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Synchronization of Mimas-Tethys System with Driven Damped Pendulum Using a Robust Adaptive Sliding Mode Controller

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Abstract In this paper, we have synchronized the Mimas-Tethys system (MTS: Natural moons of Saturn) with driven damped pendulum (DDP) using a robust adaptive sliding mode controller (RASMC) together with uncertainties, external disturbances and fully unknown parameters. A simple suitable sliding surface, which includes synchronization errors, is constructed and appropriate update laws are used to tackle the uncertainties, external disturbances and unknown parameters. All simulations to achieve the synchronization for the proposed technique for the two non-identical systems under consideration are being done using *Mathematica*.

Keywords MTS, Synchronization, RASMC

1. Introduction

Since more than the last two decades, the control and stabilization of chaos synchronization of chaotic systems has become more and more popular in recent years and received a considerable interest among nonlinear scientists. Chaos is a phenomenon of nonlinear dynamics that has some specific characteristics such as extraordinary sensitivity to initial conditions and system parameter variations, broad Fourier transform spectra and fractal properties of the motion in the phase space. Due to these especial characteristics, chaos has been used in many practical engineering areas such as chemical reactions, power converters, secure communications, information processing, biological systems and mechanical systems[1–3] and various control techniques have been proposed for controlling and synchronizing of chaotic systems, including sliding mode control, optimal control, adaptive control, nonlinear feedback control, backstepping method, passive control, fuzzy logic control, PID control, etc[4–11]. Most of the above mentioned works on chaos synchronization have focused on chaotic systems without model uncertainties and external disturbances. It has been observed practically that structural variations of the systems and un-modeled dynamical uncertainties are present in the chaotic system dynamics due to the modeling errors. So, synchronization of chaotic systems with uncertainties and external disturbances is effectively significant in the applications. In this direction, some researchers have proposed a number of techniques for synchronization of uncertain identical as well as non-identical chaotic systems that includes nonlinear feedback control, sliding mode control, backstepping procedure, linear state feedback control, active control and neural fuzzy control[12–21].

Nevertheless, the previous techniques have studied chaotic systems with fully (or partially) known parameters. While, in practice, it is hard to exactly determine the values of the system parameters in priori. Therefore, synchronization of chaotic systems with unknown parameters is essential and useful in real-life applications. Consequently, some approaches, such as sliding mode control, finite-time based control, adaptive control, optimal control, fuzzy & backstepping control have been developed for synchronization of two identical as well as nonidentical chaotic systems with unknown parameters[22–34]. Sliding mode control[35] is a robust control method which has many interesting features such as low sensitivity to external disturbances and robustness to the plant uncertainties due to structural variations and un-modeled dynamics. The sliding mode controller is composed of an equivalent control part that describes the behavior of the system when the trajectories stay over the sliding surface and a variable structure control part that enforces the trajectories to reach the sliding surface and remain on it evermore. Adaptive control is a suitable approach to overcome system uncertainties, especially uncertainties derived from uncertain parameters. Adaptive sliding mode control has the

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advantages of combining the robustness of the sliding mode control with the tracking facilities of the adaptive control.

The main purpose of this paper is to study the synchronization phenomenon of a MTS with DDP using a robust adaptive sliding mode controller (RASMC) in the presence of uncertainties, external disturbances and fully unknown parameters in both master and slave chaotic systems[36] together with the assumption that the bounds of the uncertainties and external disturbances are unknown in advance. A simple suitable sliding surface, which includes synchronization errors, is constructed. Appropriate update laws are derived to tackle the uncertainties, external disturbances and unknown parameters. Then, on the basis of the update laws, the RASMC is designed to guarantee the existence of the sliding motion. The stability and robustness of the proposed RASMC is proved using Lyapunov stability theory for the MTS and DDP.

2. Description of RASMC

For the n-dimensional master and slave systems[36] with uncertainties, external disturbances and unknown parameters are given as follows:

Master system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x})\boldsymbol{\theta} + \Delta\boldsymbol{f}(\boldsymbol{x},t) + \boldsymbol{d}^m(t). \quad (2.1)$$

Slave System:

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{G}(\boldsymbol{y})\boldsymbol{\psi} + \Delta\boldsymbol{g}(\boldsymbol{y},t) + \boldsymbol{d}^s(t) + \boldsymbol{u}(t). \quad (2.2)$$

Where $\boldsymbol{x}(t) = [x_1, x_2, ..., x_n]^T$ are the state vectors, $\boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), ..., f_n(\boldsymbol{x})]^T$ are the continuous nonlinear functions, $F_i(\boldsymbol{x}), i = 1, 2, ..., n$, is i^{th} row of an $n \times n$ matrix $(\boldsymbol{F}(\boldsymbol{x}))$ whose elements are continuous nonlinear functions, $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_n]^T$ are the unknown vector parameters, $\Delta\boldsymbol{f}(\boldsymbol{x},t) = [\Delta f_1(\boldsymbol{x},t), \Delta f_2(\boldsymbol{x},t), ..., \Delta f_n(\boldsymbol{x},t)]^T$ and $\boldsymbol{d}^m(t) = [d_1^m(t), d_2^m(t), ..., d_n^m(t)]^T$ are the vectors of unknown uncertainties and external disturbances of the master system respectively. $\boldsymbol{y}(t) = [y_1, y_2, ..., y_n]^T$ are the state vectors, $\boldsymbol{g}(\boldsymbol{y}) = [g_1(\boldsymbol{y}), g_2(\boldsymbol{y}), ..., g_n(\boldsymbol{y})]^T$ are the continuous nonlinear functions, $G_i(\boldsymbol{y}), i = 1, 2, ..., n$, is i^{th} row of an $n \times n$ matrix $(\boldsymbol{G}(\boldsymbol{y}))$ whose elements are continuous nonlinear functions, $\boldsymbol{\psi} = [\psi_1, \psi_2, ..., \psi_n]^T$ are the unknown vector parameters, $\Delta\boldsymbol{g}(\boldsymbol{y},t) = [\Delta g_1(\boldsymbol{y},t), \Delta g_2(\boldsymbol{y},t), ..., \Delta g_n(\boldsymbol{y},t)]^T$ and $\boldsymbol{d}^s(t) = [d_1^s(t), d_2^s(t), ..., d_n^s(t)]^T$ are the vectors of unknown uncertainties and external disturbances of the slave system, respectively, and $\boldsymbol{u}(t) = [u_1(t), u_2(t), ..., u_n(t)]^T$ is the vector of control inputs.

Assumption 1: Since the trajectories of chaotic systems are always bounded, then the unknown uncertainties $\Delta\boldsymbol{f}(\boldsymbol{x},t)$ and $\Delta\boldsymbol{g}(\boldsymbol{y},t)$ are assumed to be bounded. Therefore, there exist appropriate positive constants α_i^m and $\alpha_i^s, i = 1, 2, ..., n$ such that

$$|\Delta f_i(\boldsymbol{x},t)| < \alpha_i^m \text{ and } |\Delta g_i(\boldsymbol{y},t)| < \alpha_i^s, \qquad i = 1, 2, ...n \quad (2.3)$$

$$\Rightarrow |\Delta f_i(\boldsymbol{x},t) - \Delta g_i(\boldsymbol{y},t)| < \alpha_i, \qquad i = 1, 2, ..., n, \text{ where } \alpha_i \text{ are unknown constants} \quad (2.4)$$

Assumption 2: In general, it is assumed that the external disturbances are norm-bounded in C^1, i.e. $|d_i^m(t)| < \beta_i^m$

$$\text{and } |d_i^s(t)| < \beta_i^s, \qquad i = 1, 2, ..., n \quad (2.5)$$

$$\Rightarrow |d_i^m(t) - d_i^s(t)| < \beta_i, \qquad i = 1, 2, ..., n, \text{ where } \beta_i \text{ are unknown constants} \quad (2.6)$$

To solve the synchronization problem, the error between the master system (2.1) and slave systems (2.2) can be defined as $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{y}(t)$. Then from (2.1) and (2.2), the error dynamics can be written as:

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{F}(\boldsymbol{x})\boldsymbol{\theta} + \Delta\boldsymbol{f}(\boldsymbol{x},t) + \boldsymbol{d}^m(t) - \boldsymbol{g}(\boldsymbol{y}) - \boldsymbol{G}(\boldsymbol{y})\boldsymbol{\psi} - \Delta\boldsymbol{f}(\boldsymbol{y},t) - \boldsymbol{d}^s(t) - \boldsymbol{u}(t). \quad (2.7)$$

It is clear that the synchronization problem can be transformed to the equivalent problem of stabilizing the error system (2.7). The objective of this paper is that for any given master chaotic system (2.1) and slave chaotic system (2.2) with the uncertainties, external disturbances and unknown parameters a suitable feedback control law $\boldsymbol{u}(t)$ is designed such that the asymptotical stability of the resulting error system (2.7) can be achieved in the sense that $\lim_{t\to\infty} |\boldsymbol{x}(t) - \boldsymbol{y}(t)| \to 0$ for the systems under consideration.

Let us consider now, the appropriate sliding surface with the desired behavior. Therefore, the sliding surface suitable for the technique can be designed as:

$$s_i(t) = \lambda_i e_i(t), \quad i = 1, 2, ..., n \tag{2.8}$$

where $s_i(t) \in \mathbb{R}\left(\boldsymbol{s}(t) = [s_1(t), s_2(t), ..., s_n(t)]\right)$ and the sliding surface parameters λ_i are positive constants.

After designing the suitable sliding surface, let us determine the input control signal $\boldsymbol{u}(t)$ to guarantee that the error system trajectories reach to the sliding surface $\boldsymbol{s}(t) = 0$ (i.e. to satisfy the reaching condition $\boldsymbol{s}(t)\dot{\boldsymbol{s}}(t) < 0$) and stay on it, forever. Therefore, to ensure the existence of the sliding motion a discontinuous control law is proposed as:

$$u_i(t) = f_i(\boldsymbol{x}) - g_i(\boldsymbol{y}) + F_i(\boldsymbol{x})\hat{\theta}_i - G_i(\boldsymbol{y})\hat{\psi}_i + \left(\alpha_i + \beta_i\right)\operatorname{sgn}(s_i) + k_i \operatorname{sgn}(s_i), \qquad \text{for } i = 1, 2, ..., n \tag{2.9}$$

Where $\hat{\theta}_i$, $\hat{\psi}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$ are estimations for θ_i, ψ_i, α_i, β_i respectively and $k_i > 0$, $i = 1, 2, ..., n$ are the switching gain constant.

To tackle the uncertainties, external disturbances and unknown parameters, appropriate update laws are defined as:

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}} &= [\boldsymbol{F}(\boldsymbol{x})]^T \gamma, & \boldsymbol{\theta}(0) &= \boldsymbol{\theta}_0, \\ \dot{\hat{\boldsymbol{\psi}}} &= -[\boldsymbol{G}(\boldsymbol{y})]^T \gamma, & \boldsymbol{\psi}(0) &= \boldsymbol{\psi}_0, \\ \dot{\hat{\alpha}}_i = \dot{\hat{\beta}}_i &= \lambda_i |s_i|, & \alpha_i(0) &= \alpha_{i0} \,\&\, \beta_i(0) = \beta_{i0}. \end{aligned} \tag{2.10}$$

where $\gamma = [\lambda_1 s_1, \lambda_2 s_2, ..., \lambda_n s_n]^T$ and $\hat{\boldsymbol{\theta}}_0$, $\hat{\boldsymbol{\psi}}_0$, $\hat{\alpha}_{i0}$ and $\hat{\beta}_{i0}$ are the initial values of the update parameters $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\psi}}$, $\hat{\alpha}_i$ and $\hat{\beta}_i$ respectively.

In order to minimize the chattering effect due to signum function, we are replacing the $\operatorname{sgn}(s_i)$ by $\tanh(\varepsilon s_i)$ for $\varepsilon > 1$. Figure 1 shows that our results will not be affected. Therefore the new control input can be written as:

$$u_i(t) = f_i(\boldsymbol{x}) - g_i(\boldsymbol{y}) + F_i(\boldsymbol{x})\hat{\theta}_i - G_i(\boldsymbol{y})\hat{\psi}_i + \left(\alpha_i + \beta_i\right)\operatorname{sgn}(s_i) + k_i \tanh(\varepsilon s_i), \quad \text{for } i = 1, 2, ..., n \tag{2.11}$$

Based on the control input in (2.9) and update laws in (2.10), to guarantee the reaching condition $\boldsymbol{s}(t)\dot{\boldsymbol{s}}(t) < 0$ and to ensure the occurrence of the sliding motion, we have the following theorem.

Theorem 1: Consider the error dynamics (2.7), this system is controlled by $\boldsymbol{u}(t)$ in (2.9) with update laws in (2.10). Then the error system trajectories will converge to the sliding surface $\boldsymbol{s}(t) = 0$.

In this regard, we consider a Lyapunov function (that is a positive definite function also) as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{n}\left[s_i^2 + \left(\hat{\alpha}_i - \alpha_i\right)^2 + \left(\hat{\beta}_i - \beta_i\right)^2\right] + \frac{1}{2}\left\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right\|^2 + \frac{1}{2}\left\|\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right\|^2 \tag{2.12}$$

Figure 10 shows that the derivative of Lyapunov function (2.12) is less than or equal to zero for t is bigger than zero (i.e. $\dot{V}(t) \le 0$ for $t \ge 0$).

3. Description of the Systems

Elliptically orbiting planar oscillations of satellites in the solar system make an interesting study. In this paper, we synchronize the dynamical models of MTS with DDP using RASMC. The Hamiltonian (H) of the nonlinear dynamical model of the Mimas-Tethys system is written from[37] in order to study the synchronization behavior.

$$H = H_0 + H_1$$

$$H = \frac{-\dot{\phi}^2}{2} + \omega_0^2 \cos\phi + \varepsilon_1 \cos\left(\frac{3\phi}{2} + ft + \sigma_0\right) + \varepsilon_2 \cos\left(\frac{\phi}{2} + ft + \sigma_0\right) + \varepsilon_3 \cos\left(\frac{\phi}{2} - ft - \sigma_0\right) \tag{3.1}$$

where ϕ is the argument depending on the mean longitude of Mimas, ε_i are the parameters depending on the tidal force (for $i = 1, 2, 3$), f is the main frequency, ω_0 is the mass distribution parameter and σ_0 is a constant.

Using Hamilton's canonical equations $\left(\dfrac{d\phi}{dt} = \dfrac{\partial H}{\partial \dot{\phi}} \,\&\, \dfrac{d\dot{\phi}}{dt} = -\dfrac{\partial H}{\partial \phi}\right)$, (3.1) can be written as

$$\frac{d^2\phi}{dt^2} = -\omega_0^2 \sin\phi - \frac{3\varepsilon_1}{2}\sin\left(\frac{3\phi}{2} + ft + \sigma_0\right) - \frac{\varepsilon_2}{2}\sin\left(\frac{\phi}{2} + ft + \sigma_0\right) - \frac{\varepsilon_3}{2}\sin\left(\frac{\phi}{2} - ft - \sigma_0\right) \tag{3.2}$$

Introducing the two variables: $\phi = x_1$ and $\dot{x}_1 = x_2$, the equation (3.1) can be written as a following system of two first order nonlinear differential equations which is treated as master system.

$$\text{MTS}: \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\omega_0^2 \sin x_1 + h(x_1, t) \end{cases} \tag{3.3}$$

Where

$$h(x_1,t) = -\frac{3\varepsilon_1}{2}\sin\left(\frac{3x_1}{2} + ft + \sigma_0\right) - \frac{\varepsilon_2}{2}\sin\left(\frac{x_1}{2} + ft + \sigma_0\right) - \frac{\varepsilon_3}{2}\sin\left(\frac{x_1}{2} - ft - \sigma_0\right)$$

The equation of motion of DDP is given by

$$\frac{d^2\theta}{dt^2} = -\frac{g}{R}\sin\theta - \frac{b}{mR^2}\dot{\theta} + \frac{A}{mR^2}\cos(kt) \tag{3.4}$$

Introducing the two variables: $\theta = y_1$ and $\dot{y}_1 = y_2$, the equation (3.4) can be written as a following system of two first order nonlinear differential equations which is treated as slave system.

$$\text{DDP}: \begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = a_1 y_2 + b_1 \sin y_1 + c_1 \cos(kt) \end{cases} \tag{3.5}$$

Where,

$$a_1 = \frac{-b}{mR^2},\ b_1 = \frac{-g}{R} \text{ and } c_1 = \frac{A}{mR^2}.$$

In order to apply the RASMC to synchronize the MTS and DDP with uncertainties (0 & $h(x_1,t)$ and 0 & $-h(y_1,t)$ for MTS and DDP respectively), external disturbances (0 & $-c_1 \cos kt$ and 0 & $c_1 \cos kt$ for MTS and DDP respectively) and unknown parameters (as per equation (2.10)), it is assumed that the MTS drives the DDP. The master and slave systems can be rewritten in the form of (2.1) and (2.2) as follows:

$$\dot{\boldsymbol{x}} = \underbrace{\begin{bmatrix} 0 \\ -\omega_0^2 \sin x_1 \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} x_2 & 0 \\ 0 & 0 \end{bmatrix}}_{F(x)} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\theta} + \underbrace{\begin{bmatrix} 0 \\ h(x_1,t) \end{bmatrix}}_{\Delta f(x,t)} + \underbrace{\begin{bmatrix} 0 \\ -c_1 \cos kt \end{bmatrix}}_{d^m(t)} \tag{3.6}$$

$$\dot{\boldsymbol{y}} = \underbrace{\begin{bmatrix} 0 \\ b_1 \sin y_1 \end{bmatrix}}_{g(y)} + \underbrace{\begin{bmatrix} \begin{bmatrix} y_2 & 0 \\ 0 & y_2 \end{bmatrix} \end{bmatrix}}_{G(y)} \underbrace{\begin{bmatrix} 1 \\ a_1 \end{bmatrix}}_{\psi} + \underbrace{\begin{bmatrix} 0 \\ -h(y_1,t) \end{bmatrix}}_{\Delta g(y,t)} + \underbrace{\begin{bmatrix} 0 \\ c_1 \cos kt \end{bmatrix}}_{d^s(t)} + \underbrace{\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}}_{u(t)} \tag{3.7}$$

Therefore, using (2.7), the error dynamics can be expressed as:

$$\begin{aligned} \dot{e}_1 &= e_2 - u_1(t), \\ \dot{e}_2 &= -\omega_0^2 \sin x_1 - b_1 \sin y_1 - a y_2 + h(x_1,t) + h(y_1,t) - 2c_1 \cos kt - u_2(t). \end{aligned} \tag{3.8}$$

Where $u_i(t)$ for $i = 1,2$ is defined as per the equation (2.11).

4. Numerical Simulation

In the proposed study, we have chosen $\omega = 0.6$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.5$, $\varepsilon_3 = 0.9$, $\sigma_0 = 0.15$, $f = 0.8$, $a_1 = 0.1$, $b_1 = 0.2$, $c_1 = 0.3$, $k = \pi$ and the initial values of the update vector parameters $\hat{\boldsymbol{\theta}}_0$, $\hat{\boldsymbol{\psi}}_0$, $\hat{\alpha}_{i0}$ and $\hat{\beta}_{i0}$ are $[0.1, 0.2]$, $[0.1, 0.3]$, $[0.2, 0.4]$ and $[0.3, 0.5]$ respectively. Furthermore, The vector of switching gains $k_1 = 10$, $k_2 = 10$, the coefficient $\varepsilon = 10$ and the sliding surfaces are $s_1 = 10e_1$ and $s_2 = 5e_2$. The MTS and DDP are started with the initial conditions as follows: $x_1(0) = 0.2$, $x_2(0) = 0.9$, $y_1(0) = 0$ and $y_2(0) = 0.5$. Figure 2 illustrates the synchronization errors of the MTS and DDP, as one can see the synchronization errors converge to the zero, which implies that the chaos synchronization between the MTS and DDP is realized. The time responses of the update vector parameters $\hat{\theta}$, $\hat{\psi}$, $\hat{\alpha}_i$ and $\hat{\beta}_i$ are depicted in Figures 3–6, respectively. It is very well clear that all of the update parameters approach to some constants. Furthermore, figures 7 & 8 depict the time series of state vectors of the master and slave systems and phase plots of master and slave systems have been taken in figure 9, all of that also confirm the robust synchronization between the systems under consideration. Lastly in figure 10, we have plotted the graph of the derivative of Lyapunov function (2.12) which is less than or equal to zero for t is bigger than or equal to zero (i.e. $\dot{V}(t) \le 0$ for $t \ge 0$).

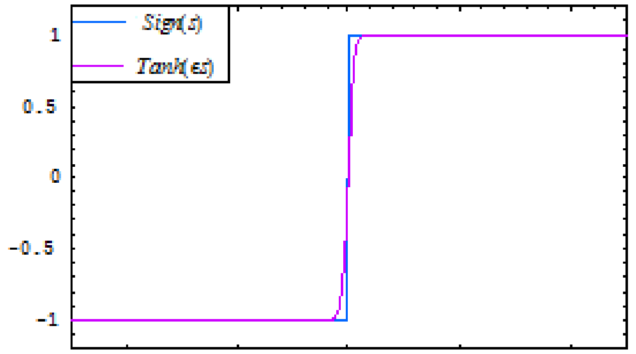

Figure 1. Comparison of sign(s) & Tanh(εs) for ε>1

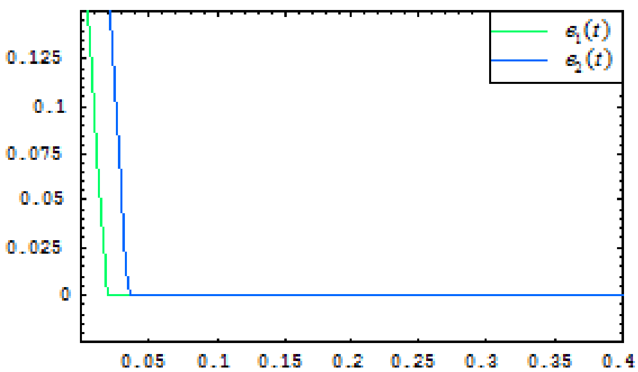

Figure 2. Time series of e_1 & e_2

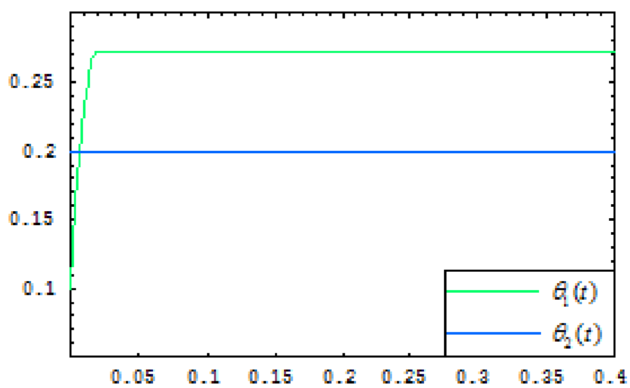

Figure 3. Time series of θ_1 & θ_2

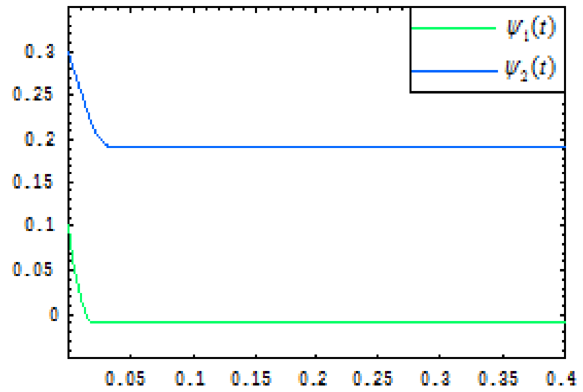

Figure 4. Time series of $\Psi1$&$\Psi2$

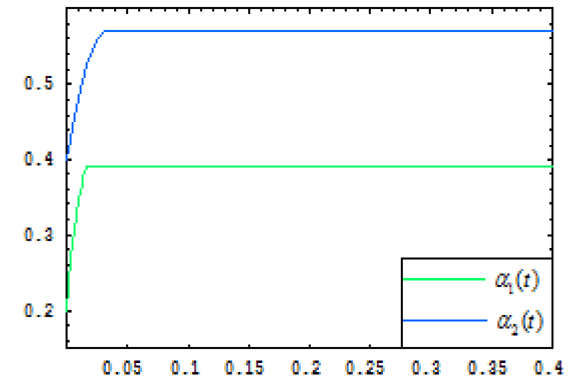

Figure 5. Time series of α_1&α_2

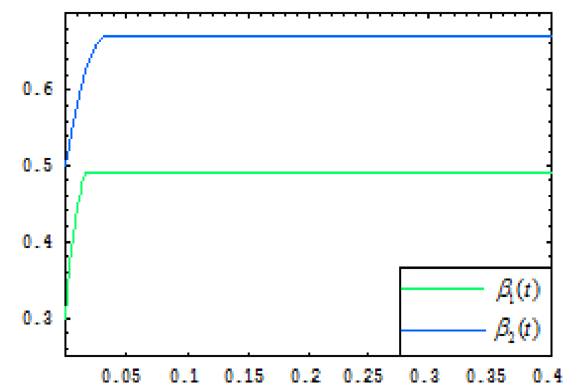

Figure 6. Time series of $\beta1$ &$\beta2$

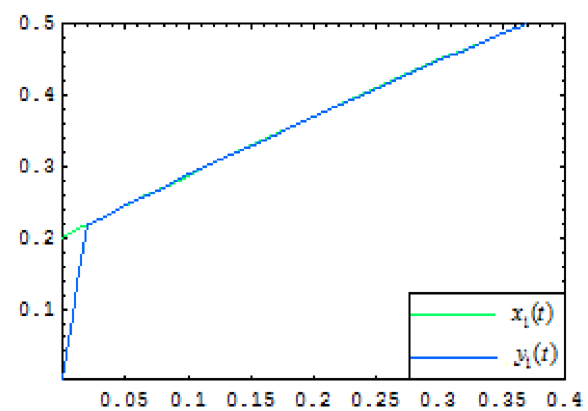

Figure 7. Time series of χ_1 & y_1

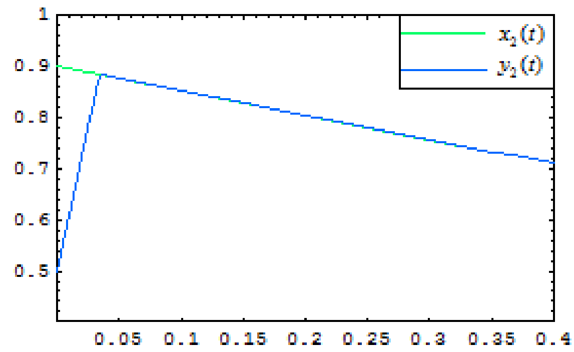

Figure 8. Time series of x_2 & y_2

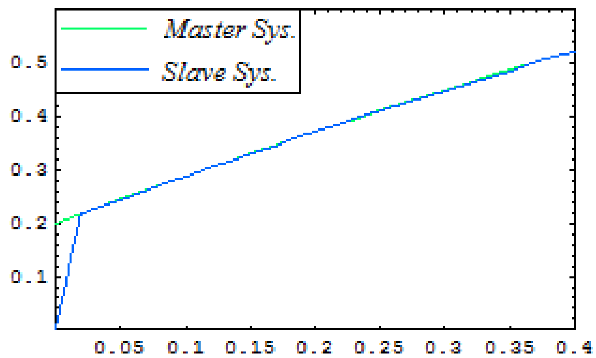

Figure 9. Phase plots of master & slave systems

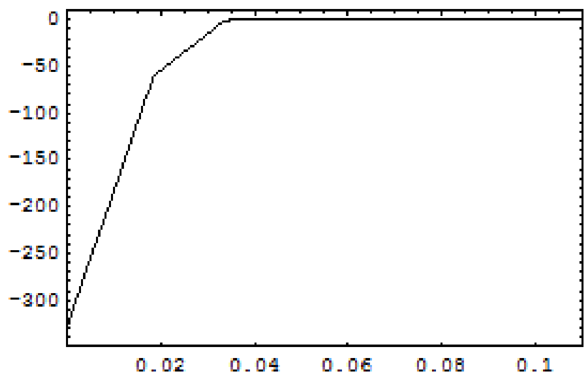

Figure 10. Time Series of $\dot{V}(t)$

5. Conclusions

In this paper, the problem of practical synchronization of chaotic systems is done using RAMSC under the effects of the model uncertainties, external disturbances and unknown parameters in synchronizing the two nonidentical chaotic systems (MTS & DDP). Numerical simulations are presented to show the applicability and feasibility of the proposed study using *Mathematica*. We conclude the following three remarkable features of our proposed study.

(1) It is robust with respect to the model uncertainties, external disturbances and unknown parameters.

(2) It can be easily realized and implemented in real world applications without requiring the bounds of the model uncertainties, external disturbances and unknown parameters to be known in advance.

(3) It is well applicable for practical synchronization of two non-identical chaotic systems even when both master and slave chaotic systems are disturbed by the model uncertainties, external disturbances and unknown parameters.

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