International Journal of Control Science and Engineering 2016, 6(1): 12-21
DOI: 10.5923/j.control.20160601.02

Fuzzy Observer Design for a Class of Discrete-Time Takagi-Sugeno Descriptor Systems

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Abstract This paper investigates the observer design problem of a class of discrete-time nonlinear descriptor systems described by Takagi-Sugeno (T-S) structure. Both measurable and unmeasurable premise variables cases are considered. The approach is based on the separation between dynamic and static relations in the T-S descriptor model. The convergence of the state estimation error is studied by using the Lyapunov theory and the stability conditions are given in terms of linear matrix inequalities (LMIs). Moreover, for reasons of ease of the implementation, the main result of this paper consists in showing that the state estimation problem for a class of considered descriptor systems can be achieved by using a fuzzy observer having only a recurrence structure. Finally, an application to a model of a heat exchanger pilot process is given to illustrate the proposed approach.

Keywords Takagi-Sugeno Model, Discrete-Time Descriptor System, Fuzzy Observer, Linear Matrix Inequality (LMI)

1. Introduction

It is well known that to apply state feedback control it is necessary to know all the states of the system. In real application, not all the states are available to be measured, thus the necessity to use an observer arises. In this paper, we are interested in the design of state observer of discrete-time nonlinear descriptor systems represented by T-S models. Recall that, the interest in using the approach based on the representation of the nonlinear systems by T-S models [1-3] relies on the fact that once the T-S fuzzy models are obtained, some analysis and design tools developed in the linear system theory can be used [4-6], which facilitates observer or/and controller synthesis for complex nonlinear systems.

However, many works have been carried out to investigate the problem of the nonlinear observer synthesis and its application for dynamical systems described by T-S fuzzy models. More specifically, the T-S fuzzy observer problem for dynamic explicit models in the continuous-time and the discrete-time has been addressed in [3], [7-10]. In implicit cases, several works exist for continuous-time case [11-15] and for discrete-time one [16], [17]. Moreover, many other works dealing with observer design in explicit form called Proportional Integral Observer were also proposed for implicit T-S models. These results are based on the singular value decomposition approach and a generalized inverse matrix and consider the output matrix without nonlinear terms see for example [18], [19] and many references herein. Notice that, generally an interesting way to solve the various fuzzy observer problems raised previously is to write the convergence conditions on the LMI form [20].

In this paper, we deal with nonlinear descriptor models. The dynamics of descriptor systems variously called singular systems or implicit systems are governed by a mixture of algebraic and differential (difference) equations. These models have been receiving a great deal of attention for many decades as a representation of dynamical systems [21-24]. This formulation includes both dynamic and static relations. In fact, this formalism is much more general than the usual one and can model physical constraints or impulsive behavior due to an improper part of the system. Recall that many physical systems are naturally modeled as descriptor systems such as chemical, electrical and mechanical engineering systems. The numerical simulation of implicit models usually combines a resolution routine of an ordinary equation together with an optimization algorithm.

This work presents a new observer structure for a class of discrete-time T-S descriptor systems. Both measurable and unmeasurable premise variables cases are considered. The approach is based on the separation between difference and algebraic equations in the T-S descriptor model. The main contribution of this paper consists in showing that the observer design problem for a class of discrete-time T-S descriptor systems can be achieved by using a fuzzy observer without the use of an optimization algorithm. Using the Lyapunov theory, stability conditions are obtained and given

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Published online at http://journal.sapub.org/control

in terms of LMIs.

The rest of the paper is structured as follows. The class of the fuzzy T-S structure of the nonlinear implicit system is defined in the Section 2. The main results about fuzzy observer design for a class of discrete-time T-S descriptor systems are exposed in Section 3. The first result is devoted to the case of measurable premise variables and the second one concerns the unmeasurable premise variables. Finally, to reinforce the theoretical results, we present an application with simulation in Section 4.

In this paper, some notations used are fairly standard. For example, $X > 0$ means the matrix X is symmetric and positive definite. X^T denotes the transpose of X. The symbol I (or 0) represents the identity matrix (or zero matrix) with appropriate dimension.

$\sum_{i,j=1}^{q} \lambda_i \lambda_j = \sum_{i=1}^{q} \sum_{i=1}^{q} \lambda_i \lambda_j$ and $\begin{pmatrix} X & * \\ Z & Y \end{pmatrix} = \begin{pmatrix} X & Z^T \\ Z & Y \end{pmatrix}$.

2. Discrete-Time T-S Descriptor Systems

Let us consider the class of discrete-time nonlinear descriptor systems:

$$\begin{cases} Ex_{k+1} = f(x_k) + g(x_k)u_k \\ y_k = \mathrm{h}(x_k) \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input, $y_k \in \mathbb{R}^p$ is the measured output. f, g and h are nonlinear functions. $E \in \mathbb{R}^{n \times n}$ such that $rank(E) = r$ is a real known constant matrix with:

$$E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

To design a T-S fuzzy observer, we need a T-S fuzzy model for the discrete-time nonlinear descriptor systems (1). In general, there are two approaches for constructing fuzzy models: identification (fuzzy modeling) using input-output data and derivation from given nonlinear system equations.

We use the second approach which derives a fuzzy model from given nonlinear dynamical models (1).

By the sector nonlinearity approach [3], the nonlinear descriptor system (1) can be exactly represented by the T–S fuzzy descriptor systems:

$$\begin{cases} Ex_{k+1} = \sum_{i=1}^{q} \lambda_i(\mathrm{z}_k)(A_i x_k + B_i u_k) \\ y_k = \sum_{i=1}^{q} \lambda_i(\mathrm{z}_k) C_i x_k \end{cases} \tag{3}$$

where $x_k = [X_k^{1T} X_k^{2T}]^T \in \mathbb{R}^n$ is the state vector with $X_k^1 \in \mathbb{R}^r$ is the vector of difference variables, $X_k^2 \in \mathbb{R}^{n-r}$ is the vector of algebraic variables, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, $C_i \in \mathbb{R}^{p \times n}$ are real known constant matrices assumed to be of the form:

$$\begin{cases} A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix}, \ B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \\ C_i = \begin{pmatrix} C_{1i} & C_{2i} \end{pmatrix} \end{cases} \tag{4}$$

and constant matrices A_{22i} are supposed invertible ($rank(A_{22i}) = n - r$).

q is the number of sub-models.

z_k is the vector containing the premise variables.

The $\lambda_i(\mathrm{z}_k)$ are the weighting functions that ensure the transition between the contribution of each sub-model:

$$\begin{cases} Ex_{k+1} = A_i x_k + B_i u_k \\ y_k = C_i x_k \end{cases} \tag{5}$$

These functions verify the following properties:

$$\begin{cases} \sum_{i=1}^{q} \lambda_i(\mathrm{z}_k) = 1 \\ 0 \le \lambda_i(\mathrm{z}_k) \le 1 \quad i = 1, \ldots, q \end{cases} \tag{6}$$

Remark 1: The dependence of the premise variables on the state variables makes it necessary to consider two cases for fuzzy observer design:

Case 1: z_k do not depend on the state variables estimated by a fuzzy observer (measurable premise variables case).

Case 2: z_k depend on the state variables estimated by a fuzzy observer (unmeasurable premise variables case).

For the *rest of the paper we consider* the Class of Discrete-Time Takagi-Sugeno Descriptor Systems (3).

Then, before giving the main results, let us make the following assumption [24]:

Assumption A1: (E, A_i) is regular, i.e. $det(zE - A_i) \neq 0 \ \forall z \in \mathbb{C}$

Assumption A2: All sub-models (5) are impulse observable and detectable.

In order to investigate the observer design for discrete-time T-S descriptor system (3), the approach is based on the separation between difference and algebraic equations in each sub-model (5) and the global fuzzy model is obtained by aggregation of the resulting sub-models.

So, using (4), system (5) can be rewritten as follows:

$$\begin{cases} X_{k+1}^1 = A_{11i}X_k^1 + A_{12i}X_k^2 + B_{1i}u_k \\ 0 = A_{21i}X_k^1 + A_{22i}X_k^2 + B_{2i}u_k \\ y_k = C_{1i}X_k^1 + C_{2i}X_k^2 \end{cases} \tag{7}$$

The form (7) for system (5) is also known as the second equivalent form [24].

From (7) and using the fact that A_{22i}^{-1} exist, the algebraic equations can be solved directly for algebraic variables, to obtain:

$$X_k^2 = -A_{22i}^{-1}A_{21i}X_k^1 - A_{22i}^{-1}B_{2i}u_k \tag{8}$$

Substitution the resulting expression of X_k^2 (equation (8)) in equation (7) yields the following model:

$$\begin{cases} X_{k+1}^1 = M_iX_k^1 + N_iu_k \\ X_k^2 \ = Q_iX_k^1 + R_iu_k \\ y_k \ = S_iX_k^1 + G_iu_k \end{cases} \tag{9}$$

where

$$\begin{cases} M_i = A_{11i} - A_{12i}A_{22i}^{-1}A_{21i} \\ N_i = B_{1i} - A_{12i}A_{22i}^{-1}B_{2i} \\ Q_i = -A_{22i}^{-1}A_{21i} \\ R_i = -A_{22i}^{-1}B_{2i} \\ S_i = C_{1i} - C_{2i}A_{22i}^{-1}A_{21i} \\ G_i = -C_{2i}A_{22i}^{-1}B_{2i} \end{cases} \tag{10}$$

The weighting functions $\lambda_i(\mathrm{z}_k)$, $i = 1,\ldots,q$ can be rewritten as

$$\begin{aligned} \lambda_i(\mathrm{z}_k) &= \lambda_i(X_k^1, X_k^2 = Q_iX_k^1 + R_iu_k) \\ &= \lambda_i(X_k^1, u_k) = \lambda_i(\xi_k) \end{aligned} \tag{11}$$

with $\xi_k = [X_k^{1T}u_k^T]^T$.

In descriptor form, subsystem (9) takes the following equivalent form of submodel (5):

$$\begin{cases} Ex_{k+1} = \bar{M}_ix_k + \bar{N}_iu_k \\ \ y_k = \bar{S}_ix_k + G_iu_k \end{cases} \tag{12}$$

where

$$\bar{M}_i = \begin{pmatrix} M_i & 0 \\ Q_i & -I \end{pmatrix}; \quad \bar{N}_i = \begin{pmatrix} N_i \\ R_i \end{pmatrix}; \quad \bar{S}_i = (S_i \quad 0) \tag{13}$$

Then, the fuzzy descriptor system (3) can be represented by the following equivalent form:

$$\begin{cases} Ex_{k+1} = \sum_{i=1}^{q} \mu_i(\mathrm{z}_k)(\bar{M}_ix + \bar{N}_iu) \\ y_k = \sum_{i=1}^{q} \mu_i(\mathrm{z}_k)(\bar{S}_ix_k + G_iu) \end{cases} \tag{14}$$

So, using (6), (11) and (13), system (14) can be rewritten as follows:

$$\begin{cases} X_{k+1}^1 = \sum_{i=1}^{q} \lambda_i(\xi_k)(M_iX_k^1 + N_iu_k) \\ X_k^2 = \sum_{i=1}^{q} \lambda_i(\xi_k)(Q_iX_k^1 + R_iu_k) \\ y_k = \sum_{i=1}^{q} \lambda_i(\xi_k)(S_iX_k^1 + G_iu_k) \end{cases} \tag{15}$$

3. Main Results

In this section, the purpose is to suggest a new method for the observer design of discrete-time T-S descriptor model (3). Both measurable and unmeasurable premise variables cases are considered. The first result devoted to the case of measurable premise variables is exposed in subsection 3.1 and the second one which concerns the unmeasurable premise variables case is established in subsection 3.2.

3.1. Observer Design with Measurable Premise Variables

Based on the transformation of the discrete-time T-S descriptor system (3) into the equivalent form (15), the proposed observer is given by the following equations:

$$\begin{cases} \hat{X}_{k+1}^1 = \sum_{i=1}^{q} \lambda_i(\xi_k)(M_i\hat{X}_k^1 + N_iu_k + L_i(y_k - \hat{y}_k)) \\ \hat{X}_k^2 = \sum_{i=1}^{q} \lambda_i(\xi_k)(Q_i\hat{X}_k^1 + R_iu_k) \\ \hat{y}_k = \sum_{i=1}^{q} \lambda_i(\xi_k)(S_i\hat{X}_k^1 + G_iu_k) \end{cases} \tag{16}$$

where $(\hat{X}_k^1, \hat{X}_k^2)$ and $\hat{y}_k$ denote the estimated state vector and output vector respectively. The activation functions $\lambda_i(\xi_k)$ are the same than those used in the T-S model (15). The local gain L_i can be determined by theorem 1.

Denoting the state estimation error by:

$$e_k = \begin{pmatrix} e_k^1 \\ e_k^2 \end{pmatrix} = \begin{pmatrix} X_k^1 - \hat{X}_k^1 \\ X_k^2 - \hat{X}_k^2 \end{pmatrix} \tag{17}$$

It follows from (15) and (16) that the dynamics of the estimation error e_k is given by the following equations:

$$\begin{cases} e^1_{k+1} = \sum_{i,j=1}^{q} \lambda_i(\xi_k)\lambda_j(\xi_k)\Gamma_{ij}e^1_k \\ e^2_k = \sum_{i=1}^{q} \lambda_i(\xi_k)Q_i e^1_k \end{cases} \tag{18}$$

where

$$\Gamma_{ij} = M_i - L_i S_j \tag{19}$$

Note that to prove the convergence of the estimation error e_k toward zero, it suffices to prove from (18), that e^1_k converges toward zero.

Then, the following result can be stated.

Theorem 1. Under the above assumptions, the state estimation error between the T-S descriptor model (3) and its observer (16) converges asymptotically towards zero, if there exist matrices $P > 0$, K_i, $i = 1,...,q$ verifying the following LMIs:

$$\begin{cases} \begin{pmatrix} -P & * \\ PM_i - K_iS_i & -P \end{pmatrix} < 0 \quad i \in I_p = \{1,...,q\} \\ \begin{pmatrix} -P & * \\ \dfrac{PM_i - K_iS_j + PM_j - K_jS_i}{2} & -P \end{pmatrix} < 0 \\ \forall (i,j) \in I_p^2,\ i < j \ s.t\ \lambda_i(\xi_k)\lambda_j(\xi_k) \neq 0 \end{cases} \tag{20}$$

The gains of the observer (16) are then computed by:

$$L_i = P^{-1}K_i \tag{21}$$

Proof: Let us consider the following quadratic Lyapunov function:

$$V_k = (e^1_k)^T P e^1_k, \quad P > 0 \tag{22}$$

Estimation error convergence is ensured if the following condition is guaranteed:

$$V_{k+1} - V_k = (e^1_{k+1})^T P e^1_{k+1} - (e^1_k)^T P e^1_k < 0 \tag{23}$$

By using (18), the condition (23) can be written as:

$$V_{k+1} - V_k = \sum_{i,j=1}^{q} \lambda_i(\xi_k)\lambda_j(\xi_k)(e^1_k)^T(\Gamma^T_{ij}P\Gamma_{ij} - P)e^1_k < 0 \tag{24}$$

Then, the negativity of $V_{k+1} - V_k$ is assured if:

$$(e^1_k)^T(\Gamma^T_{ij}P\Gamma_{ij} - P)e^1_k < 0 \tag{25}$$

which is equivalent to the following conditions:

$$\begin{cases} L_e(\Gamma_{ii}, P) < 0 \qquad i \in I_p = \{1,...,q\} \\ L_e(\Gamma_{ij}, P) < 0 \\ \forall (i,j) \in I_p^2,\ i < j \ s.t\ \lambda_i(\xi_k)\lambda_j(\xi_k) \neq 0 \end{cases} \tag{26}$$

where

$$L_e(\Gamma_{ij}, P) = \left(\frac{\Gamma_{ij} + \Gamma_{ji}}{2}\right)^T P\left(\frac{\Gamma_{ij} + \Gamma_{ji}}{2}\right) - P \tag{27}$$

Then from (19), we can establish the LMI conditions (20) of Theorem 1 by using the Schur complement [19] and the following change of variables:

$$K_i = PL_i \tag{28}$$

Thus, from the Lypunov stability theory, if the LMI conditions (20) are satisfied, the system (18) is exponentially asymptotically stable. This completes the proof of Theorem 1.

3.2. Observer Design with Unmeasurable Premise Variables

In this subsection, the aim is to extend the above observer design method presented in subsection 3.1 to a class of discrete-time T-S descriptor systems (3) when the premise variables are supposed depend on the unmeasurable state variables. In this case, the proposed fuzzy observer is described as:

$$\begin{cases} \hat{X}^1_{k+1} = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(M_i\hat{X}^1_k + N_iu_k + L_i(y_k - \hat{y}_k)) \\ \hat{X}^2_k = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(Q_i\hat{X}^1_k + R_iu_k) \\ \hat{y}_k = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(S_i\hat{X}^1_k + G_iu_k) \end{cases} \tag{29}$$

where $(\hat{X}^1_k, \hat{X}^2_k)$, $\hat{y}_k$ and $\hat{\xi}_k$ denote the estimated state vector, the output vector and the decision variable vector respectively. The observer gains L_i, $i = 1,\dots,q$ are the unknown parameters to be determined.

In order to establish the conditions for the asymptotic convergence of the observer (29), we define the state estimation errors:

$$e_k = \begin{pmatrix} e^1_k \\ e^2_k \end{pmatrix} = \begin{pmatrix} X^1_k - \hat{X}^1_k \\ X^2_k - \hat{X}^2_k \end{pmatrix} \tag{30}$$

Substituting (15) and (29) in (30) yields:

$$\begin{cases} e_{k+1}^1 = \sum_{i=1}^{q} \lambda_i(\xi_k)(M_i X_k^1 + N_i u_k) \\ \qquad - \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(M_i \hat{X}_k^1 + N_i u_k) + L_i(y_k - \hat{y}_k)) \\ e_k^2 = \sum_{i=1}^{q} \lambda_i(\xi_k)(Q_i X_k^1 + R_i u_k) - \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(Q_i \hat{X}_k^1 + R_i u_k) \end{cases} \tag{31}$$

which is equivalent to the following conditions:

$$\begin{cases} e_{k+1}^1 = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(M_i e_k^1 - L_i(y_k - \hat{y}_k)) \\ \qquad + \sum_{i=1}^{q} (\lambda_i(\xi_k) - \lambda_i(\hat{\xi}_k))(M_i X_k^1 + N_i u_k) \\ e_k^2 = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k) Q_i e_k^1 + \sum_{i=1}^{q} (\lambda_i(\xi_k) - \lambda_i(\hat{\xi}_k))(Q_i X_k^1 + R_i u_k) \end{cases} \tag{32}$$

Note that:

$$\sum_{i=1}^{q} (\lambda_i(\xi_k) - \lambda_i(\hat{\xi}_k)) X_i = \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(X_i - X_j) \tag{33}$$

with $X_i = M_i, N_i, Q_i, R_i$.

Hence, the equation (32) becomes:

$$\begin{cases} e_{k+1}^1 = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k)(M_i e_k^1 - L_i(y_k - \hat{y}_k)) \\ \qquad + \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(\Delta M_{ij} X_k^1 + \Delta N_{ij} u_k) \\ e_k^2 = \sum_{i=1}^{q} \lambda_i(\hat{\xi}_k) Q_i e_k^1 + \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(\Delta Q_{ij} X_k^1 \Delta R_{ij} u_k) \end{cases} \tag{34}$$

where $\Delta M_{ij} = M_i - M_j$, $\Delta N_{ij} = N_i - N_j$, $\Delta Q_{ij} = Q_i - Q_j$ and $\Delta R_{ij} = R_i - R_j$.

Since $\sum_{i=1}^{q} \lambda_i(\xi) = 1$, equality (34) can be written as follows:

$$\begin{cases} e_{k+1}^1 = \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(M_j e_k^1 - L_j(y_k - \hat{y}_k)) \\ \qquad + \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(\Delta M_{ij} X_k^1 + \Delta N_{ij} u_k) \\ e_k^2 = \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(Q_j e_k^1 + \Delta Q_{ij} X_k^1 + \Delta R_{ij} u_k) \end{cases} \tag{35}$$

Similarly, y_k and $\hat{y}_k$ can be written as follows:

$$\begin{cases} y_k = \sum_{i,h=1}^{q} \lambda_i(\xi_k) \lambda_h(\hat{\xi}_k)[(S_h + \Delta S_{ih}) X_k^1 + (G_h + \Delta G_{ih}) u_k] \\ \hat{y}_k = \sum_{i,h=1}^{q} \lambda_i(\xi_k) \lambda_h(\hat{\xi}_k)(S_h \hat{X}_k^1 + G_h u_k) \end{cases} \tag{36}$$

where $\Delta S_{ih} = S_i - S_h$ and $\Delta G_{ih} = G_i - G_h$.

By substituting (36) in (35), we obtain:

$$\begin{cases} e_{k+1}^1 = \sum_{i,j,h=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k) \lambda_h(\hat{\xi}_k)(\Gamma_{jh} e_k^1 + \Phi_{ijh} X_k^1 + \Omega_{ijh} u_k) \\ e_k^2 = \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(Q_j e_k^1 + \Delta Q_{ij} X_k^1 + \Delta R_{ij} u_k) \end{cases} \tag{37}$$

where

$$\begin{cases} \Gamma_{jh} = M_j - L_j S_h \\ \Phi_{ijh} = \Delta M_{ij} - L_j \Delta S_{ih} \\ \Omega_{ijh} = \Delta N_{ij} - L_j \Delta G_{ih} \\ i, j, h \in \{1, \ldots, q\} \end{cases} \tag{38}$$

Let $\overline{e}_k^1 = [e_k^{1T} X_k^{1T}]^T$ and $\overline{e}_k^2 = [e_k^{2T} X_k^{2T}]^T$, we have:

$$\begin{cases} \overline{e}_{k+1}^1 = \sum_{i,j,h=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k) \lambda_h(\hat{\xi}_k)(\widetilde{M}_{ijh} \overline{e}_k^1 + \widetilde{N}_{ijh} u_k) \\ \overline{e}_k^2 = \sum_{i,j=1}^{q} \lambda_i(\xi_k) \lambda_j(\hat{\xi}_k)(\widetilde{K}_{ij} \overline{e}_k^1 + \widetilde{L}_{ij} u_k) \end{cases} \tag{39}$$

where

$$\begin{cases} \widetilde{M}_{ijh} = \begin{pmatrix} \Gamma_{jh} & \Phi_{ijh} \\ 0 & M_i \end{pmatrix} \\ \widetilde{N}_{ijh} = \begin{pmatrix} \Omega_{ijh} \\ N_i \end{pmatrix} \\ \widetilde{K}_{ij} = \begin{pmatrix} Q_i & \Delta Q_{ij} \\ 0 & Q_i \end{pmatrix} \\ \widetilde{L}_{ij} = \begin{pmatrix} \Delta R_{ij} \\ R_i \end{pmatrix} \end{cases} \tag{40}$$

Note that to prove the convergence of the estimation error e_k toward zero, it suffices to prove from (39) that e_k^1 converges toward zero. Then, the following result can be stated.

Theorem 2. The system (39) is globally asymptotically stable if there exist matrices $P_1 > 0$, $P_2 > 0$, K_j, $j = 1, \ldots, q$ verifying the following LMIs:

$$\begin{cases} \Sigma_{ijh} = \begin{pmatrix} m_{11} & * & * & * \\ m_{21} & m_{22} & * & * \\ m_{31} & m_{32} & m_{33} & * \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} < 0 \\ \forall i,j,h \in \{1,\dots,q\} \end{cases} \tag{41}$$

where

$$\begin{cases} m_{11}=M_j^T P_1 M_j - M_j^T K_j S_h - S_h^T K_j^T M_j - P_1 \\ m_{21}=\Delta M_{ij}^T P_1 M_j - \Delta M_{ij}^T K_j S_h - \Delta S_{ih}^T K_j^T M_j \\ m_{22}=\Delta M_{ij}^T P_1 \Delta M_{ij} - \Delta M_{ij}^T K_j \Delta S_{ih} \\ \quad -\Delta S_{ih}^T K_j^T \Delta M_{ij} + M_i^T P_2 M_i - P_2 \\ m_{31}=\Delta N_{ij}^T P_1 M_j - \Delta N_{ij}^T K_j S_h - \Delta G_{ih}^T K_j^T M_j \\ m_{32}=\Delta N_{ij}^T P_1 \Delta M_{ij} - \Delta N_{ij}^T K_j \Delta S_{ih} \\ \quad -\Delta G_{ih}^T K_j^T \Delta M_{ij} + N_i^T P_2 M_i \\ m_{33}=\Delta N_{ij}^T P_1 \Delta N_{ij} - \Delta N_{ij}^T K_j \Delta G_{ih} \\ \quad -\Delta G_{ih}^T K_j^T \Delta N_{ij} + N_i^T P_2 N_i \\ m_{41}=K_j S_h \\ m_{42}=K_j \Delta S_{ih} \\ m_{43}=K_j \Delta G_{ih} \\ m_{44}=-P_1 \end{cases} \tag{42}$$

The gains L_j, $i = 1,\dots,q$ of the observer (29) are then computed by:

$$L_j = P_1^{-1} K_j \tag{43}$$

Proof: Considering the following quadratic Lyapunov function:

$$V_k = (\overline{e}_k^1)^T P \overline{e}_k^1 \quad , \quad P > 0 \tag{44}$$

The variation of V_k along the trajectory of (39) is given by:

$$\Delta V = V_{k+1} - V_k \tag{45}$$

Using (39), we have:

$$\Delta V = \sum_{i,j,h=1}^{q} \lambda_i(\xi_k)\lambda_j(\hat{\xi}_k)\lambda_h(\hat{\xi}_k)[\Psi_{ijh}^T P \Psi_{ijh} - \overline{e}_k^1 P \overline{e}_k^1] \tag{46}$$

Where

$$\Psi_{ijh} = (\widetilde{M}_{ijh} \overline{e}_k^1 + \widetilde{N}_{ijh} u_k) \tag{47}$$

ΔV can be written in the form:

$$\Delta V = \sum_{i,j,h=1}^{q} \lambda_i(\xi_k)\lambda_j(\hat{\xi}_k)\lambda_h(\hat{\xi}_k)\left((\overline{e}_k^1)^T \quad u_k^T\right)\Sigma_{ijh}\begin{pmatrix} \overline{e}_k^1 \\ u_k \end{pmatrix} \tag{48}$$

where

$$\Sigma_{ijh} = \begin{pmatrix} \widetilde{M}_{ijh}^T P \widetilde{M}_{ijh} - P & \widetilde{M}_{ijh}^T P \widetilde{N}_{ijh} \\ \widetilde{N}_{ijh}^T P \widetilde{M}_{ijh} & \widetilde{N}_{ijh}^T P \widetilde{N}_{ijh} \end{pmatrix} \tag{49}$$

Let us choose the following structure for the matrix P:

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \tag{50}$$

By using (40) and (50), Σ_{ijh} can be reduced as follows:

$$\Sigma_{ijh} = \begin{pmatrix} \Gamma_{jh}^T P_1 \Gamma_{jh} - P_1 & \Gamma_{jh}^T P_1 \Phi_{ijh} & \Gamma_{jh}^T P_1 \Omega_{ijh} \\ \Phi_{ijh}^T P_1 \Gamma_{jh} & \Sigma_{22} & \Sigma_{23} \\ \Omega_{ijh}^T P_1 \Gamma_{jh} & \Sigma_{32} & \Sigma_{33} \end{pmatrix} \tag{51}$$

where

$$\begin{cases} \Sigma_{22} = \Phi_{ijh}^T P_1 \Phi_{ijh} + A_i^T P_2 A_i - P_2 \\ \Sigma_{23} = \Phi_{ijh}^T P_1 \Omega_{ijh} + A_i^T P_2 B_i \\ \Sigma_{32} = \Omega_{ijh}^T P_1 \Phi_{ijh} + B_i^T P_2 A_i \\ \Sigma_{33} = \Omega_{ijh}^T P_1 \Omega_{ijh} + B_i^T P_2 B_i \end{cases} \tag{52}$$

The negativity of ΔV is guaranteed if:

$$\Sigma_{ijh} < 0 \qquad \forall i,j,h \in \{1,\dots,q\} \tag{53}$$

Then from (38), we can establish the LMI conditions (41) of Theorem 2 by using the Schur complement [19] and the following change of variables:

$$K_j = P_1 L_j \tag{54}$$

Thus, from the Lypunov stability theory, if the LMI conditions (41) are satisfied, the system (39) is exponentially asymptotically stable. This completes the proof of Theorem 2.

4. Application to a Heat Exchanger System

In this section, the proposed fuzzy observer design (29) is applied to a heat exchanger pilot process in order to estimate on-line these unknown states. The following discrete-time T-S descriptor model with unmeasurable premise variables that we consider here is obtained by Euler discretisation of the model given in [15]. It takes the form:

$$\begin{cases} Ex_{k+1} = \sum_{i=1}^{4} \lambda_i(x_k)(A_i x_k + Bu_k) \\ y_k = Cx_k \end{cases} \qquad (55)$$

where

$x_k = \left(x_{1k}, \cdots, x_{8k}\right)^T$ is the state vector, $u_k = (u_{1k}, u_{2k})^T$ is the control vector, $y_k = (x_{1k}, x_{4k})^T$ is the vector of output measurements.

$$A_1 = \begin{pmatrix} 1 & 403.39 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 1 & 14.74 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.01 \\ 0 & -39.48 & -8.80 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -39.48 & -8.80 & 0 & -1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 403.39 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.99 & 14.74 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.01 \\ 0 & -39.48 & -8.80 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -39.48 & -8.80 & 0 & -1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0.91 & 403.39 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 1 & 14.74 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.01 \\ 0 & -39.48 & -8.80 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -39.48 & -8.80 & 0 & -1 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 0.91 & 403.39 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.99 & 14.74 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.01 \\ 0 & -39.48 & -8.80 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -39.48 & -8.80 & 0 & -1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 36.72 & 0 \\ 0 & 36.72 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The membership functions are given by:

$$\begin{cases} \lambda_1(x_k) & = & (1-\alpha_1 x_{2k})(1-\alpha_1 x_{5k}) \\ \lambda_2(x_k) & = & (1-\alpha_1 x_{2k})(\alpha_3+\alpha_1 x_{5k}) \\ \lambda_3(x_k) & = & (\alpha_2+\alpha_1 x_{2k})(1-\alpha_1 x_{5k}) \\ \lambda_4(x_k) & = & (\alpha_2+\alpha_1 x_{2k})(\alpha_3+\alpha_1 x_{5k}) \end{cases} \qquad (56)$$

with $\alpha_1 = 66.6667$, $\alpha_2 = 6.6667*10^{-9}$, $\alpha_3 = -0.1378$.

Note that, the application of the proposed observer (29) for heat exchanger pilot process requires that the above model (55) takes the form (15). To do so, considering the following:

$X_k^1 = \left[x_{1k} \ x_{2k} \ x_{3k} \ x_{4k} \ x_{5k} \ x_{6k}\right]^T$,

$X_k^2 = \left[x_{7k} \ x_{8k}\right]^T$.

$E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ with $rank(E) = 6$.

For $i = 1, 2, 3, 4$:

$$A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix} = \begin{pmatrix} A_i(1:6,1:6) & A_i(1:6,7:8) \\ A_i(7:8,1:6) & A_i(7:8,7:8) \end{pmatrix}$$

$$B_i = B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} B(1:6,1:2) \\ B(7:8,1:2) \end{pmatrix};$$

$$C_i = C = \begin{pmatrix} C_1 & C_2 \end{pmatrix} = \begin{pmatrix} C(1:2,1:6) & C(1:2,7:8) \end{pmatrix}.$$

This shows that the model (55) is a particular case of the system (3).

Hence, the fuzzy descriptor system (55) can be rewritten in the following equivalent form:

$$\begin{cases} X_{k+1}^1 & = & \sum_{i=1}^{4} \lambda_i(x_k)(M_i X_k^1 + N_i u_k) \\ X_k^2 & = & \sum_{i=1}^{4} \lambda_i(x_k)(Q_i X_k^1 + R_i u_k) \\ y_k & = & \sum_{i=1}^{4} \lambda_i(x_k)(S_i X_k^1 + G_i u_k) \end{cases} \qquad (57)$$

where M_i, N_i, Q_i, R_i, S_i and G_i are given in the above equation (10).

Using Theorem 2, we obtain the following observer gains:

$$L_1 = \begin{pmatrix} 1.1688 & 0.0028 \\ 0.0005 & 0.0000 \\ -0.0004 & 0.0000 \\ 0.0010 & 1.0195 \\ 0.0000 & 0.0006 \\ 0.0000 & -0.0014 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 1.1688 & 0.0030 \\ 0.0005 & -0.0000 \\ -0.0004 & -0.0000 \\ 0.0010 & 1.0033 \\ -0.0000 & 0.0006 \\ -0.0000 & -0.0014 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} 1.0843 & 0.0030 \\ 0.0005 & -0.0000 \\ -0.0004 & -0.0000 \\ 0.0010 & 1.0195 \\ -0.0000 & 0.0006 \\ -0.0000 & -0.0014 \end{pmatrix}, \quad L_4 = \begin{pmatrix} 1.0843 & 0.0027 \\ 0.0005 & 0.0000 \\ -0.0004 & 0.0000 \\ 0.0010 & 1.0033 \\ 0.0000 & 0.0006 \\ 0.0000 & -0.0014 \end{pmatrix}$$

The numerical values of control vector are:

$u_{1k} = 0.012$ and $u_{2k} = 0.014$.

Simulation results with initial conditions:

$$x_0 = [73\ \ 0\ \ 0\ \ 18\ \ 0\ \ 0\ \ 0.2203\ \ 0.4406]^T,$$

$$\hat{x}_0 = [73\ \ 0.002\ \ 0.002\ \ 18\ \ 0.002\ \ 0.002\ \ 0.1720\ \ 0.3923]^T$$

are given in figures 1 to 6.

These results of the simulation show the performances of the proposed observer (29) with the parameters L_i, $i = 1, 2, 3, 4$ where the dotted lines denote the state variables estimated by the fuzzy observer. This simulation shows that the estimated states converge to their corresponding state variables.

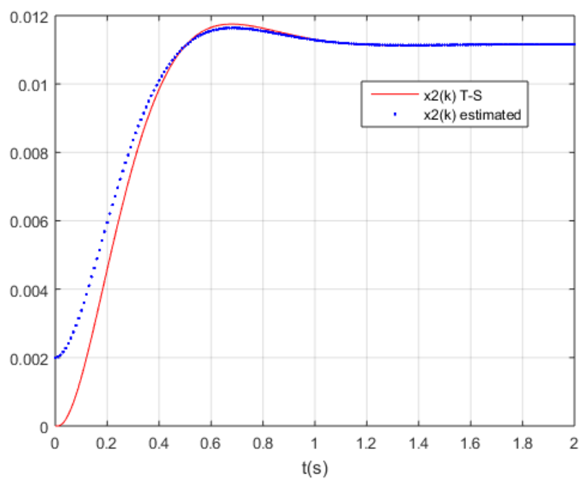

Figure 1. x_{2k} and its estimate

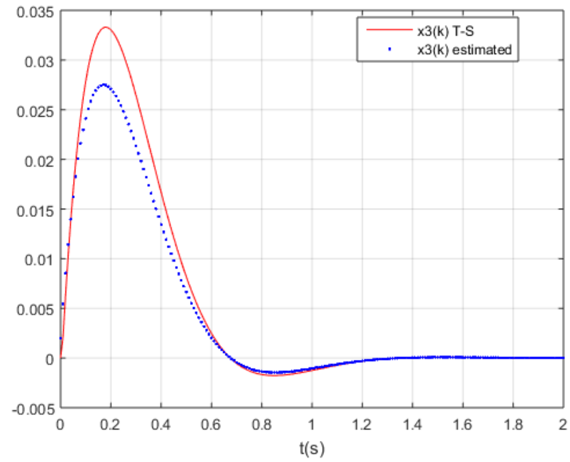

Figure 2. x_{3k} and its estimate

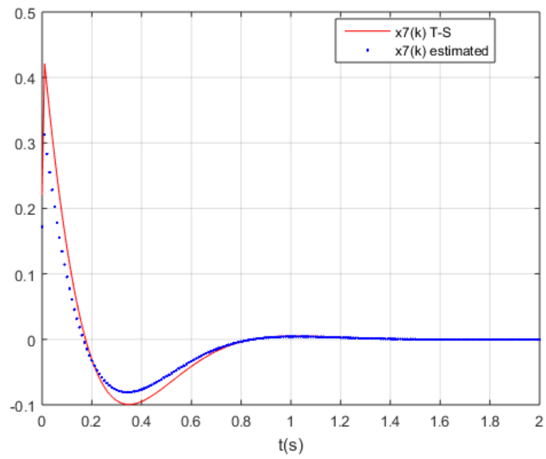

Figure 3. x_{7k} and its estimate

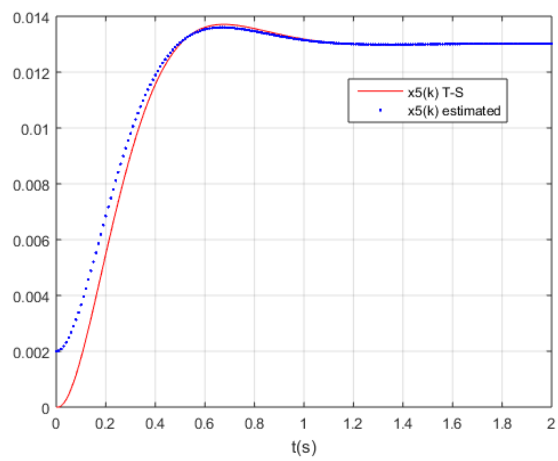

Figure 4. x_{5k} and its estimate

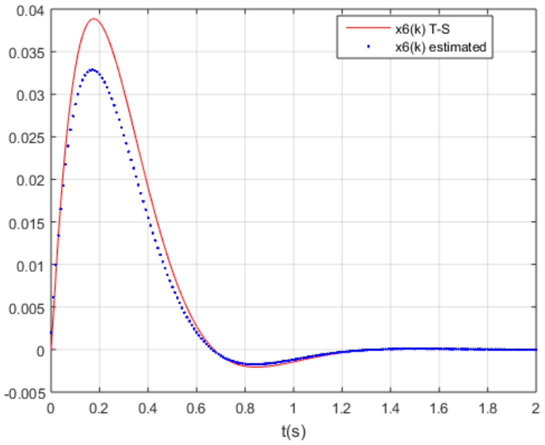

Figure 5. x_{6k} and its estimate

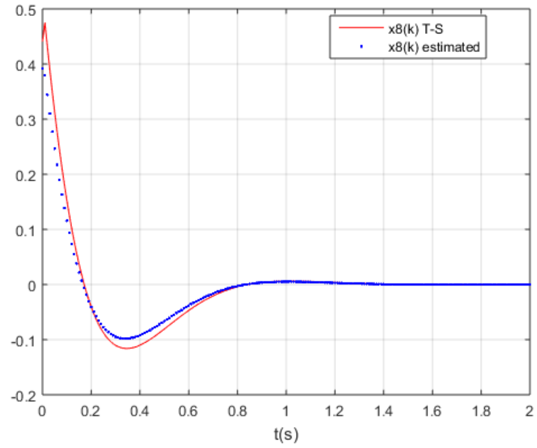

Figure 6. x_{8k} and its estimate

5. Conclusions

Based on the separation approach between recurrence and algebraic equations in the descriptor model, a new method to design fuzzy observer for discrete-time T-S descriptor systems is proposed in this paper. Both measurable and unmeasurable premise variables cases are considered. The convergence conditions are obtained by using Lyapunov theory. The existence of conditions ensuring the convergence of the state estimation error is expressed in terms of LMIs. To illustrate the proposed methodology with unmeasurable premise variables, a discrete-time T-S descriptor model of a heat exchanger is considered. The effectiveness of the proposed fuzzy observer for the on-line estimation of unknown states of the used model is verified by numerical simulation.

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