 359-363.1979. (with M. Maji, S. Sen and B.P. Pal).

[15] Mukhopadhyay, A. *et al.*. A mechanism of stress accumulation near a strike-slip fault. Indian Journal of Meteorology, Hydrology and Geophysics (Mausam), vol.30, p;. 367-372 .1979. (with B.P. Pal and S. Sen).

[16] Mukhopadhyay, A. *et al.* On stress accumulation in a visco-elastic lithosphere containing a continuously slipping fault. Bull. Soc. Earthquake Technology, vol.17, No.1, pp. 1-10 .1980. (with S. Sen and B.P. Pal).

[17] Mukhopadhyay, A. *et al.* On stress accumulation near a continuously slipping fault in a two-layer model of the lithosphere. Bull. Soc. Earthquake Tech., vol.17, No.4, pp. 29-38 .1980. (with B.P. Pal and S. Sen).

[18] Mukhopadhyay, A. *et al.* On two interacting creeping vertical surface-breakingstrike-slip faults in the lithosphere. Bull. Soc. Earthquake Tech., vol.21, pp. 163-191 .1984. (with P. Mukherji).

[19] Mukhopadhyay, A. *et al.* On two aseismically creeping and interacting buried vertical strike-slip faults in the lithosphere. Bull. Soc. Earthquake Tech., vol.23, pp. 91-117 .1986. (with P. Mukherji).

[20] Mukhopadhyay, A. *et al.* On two aseismically creeping and interacting vertical strike-slip faults one buried and the other surface breaking. Bull. Soc. Earthquake Tech., vol.25, No.2, pp. 49-71. 1988.

[21] Rybicki. K.: The elastic residual field of a very long strike-slip fault in the presence of a discontinuity,Bull.Seis.Soc.Am.61

[22] Segal, P.: Earthquake and volcano deformation : Princeton University Press. 2010

[23] Sen, S. Sarker, S. and Mukhopadhyay, A. A Creeping and surface breaking long strike -slip fault inclined to the vertical in a viscoelastic half space, Mausam, 44, 4, 365-4, 372. 1993

[24] SEN.S, DEBNATH.S.K:A creeping vertical strike-slip fault of finite length in a viscoelastic half-space model of the lithosphere. International journal of computing.vol-2, issue-3,P687-697.2012.

[25] SEN.S, DEBNATH. S. K: Longdip-slip fault in a viscoelastic half-space model of the lithosphere. American journal of computational and applied mathematics.Vol-2.No-6, p-249-256.2012.