

Multiobjective Optimization of an Operational Amplifier by the Ant Colony Optimisation Algorithm

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Abstract The Ant Colony Optimization (ACO) algorithm is used as a multi-objective optimization technique to size a most popular analog circuit, the CMOS operational amplifier (Op-Amp). The work consist of finding the more convenient transistors sizes, including the channel widths and lengths, in order to meet or reach the specified requirements such as the voltage gain A_v , the Common Mode Rejection Ratio CMRR, the die area A, the power consumption P and the Slew Rate SR. SPICE simulations are used to strengthen and to validate the obtained sizing/performances.

Keywords Ant Colony Optimization, Multi-objective optimization, CMOS, Op-Amp

1. Introduction

Over the past decade, significant progress has been realized with the appearance of a new generation of powerful and approximate optimization methods, known as metaheuristics[1]. Such methods are used to solve real-world problems within a reasonable amount of time. They always offer 'good' approximation of the 'best' solutions for optimization problems[2]. One of the incoming problems to be resolved in the nearer future is the sizing of electronic circuits given the continuous increase of the integration densities. So, the designers and electronic engineers had a good and exciting challenge to reach, that is to find a technique, which can easily determine the components sizing, taken into account a well determined specifications. Some (meta-) heuristics were proposed in the literature and were used by some designers to optimize the sizing of the analog components automatically, such as Tabu Search[3,4], Genetic Algorithms (GA)[5], local search (LS)[6], Wasp Nets (WN)[7], Bacterial Foraging Optimization (BFO)[8], Particle Swarm Optimization (PSO)[9] and recently Ant Colony Optimization (ACO)[10, 11].

These algorithms have often dealt with single-objective optimizations; however, the optimization of analog circuits is generally a multi-objective problem. They are always formed by at least two conflicting performance functions.

That means that improving one performance results automatically into the degradation of another one. In this way a set of several meta-heuristics algorithms have been developed, such as Multi-objective Optimization Genetic

Algorithm (MOGA)[12], Multi-objective Optimization Particle Swarm Optimization (MOPSO)[13].

In the domain of meta-heuristic methods, an important interest has been paid to the Ant Colony Optimization algorithm. The basic idea is to imitate the cooperative behaviour of ant colonies in order to solve combinatorial optimization problems within a reasonable amount of time. The ACO is actually recognized as one of the most successful strands of swarm intelligence[14]. Some ACO-based algorithms have been proposed such as the multi-objective optimization problems MOACO[15], the Multiple Objective Ant-Q Algorithm (MOAQ)[16], the Pareto-Ant Colony Optimization (P-ACO)[17] and the Ant Algorithm for Bi-criterion Optimization Problems[18].

In a previous work, we have adapted and used the MOACO algorithm for a two objectives electronic circuit optimization namely the Second Generation Current Conveyors[19]. At the present we propose to use the same algorithm in order to optimize the sizing of the CMOS Op-Amp, which is a more popular analog circuit requiring many highly interdependent performances.

Thus the subsequent section will present an overview of the ACO technique followed by with a presentation of the proposed adaptation of the ACO technique for analog circuits optimization and introduces the MOACO. The third deals with the optimal sizing of the CMOS Operational Amplifier (Op-Amp) and presents the main results and simulations. Finally, the paper will be concluded in the final section.

2. Algorithm Presentation

2.1. Ant Colony Optimization

The ACO technique is inspired by the collective behavior

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of deposition and monitoring of some traces as it is observed in insect colonies [20,21], such as ants. It is for example well known that ants deposit pheromone on the ground in order to mark some favourable paths that should be followed by other members of the colony. Fig. 1 shows an illustration of the ability of ants to find the shortest path between food and their nest. Ants communicate indirectly through dynamic changes in their environment (pheromone trails).

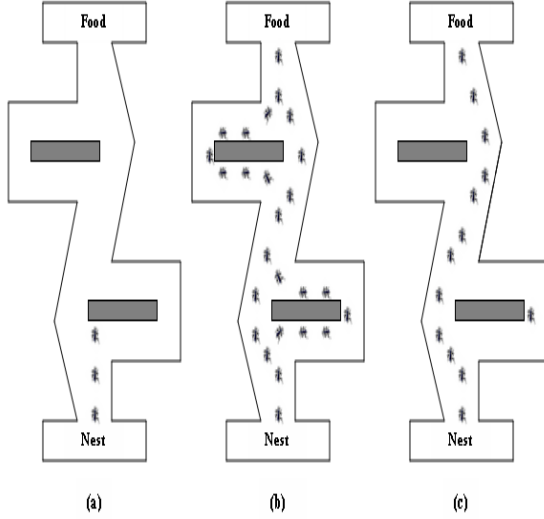


Figure 1. Self-adaptive behaviour of a real ant colony, (a) the ants go in the search of food; (b) ants follow a path between nest and food source; ants choose, with equal probability, whether to shortest or longest path; (c) the majority of ants have chosen the shortest path

The ACO was initially used to solve graph related problems, such as the travelling salesman problem[22], vehicle routing problem[23], Optical networks routing[24], and bioinformatics problems[25]. A graph is composed of vertices and edges. Each ant constructs its own path from the starting to the final vertex by “walking” along edges connecting the vertices by depositing a certain amount of pheromones (a chemical substance) that evaporates during the time, unless it is reinforced by another ant ‘walking’ along the same edge. Thus, the ‘best’, i.e. the shortest, path is determined on the base of these pheromones. Besides, movement of the ants is highly conditioned by their visibility regarding the final objective.

For solving such problems, ants randomly select the vertex to be visited. When an ant k is in the vertex i , the probability for going to the vertex j is given by the following expression[26,27]:

$$P_{ij}^k = \begin{cases} \frac{(\tau_{ij})^\alpha \cdot (\eta_{ij})^\beta}{\sum_{l \in J_i^k} (\tau_{il})^\alpha \cdot (\eta_{il})^\beta} & \text{if } j \in J_i^k \\ 0 & \text{if } j \notin J_i^k \end{cases} \quad (1)$$

where J_i^k is the set of neighbours of the vertex i of the k th ant, τ_{ij} is the amount of pheromone trail on the edge (i, j) , α and β are weightings that control the pheromone trail τ_{ij} and the visibility value, η_{ij} given by:

$$\eta_{ij} = \frac{1}{d_{ij}} \quad (2)$$

Where d_{ij} is the distance between vertices i and j .

The pheromone rate values are updated during each iteration by all the ants that have built a solution in the iteration itself. The pheromone rate τ_{ij} , which is associated with the edge joining vertices i and j , is updated as follows:

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k \quad (3)$$

where ρ is the evaporation rate, m is the number of ants, and $\Delta \tau_{ij}^k(t)$ is the quantity of pheromone laid ‘deposited’, or dropped of on edge (i, j) by ant k :

$$\Delta \tau_{ij}^k = \begin{cases} \frac{Q}{L^k} \\ 0 \end{cases}$$

if ant k used edge (i, j) in its tour, otherwise

Q is a constant and L_k is the length of the tour constructed by the ant k .

The ACO approach attempts to solve an optimization problem by iterating the following two steps:

- The Candidate solutions are constructed using a pheromone model, that is, a parameterized probability distribution over the solution space;
- The candidate solutions are used to modify the pheromone values in a way that is deemed to bias future sampling toward high quality solutions.

2.2. Adaptation

The main idea consists of constructing a graph that imitates the movement of the ants[28,29]. Then, we construct a graph composed of the discretized variable vectors, corresponding to the graph vertices. Thus, each ant will construct its path by a random displacement from a variable value to another, as it is shown in Fig. 2; $V_1, V_2, V_3 \dots V_N$ constitute the discrete variable vectors.

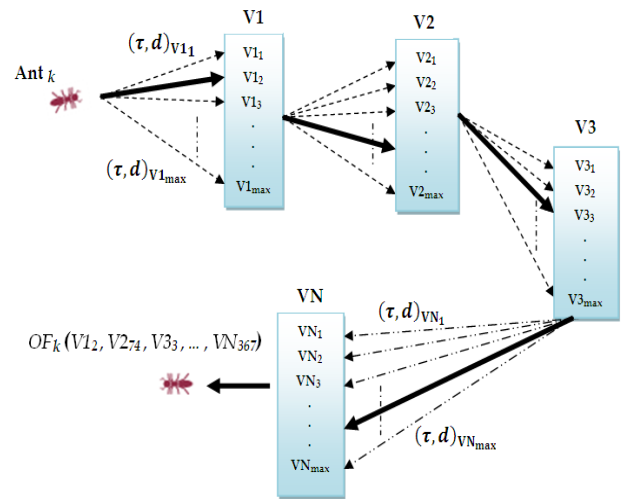


Figure 2. A pictorial graph showing the movement of the ants

In short, each ant k will randomly chose a path (values of

$V_1, V_2 \dots$), according to the probability given by expression (1), and form a non-connected directed graph while randomly generating a rate of pheromone at the formed graph edges. At each iteration, the path giving the minimum value of the objective function (OF) sees its pheromone rate increasing, in contrast with the other paths, for which the pheromone rates start to evaporate with respect to expression (3).

In the multi-objective problem, we seek to optimize several functions that are usually interdependent. So, the concept of Pareto optimality is used[30,31]. This approach consists, of the following, with n parameters (decision variables) and K objectives:

$$\text{Min } f(x) = (f_1(x), f_2(x), \dots, f_k(x)), \text{ with } x = (x_1, x_2, \dots, x_n) \in X.$$

A decision vector $a \in X$ dominates another $b \in X$ ($a > b$) if, and only if:

$$\forall i \in 1, 2, \dots, k \mid f_i(a) \leq f_i(b) \wedge \exists j \in 1, 2, \dots, k \mid f_j(a) < f_j(b)$$

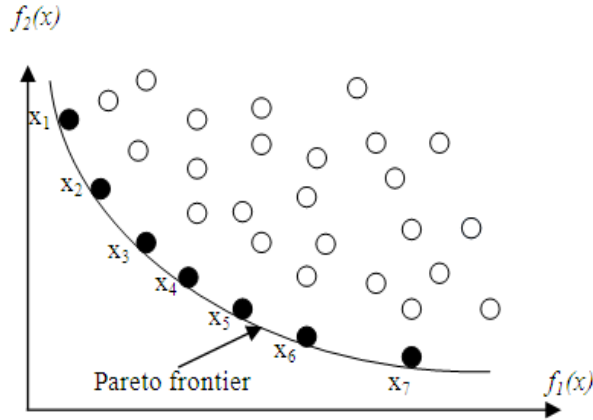


Figure 3. Non-dominated solutions for two-objective optimization problem

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Random initialization of the pheromone value
Do
    For each iteration
        For each ant
            Create the set X of the ant;
            Label: Determine the objective weight  $P_k$  for
                each objective  $k$  randomly;
            For each project  $k$ 
                -select a project;
                -add it to  $X$ ;
            End
            If set  $X$  is feasible and efficiencies
                -store set  $X$  and remove dominated ones;
                If Not Goto Label
                    End if
                For each project  $k$ 
                    -determine the best solution and update
                        pheromone;
                End
            End
        End
    End
End
    
```

Figure 4. Pseudo code for the proposed MOACO algorithm

In the multi-objective optimization problem, a set of non-dominated solutions form the Pareto frontier. An example is shown in Figure 3, where the solid (filled) circles represent the non-dominated solutions which form the Pareto frontier, while the open circles represent the dominated solutions. This result corresponds to a two-objective optimization problem, where the goal was the minimization of the two objectives, i.e. to search for the non-dominated solutions located along the Pareto frontier.

To resolve the multiobjective problems, we proposed the algorithm shown in Figure 4. In the initialization phase: Ants are generated each starting with a set X , the objective weights P_k is determined randomly for each ant. In the construction phase of the algorithm, each ant tries to construct a feasible set X by using a pseudo-random proportional rule. After a set has been constructed, its feasibility and efficiency is determined. Pheromone updating is performed by using the best solution X_k of the current iteration for each objective k .

3. Operational Amplifier Optimization

The two-stage CMOS operational amplifier (Op-Amp) shown in Figure 5, is considered as an example for the validation of our proposed algorithm. In fact the design of the Op-Amp continues to pose a challenge as transistor channel lengths scale down with each generation of CMOS technologies[32].

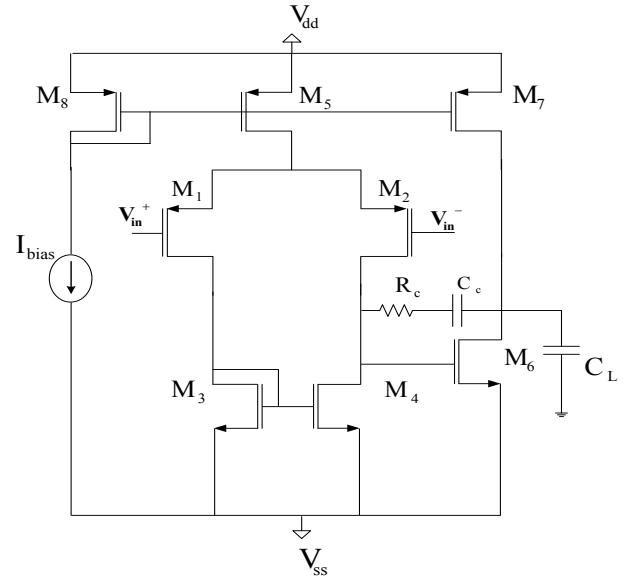


Figure 5. A two stage CMOS operational amplifier

Performances of an Op-Amp are evaluated via several parameters such as:

- The open-loop voltage gain Av :

$$Av = \frac{2C_{ox}}{(\lambda_n + \lambda_p)^2} \sqrt{\frac{\mu_n \mu_p}{I_1 I_7} \left(\frac{W}{L}\right)_2 \left(\frac{W}{L}\right)_6} \quad (5)$$

- The power dissipation P :

$$P = (V_{dd} - V_{ss})(I_{bias} + I_5 + I_7) \quad (6)$$

- The Common Mode Rejection Ratio $CMRR$:

$$CMRR = \frac{2C_{ox}}{(\lambda_n + \lambda_p)\lambda_p} \sqrt{\frac{\mu_n \mu_p}{I_5^2} \left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_3} \quad (7)$$

- The die Area A :

$$A \approx \sum_{i=1}^8 W_i L_i \quad (8)$$

- The Slew Rate SR :

$$SR = \min \left\{ \frac{2I_1}{C_c}, \frac{I_7}{C_c + C_{TL}} \right\} \quad (9)$$

Those expressions (5) and (7), were obtained by considering the small signal equivalent transistor's models. V_{dd} and V_{ss} are respectively the positive and the negative supply voltages; W_1 - W_8 and L_1 - L_8 are the gates widths and the channels lengths of the transistors M_1 - M_8 respectively. I_{bias} is the bias current, gm refers to the transconductance of the MOS transistor, C_{ox} , λ_n , λ_p , μ_n and μ_p are technological parameters. C_c is a compensation capacitor and C_{TL} is the total capacitance at the output node which can be expressed as:

$$C_{TL} = C_L + Cgd_6 + Cgd_7 \quad (10)$$

Cgd_6 and Cgd_7 denote to the parasitic grid to drain capacitance for transistor M_6 and M_7 respectively.

Determining the optimal dimensions of the transistors for a specific design involves a tradeoff among all these performance measures. Each transistor must be in saturation. Expressions (11)-(14) give the corresponding constraints, that have to be satisfied when computing optimal sizes of the transistors M_1 (and M_2), M_5 , M_6 and M_7 respectively.

$$V_{cm,min} - V_{ss} - V_{TP} - V_{TN} \geq \sqrt{\frac{2I_1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_3}} \quad (11)$$

$$V_{dd} - V_{cm,max} + V_{TP} \geq \sqrt{\frac{2I_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_1}} + \sqrt{\frac{2I_5}{\mu_p C_{ox} \left(\frac{W}{L}\right)_5}} \quad (12)$$

$$V_{out,min} - V_{ss} \geq \sqrt{\frac{2I_7}{\mu_n C_{ox} \left(\frac{W}{L}\right)_6}} \quad (13)$$

$$V_{dd} - V_{out,max} \geq \sqrt{\frac{2I_7}{\mu_p C_{ox} \left(\frac{W}{L}\right)_7}} \quad (14)$$

where $I_5 = \left(\frac{W}{L}\right)_5 I_{bias}$, $I_7 = \left(\frac{W}{L}\right)_7 I_{bias}$, and $I_1 = \frac{I_5}{2}$ while

respecting the expression (15):

$$\frac{\left(\frac{W}{L}\right)_3}{\left(\frac{W}{L}\right)_6} = \frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_6} = \frac{1}{2} \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_7} \quad (15)$$

where, V_{tp} and V_{tn} are the PMOS and the NMOS threshold voltages, respectively.

This optimization belongs to the family of NP hard problems; in fact, there are 11 design parameters to optimize for the two-stage Op-Amp; the widths and lengths of all tran-

sistors, (W_1 - W_8 and L), the bias current I_{bias} and the value of the compensation capacitor C_c , in addition to the various constraints of the problem. Note that all the channel length L is considered the same for all the transistors.

The considered optimization problem is a typical multi-objective one, consisting of minimizing two objective functions (the die area and the consumed power), and maximizing the other performances.

Table 1. Parameters of ACO algorithm

Number of iterations	10000
Number of ants (m)	50
Number of projects	100
Evaporation rate (ρ)	0.1
Quantity of deposit pheromone by the best ant (Q)	0.2
Pheromone factor (α)	1
Heuristics factor (β)	1

Table 2. Optimal device sizing and performances

	1	2	3	4	5
W_{12} (μm)	102.31	100.00	115.43	100.00	107.37
W_{34} (μm)	75.22	84.98	74.48	82.38	78.12
W_5 (μm)	15.24	18.27	18.81	13.61	17.21
W_6 (μm)	105.41	119.02	115.87	109.18	123.54
W_7 (μm)	10.68	12.80	14.63	09.01	13.60
W_8 (μm)	15.07	17.98	14.48	19.95	20.13
L (μm)	0.82	0.95	0.88	0.90	0.92
I_o (μA)	50.53	32.11	80.00	10.00	24.43
C_c (pF)	04.68	12.05	15.00	05.42	08.10
Av (dB)	94.58	98.21	88.13	105.29	101.24
$CMRR$ (dB)	91.97	93.91	82.97	101.18	96.02
P (mW)	1.37	0.87	2.57	0.23	0.33
A (μm^2)	411.20	511.13	478.38	464.86	501.82
SR (V/ μs)	119.34	76.18	257.08	16.62	45.79

Table 3. SPICE performance results

	1	2	3	4	5
Av (dB)	92.32	95.46	86.07	103.44	99.31
$CMRR$ (dB)	88.45	89.83	77.50	96.55	91.78
P (mW)	1.56	1.03	3.12	0.46	0.82
A (μm^2)	-----	-----	-----	-----	-----
SR (V/ μs)	112.40	72.33	241.45	13.28	40.47

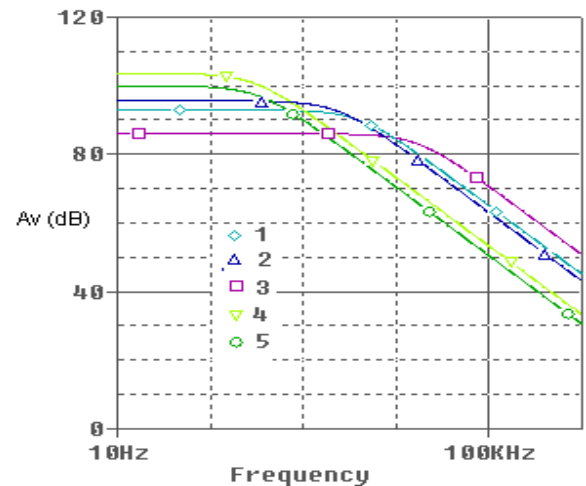


Figure 6. Spice simulation results for Gain

The MOACO algorithm when applied to our optimization problem, using the parameter values given in Table 1, gives 43 optimal parameter designs. Table 2 present five of these parameter designs and their corresponding performances. Also, it is to be noted that the computing time equals 184s.

SPICE simulation results performed, using AMS 0.35 μ m technology, Voltage power supply is $V_{dd}/V_{ss}=+2.5V/-2.5V$, are presented in the Table 3; they show the good agreement with the expected ones.

Figure 6 shows SPICE simulation results for the five optimal designs, of the gain. The choice between the sets of the determined optimal parameters, given by this MOACO algorithm, will depend on the desiderata of the designer.

4. Conclusions

The presented work proposes an adaptation of the ant colony optimization technique to the optimal sizing of analog circuits. We show the practical applicability of the ACO to optimize performances of electronics integrated circuits and its suitability for solving a multiobjective optimization problem. The proposed algorithm is validated by the Operational Amplifier performances optimization. Viability of the technique was proved via SPICE simulations.

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