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A Study on Separabilitybased on Realignment Criteria

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Abstract A beautiful quantumevent is the existence of inseparability in many body systems. Quantum systemcontaining of two subsystems is separable if its density matrix can be expressed as $\rho_{AB}=\sum_i p_i\rho_A\otimes\rho_B$ where ρ_A and ρ_B are density matrices for two subsystems where $0\le p_i\le 1$ and $\sum_i p_i=1$.Now people are involving in deep study last years various types of detection criterion and make implementation to the various states for detecting whether they are separable or not. In this work we just try to make a comparison of some separability criterion in a particular state and obtained a little bit consequences.

Keywords Separability, Entanglement , Incomparability,Warner State, PPT, Cross norm

1. Introduction

In the fundamental level nature requires quantum description rather than classical one. Now a days our interest spontaneously go to the entanglement in many bodyquantum systems. It is known to everybody that entanglement is a characteristic feature and powerful resource of quantum mechanics and is the basic ingredient of the foundation of the theory. To find the states which are useful for quantum information tasks, characterization of entangled states is of our great interest, this is called separability problem. This is one of the unsolved problems of quantum mechanics. Quantum inseparability first recognized by Einstein, Podolsky and Rosen[1] and Schrödinger[2,14] is one of the most astonishing features of quantum formalism. Recently people give their deep concentration to find out separability criteria in some particular states basically pure states and dynamical development of experimental methods and non-locality of quantum mechanics together inspire people to find out on the inseparability problems in quantum mechanics. Now there available several criterions to detect quantum entanglement of composite quantum states. But in the mixed states there no rigorous criteria for finding whether a state is entangled or not. Here we want to confined ourselves in positive partial transpose (PPT) criteria[4], computable cross norm or realignment criteria[5] and an other computable separability criteria[5]. According to Peres and Horodechi[3],PPT is a strong sufficient condition for detecting entanglement for 2×2 and 2×3 systems. On the way of searching some powerful criterion which will perform better than PPT criterion, some desirable results for detecting entanglement [6,7]. In this paper, we have performed a comparative study of these three criterions for a most general state of a composite quantum bipartite system.

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2. Definitions and Explanations

Bipartite Pure States:

Let A and B be two non-interacting systems and H_A and H_B be their respective Hilbert spaces. The Hilbert space of the composite system is the tensor product H_A and H_B .If the system A is in the state $|\psi_A\rangle$ and system B is in the state $|\Phi_B\rangle$, then the composite system is in the state $|\psi_A\rangle\otimes|\Phi_B\rangle$ .The state of a composite system white can be represented in this form are called seperable states or product states. If there are states which can not be represented in this form are called entangled states. Let a projector $|\psi_{AB}\rangle\langle\psi_{AB}|$ on a vector $|\psi_{AB}\rangle H_A\in\otimes H_B$ is a pure state if the local subsystems are pure states. For a pure state $|\psi_{AB}\rangle H_A\in\otimes H_B$ can be represented as

$$\left|\psi_{AB}\right\rangle = \sum_i^N C_i\left|\alpha_j\right\rangle\left\langle\beta_j\right|$$

Where $|\alpha_i\rangle$ and $|\beta_i\rangle$ are the basis for H_A and H_B respectively, $C_i>1$ and $\sum_i^N C_i=1$ where $N=\min(\dim$

H_A ,dim H_B). This decomposition of $|\psi_{AB}\rangle$ is called Schmidt decomposition and C_i 's are called Schmidt coefficients.

Bipartite Mixed states[8-10,18,19]:

Let $\rho_i^A = |\alpha_i\rangle\langle\alpha_i|$ and $\rho_i^B = |\beta_i\rangle\langle\beta_i|$ be respectively the states of the systems H_A and H_B.Then combined state of their joint system can be written as

$$|\alpha_i\rangle\langle\alpha_i| \otimes |\beta_i\rangle\langle\beta_i| \quad (i=1,2,3,\ldots M)$$

A necessary and sufficient condition of the mixed state ρ AB to be separable is that it can be represented as a convex combination of the product of projectors on local states as

$$\rho_{AB} = \sum_i^M |\alpha_i\rangle\langle\alpha_i| \otimes |\beta_i\rangle\langle\beta_i|$$

Otherwise, the mixed state is said to be entangled state.

3. Partial Transposition Criterion[11,12]

A bipartite density matrix ρ_{ABt} can be expressed as

$\rho_{AB} = \sum_{i,j,k,l,} |ij\rangle\langle kl|$ Where $\{|i\rangle\}$ and $\{|k\rangle\}$ ($i, j = 1,2,3,\ldots, N; N \le \dim H_A$) and $\{|j\rangle\}$ and $\{|l\rangle\}$ ($j, l = 1,2,3,\ldots M; M \le \dim H_B$) are orthogonal bases of H_A and H_B respectively.

The partial transposition ρ_{AB}^{T} of ρ_{AB} with respect to system A, is defined as

$$\rho_{AB}^{T_A} = \rho_{kjil}$$

$$= \rho_{ijkl}\, |kj\rangle\langle il|$$

If $\rho_{AB}^{T_A} \ge 0$,that is the eigen values of $\rho_{AB}^{T_A}$ are non-negative then the state has positive partial transpose(PPT). Otherwise the state has non-positive partial transposition(NPT).

Detection of entanglement:

If any one of the eigen values of partial transposition of ρ AB be negative then the state is entangled.

4. Crossnorm or Realignment Criteria

For a bipartite density matrix

$$\rho_{AB} = \sum_{ijkl} \rho_{ijkl}\, |ij\rangle\langle kl|$$

The realignment operation R (ρ_{AB}) can be defined as[15-17]

$$R(\rho_{AB}) = \sum_{ijkl} |ik\rangle\langle jl|$$

Detection of entanglement:

If the sum of all the eigen values of R(ρ_{AB}) be less than or equal to 1 then the state is separable otherwise it is entangled.

5. A Computable Separability Criterion

We consider Hilbert spaces H_A and H_B of the systems A and B respectively. Then there is a correspondence between states $|\psi\rangle \in H_A \otimes H_B$ and Hilbert-Schmidt operators

$$T : H_A \to H_B$$

According to the rule :

if $|\psi\rangle = \sum_{ij} C_{ij} |ij\rangle$ be decomposition of $|\psi\rangle$ in terms of orthogonal bases $\{|i\rangle\}$ and $\{|j\rangle\}$ of H_A and H_B respectively. Then $T(\psi)$ is given by

$$T(\psi) = \sum_{ij} C_{ij} |i\rangle\langle j*|$$

Conversely, if $T(\psi) = \sum_{ij} C_{ij} |i\rangle\langle j|$ for some orthogonal bases $\{|i\rangle\}$ and $\{\langle j|\}$ of H_A and H_B respectively,then

$$|\psi\rangle = \sum_{ij} C_{ij} |i\rangle\langle j*|.$$

Theorem: If H_A and H_B be finite dimentional Hilbert spaces and KA =HS(H_A) $\approx C^n$ and KB=HS(H_B) $\approx C^m$ be the spaces of Hilbert-Schmidt operators on H_A and H_B respectively.

Then there exists a one to one correspondence between Hilbert-Schmidt operators T $\in$ HS($H_A \otimes H_B$) and Hilbert-Schimdt operators $\Im(T)$=HS (H_A) $\to$ HS(H_B)

Detection of entanglement:

We know that, for a finite dimentional Hilbert space H , if $\rho \in$ T($H \otimes H$) be a density operator then ρ is separable imples the sum of squares of the eigen values $\Gamma(\Im(\rho)) \le 1$.

6. Results in Details

We consider a most general state

$$\rho = p \left[\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right] + \left(\frac{1-p}{4}\right) I_A \otimes I_B$$

For this state, we are going to test where it is entangled and where separable using different types of entanglement detection criteria.

At first for partial transpose criterion, the state is

$$\rho = p\left[\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right] + \left(\frac{1-p}{4}\right) I_A \otimes I_B$$

Taking partial transpose on the party B, we have

$$\rho_{AB}^{T_B} = \begin{pmatrix} \frac{1-p}{4} & 0 & 0 & -\frac{p}{2} \\ 0 & \frac{1+p}{4} & 0 & 0 \\ 0 & 0 & \frac{1+p}{4} & 0 \\ -\frac{p}{2} & 0 & 0 & \frac{1-p}{4} \end{pmatrix}$$

Eigen values of this matrix are given by

$$\frac{1+p}{4}, \frac{1+p}{4}, \frac{1+p}{4}, \frac{1-3p}{4}$$

For the state to be separable , we have

$$\frac{1+p}{4} \geq 0$$

$$\Rightarrow p \geq -1$$

And

$$\frac{1-3p}{4} \geq 0$$

$$\Rightarrow p \leq \frac{1}{3}$$

Since $p \geq 0$, the physical range of separability of p is

$$0 \leq p \leq \frac{1}{3},$$

That is when $\frac{1}{3} < p \leq 1$, the state is entangled.

Now, we test for cross norm or realignment criterion, the realignment operator $R(\rho)$ of the state

$$\rho = p\left[\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right] + \left(\frac{1-p}{4}\right) I_A \otimes I_B$$

$$= \frac{(1-p)}{4}|00\rangle\langle 00| + \left(\frac{p}{2} + \frac{1-p}{4}\right)|01\rangle\langle 01| - \frac{p}{2}|01\rangle\langle 10|$$
$$+ \left(\frac{p}{2} + \frac{1-p}{4}\right)|10\rangle\langle 01| - \frac{p}{2}|10\rangle\langle 01| + \frac{(1-p)}{4}|11\rangle\langle 11|$$

is given by

$$R(\rho) = \frac{(1-p)}{4}|00\rangle\langle 00| + \left(\frac{p}{2} + \frac{1-p}{4}\right)|00\rangle\langle 11| - \frac{p}{2}|01\rangle\langle 10| + \left(\frac{p}{2} + \frac{1-p}{4}\right)|11\rangle\langle 00|$$
$$- \frac{p}{2}|10\rangle\langle 01| + \frac{(1-p)}{4}|11\rangle\langle 11|.$$

Now theeigen values are obtained as $-\frac{p}{2}, -\frac{p}{2}, \frac{p}{2}, \frac{1}{2}$.

Sum of all the eigen values is $= \frac{1}{2} - \frac{p}{2} = \frac{1-p}{2}$

The is separable when $\frac{1-p}{2} \leq 1$, i.e, $p \geq -1$

but the physical range of p is $[0,1]$ which implies that the state is separable for any value of p.

Now for another computable separability criterion, we consider the same state

$$\rho = p\left[\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right] + \left(\frac{1-p}{4}\right) I_A \otimes I_B$$

$$= \frac{p}{2}\left[|0\rangle\langle 0| \otimes |0\rangle\langle 1| - |0\rangle\langle 1| \otimes |1\rangle\langle 0| - |1\rangle\langle 0| \otimes |0\rangle\langle 1| + |1\rangle\langle 1| \otimes |0\rangle\langle 0|\right]$$
$$+ \frac{(1-p)}{4}\left[|0\rangle\langle 0| \otimes |0\rangle\langle 0| + |0\rangle\langle 0| \otimes |1\rangle\langle 1| + |1\rangle\langle 1| \otimes |0\rangle\langle 0| + |1\rangle\langle 1| \otimes |1\rangle\langle 1|\right]$$

$$= \frac{(1-p)}{4}|E_{00}\rangle\langle E_{11}| - \frac{p}{2}|E_{01}\rangle\langle E_{10}| - \frac{p}{2}|E_{10}\rangle\langle E_{01}| + \frac{(1+p)}{4}|E_{11}\rangle\langle E_{00}| + \frac{(1-p)}{4}|E_{00}\rangle\langle E_{00}| + \frac{(1-p)}{4}|E_{11}\rangle\langle E_{11}|$$

We write this state in matrix form as

$$= \begin{pmatrix} \frac{1-p}{4} & 0 & 0 & \frac{1+p}{4} \\ 0 & 0 & -\frac{p}{2} & 0 \\ 0 & -\frac{p}{2} & 0 & 0 \\ \frac{1+p}{4} & 0 & 0 & \frac{1-p}{4} \end{pmatrix}$$

Therefore, we have

$$\Im*(\rho)\Im(\rho) = \frac{1}{4}\begin{pmatrix} 2(1+p2) & 0 & 0 & 2(1-p2) \\ 0 & p2 & 0 & 0 \\ 0 & 0 & p2 & 0 \\ 2(1-p2) & 0 & 0 & 2(1+p2) \end{pmatrix}$$

The eigen values of the above matrix are

$$p^2, p^2, 4, p^2$$

Now, adding the absolute values of the square root of all eigen values, we arrive at

$$\Gamma(\Im(\rho)) = 4p + 2 > 1 \text{ for } 0 \le p \le 1$$

Hence we see that for all physical values of p, the state is entangled.
In short, we represent the above results in tabular form as
Let us consider a state

$$\rho_{AB} = \frac{1}{N}\left[PI + |\psi\rangle\langle\psi|\right]$$

where $|\psi\rangle = \sqrt{\alpha_1}|11\rangle + \sqrt{\alpha_2}|22\rangle + \sqrt{\alpha_3}|33\rangle$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$

Table 1. Representation of different separability criterian for the warner state

State	Entanglement detection criteria	Range where the state is entangled	Range where the state is separable
$\rho = p\left[\frac{\|01\rangle - \|10\rangle}{\sqrt{2}}\right] + \left(\frac{1-p}{4}\right) I_A \otimes I_B$	PPT Criteria	$\frac{1}{3} < p \le 1$	$0 \le p \le \frac{1}{3}$
	Crossnorm or Realignment criteria	×	$0 \le p \le 1$
	Another computable criteria	$0 \le p \le 1$	×

$$= \frac{1}{N}\Big[p\big(|11\rangle\langle 11| + |22\rangle\langle 22| + |33\rangle\langle 33| + |12\rangle\langle 12| + |13\rangle\langle 13| + |21\rangle\langle 21| + |23\rangle\langle 23| + |31\rangle\langle 31| + |32\rangle\langle 32|\big) + \alpha_1|11\rangle\langle 11| + \sqrt{\alpha_1\alpha_2}|11\rangle\langle 22| + \sqrt{\alpha_1\alpha_3}|11\rangle\langle 33| + \sqrt{\alpha_1\alpha_2}|22\rangle\langle 11| + \alpha_2|22\rangle\langle 22| + \sqrt{\alpha_2\alpha_3}|22\rangle\langle 33| + \sqrt{\alpha_1\alpha_3}|33\rangle\langle 11| + \sqrt{\alpha_2\alpha_3}|33\rangle\langle 22| + \alpha_3|33\rangle\langle 33|\Big]$$

For normality, we have

$$\frac{9p+\alpha_1+\alpha_2+\alpha_3}{N}=1 \quad \Rightarrow N=9p+1$$

$\therefore$ the state becomes,

$$\begin{aligned}\rho_{AB} = \frac{1}{9p+1}\Big[&(p+\alpha_1)|11\rangle\langle 11| + (p+\alpha_2)|22\rangle\langle 22| + (p+\alpha_3)|33\rangle\langle 33| \\ &+ p|11\rangle\langle 22| + p|11\rangle\langle 33| + p|22\rangle\langle 11| + p|22\rangle\langle 33| + p|33\rangle\langle 11| \\ &+ p|33\rangle\langle 22| + \sqrt{\alpha_1\alpha_2}|11\rangle\langle 22| + \sqrt{\alpha_1\alpha_3}|11\rangle\langle 33| \\ &+ \sqrt{\alpha_2\alpha_1}|22\rangle\langle 11| + \sqrt{\alpha_2\alpha_3}|22\rangle\langle 33| + \sqrt{\alpha_3\alpha_1}|33\rangle\langle 11| \\ &+ \sqrt{\alpha_3\alpha_2}|33\rangle\langle 22|\end{aligned}$$

The realignment operator R (ρ_{AB}) can be written as

$$\begin{aligned}R(\rho_{AB}) = \frac{1}{9p+1}\Big[&(p+\alpha_1)|11\rangle\langle 11| + (p+\alpha_2)|22\rangle\langle 22| + (p+\alpha_3)|33\rangle\langle 33| \\ &+ p|11\rangle\langle 22| + p|11\rangle\langle 33| + p|22\rangle\langle 11| + p|22\rangle\langle 33| + p|33\rangle\langle 11| \\ &+ p|33\rangle\langle 22| + \sqrt{\alpha_1\alpha_2}|12\rangle\langle 12| + \sqrt{\alpha_1\alpha_3}|13\rangle\langle 13| + \sqrt{\alpha_2\alpha_1}|21\rangle\langle 21| \\ &+ \sqrt{\alpha_2\alpha_3}|23\rangle\langle 23| + \sqrt{\alpha_3\alpha_1}|31\rangle\langle 31| + \sqrt{\alpha_3\alpha_2}|32\rangle\langle 32| \quad \Big]\end{aligned}$$

$$=\frac{1}{9p+1}\begin{pmatrix} p+\alpha_1 & p & p & 0 & 0 & 0 & 0 & 0 & 0 \\ p & p+\alpha_2 & p & 0 & 0 & 0 & 0 & 0 & 0 \\ p & p & p+\alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha_1\alpha_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\alpha_3\alpha_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\alpha_2\alpha_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\alpha_2\alpha_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\alpha_1\alpha_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\alpha_3\alpha_2} \end{pmatrix}$$

Sum of all the eigen values of the above matrix is

$$\frac{3p+1}{N}+\frac{2\sqrt{\alpha_1\alpha_2}}{N}+\frac{2\sqrt{\alpha_2\alpha_3}}{N}+\frac{2\sqrt{\alpha_3\alpha_1}}{N}$$

$$= \frac{1}{9p+1}\left[3p+1+2\left(\sqrt{\alpha_1\alpha_2}+\sqrt{\alpha_2\alpha_3}+\sqrt{\alpha_3\alpha_1}\right)\right]$$

To find Min

$$\left(\sqrt{\alpha_1\alpha_2}+\sqrt{\alpha_2\alpha_3}+\sqrt{\alpha_3\alpha_1}\right),$$

where $\alpha_1+\alpha_2+\alpha_3=1$

and we assume that

$$\alpha_1>\alpha_2>\alpha_3$$

Then Min

$$\left(\sqrt{\alpha_1\alpha_2}+\sqrt{\alpha_2\alpha_3}+\sqrt{\alpha_3\alpha_1}\right)=\sqrt{\alpha_1\alpha_2}$$

and Max

$$\left(\sqrt{\alpha_1\alpha_2}+\sqrt{\alpha_2\alpha_3}+\sqrt{\alpha_3\alpha_1}\right)=\sqrt{\alpha_2\alpha_3}$$

The state must be separable if

$$\frac{3p+1+2\sqrt{\alpha_2\alpha_3}}{9p+1}\leq 1$$

$$\Rightarrow \alpha_1\alpha_2 \leq 9p^2$$

The state must be entangled if

$$\frac{3p+1+2\sqrt{\alpha_1\alpha_2}}{9p+1}>1$$

$$\Rightarrow \alpha_1\alpha_2 > 9p^2$$

Partial Traspose Criteria:

$$\rho_{AB}=\frac{1}{9p+1}\Big[(p+\alpha_1)|11\rangle\langle 11|+(p+\alpha_2)|22\rangle\langle 22|+(p+\alpha_3)|33\rangle\langle 33|$$
$$+p|11\rangle\langle 22|+p|11\rangle\langle 33|+p|22\rangle\langle 11|+p|22\rangle\langle 33|+p|33\rangle\langle 11|$$
$$+p|33\rangle\langle 22|+\sqrt{\alpha_1\alpha_2}|11\rangle\langle 22|+\sqrt{\alpha_1\alpha_3}|11\rangle\langle 33|$$
$$+\sqrt{\alpha_2\alpha_1}|22\rangle\langle 11|+\sqrt{\alpha_2\alpha_3}|22\rangle\langle 33|+\sqrt{\alpha_3\alpha_1}|33\rangle\langle 11|$$
$$+\sqrt{\alpha_3\alpha_2}|33\rangle\langle 22|\ \Big]$$

Taking Partial transpose on the party B, we get

$$\rho_{AB}=\frac{1}{9p+1}\Big[(p+\alpha_1)|11\rangle\langle 11|+(p+\alpha_2)|22\rangle\langle 22|+(p+\alpha_3)|33\rangle\langle 33|$$
$$+p|12\rangle\langle 12|+p|13\rangle\langle 13|+p|21\rangle\langle 21|+p|23\rangle\langle 23|+p|31\rangle\langle 31|$$
$$+p|32\rangle\langle 32|+\sqrt{\alpha_1\alpha_2}|21\rangle\langle 12|+\sqrt{\alpha_1\alpha_3}|31\rangle\langle 13|+\sqrt{\alpha_1\alpha_2}|12\rangle\langle 21|$$
$$+\sqrt{\alpha_2\alpha_3}|32\rangle\langle 23|+\sqrt{\alpha_3\alpha_1}|13\rangle\langle 31|+\sqrt{\alpha_3\alpha_2}|23\rangle\langle 32|$$

Now the eigen values of the above matrix are

$$\frac{p+\alpha_1}{9p+1},\frac{p+\alpha_2}{9p+1},\frac{p+\alpha_3}{9p+1},\frac{p\pm\sqrt{\alpha_1\alpha_2}}{9p+1},\frac{p\pm\sqrt{\alpha_2\alpha_3}}{9p+1},\frac{p\pm\sqrt{\alpha_3\alpha_1}}{9p+1}$$

All of this eigen values are non-negative if

$$p>\sqrt{\alpha_1\alpha_2}$$

Therefore the state must be separable if

$$p>\sqrt{\alpha_1\alpha_2}$$

Which includes the separablelity condition for realignment criteria that is

$$p\geq\frac{\sqrt{\alpha_1\alpha_2}}{3}$$

and $p>\sqrt{\alpha_1\alpha_2}\geq\frac{\sqrt{\alpha_1\alpha_2}}{3}$.

7. Conclutions

Now we come to the conclusion about the detection of entanglement of a general state using three strong entanglement detection criteria, we see that through the state is entangled when formed, PPT criteria shows that when p lies on[0, $\frac{1}{3}$], the state is separable and entangled on

$(\frac{1}{3},1]$. But realignment criteria shows that the state is always separable whatever be the value of p. And the other criteria shows that the state is always entangled for any value of p. These conflicting results do not help us to reach to any concrete decision which one is stronger without any hesitations. So we can decide that PPT criteria is always strong criteria than others in $2\otimes 2$ and $2\otimes 3$ systems. Here PPT criteria exhibit more explicit and clear range of separability with respect to the other two methods. So in this paper through a detailed numerical search of the well known Warner state, it is clearly established that PPT is the reliable excellent criteria in the $2\otimes 2$ and $2\otimes 3$ systems.

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