

Separation Principle for a Class of Takagi-Sugeno Descriptor Models

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Abstract This paper studies the design problem of an observer-based controller for a class of nonlinear descriptor systems described by Takagi-Sugeno (T-S) fuzzy structure [1, 2] where the premise variables are unmeasurable. Based on the combination of the control law established in [3] and the fuzzy observer in explicit structure given in [4], a new procedure to investigate the separation principle problem for the considered class of Takagi-Sugeno descriptor models (TSDMs) is then proposed. The guarantee of the global asymptotic stability of the closed-loop augmented system is proved by using the Lyapunov theory and the conditions of asymptotic stability are given in terms of linear matrix inequalities (LMIs). Finally, an application based on rolling disc process is presented as an illustrative example to show the performance of the proposed dynamic controller design.

Keywords Takagi-Sugeno descriptor model, Fuzzy observer, State feedback controller, Lyapunov method, LMI technique, Separation principle

1. Introduction

It is well-known that in order to realize in practice a stabilisation by static state feedback for most industrial applications, the separation principle variously called dynamic output feedback controller or observer-based controller plays a key role since instead of a full-state feedback, we use signals reconstructed by a state observer from the on-line measurements of the input and output of the process. The aim of the paper consists to design an observer-based controller for a class of nonlinear descriptor systems described by T-S representation. Notice that, descriptor models representation has been widely used in the dynamic modeling of many chemical and physical processes [5-8]. The numerical simulation of such dynamic models usually combines an ODE numerical method together with an optimization algorithm. In the literature, there have been several studies concerning the issue of stability and the stabilisation of T-S models [9-11]. Likewise, many works have been carried out to investigate the observer design of T-S models [12-15]. The separation property of T-S observers and controllers was discussed by [16-20]. For TSDMs which are defined by extending the ordinary T-S representation [3, 21], several research works

concerning the problem of control and observation design and applications have been developed [3, 4, 21-25]. Notice that, generally an interesting way to solve the various fuzzy observer and controller problems raised previously is to write the convergence conditions on the LMI form [26].

Based on the use of the state feedback given in [3] and the fuzzy observer proposed in [4], the main contribution of this paper consists to propose a new result of separation principle for a class of TSDMs with unmeasurable premise variables. The global asymptotic stability of the closed-loop system is studied by using the Lyapunov theory and the stability conditions are given in terms of LMIs. Moreover, the proposed result is given without the use of an optimization algorithm.

The outline of the paper is structured as follows. In Section 2, the considered class of TSDMs with unmeasurable premise variables is presented. In Section 3, the main result about fuzzy observer-based controller design is established. The control and observer gains are found directly from LMI formulation. Finally, in Section 4, an illustrative application to show the good performance of the proposed result is given.

Throughout the paper, some notations used are fair standard. For example, $X > 0$ means the matrix X is symmetric and positive definite. X^T denotes the transpose of X . The symbol I (or 0) represents the identity matrix (or zero matrix) with appropriate dimension.

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$$\sum_{i,j=1}^q \mu_i \mu_j = \sum_{i=1}^q \sum_{j=1}^q \mu_i \mu_j \quad \text{and} \quad \begin{pmatrix} X & * \\ Y & Z \end{pmatrix} = \begin{pmatrix} X & Y^T \\ Y & Z \end{pmatrix}.$$

At first, we recall some basic lemmas that are frequently used in the setting the proof of the main result of this paper.

Lemma 1 (Congruence): Let two matrices P and Q , if P is positive definite and if Q is a full column rank matrix, then the matrix QPQ^T is positive definite.

Lemma 2 (Young's inequality): For any matrices S and T with appropriate dimensions, the following property holds for any invertible matrix J and scalar $\alpha > 0$:

$$S^T T + T^T S \leq \alpha S^T J S + \alpha^{-1} T^T J^{-1} T \quad (1)$$

Lemma 3: Considering matrices X , $\Pi < 0$ and a scalar $\gamma > 0$, the following holds:

$$X^T \Pi X \leq -\gamma (X^T + X) - \gamma^2 \Pi^{-1} \quad (2)$$

2. Takagi-Sugeno Descriptor Models

The following class of TSDMs is considered in this paper:

$$\begin{cases} E\dot{x} = \sum_{i=1}^q \mu_i(\xi)(A_i x + B_i u) \\ y = Cx \end{cases} \quad (3)$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, $y \in R^p$ is the measured output. $A_i \in R^{n \times n}$, $B_i \in R^{n \times m}$, $C \in R^{p \times n}$ and $E \in R^{n \times n}$ such that $\text{rang}(E) = r < n$, are real known constant matrices. q is the number of sub-models. ξ is the premise variable which can be here depend on completely or partially unmeasured state of the system and the $\mu_i(\xi)$ ($i=1, \dots, q$) are the weighting functions that ensure the transition between the contribution of each sub model:

$$\begin{cases} E\dot{x} = A_i x + B_i u \\ y = Cx \end{cases} \quad (4)$$

They verify the so-called convex sum properties:

$$\begin{cases} 0 \leq \mu_i(\xi) \leq 1 \\ \sum_{i=1}^q \mu_i(\xi) = 1 \end{cases} \quad (5)$$

The main object of this paper is to design an observer-based controller for the class of TSDMs (3). For this objective, we suppose that [5]:

- H₁) (E, A_i) is regular, i.e. $\det(sE - A_i) \neq 0 \quad \forall s \in \mathbb{C}$
- H₂) Sub-models (4) are impulse controllable and stabilisable.
- H₃) Sub-models (4) are impulse observable and detectable.

Moreover, the following hypothesis which is necessary for the observer design given in [4] is assumed to be verified:

$$H_4) \quad \text{rang} \begin{pmatrix} E \\ C \end{pmatrix} = n$$

Note that from the consideration of the hypothesis H₄), there exists a non-singular matrix $\begin{pmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_3 & \Lambda_4 \end{pmatrix}$ such that:

$$\begin{cases} \Lambda_1 E + \Lambda_2 C = I \\ \Lambda_3 E + \Lambda_4 C = 0 \end{cases} \quad (6)$$

where $\Lambda_1 \in R^{n \times n}$, $\Lambda_2 \in R^{n \times p}$, $\Lambda_3 \in R^{p \times n}$,

$\Lambda_4 \in R^{p \times p}$ are constant matrices which can be found by the singular value decomposition of $\begin{pmatrix} E \\ C \end{pmatrix}$.

In order to investigate the separation principle problem for the considered class of TSDMs (3), firstly the following control law established in [3] is adopted:

$$u(x) = -\sum_{i=1}^q \mu_i(\xi) F_i x \quad (7)$$

where the gains F_i , $i=1, \dots, q$ can determined in order that the closed-loop system (3) is asymptotically stable.

The following result has been studied in [3].

Theorem 1 [3]: The closed-loop system (3)-(7) is globally exponentially stable if there exist matrices $Y_1 > 0$, U_i , $i=1, \dots, q$ verifying the following LMIs:

$$\begin{cases} Y_1^T E^T = E Y_1 \geq 0 \\ Y_{ii} < 0 & i=1, \dots, q \\ Y_{ij} + Y_{ji} < 0 & i < j \text{ s.t. } \mu_i \cap \mu_j \neq \emptyset \end{cases} \quad (8)$$

where

$$Y_{ij} = A_i Y_1 + Y_1 A_i^T - B_i U_j - U_j^T B_i^T \quad (9)$$

The fuzzy local feedback gains F_i , $i=1, \dots, q$ are given by:

$$F_i = U_i Y_1^{-1} \quad (10)$$

On the other hand, the following fuzzy observer given in [4] is employed:

$$\begin{cases} \dot{\hat{z}} = \sum_{i=1}^q \mu_i(\hat{\xi})(N_i \hat{z} + L_{1i} y + L_{2i} y + G_i u) \\ \hat{x} = \hat{z} + (\Lambda_2 + K \Lambda_4) y \end{cases} \quad (11)$$

where $\hat{z} \in R^n$ is the state of fuzzy observer and $\hat{x} \in R^n$ is the estimate of x . $\hat{\xi} \in R^q$ denotes the estimated premise variables vector. The matrices N_i , L_{1i} , L_{2i} , G_i and K are unknown matrices of appropriate dimensions to be determined such that \hat{x} converges asymptotically to x .

Λ_2 and Λ_4 are such that equation (6) is satisfied.

The convergence condition of the observer (11) can be formulated by the following Theorem.

Theorem 2 [4]: The state error between the T-S descriptor model (3) and its observer (11) converges asymptotically towards zero, if there exist matrices $Y_2 > 0$, $Y_3 > 0$, Q and W_j , $j = 1, \dots, q$ such that the following LMIs hold :

$$\begin{pmatrix} \Delta_j + \Delta_j^T & * & * \\ \Theta_1 & A_i^T Y_3 + Y_3 A_i & * \\ \Theta_2 & B_i^T Y_3 & 0 \end{pmatrix} < 0 \quad \forall i, j \in \{1, \dots, q\} \quad (12)$$

where

$$\Delta_j = Y_2 \Lambda_1 A_j + Q \Lambda_3 A_j - W_j C \quad (13)$$

and

$$\begin{cases} \Theta_1 = (A_i - A_j)^T \Lambda_1^T Y_2 + (A_i - A_j)^T \Lambda_3^T Q^T \\ \Theta_2 = (B_i - B_j)^T \Lambda_1^T Y_2 + (B_i - B_j)^T \Lambda_3^T Q^T \end{cases} \quad (14)$$

The observer gains N_i , L_{1i} , L_{2i} , G_i , K are given by:

$$\begin{cases} K = Y_2^{-1} Q \\ G_i = (\Lambda_1 + K \Lambda_3) B_i \\ L_{2i} = Y_2^{-1} W_i \\ N_i = (\Lambda_1 + K \Lambda_3) A_i - L_{2i} C \\ L_{1i} = N_i (\Lambda_2 + K \Lambda_4) \end{cases} \quad (15)$$

3. Main Result

As mentioned above, based on the use of the state feedback given by (7) and the fuzzy observer (11), this section studies the problem of observer-based controller design for a class of TSDMs (3). Indeed, the following augmented system in closed-loop is considered:

$$\begin{cases} E \dot{\hat{x}} = \sum_{i=1}^q \mu_i(\xi) (A_i \hat{x} + B_i u(\hat{x})) \\ \dot{z} = \sum_{i=1}^q \mu_i(\hat{\xi}) (N_i z + L_{1i} y + L_{2i} y + G_i u(\hat{x})) \\ \hat{x} = z + (\Lambda_2 + K \Lambda_4) y \\ u(\hat{x}) = - \sum_{i=1}^q \mu_i(\hat{\xi}) F_i \hat{x} \end{cases} \quad (16)$$

Note that:

In order to establish the conditions for the asymptotic convergence of the system (16), we define the state estimation error:

$$\varepsilon = x - \hat{x} \quad (17)$$

Then, by substituting (6) and (16) into (17) we obtain:

$$\varepsilon = (\Lambda_1 + K \Lambda_3) E x - z \quad (18)$$

So, error dynamics will be:

$$\dot{\varepsilon} = (\Lambda_1 + K \Lambda_3) E \dot{x} - \dot{z} \quad (19)$$

It follows from (16) and (17) that the error dynamics can be written as:

$$\begin{aligned} \dot{\varepsilon} = & \sum_{i,j=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) (\Lambda_1 + K \Lambda_3) (\Gamma_{ij} x + B_i F_j \varepsilon) \\ & - \sum_{i,j=1}^q \mu_i(\hat{\xi}) \mu_j(\hat{\xi}) (N_i z + L_{1i} y + L_{2i} y - G_i F_j x + G_i F_j \varepsilon) \end{aligned} \quad (20)$$

where

$$\Gamma_{ij} = A_i - B_i F_j \quad (21)$$

By substituting (18), equation (20) becomes:

$$\begin{aligned} \dot{\varepsilon} = & \sum_{i,j=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) (\Lambda_1 + K \Lambda_3) (\Gamma_{ij} x + B_i F_j \varepsilon) \\ & + \sum_{i,j=1}^q \mu_i(\hat{\xi}) \mu_j(\hat{\xi}) (N_i \varepsilon - \Pi_{ij} x - G_i F_j \varepsilon) \end{aligned} \quad (22)$$

where

$$\Pi_{ij} = N_i (\Lambda_1 + K \Lambda_3) E + L_{1i} C + L_{2i} C - G_i F_j \quad (23)$$

Provided the matrices gains N_i , L_{1i} , L_{2i} , G_i and K satisfy:

$$\Pi_{ij} = (\Lambda_1 + K \Lambda_3) \Gamma_{ij} \quad (24)$$

$$G_i = (\Lambda_1 + K \Lambda_3) B_i \quad (25)$$

Then, from (6), (24) and (25), we have:

$$N_i = (\Lambda_1 + K \Lambda_3) A_i - L_{2i} C + (N_i (\Lambda_2 + K \Lambda_4) - L_{1i}) C \quad (26)$$

Take:

$$L_{1i} = N_i (\Lambda_2 + K \Lambda_4) \quad (27)$$

Then:

$$N_i = (\Lambda_1 + K \Lambda_3) A_i - L_{2i} C \quad (28)$$

It follows that the system (22) reduces to:

$$\dot{\varepsilon} = \sum_{i,j=1}^q (\mu_i(\xi) - \mu_i(\hat{\xi})) \mu_j(\hat{\xi}) (\Lambda_1 + K \Lambda_3) (\Gamma_{ij} x + B_i F_j \varepsilon) + \sum_{i,j=1}^q \mu_i(\hat{\xi}) \mu_j(\hat{\xi}) N_i \varepsilon \quad (29)$$

$$\begin{cases} \sum_{i,j=1}^q (\mu_i(\xi) - \mu_i(\hat{\xi})) \mu_j(\hat{\xi}) A_i = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) \Delta A_{ik} \\ \sum_{i,j=1}^q (\mu_i(\xi) - \mu_i(\hat{\xi})) \mu_j(\hat{\xi}) B_i = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) \Delta B_{ik} \end{cases} \quad (30)$$

where $\Delta A_{ik} = A_i - A_k$ and $\Delta B_{ik} = B_i - B_k$.

Then, from (21), the equation (29) becomes:

$$\dot{\varepsilon} = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) (\Lambda_1 + K \Lambda_3) ((\Delta A_{ik} - \Delta B_{ik} F_j) x + \Delta B_{ik} F_j \varepsilon) + \sum_{j,k=1}^q \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) N_k \varepsilon \quad (31)$$

By (5), equation (31) can be reduced to the following:

$$\dot{\varepsilon} = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) (\Phi_{ijk} x + \Psi_{ijk} \varepsilon) \quad (32)$$

where

$$\begin{cases} \Phi_{ijk} = (\Lambda_1 + K \Lambda_3) (\Delta A_{ik} - \Delta B_{ik} F_j) \\ \Psi_{ijk} = (\Lambda_1 + K \Lambda_3) \Delta B_{ik} F_j + N_k \end{cases} \quad (33)$$

Thus, the global asymptotic stability of system (16) can be proved in an equivalent fashion for the following system:

$$\begin{cases} E \dot{x} = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) (\Gamma_{ij} x + B_i F_j \varepsilon) \\ \dot{\varepsilon} = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) (\Phi_{ijk} x + \Psi_{ijk} \varepsilon) \end{cases} \quad (34)$$

which can be rewritten as follows:

$$\bar{E} \dot{x}_a = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) \Omega_{ijk} x_a \quad (35)$$

where

$$\begin{cases} x_a = \begin{pmatrix} x \\ \varepsilon \end{pmatrix}, \quad \bar{E} = \begin{pmatrix} E & 0 \\ 0 & I \end{pmatrix} \\ \Omega_{ijk} = \begin{pmatrix} \Gamma_{ij} & B_i F_j \\ \Phi_{ijk} & \Psi_{ijk} \end{pmatrix} \end{cases} \quad (36)$$

The following Theorem constitutes the main result of the paper.

Theorem 3: The system (35) is globally asymptotically stable if for predefined scalars $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_3 > 0$, $\alpha_4 > 0$, $\gamma > 0$; there exist matrices $Y_1 > 0$, $Y_2 > 0$, Q , U_i , W_i , $i = 1, \dots, q$, verifying the following LMIs:

$$\Pi_{ijk} = \begin{pmatrix} -2\gamma Z & * & * \\ \gamma I & \Pi_{ijk}^1 & * \\ \Pi_{ijk}^2 & 0 & -\alpha_4^{-1} I \end{pmatrix} < 0 \quad \forall i, j, k \in \{1, \dots, q\} \quad (37)$$

where

$$\left\{ \begin{array}{l} \Pi_{ijk}^1 = \begin{pmatrix} \Pi^{11} & * & * & * & * \\ \Pi^{12} & \Phi_1 & * & * & * \\ \Pi^{13} & 0 & \Phi_2 & * & * \\ \Pi^{14} & 0 & 0 & \Phi_3 & * \\ \Pi^{15} & 0 & 0 & 0 & -\alpha_4 I \end{pmatrix} \\ \Pi_{ijk}^2 = (\Pi^{21} \ 0 \ 0 \ 0 \ 0) \\ Z = \text{diag}(X, X, X, X, I) \\ X = \text{diag}(Y_1, Y_1) \end{array} \right. \quad (38)$$

with

$$\left\{ \begin{array}{l} \Phi_1 = \begin{pmatrix} -\alpha_1 I & 0 \\ 0 & -\alpha_1^{-1} I \end{pmatrix} \\ \Phi_2 = \begin{pmatrix} -\alpha_2 I & 0 \\ 0 & -\alpha_2^{-1} I \end{pmatrix} \\ \Phi_3 = \begin{pmatrix} -\alpha_3 Y_1 & 0 \\ 0 & -\alpha_3^{-1} Y_1 \end{pmatrix} \end{array} \right. \quad (39)$$

and

$$\left\{ \begin{array}{l} \Pi^{11} = \begin{pmatrix} Y_{ij} & 0 \\ 0 & \Delta_k + \Delta_k^T \end{pmatrix} \\ \Pi^{12} = \begin{pmatrix} 0 & \Lambda_1^T Y_2 + \Lambda_3^T Q^T \\ \Delta A_{ik} Y_1 & 0 \end{pmatrix} \\ \Pi^{13} = \begin{pmatrix} 0 & -(\Lambda_1^T Y_2 + \Lambda_3^T Q^T) \\ \Delta B_{ik} U_j & 0 \end{pmatrix} \\ \Pi^{14} = \begin{pmatrix} U_j^T B_i^T & 0 \\ 0 & I \end{pmatrix} \\ \Pi^{15} = (0 \ \Lambda_1^T Y_2 + \Lambda_3^T Q^T) \\ \Pi^{21} = (0 \ \Delta B_{ik} U_j) \end{array} \right. \quad (40)$$

with Y_{ij} and Δ_k are defined in (9) and (13) respectively.

The dynamic controller gains $F_i, N_i, L_{1i}, L_{2i}, G_i$ and K are given by:

$$\left\{ \begin{array}{l} F_i = U_i Y_1^{-1} \\ K = Y_2^{-1} Q \\ G_i = (\Lambda_1 + K \Lambda_3) B_i \\ L_{2i} = Y_2^{-1} W_i \\ N_i = (\Lambda_1 + K \Lambda_3) A_i - L_{2i} C \\ L_{1i} = N_i (\Lambda_2 + K \Lambda_4) \end{array} \right. \quad (41)$$

where $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4$ are such that (6) is satisfied.

Proof of Theorem 3: To prove the convergence to zero of the state variable system (35), let us consider the candidate Lyapunov function as follows:

$$V(x_a) = x_a^T \bar{E} P x_a, \quad P > 0, \quad \bar{E}^T P = P^T \bar{E} \geq 0 \quad (42)$$

with

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \quad (43)$$

The time derivative of the Lyapunov function (42) along the trajectories of the system (35) is obtained as:

$$\dot{V}(x_a) = \sum_{i,j,k=1}^q \mu_i(\xi) \mu_j(\hat{\xi}) \mu_k(\hat{\xi}) x_a^T (\Omega_{ijk}^T P + P \Omega_{ijk}) x_a \quad (44)$$

Therefore, we have the following stability conditions:

$$\Omega_{ijk}^T P + P \Omega_{ijk} < 0, \quad \forall i, j, k \in \{1, \dots, q\} \quad (45)$$

which become from (36) and (43):

$$\begin{pmatrix} \Gamma_{ij}^T P_1 + P_1 \Gamma_{ij} & * \\ P_2 \Phi_{ijk} + F_j^T B_i^T P_1 & \Psi_{ijk}^T P_2 + P_2 \Psi_{ijk} \end{pmatrix} < 0, \quad \forall i, j, k \in \{1, \dots, q\} \quad (46)$$

Applying Lemma 1 to (46) by pre-multiplying and post-multiplying $\begin{pmatrix} P_1^{-1} & 0 \\ 0 & I \end{pmatrix}$ to both sides and defining new variables

$Y_1 = P_1^{-1}$ and $Y_2 = P_2$, thus we obtain:

$$\Theta = \begin{pmatrix} Y_1 \Gamma_{ij}^T + \Gamma_{ij} Y_1 & * \\ Y_2 \Phi_{ijk} Y_1 + F_j^T B_i^T & \Psi_{ijk}^T Y_2 + Y_2 \Psi_{ijk} \end{pmatrix} < 0, \quad \forall i, j, k \in \{1, \dots, q\} \quad (47)$$

which become from (33):

$$\Theta = \Sigma + \Sigma_1 + \Sigma_2 < 0 \quad \forall i, j, k \in \{1, \dots, q\} \quad (48)$$

where

$$\begin{cases} \Sigma = \begin{pmatrix} Y_1 \Gamma_{ij}^T + \Gamma_{ij} Y_1 & 0 \\ 0 & N_k^T Y_2 + Y_2 N_k \end{pmatrix} \\ \Sigma_1 = \begin{pmatrix} 0 & * \\ Y_2 (\Lambda_1 + K \Lambda_3) \Delta A_{ik} Y_1 - Y_2 (\Lambda_1 + K \Lambda_3) \Delta B_{ik} F_j Y_1 + F_j^T B_i^T & 0 \end{pmatrix} \\ \Sigma_2 = \begin{pmatrix} 0 & 0 \\ 0 & Y_2 (\Lambda_1 + K \Lambda_3) \Delta B_{ik} F_j + F_j^T \Delta B_{ik}^T (\Lambda_1 + K \Lambda_3)^T Y_2 \end{pmatrix} \end{cases} \quad (49)$$

By considering the following:

$$\begin{cases} X_1 = \begin{pmatrix} 0 & (\Lambda_1 + K \Lambda_3)^T Y_2 \end{pmatrix} \\ Z_1 = \begin{pmatrix} \Delta A_{ik} Y_1 & 0 \end{pmatrix} \\ X_2 = \begin{pmatrix} 0 & -(\Lambda_1 + K \Lambda_3)^T Y_2 \end{pmatrix} \\ Z_2 = \begin{pmatrix} \Delta B_{ik} F_j Y_1 & 0 \end{pmatrix} \\ X_3 = \begin{pmatrix} F_j^T B_i^T & 0 \end{pmatrix} \\ Z_3 = \begin{pmatrix} 0 & I \end{pmatrix} \\ X_4 = \begin{pmatrix} 0 & (\Lambda_1 + K \Lambda_3)^T Y_2 \end{pmatrix} \\ Z_4 = \begin{pmatrix} 0 & \Delta B_{ik} F_j \end{pmatrix} \end{cases} \quad (50)$$

Matrix Θ can be rewritten again as:

$$\Theta = \Sigma + X_1^T Z_1 + Z_1^T X_1 + X_2^T Z_2 + Z_2^T X_2 + X_3^T Z_3 + Z_3^T X_3 + X_4^T Z_4 + Z_4^T X_4 \quad (51)$$

Applying Lemma 2, the equation (51) becomes:

$$\Theta \leq \Sigma + \Omega_1 \Phi_1^{-1} \Omega_1^T + \Omega_2 \Phi_2^{-1} \Omega_2^T + \Omega_3 \Phi_3^{-1} \Omega_3^T + \Omega_4 \Phi_4^{-1} \Omega_4^T \quad (52)$$

where

$$\begin{cases} \Omega_1 = \begin{pmatrix} 0 & Y_1 \Delta A_{ik}^T \\ Y_2 (\Lambda_1 + K \Lambda_3) & 0 \end{pmatrix} \\ \Omega_2 = \begin{pmatrix} 0 & Y_1 F_j^T \Delta B_{ik}^T \\ -Y_2 (\Lambda_1 + K \Lambda_3) & 0 \end{pmatrix} \\ \Omega_3 = \begin{pmatrix} B_i F_j Y_1 & 0 \\ 0 & I \end{pmatrix} \\ \Omega_4 = \begin{pmatrix} 0 & 0 \\ Y_2 (\Lambda_1 + K \Lambda_3) & F_j^T \Delta B_{ik}^T \end{pmatrix} \\ \Phi_4 = \begin{pmatrix} -\alpha_4 I & 0 \\ 0 & -\alpha_4^{-1} I \end{pmatrix} \end{cases} \quad (53)$$

and Φ_1, Φ_2, Φ_3 are defined in (39).

Hence, using the Schur complement [26], the inequality $\Theta < 0$ hold if the following matrix inequality is satisfied:

$$M = \begin{pmatrix} \Sigma & * & * & * & * \\ \Omega_1^T & \Phi_1 & * & * & * \\ \Omega_2^T & 0 & \Phi_2 & * & * \\ \Omega_3^T & 0 & 0 & \Phi_3 & * \\ \Omega_4^T & 0 & 0 & 0 & \Phi_4 \end{pmatrix} < 0 \quad (54)$$

Now, in order to establish the LMI conditions (37) of Theorem 3, we rewrite the inequality (54) as follows:

$$M = \begin{pmatrix} M_1 & * \\ M_2 & -\alpha_4^{-1} I \end{pmatrix} < 0 \quad (55)$$

where

$$\begin{cases} M_1 = \begin{pmatrix} \Sigma & * & * & * & * \\ \Omega_1^T & \Phi_1 & * & * & * \\ \Omega_2^T & 0 & \Phi_2 & * & * \\ \Omega_3^T & 0 & 0 & \Phi_3 & * \\ \Omega_4^T & 0 & 0 & 0 & -\alpha_4 I \end{pmatrix} \\ M_2 = (\Omega_4^{2T} \ 0 \ 0 \ 0 \ 0) \end{cases} \quad (56)$$

with

$$\begin{cases} \Omega_4^{1T} = (0 \ (\Lambda_1 + K \Lambda_3)^T Y_2) \\ \Omega_4^{2T} = (0 \ \Delta B_{ik} F_j) \end{cases} \quad (57)$$

and we consider the following matrix W defined by:

$$W = \text{diag}(Z, I) \quad (58)$$

where

$$Z = \text{diag}(X, X, X, X, I) \quad (59)$$

with

$$X = \text{diag}(Y_1, Y_1) \quad (60)$$

Applying Lemma 1, $M < 0$ is equivalent to:

$$W^T M W < 0 \quad (61)$$

which can be rewritten as follows:

$$\begin{pmatrix} Z M_1 Z & * \\ M_2 Z & -\alpha_4^{-1} I \end{pmatrix} < 0 \quad (62)$$

Hence, applying Lemma 3 on $Z M_1 Z$ leads to:

$$\begin{pmatrix} -2\gamma Z - \gamma^2 M_1^{-1} & * \\ M_2 Z & -\alpha_4^{-1} I \end{pmatrix} < 0 \quad (63)$$

where $\gamma > 0$.

Using the Schur complement, we obtain:

$$\Pi_{ijk} = \begin{pmatrix} -2\gamma Z & * & * \\ \gamma I & M_1 & * \\ M_2 Z & 0 & -\alpha_4^{-1} I \end{pmatrix} < 0 \quad (64)$$

Then, the replacement of the various terms given in (64) by their expressions given before and the use of the changes of variables:

$$\begin{cases} U_i = F_i Y_1 \\ Q = Y_2 K \\ W_i = Y_2 L_{2i} \end{cases} \quad (65)$$

We establish the LMI conditions given in Theorem 3. Finally, the dynamic controller gains given by (41) can be determined from (25), (27), (28) and (65).

4. Illustrative Example

To illustrate the efficiency of the proposed dynamic controller, we consider a rolling disc process described by the following TSDM with unmeasurable premise variable given in [25] such that the hypothesis H_4 is satisfied:

$$\begin{cases} E \dot{x} = \sum_{i=1}^2 \mu_i(\xi) (A_i x + B u) \\ y = C x \end{cases} \quad (66)$$

where $x = (x_1 \ x_2 \ x_3 \ x_4)^T$ is the state vector with x_1 is the position of the center of the disc, x_2 is the translational velocity of the same point, x_3 is the angular velocity of the disc, x_4 is the contact force between the disc and the surface. u is the applied input force to the disc and y is the vector of output measurement.

$$\begin{aligned}
A_1 &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ \xi_{\max} & -\frac{b}{m} & 0 & \frac{1}{m} \\ 0 & 1 & -r & 0 \\ \xi_{\max} & -\frac{b}{m} & 0 & \frac{r^2}{J} + \frac{1}{m} \end{pmatrix}, \\
A_2 &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ \xi_{\min} & -\frac{b}{m} & 0 & \frac{1}{m} \\ 0 & 1 & -r & 0 \\ \xi_{\min} & -\frac{b}{m} & 0 & \frac{r^2}{J} + \frac{1}{m} \end{pmatrix}, \\
E &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{r}{J} \end{pmatrix}, \\
C &= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}
\end{aligned}$$

The weighting functions are defined by:

$$\begin{cases} \mu_1(\xi) = \frac{\xi - \xi_{\min}}{\xi_{\max} - \xi_{\min}} \\ \mu_2(\xi) = \frac{\xi_{\max} - \xi}{\xi_{\max} - \xi_{\min}} \end{cases} \quad (67)$$

where ξ is the premise variable having for expression:

$$\xi = \frac{-k_1 - k_2 x_1^2}{m} \in [\xi_{\min}, \xi_{\max}] \quad (68)$$

The physical parameters definition and their numerical values are given in [25].

As mentioned previously, the objective of this section is to use the TSDM (66) as an illustrative example to demonstrate the ability of Theorem 3 to be applied.

Therefore, the following augmented system in closed-loop is considered:

$$\begin{cases} E\dot{\hat{x}} = \sum_{i=1}^2 \mu_i(\xi)(A_i \hat{x} + B u(\hat{x})) \\ \dot{z} = \sum_{i=1}^2 \mu_i(\hat{\xi})(N_i z + L_{1i} y + L_{2i} y + G_i u(\hat{x})) \\ \hat{x} = z + (\Lambda_2 + K \Lambda_4) y \\ u(\hat{x}) = -\sum_{i=1}^2 \mu_i(\hat{\xi}) F_i \hat{x} \end{cases} \quad (69)$$

where F_i , N_i , L_{1i} , L_{2i} , G_i , $i=1,2$ and K are the gains of the proposed dynamic controller. Notice that to apply the

theorem 3 in order to calculate such dynamic controller gains; it suffices to rewritten system (69) into its equivalent representation (35) as mentioned in Section 3.

Thus, in a first time, we calculate the matrices Λ_1 , Λ_2 , Λ_3 and Λ_4 satisfying (6) from singular value decomposition of $\begin{pmatrix} E \\ C \end{pmatrix}$. Indeed, under hypothesis H_4), the following numerical expressions of Λ_i were obtained:

$$\begin{aligned}
\Lambda_1 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \Lambda_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
\Lambda_3 &= \begin{pmatrix} 0 & 0 & 0.71 & -0.71 \\ 0 & 0 & -0.71 & -0.71 \end{pmatrix}, \quad \Lambda_4 = 10^{-15} \begin{pmatrix} 0 & -0.12 \\ 0 & -0.05 \end{pmatrix}
\end{aligned}$$

In a second time, we resolve the LMIs (37) in $Y_1 > 0$, $Y_2 > 0$, Q , U_i , W_i . Indeed, with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.1$ and $\gamma = 1$, we obtain the following dynamic controller gains calculated by (41):

$$F_1 = (-153.62 \quad -240.44 \quad -0.23 \quad -9.95), \quad F_2 = (-153.62 \quad -240.44 \quad -0.23 \quad -9.95)$$

$$N_1 = \begin{pmatrix} -3.58 & -0.55 & 1.57 & 0.19 \\ -7.02 & -5.01 & 5.14 & 2.99 \\ 3.08 & 0.55 & -2.07 & -0.19 \\ 7.02 & 4.51 & -5.14 & -3.49 \end{pmatrix}, \quad N_2 = \begin{pmatrix} -3.84 & -0.58 & 1.72 & 0.17 \\ -7.56 & -5.06 & 5.41 & 2.94 \\ 3.34 & 0.58 & -2.22 & -0.17 \\ 7.56 & 4.56 & -5.41 & -3.44 \end{pmatrix}$$

$$L_{11} = \begin{pmatrix} 1.57 & 0.19 \\ 5.14 & 2.99 \\ -2.07 & 0.19 \\ -5.14 & -3.49 \end{pmatrix}, \quad L_{12} = \begin{pmatrix} 1.72 & 0.17 \\ 5.41 & 2.94 \\ -2.22 & -0.17 \\ -5.41 & -3.44 \end{pmatrix},$$

$$L_{21} = \begin{pmatrix} -1.55 & -0.04 \\ -3.29 & -2.73 \\ 2.05 & 0.04 \\ 3.29 & 3.23 \end{pmatrix}, \quad L_{22} = \begin{pmatrix} -1.70 & -0.01 \\ -3.55 & -2.68 \\ 2.20 & 0.01 \\ 3.55 & 3.18 \end{pmatrix},$$

$$G_1 = \begin{pmatrix} -0.26 \\ -0.39 \\ 0.26 \\ 0.39 \end{pmatrix}, \quad G_2 = \begin{pmatrix} -0.26 \\ -0.39 \\ 0.26 \\ 0.39 \end{pmatrix}, \quad K = \begin{pmatrix} -1.49 & -1.42 \\ -5.49 & 1.08 \\ 1.49 & 1.42 \\ 5.49 & -1.08 \end{pmatrix}$$

In order to illustrate the performances of the proposed observer-based controller, numerical simulations of system (69) were performed using a Runge-Kutta method combined with the Newton-Raphson algorithm.

The following initial conditions of system (69) were used: $x_0 = [0.10 \quad 0.30 \quad 0.75 \quad -12.67]^T$, $\hat{x}_0 = [0.05 \quad 0.45 \quad 0.80 \quad -12.82]^T$

The simulation results of the system (69) with the dynamic controller gains F_i , N_i , L_{1i} , L_{2i} , G_i and K are given in Figures 1, 2, 3 and 4 where the dashed lines denote the state variables estimated by the fuzzy observer.

These simulation results show the good performance of the proposed observer-based controller designed. Indeed, the

global asymptotic stability of the augmented system (69) in closed-loop with the proposed observer-based controller is satisfied.

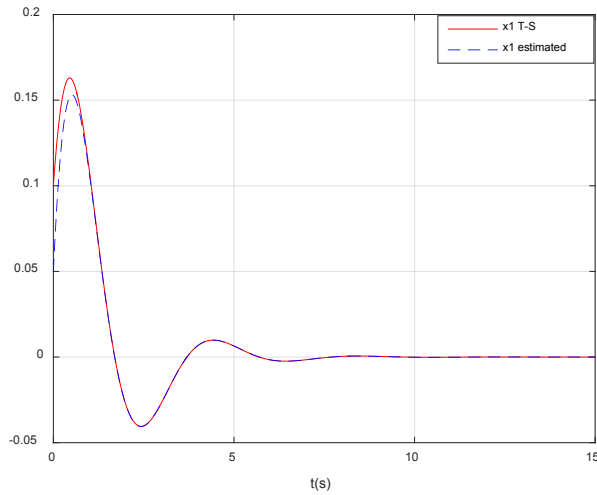


Figure 1. x_1 & \hat{x}_1 with fuzzy observer-based controller

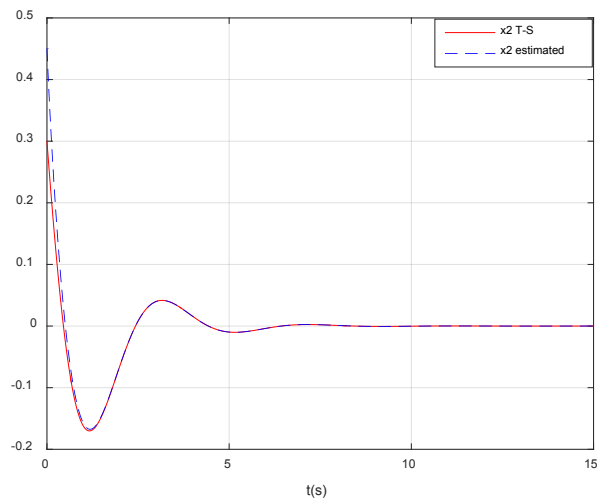


Figure 2. x_2 & \hat{x}_2 with fuzzy observer-based controller

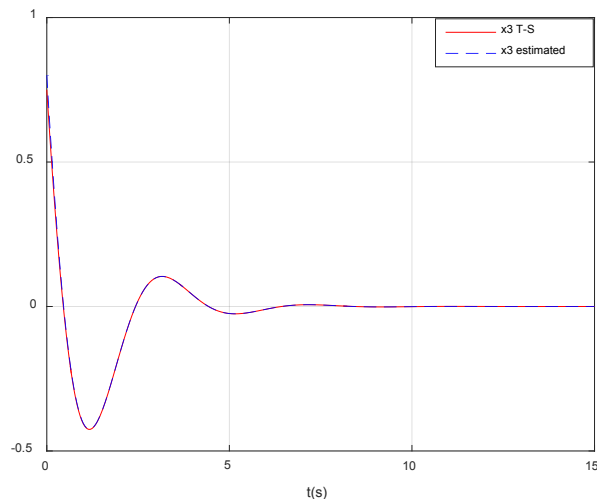


Figure 3. x_3 & \hat{x}_3 with fuzzy observer-based controller

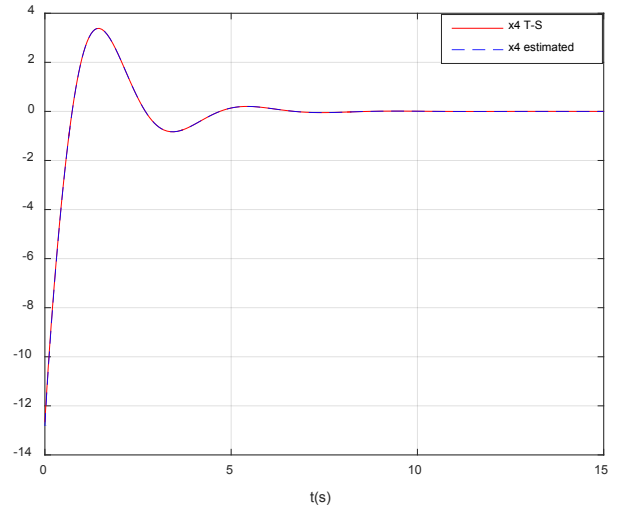


Figure 4. x_4 & \hat{x}_4 with fuzzy observer-based controller

Moreover, these results of simulation show a good estimation of the unknown states of the process which is necessary for the real-time implementation of the control law (7).

5. Conclusions

In this paper, a new result of separation principle for a class of TSDMs with unmeasurable premise variables is suggested. More precisely, the proposed result concerns the real-time implementation without the use of an optimization algorithm of the static state feedback given in [3] by using the fuzzy observer proposed in [4]. The global asymptotic stability of the augmented system in closed-loop is studied by using the Lyapunov method and the stability conditions are given in terms of LMIs. The good performance of the proposed observer-based controller design is illustrated in simulation through a rolling disc process as an illustrative example.

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