

# A Direct Method for Transmission Loss Allocation

S. V. N. L. Lalitha<sup>1,\*</sup>, Maheswarapu Sydulu<sup>2</sup>

<sup>1</sup>EEE Department, K L University, Guntur, Andhra Pradesh, 522502, India

<sup>2</sup>EEE Department, National Institute of Technology, Warangal, Andhra Pradesh, 506004, India

**Abstract** Fair allocation of transmission loss among market participants is essential in the present restructured electricity markets. This paper proposes a direct method to find the loss allocation. The methodology is based on simple circuit laws and does not involve any assumptions. Considering the real power injection and real power loss contribution factors loss allocation can be done. Case study of the proposed loss allocation methodology is conducted on an IEEE 14 bus system. Results are compared with the results of the existing methods in the literature.

**Keywords** Load flow, network usage, Transmission Pricing,  $Z_{bus}$ , transmission loss allocation

## 1. Introduction

Transmission loss allocation is important in restructured electricity markets. Since generators and demands are all connected to the same network, actions by one participant can have significant effects on others making it difficult to investigate the cost, each participant is responsible for[1]. It is difficult to achieve an efficient transmission loss allocation scheme that could fit all market structures in different locations. The ongoing research on transmission pricing indicates that there is no generalized agreement on pricing methodology. In practice, each restructuring model has chosen a method that is based on a particular characteristic of its network[2].

The most common and simplest approach reported in the literature for transmission loss is the so called postage – stamp method, depends only on the amount of power moved and the duration of its use, irrespective of the supply and delivery points, distance of transmission usage. A participant, who uses the transmission system lightly, i.e. at a shorter electrical distance, actually subsidizes others who use the system heavily[1]. Contract path method proposed for minimizing transmission charges does not reflect the actual flows through the transmission grid[2]. As an alternative, MW-Mile methodology was introduced in which different users are charged in proportion to their utilization of the grid[1,2]. The key feature in MW-Mile method is to find the contribution or share of each generator and demand in every line flow and hence the loss.

In the flow based methods, J.Bailek *et al* proposed a method for loss allocation, where in considering merit order approach generation dispatch and nodal clearing prices are

determined initially neglecting transmission losses and later on loss allocation is done among generators and demands[3]. The flow based methods use the proportional sharing principle, which implies that any active power flow leaving a bus is proportionally made up of the flows entering that bus, such that Kirchhoff's current law is satisfied. For the loss allocation, the share of generators and demands must be specified such as 50% loss among generators and 50% loss among loads. Equivalent bilateral exchanges method proposed for loss allocation does not require the choice of an arbitrary slack bus and is flow based in the sense that the loss allocated to individual agents takes into account their relative network positions[4]. Incremental transmission loss (ITL) coefficients which can be positive or negative have been used for transmission loss allocation[5]. A.J.Conejo *et al* identified a natural mathematical separation of the system losses among the various network buses and exploited  $Z_{bus}$  and proposed a loss allocation methodology[6]. A  $Z_{bus}$  based method which gives the contributions of bus currents to complex line flow is established in[7]. J.S. Daniel *et al* modified the  $Y_{bus}$  to include the effect of transmission loss and proposed a method for loss allocation[8]. Qieng Ding and Ali Abur derived a quadratic loss expression and allocated the transmission loss with the help of a bus loss matrix[9]. A unified approach for transmission loss allocation for allocation among buses and multiple transactions is proposed in[10].

Reflection of the transmission loss in the spot pricing is rarely done, due to the complicated aspects of loss allocation such as nonlinearity, path-dependency, and non-uniqueness of the solution. The most important issue is, to reduce the allocation error, i.e., the discrepancy between the sum of theoretically allocated losses and the actual system loss[11]. The system loss is decomposed by the sum of bus-wise partial integrals, each of which represents the bus-wise loss allocation in[12] to make the sum of the losses allocated among the buses exactly equal to the actual system loss. S.M. Abdul Khader decomposed transmission loss into three

\* Corresponding author:

svnlalitha@gmail.com (S.V.N.L. Lalitha)

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components such as the current flow from generators to loads, circulating current between generators and the contribution of network structure and controls to increase or decrease transmission losses[13]. Keshmiri and Ehsan proposed loss weight factors obtained from the square current magnitude and Z-bus matrix elements based on power flow tracing for loss allocation[14]. Similar concept was used for loss allocation in case of bilateral transactions in[15]. Assuming that more power transfer is responsible for larger losses and based on actual loss formula, loss allocation for mixed pool and bilateral markets was proposed in[16]. Using AC load flow loss factors were calculated for loss allocation based on incremental methods in[17]. Using a bus-branch flow direction matrix determined using power flow tracing loss allocation was done in[18].

This paper proposes a direct method to find the transmission loss allocation factors as an extension to the  $Z_{bus}$  method proposed by A.J.Conejo *et al* in[7]. The methodology is based on simple circuit laws and does not involve any assumptions. Considering the real power injection and real power loss contribution factors of the buses and transmission lines transmission loss allocation can be done. Case study of the proposed loss allocation methodology is conducted on an IEEE 14 bus system. The method can also be extended for developing a cost allocation methodology. A significant contribution of the paper lies in the fact that it allows the determination of the loss allocation factors directly. It does not require any proportionality assumption. The proposed loss allocation method presents very interesting results when compared with the other methods existing in the literature.

The paper is organized as follows. In Section 2, starting from a converged load flow solution, determination of the contributions of bus currents to the complex line flow is presented. Based on the contributions, transmission loss allocation methodology is established in section 3. With the help of a six bus example the methodology is explained. Results of IEEE 14 bus system loss allocation are presented in section 4. Possible conclusions made from the proposed methodology are given in section 5.

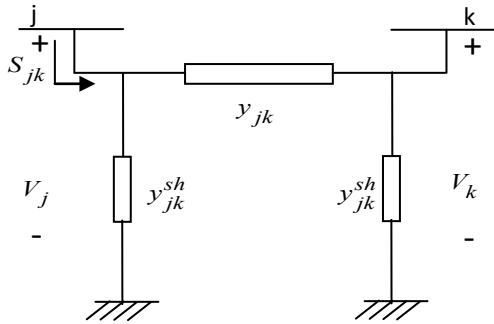


Figure 1.  $\Pi$ -Equivalent circuit of line  $j \rightarrow k$

## 2. Contributions of Bus Currents

Entire data related to the network such as bus voltages, complex line flows, slack bus power generation etc are ob-

tained from a converged load flow solution. This section presents a direct methodology that finds the coefficients of the bus currents in the complex line flow. Once the coefficients are determined, next step is to find the allocation of transmission loss and transmission cost pertaining to individual buses.

The  $\Pi$  equivalent circuit of a line having a line admittance  $y_{jk}$  and half line charging susceptance  $y_{jk}^{sh}$  connected between the buses  $j$  and  $k$  is shown in Fig. 1.  $V_j$  and  $V_k$  represent the nodal voltages of buses  $j$  &  $k$  respectively.

From the load flow solution we can write expression for the complex line flow  $S_{jk}$  in terms of the node voltage  $V_j$  and the line current  $I_{jk}$  through the line  $j \rightarrow k$  as

$$S_{jk} = V_j I_{jk}^* \quad (1)$$

From the  $Z_{bus}$  based system equations, the voltage at node  $j$  is given by

$$V_j = \sum_{i=1}^n Z_{ji} I_i \quad (2)$$

The current through the line  $j \rightarrow k$  is obtained as

$$I_{jk} = (V_j - V_k) y_{jk} + V_j y_{jk}^{sh} \quad (3)$$

Substituting (2) in (3) and rearranging

$$I_{jk} = \sum_{i=1}^n [(Z_{ji} - Z_{ki}) y_{jk} + Z_{ji} y_{jk}^{sh}] I_i \quad (4)$$

Substituting the values  $I_{jk}$  from (4) in (1) and rearranging

$$S_{jk} = \sum_{i=1}^n Factor1_{jk}^i I_i^* \quad (5)$$

where

$$Factor1_{jk}^i = V_j [(Z_{ji} - Z_{ki}) y_{jk} + Z_{ji} y_{jk}^{sh}] I_i^* \quad (6)$$

Thus, the complex power flow  $S_{jk}$  through any line  $j \rightarrow k$  is represented as a function of the all bus currents;  $i = 1, 2, 3, \dots, n$ .  $Factor1_{jk}^i$  represents contribution of  $i^{th}$  bus to  $j \rightarrow k$  line power flow.

## 3. Transmission Loss Allocation

Following the same procedure explained in the previous section, complex line flow from bus  $k$  to bus  $j$  i.e.  $S_{kj}$  can be expressed as

$$S_{kj} = \sum_{i=1}^n Factor2_{jk}^i I_i^* \quad (7)$$

where

$$Factor2_{jk}^i = V_k [(Z_{ki} - Z_{ji}) y_{jk} + Z_{ki} y_{jk}^{sh}] I_i^* \quad (8)$$

$Factor2_{jk}^i$  represents contribution of  $i^{th}$  bus to  $k \rightarrow j$  line complex power flow i.e counter flow. It is a well established fact that, Complex line loss in any line is the algebraic sum of active and counter complex line flows. Hence, we can write

$$S_{lineloss} = S_{jk} + S_{kj} = \sum_{i=1}^n Factor^i_{jk} I_i^* \quad (9)$$

where

$$Factor_{jk}^i = Factor1_{jk}^i + Factor2_{jk}^i \quad (10) \quad \text{manner.}$$

The significance of  $Factor_{jk}^i$  is that it represents the contribution of  $i^{th}$  bus to the  $j \rightarrow k$  line loss and also the contribution of line  $j \rightarrow k$  to the power injection at bus  $i$ . For a given power flow solution this quantity is a constant.

#### Worked Example

The methodology is explained with the help of a Six bus system [10]. Two generators (located at buses 1 and 2) supply the power demand (located at buses 3, 5 and 6), while bus 4 is a zero injection (transfer) bus.

The total real power loss in the system 8.37 MW is allocated among all 6 buses using the proposed method and its results are compared with those of the results obtained from existing methods in the literature.

Base case bus voltages are obtained from N-R load flow solution for a six bus system whose line and bus data is shown in Tables 1 & 2

At this stage, a new table is formed as shown in Table 3 and designated as  $[B]$  matrix.

From close observation of the elements of Table 3, it is noticed that the algebraic sum of all elements in *any row*, say  $i^{th}$  row, gives the “*real power injection*” at that bus i.e.  $i^{th}$  bus power injection. Here, positive sign indicates the power generation and negative sign indicates the power demand. Further, it is to be noted that algebraic sum of all elements in *any column*, say  $l^{th}$  column, gives the *real power loss* ( $rloss(l)$ ) in the transmission line corresponding to that column i.e.  $l^{th}$  line. This table is useful for dual purposes i.e. transmission loss as well as cost allocations. It has been arrived at without any assumptions and the results are highly reliable.

Now, using the  $[B]$  matrix, transmission line real power loss allocation among all buses can be done in the following

**Table 1.** Transmission Line Data of Six Bus System

S no	from bus	to bus	r p.u.	x p.u.	B <sub>sh</sub> /2 p.u.	tap ratio
1	1	4	0.08	0.37	0.015	1
2	1	6	0.123	0.518	0.021	1
3	2	3	0.723	1.05	0	1
4	2	5	0.282	0.64	0	1
5	3	4	0	0.133	0	1.041
6	4	6	0.097	0.407	0.015	1
7	5	6	0	0.3	0	1.049

**Table 2.** Bus Data and Loss Allocation of Six Bus System

Sl	Bus voltage		P in MW		Q in MVAR		TLA MW
	p.u	deg	gen	load	gen	load	
1	1.1	0	112	0	45.34	0	2.932
2	1.1	-9.92	31.37	0	15.62	0	1.374
3	1.005	-14.3	0	55	0	13	1.855
4	0.983	-10.6	0	0	0	0	0
5	0.978	-15.3	0	30	0	18	0.98
6	0.961	-13.3	0	50	0	5	1.227
	Total		143.37	135	60.96	36	8.369

a. Find the sum of the “*absolute*” contribution of all buses to the real power loss of line  $j \rightarrow k$  (say “ $l^{th}$  line”) i.e cumulative power loss “ $Cploss(l)$ ” where

$$cploss(l) = \sum_{i=1}^n |B(i,l)| \quad (11)$$

b. Now to find contribution of  $i^{th}$  bus to the real power loss of line  $j \rightarrow k$  ( $l^{th}$  line) a power loss factor is defined as  $C(i,l)$  where

$$C(i,l) = \frac{|B(i,l)|}{cploss(l)} * rloss(l) \quad (12)$$

The matrix  $[C]$  is indicated in Table 6.

**Table 3.**  $[B]$  = Real Part of  $(Factor1_{jk}^i + Factor2_{jk}^i)$  (in PU)

Line l \ bus i	1	2	3	4	5	6	7	Pinj
1	0.343	0.466	-0	0.006	0.001	0.31	-0.002	1.12
2	0.086	0.123	0.009	0.012	0	0.087	-0.002	0.32
3	-0.16	-0.23	0.008	-0.01	-0.002	-0.161	0.0026	-0.55
4	0	0	0	0	0	0	0	0
5	-0.09	-0.13	-0	0.005	0	-0.09	0.0015	-0.3
6	-0.15	-0.2	-0	0.002	0	-0.145	0	-0.5
<i>rloss</i>	0.029	0.033	0.007	0.014	0	0.001	0	
<i>Cploss</i>	0.829	1.144	0.026	0.034	0.003	0.793	0.008	

**Table 4.**  $[C]$  = Power Loss Factors  $C(i,l)$  Of Six Bus System In MW

Line l \ bus i	1	2	3	4	5	6	7	LA(i)
1	1.175	1.343	0.120	0.2383	0	0.054	0	2.932
2	0.294	0.353	0.226	0.4853	0	0.015	0	1.374
3	0.549	0.655	0.208	0.4142	0	0.028	0	1.855
4	0	0	0	0	0	0	0	0
5	0.296	0.365	0.099	0.2029	0	0.015	0	0.979
6	0.526	0.581	0.031	0.0632	0	0.025	0	1.227
<i>rloss</i>	2.841	3.299	0.685	1.4039	0	0.139	0	

c. By summing up all the individual real power loss factors  $C(i,l)$  of all lines, the total loss allocated to  $i^{\text{th}}$  bus is defined as  $LA(i)$  given by

$$LA(i) = \sum_{l=1}^{nline} C(i,l) \quad (13)$$

It is to be noted that the loss allocated to  $i^{\text{th}}$  bus in the total system losses is given by  $LA(i)$ . The power loss factors matrix  $[C]$  calculated for the six bus example is shown in Table 4 given below.

Any element  $C(i,l)$  in the above table 6 represents amount of  $l^{\text{th}}$  transmission line real power loss allocated to  $i^{\text{th}}$  bus i.e the contribution of  $i^{\text{th}}$  bus to the  $l^{\text{th}}$  transmission line real power loss.

Note that the sum of all elements in any row of Power loss factors matrix  $[C]$ , say  $i^{\text{th}}$  row, gives the total real power loss allocated to the  $i^{\text{th}}$  bus. Further, it is to be noted that the sum of all elements in any column, say  $l^{\text{th}}$  column, gives the real power loss in the transmission line corresponding to that column i.e  $l^{\text{th}}$  line.

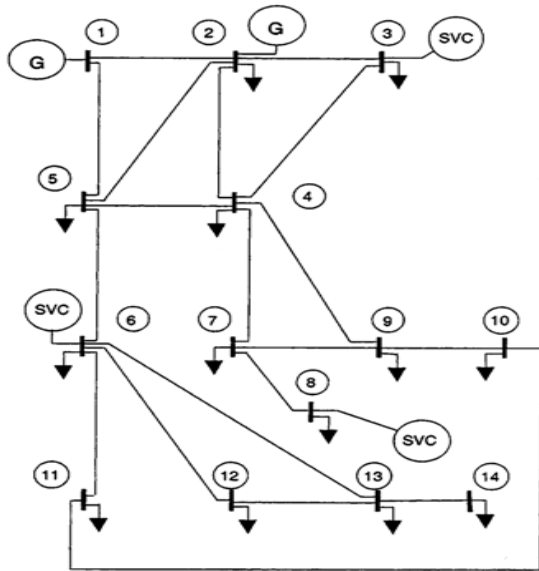


Figure 2. IEEE 14 bus system

Table 5. Loss Allocation Among Buses For IEEE 14 Bus System

Bus	Loss allocation in MW (Total loss=13.56 MW)					
	Zbus	PR	PS	ITL	LWF	Proposed
1	7.64	5.5	6.48	6.14	6.4	6.45
2	0.16	0.64	0.3	0.96	0.36	0.48
3	2.78	2.34	2.88	2.92	3.44	2.58
4	0.84	1.16	1.26	1.26	1.22	1.31
5	0.08	0.18	0.16	0.18	0.14	0.21
6	0.48	1.02	0.26	0.32	0.24	0.33
7	0	0	0	0	0	0
8	0.02	0.66	0	0	0	0.07
9	0.52	0.82	0.78	0.68	0.72	0.8
10	0.18	0.26	0.28	0.2	0.22	0.24
11	0.06	0.1	0.1	0.08	0.08	0.1
12	0.1	0.14	0.16	0.18	0.12	0.18
13	0.26	0.34	0.38	0.32	0.28	0.38
14	0.44	0.4	0.52	0.32	0.34	0.43
Total	13.56	13.56	13.56	13.56	13.56	13.56

## 4. Results

The proposed loss allocation method has been tested on IEEE 14 bus test system and the losses are allocated among the buses. Results are compared with those of the results obtained using  $Z_{bus}$  loss allocation method, Pro-rata (PR) (considering currents and powers), Proportional sharing (PS), Incremental transmission loss (ITL) co-efficients and Loss Weight Factors (LWF) method. Line and bus data of IEEE 14 bus system given in [6] is considered. Test results of loss allocation among buses for the IEEE 14 bus system using the proposed method and the other methods are presented in Table 5. Note that all methods allot zero loss to the transfer bus which has zero injection. It is highly interesting to observe that the results obtained using the proposed method significantly differs from other methods. It is to be noted that, in the proposed method, there are no assumptions and the results are purely based on circuit laws and hence the results obtained from the proposed method may be assumed to be reliable. Further, the amount of loss allocated to any bus by the proposed method is directly reflecting the amount of real power at that bus which is a point to be considered for further research.

## 5. Conclusions

In this paper, starting from a converged load flow solution, contribution factors of each complex power injection at a bus to the complex line loss in each of the transmission line are found out. At the same time, contributions of each of the transmission lines to the complex power injection at a bus are determined. These contributions allow allocation of loss among the buses based on the usage. The loss allocation factors are derived, starting from a converged load flow solution without any assumptions. In this paper a real power loss allocation methodology based on the contribution factors is established. Another significant feature of the proposed methodology is that, effect of reactive power generation and load can be easily established using the proposed methodology. Since the contribution factors derived does not involve any assumptions, there is every chance that these factors can play a vital role in the determination of Locational Marginal Prices in the present day electricity markets.

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