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New Discrete Sliding Mode Control for Nonlinear Multivariable Systems: Multi-Periodic Disturbances Rejection and Stability Analysis

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Abstract The Sliding Mode Control (SMC) is well known by its robustness to parameter uncertainties or external disturbances. The repetitive control (RC) is able to track or reject periodic and multi-periodic signals. This paper presents a new control strategy as a combination of a Sliding Mode approach and Repetitive Control to reject periodic and multi-periodic disturbances in the case of nonlinear multi-input discrete-time systems. A stability analysis of the new repetitive discrete sliding mode control is then proposed. The simulation results show the effectiveness of the proposed control to reject periodic and multi-periodic disturbances.

Keywords Nonlinear Multivariable Control Systems, Sliding Mode Control, Repetitive Control, Rejection of Periodic And Multi-Periodic Disturbances

1. Introduction

The sliding mode control (SMC) has been widely studied over several decades of years in continuous and discrete time-systems[1,2].

The continuous-time SMC is known by its robustness against uncertainties and external disturbances[3,4]. It consists of two steps. The first step is the design of a sliding surface along which the process can slide to find its desired final value. The second one is the synthesis of the control law in such away that any state outside the sliding surface is forced to reach the desired sliding manifold in finite time and stay on it. Nevertheless, the main drawback of the continuous time sliding mode control is the chattering phenomenon which appears as a source to excite unmodeled high frequency dynamics of the process. Many approaches have been proposed to solve this problem such as second and high order sliding mode control[5,6].

In the face of the development of computer and applications of digital control, discrete sliding mode control (DSMC) has become more important in academia and industries [7-11]. The important difference between continuous time and discrete time sliding mode control is that the DSMC does not possess the invariance properties found in continuous time systems due to a finite sampling data rate. A discrete quasi-sliding mode control based on new reaching law was proposed by Gao[12]. This control law ensures that system trajectory will hit the surface in finite time and thereafter undergo a zigzag motion in the vicinity of the surface. Therefore, the trajectory will stay within a specified band called the quasi-sliding mode band. However, this approach is sensitive to external disturbances. Due to the modeling errors, the nominal performance is traded off to keep robust stability. As the modeling error expands, the performance of the nominal system becomes unsatisfactory.

The dynamics of industrial processes are generally not exactly known, nonlinear, multivariable and are subject to periodic and multi-periodic disturbances, such as: optical and magnetic disk drives, electronic rectifiers, rotating machine tools and robots, etc.

In deal with the control of such processes, repetitive control (RC) is a major approach being used[13-17]. The main idea behind repetitive control is to remove errors that occur at the fundamental and harmonics frequency of the periodic signal. The RC is based on the Internal Model Principle (IMP) [18] which states that zero error tracking of any reference input, in a steady state, can be accomplished if a generator of the reference input is incorporated in a stable closed loop system. A periodic signal can be generated by a delay block with a positive feedback loop.

The repetitive control has been studied in the continuous and discrete time domain and has been used in different engineering areas. However, the RC may cause system instability. Therefore, the trade off between stability and tracking performance becomes a challenging task in the repetitive control[19,20].

Many techniques are developed to ameliorate the discrete

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sliding mode control. For discrete linear multivariable systems, we have proposed a repetitive sliding mode control with a new reaching law to reduce the problem of chattering and reject the periodic disturbances[21].

This work deals with a repetitive sliding mode control for discrete time nonlinear multivariable systems. This approach is designed to force the state trajectories to reach the surface in finite time and keep them on it, and in the same time to reject periodic and multi-periodic disturbances.

This paper is organized as follows. Section 2 describes the classical sliding mode control for discrete nonlinear multivariable systems. The development of repetitive sliding mode control to reject periodic and multi-periodic disturbances is presented in section 3. In section 4, a stability analysis of the proposed control is presented. Section 5 gives the simulation results.

2. The Classical Discrete Sliding Mode Control for Nonlinear Multivariable Systems

Consider a class of nonlinear multi-inputs discrete-time system described by the following model[22]:

$$x(k+1) = f(x(k)) + Gu(k) \tag{1}$$

where $x(k)$ and $u(k)$ are respectively the state and input vectors:

$$x(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix};\ u(k) = \begin{bmatrix} u_1(k) \\ \vdots \\ u_m(k) \end{bmatrix}$$

f is an $n \times 1$ nonlinear function and G is an $n \times m$ matrix.

The sliding vector is defined as[9,23]:

$$s(k) = Cx(k) = \begin{bmatrix} s_1(k) \\ \vdots \\ s_m(k) \end{bmatrix} \tag{2}$$

where C is a $m \times n$ matrix.

The sliding function is chosen in order to verify the following reaching law[23]:

$$s(k+1) = \Phi s(k) - \begin{bmatrix} m_1 \mathrm{sat}(s_1(k), \delta_1) \\ m_2 \mathrm{sat}(s_2(k), \delta_2) \\ \vdots \\ m_i \mathrm{sat}(s_i(k), \delta_i) \\ \vdots \\ m_m \mathrm{sat}(s_m(k), \delta_m) \end{bmatrix} \tag{3}$$

where Φ ($\Phi \in \Re^{m\times n}$) is a diagonal matrix such $0 \le \Phi_{i,i} < 1,\ \forall i = 1..m$ and m_i is a positive gain.

The term "sat" represent the saturation function which is defined by:

$$\mathrm{sat}(s_i(k), \delta_i) = \begin{cases} +1, & \text{if} \quad s_i(k) > \delta_i \\ \dfrac{s_i(k)}{\delta_i}, & \text{if} \quad |s_i(k)| \le \delta_i \\ -1, & \text{if} \quad s_i(k) < -\delta_i \end{cases}$$

Using the equations (1), (2) and (3), $s(k+1)$ can be written as:

$$s(k+1) = \Phi s(k) - \begin{bmatrix} m_1 \mathrm{sat}(s_1(k), \delta_1) \\ m_2 \mathrm{sat}(s_2(k), \delta_2) \\ \vdots \\ m_i \mathrm{sat}(s_i(k), \delta_i) \\ \vdots \\ m_m \mathrm{sat}(s_m(k), \delta_m) \end{bmatrix} = Cf(x(k)) + CGu(k)$$

Then, the control law can be expressed as follows:

$$u(k) = (CG)^{-1}(-Cf(x(k)) + \Phi s(k)) - (CG)^{-1} \begin{bmatrix} m_1 \mathrm{sat}(s_1(k), \delta_1) \\ m_2 \mathrm{sat}(s_2(k), \delta_2) \\ \vdots \\ m_i \mathrm{sat}(s_i(k), \delta_i) \\ \vdots \\ m_m \mathrm{sat}(s_m(k), \delta_m) \end{bmatrix} \tag{4}$$

The problem of this discrete sliding mode control is its sensitivity to external disturbances. In a variety of industrial processes, these disturbances are, often, periodic signals (robotic, rotating machine tools, active noise control, etc.).

Clearly, the presence of these disturbances will degrade system performances. In fact, they can excite unmodeled dynamics and cause instability. Then, the robustness of classical sliding mode control is not available when the system is subject to external periodic disturbances.

To overcome this problem, in the next section, we present a new strategy named discrete multivariable repetitive sliding mode control.

3. New Discrete Sliding Mode Control for Nonlinear Multivariable Systems: Rejection of Periodic and Multi-Periodic Disturbances

Consider a nonlinear discrete multivariable system subjected to external disturbances defined by:

$$x(k+1) = f(x(k)) + Gu(k) + p(k) \tag{5}$$

where $p(k)$ is the external disturbance vector.

Suppose the following matching condition is satisfied:

$$p(k) = Gd(k);\ d(k) = \begin{bmatrix} d_1(k) & d_2(k) & \dots & d_m(k) \end{bmatrix}^T \tag{6}$$

Then, the equation (5) is equivalent to:

$$x(k+1) = f(x(k)) + G(u(k) + d(k)) \tag{7}$$

3.1. Case of Periodic Disturbances

We suppose that the components $d_i(k), i \in [1 \quad m]$ of the disturbances' vector $d(k)$ are assumed to be periodic with the period N:

$$d_i(k) = d_i(k-N), i \in [1 \quad m]$$

The difference between $s(k+1)$ and $s(k+1-N)$ can be expressed as:

$$s(k+1) - s(k+1-N) = Cf(x(k)) + CGu(k) + CGd(k) - Cf(x(k-N)) - CGu(k-N) - CGd(k-N)$$

Or, we have:

$$d(k)=\begin{bmatrix} d_1(k) \\ \vdots \\ d_i(k) \\ \vdots \\ d_m(k) \end{bmatrix}=\begin{bmatrix} d_1(k-N) \\ \vdots \\ d_i(k-N) \\ \vdots \\ d_m(k-N) \end{bmatrix}=d(k-N)$$

Then, the control law can be calculated as:

$$u(k)=u(k-N)+(CG)^{-1}\left(s(k+1)-s(k+1-N)-Cf(x(k))+Cf(x(k-N))\right)$$

Applying the equation (3) in the last relation, the control law can be given as:

$$u(k)=u(k-N)-(CG)^{-1}\left(Cf(x(k))-Cf(x(k-N))-\Phi s(k)+s(k+1-N)\right) \\ -(CG)^{-1}\begin{bmatrix} m_1\mathrm{sat}(s_1(k),\delta_1) \\ m_2\mathrm{sat}(s_2(k),\delta_2) \\ \vdots \\ m_i\mathrm{sat}(s_i(k),\delta_i) \\ \vdots \\ m_m\mathrm{sat}(s_m(k),\delta_m) \end{bmatrix} \quad (8)$$

3.2. Case of Multi-Periodic Disturbances

We suppose now that the components $d_i(k), i\in[1 \quad m]$ of the disturbances' vector $d(k)$ are assumed to be multi- periodic:

$$P(z^{-1})d_i(k)=0,\ P(z^{-1})=\prod_{j=1}^{M}(1-z^{-N_j})$$

with $T_j=N_jT_e, j\in[1 \quad M]$ are the M periods of disturbances. $P(z^{-1})$ can be written as follows:

$$P(z^{-1})=1+\sum_{j=1}^{N}\alpha_j z^{-j} \quad (9)$$

where:

$$N=\sum_{j=1}^{M}N_j,\ \alpha_j\in\{-1,0,1\},\ j\in[1 \quad M]$$

Multiplying $s(k+1)$ by $P(z^{-1})$, we obtain:

$$\begin{aligned} P(z^{-1})s(k+1) &= P(z^{-1})\left(Cf(x(k))+CGu(k)+CGd(k)\right) \\ &= CP(z^{-1})f(x(k))+CGP(z^{-1})u(k)+CGP(z^{-1})d(k) \\ &= CP(z^{-1})f(x(k))+CGP(z^{-1})u(k) \end{aligned} \quad (10)$$

Using the last equation and the equation (9), the control law can be calculated as:

$$u(k)=-\sum_{j=1}^{N}\alpha_j u(k-j)+(CG)^{-1}\left(s(k+1)+\sum_{j=1}^{N}\alpha_j s(k+1-j)\right) \\ -(CG)^{-1}C\left(f(x(k))+\sum_{j=1}^{N}\alpha_j f(x(k-j))\right)$$

Replace $s(k+1)$ by its expression (equation (3)), the control law can be rewritten as follows:

$$u(k)=-\sum_{j=1}^{N}\alpha_j u(k-j)+(CG)^{-1}\left(\Phi s(k)+\sum_{j=1}^{N}\alpha_j s(k+1-j)\right) \\ -(CG)^{-1}C\left(f(x(k))+\sum_{j=1}^{N}\alpha_j f(x(k-j))\right) \quad (11) \\ -(CG)^{-1}\begin{bmatrix} m_1\mathrm{sat}(s_1(k),\ \delta_1) \\ m_2\mathrm{sat}(s_2(k),\ \delta_2) \\ \vdots \\ m_i\mathrm{sat}(s_i(k),\ \delta_i) \\ \vdots \\ m_m\mathrm{sat}(s_m(k),\ \delta_m) \end{bmatrix}$$

4. Stability of the New Discrete Multivariable Sliding Mode Control

By applying the control law (11) to the system (7), the sliding vector $s(k+1)$ can be written as follows:

$$\begin{aligned} s(k+1) &= Cx(k+1)=Cf(x(k))+CGu(k)+CGd(k) \\ &= Cf(x(k))-CG\sum_{j=1}^{N}\alpha_j u(k-j)+\Phi s(k)+\sum_{j=1}^{N}\alpha_j s(k+1-j) \\ &\quad -Cf(x(k))-C\sum_{j=1}^{N}\alpha_j f(x(k-j))+CGd(k) \\ &\quad -\begin{bmatrix} m_1\mathrm{sat}(s_1(k),\delta_1) \\ m_2\mathrm{sat}(s_2(k),\delta_2) \\ \vdots \\ m_i\mathrm{sat}(s_i(k),\delta_i) \\ \vdots \\ m_m\mathrm{sat}(s_m(k),\delta_m) \end{bmatrix} \end{aligned} \quad (12)$$

This last equation is equivalent to:

$$s(k+1)=\Phi s(k)+CGd(k)+CG\sum_{j=1}^{N}\alpha_j d(k-j) \\ -\begin{bmatrix} m_1\mathrm{sat}(s_1(k),\delta_1) \\ m_2\mathrm{sat}(s_2(k),\delta_2) \\ \vdots \\ m_i\mathrm{sat}(s_i(k),\delta_i) \\ \vdots \\ m_m\mathrm{sat}(s_m(k),\delta_m) \end{bmatrix}$$

We note:

$$\tilde{d}(k)=CGP(z^{-1})d(k)=CG\left(1+\sum_{j=1}^{N}\alpha_j z^{-j}\right)d(k) \\ =\left[\tilde{d}_1(k) \ \ \ldots \ \ \tilde{d}_i(k) \ \ \ldots \ \ \tilde{d}_m(k)\right]^T$$

Then:

$$s(k+1)=\Phi s(k)+\tilde{d}(k)-\begin{bmatrix} m_1\mathrm{sat}(s_1(k),\delta_1) \\ m_2\mathrm{sat}(s_2(k),\delta_2) \\ \vdots \\ m_i\mathrm{sat}(s_i(k),\delta_i) \\ \vdots \\ m_m\mathrm{sat}(s_m(k),\delta_m) \end{bmatrix} \quad (13)$$

A sufficient and necessary condition for a system to satisfy a convergent sliding mode in the sense of Lyapunov is [24]:

$$|s_i(k+1)|<|s_i(k)|,\ i\in[1 \quad m] \quad (14)$$

This last equation is equivalent to:

$$\begin{cases}(s_i(k+1)-s_i(k))\mathrm{sign}(s_i(k))<0 \\ (s_i(k+1)+s_i(k))\mathrm{sign}(s_i(k))>0\end{cases},\ i\in[1 \quad m] \quad (15)$$

where "sign" is the signum function:

$$\mathrm{sign}(s_i(k))=\begin{cases}+1, & \text{if} \quad s_i(k)\geq 0 \\ -1, & \text{if} \quad s_i(k)<0\end{cases}$$

The stability of the proposed control can be given by the following theorem:

Theorem 1. Consider the system (7) to which the discrete multivariable repetitive sliding mode control is applied (11).

This system verifies a convergent quasi-sliding mode, if the following conditions $P1$, $P2$ and $P3$ are satisfied[25,26]:

$$P1.\ \left|s_i(k+1)\right| < \left|s_i(k)\right|,\ \text{if } \left|s_i(k)\right| > \delta_i,\ i \in [1 \quad m] \tag{16}$$

$$P2.\ \left|s_i(k+1)\right| < \delta_i,\ \text{if } \left|s_i(k)\right| = \delta_i,\ i \in [1 \quad m] \tag{17}$$

$$P3.\ \left|s_i(k+1)\right| < \delta_i,\ \text{if } \left|s_i(k)\right| < \delta_i,\ i \in [1 \quad m] \tag{18}$$

where:

$$\left|\tilde{d}_i(k)\right| \le \bar{d}_i,\ 0 < \bar{d}_i < m_i,\ \Phi_{i,i} \ge \left(\frac{m_i}{\delta_i}\right) \tag{19}$$

Proof.

Firstly, we begin by the condition (16).

Consider the Case 1: $s_i(k) > \delta_i$

The difference between $s_i(k+1)$ and $s_i(k)$ is given by:

$$\begin{aligned} s_i(k+1) - s_i(k) &= (\Phi_{i,i} - 1)s_i(k) - m_i + \tilde{d}_i(k) \\ &< -m_i + \tilde{d}_i(k) \\ &< 0 \end{aligned}$$

The sum of $s_i(k+1)$ and $s_i(k)$ can be written as follows:

$$\begin{aligned} s_i(k+1) + s_i(k) &= (\Phi_{i,i} + 1)s_i(k) - m_i + \tilde{d}_i(k) \\ &> \delta_i + \Phi_{i,i}\delta_i - m_i + \tilde{d}_i(k) \\ &> \Phi_{i,i}\delta_i + \tilde{d}_i(k) \\ &\ge m_i + \tilde{d}_i(k) > 0 \end{aligned}$$

Then:

$$\left|s_i(k+1)\right| < \left|s_i(k)\right|,\ \text{if } s_i(k) > \delta_i$$

Consider the Case 2: $s_i(k) < -\delta_i$

In this case, the difference between $s_i(k+1)$ and $s_i(k)$ can be calculated as:

$$\begin{aligned} s_i(k+1) - s_i(k) &= (\Phi_{i,i} - 1)s_i(k) + m_i + \tilde{d}_i(k) \\ &> m_i + \tilde{d}_i(k) \\ &> 0 \end{aligned}$$

The sum of $s_i(k+1)$ and $s_i(k)$ can be expressed as:

$$\begin{aligned} s_i(k+1) + s_i(k) &= (\Phi_{i,i} + 1)s_i(k) + m_i + \tilde{d}_i(k) \\ &< -\delta_i - \Phi_{i,i}\delta_i + m_i + \tilde{d}_i(k) \\ &< -\Phi_{i,i}\delta_i + \tilde{d}_i(k) \\ &< -m_i + \tilde{d}_i(k) \\ &< 0 \end{aligned}$$

So:

$$\left|s_i(k+1)\right| < \left|s_i(k)\right|,\ \text{if } s_i(k) < -\delta_i$$

Using cases 1 and 2, the condition $P1$ is verified.

Secondly, we consider the condition (17).

Consider the Case 3: $s_i(k) = \delta_i$

We have:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + \tilde{d}_i(k) \\ &< \Phi_{i,i}\delta_i - m_i + \tilde{d}_i(k) \\ &< \delta_i \end{aligned}$$

The sliding function $s_i(k+1)$ can be written as follows:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + \tilde{d}_i(k) \\ &> \Phi_{i,i}\delta_i - m_i + \tilde{d}_i(k) \\ &> \tilde{d}_i(k) \\ &> -m_i \\ &\ge -\Phi_{i,i}\delta_i \\ &> -\delta_i \end{aligned}$$

Then:

$$\left|s_i(k+1)\right| < \delta_i,\ \text{if } s_i(k) = \delta_i$$

Consider the Case 4: $s_i(k) = -\delta_i$

In this case, the expression of the sliding function $s_i(k+1)$ can be given by:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + \tilde{d}_i(k) \\ &> -\delta_i \end{aligned}$$

or:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + \tilde{d}_i(k) \\ &< \tilde{d}_i(k) \\ &< m_i \\ &< \Phi_{i,i}\delta_i \\ &< \delta_i \end{aligned}$$

Thus:

$$\left|s_i(k+1)\right| < \delta_i,\ \text{if } s_i(k) = -\delta_i$$

Using cases 3 and 4, we obtain the condition $P2$.

Finally, we consider the third condition (18).

Consider the Case 5: $0 \le s_i(k) < \delta_i$

Using this condition, the sliding function $s_i(k+1)$ can be expressed as:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + \tilde{d}_i(k) \\ &< \Phi_{i,i}\delta_i - m_i + \tilde{d}_i(k) \\ &< \Phi_{i,i}\delta_i \\ &< \delta_i \end{aligned}$$

The sliding function $s_i(k+1)$ can be written as follows:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) - m_i + \tilde{d}_i(k) \\ &> \tilde{d}_i(k) \\ &> -m_i \\ &> -\Phi_{i,i}\delta_i \\ &> -\delta_i \end{aligned}$$

Then:

$$\left|s_i(k+1)\right| < \delta_i,\ \text{if } 0 \le s_i(k) < \delta_i$$

Consider the Case 6: $-\delta_i < s_i(k) \le 0$

The sliding function $s_i(k+1)$ can be calculated as:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + \tilde{d}_i(k) \\ &> -\Phi_{i,i}\delta_i + m_i + \tilde{d}_i(k) \\ &> -\Phi_{i,i}\delta_i \\ &> -\delta_i \end{aligned}$$

We have:

$$\begin{aligned} s_i(k+1) &= \Phi_{i,i}s_i(k) + m_i + \tilde{d}_i(k) \\ &< \tilde{d}_i(k) \\ &< m_i \\ &< \Phi_{i,i}\delta_i \\ &< \delta_i \end{aligned}$$

Then:

$$\left|s_i(k+1)\right| < \delta_i,\ \text{if } -\delta_i < s_i(k) \le 0$$

Using cases 5 and 6, the condition $P3$ is verified.

Using the cases 1, 2, 3, 4, 5 and 6, the three conditions (16),

(17) and (18) are verified. Then, the global system with a discrete multivariable repetitive sliding mode control is stable.

5. Simulation Example

Consider a nonlinear multivariable system described as follows:

$$x(k+1) = f(x(k)) + G(u(k) + d(k))$$

with:

$$f(x(k)) = \begin{bmatrix} \dfrac{x_1(k)}{1+(x_2(k))^2} \\ \dfrac{x_1(k)x_2(k)}{1+(x_2(k))^2} \end{bmatrix}; G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}; u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}; d(k) = \begin{bmatrix} d_1(k) \\ d_2(k) \end{bmatrix}$$

The retained synthesis parameters are:

$$m_1 = m_2 = 0.01;\ \delta_1 = \delta_2 = 0.1;\ \Phi_{1,1} = \Phi_{2,2} = 0.75$$

We suppose that the disturbances are given by:

$$\begin{cases} d_1(k) = 0 \\ d_2(k) \end{cases};\ k \le 100$$

$$\begin{cases} d_1(k) = 0.1 + 0.1\sin(\frac{2\pi k}{N}) + 0.08\cos(\frac{2\pi k}{N}) + 0.05\sin(\frac{4\pi k}{N}) + 0.01\cos(\frac{4\pi k}{N}) \\ d_2(k) = 0.15 + 0.15\sin(\frac{2\pi k}{N}) + 0.1\cos(\frac{2\pi k}{N}) + 0.07\sin(\frac{4\pi k}{N}) + 0.05\cos(\frac{4\pi k}{N}) \end{cases};\ k > 100$$

Firstly, we suppose that the period of disturbances is equal to 10 (N=10). The evolution of disturbances $d_1(k)$ and $d_2(k)$ is given in figure 1.

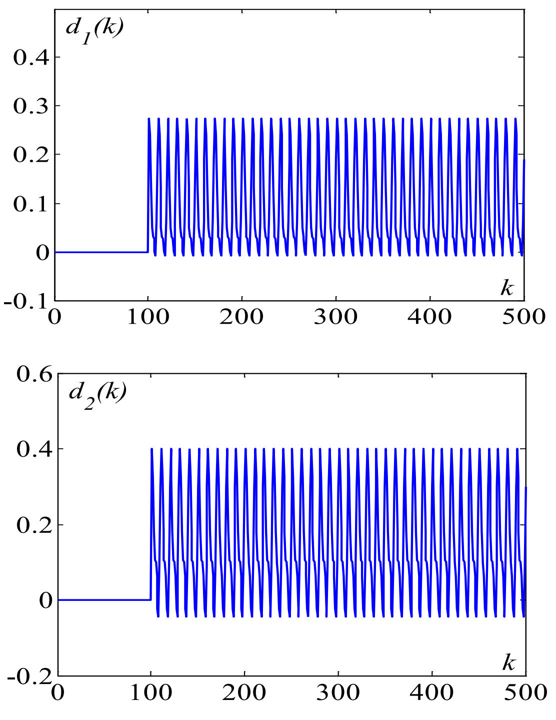

Figure 1. Evolutions of periodic disturbances $d_1(k)$ and $d_2(k)$

Firstly, we apply the control law defined by the equation (4).

Figure 2, figure 3 and figure 4 illustrate respectively the evolutions of the states $x_1(k)$ and $x_2(k)$, the inputs $u_1(k)$ and $u_2(k)$ and the sliding functions $s_1(k)$ and $s_2(k)$. The figure 2 proves that the control law (equation (4)) force the states to converge to zero in the absence of external disturbances (k<100). For k>100, the discrete sliding mode control (equation (4)) is not able to reject periodic disturbances.

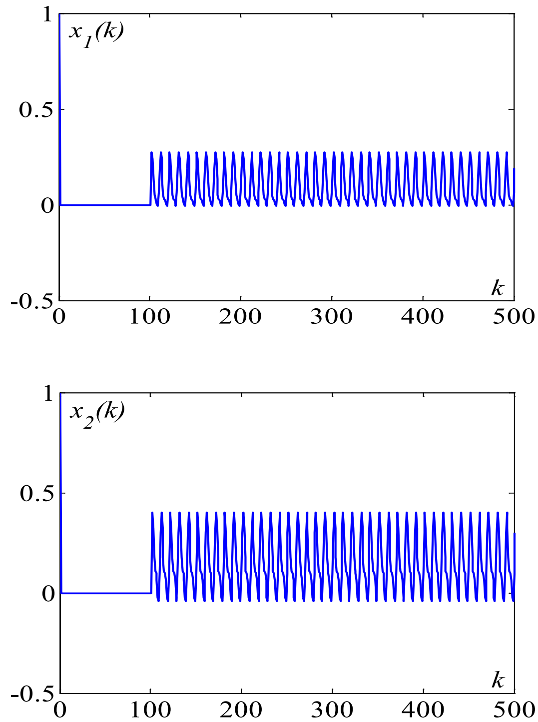

Figure 2. Evolutions of the states $x_1(k)$ and $x_2(k)$ (classical sliding mode control)

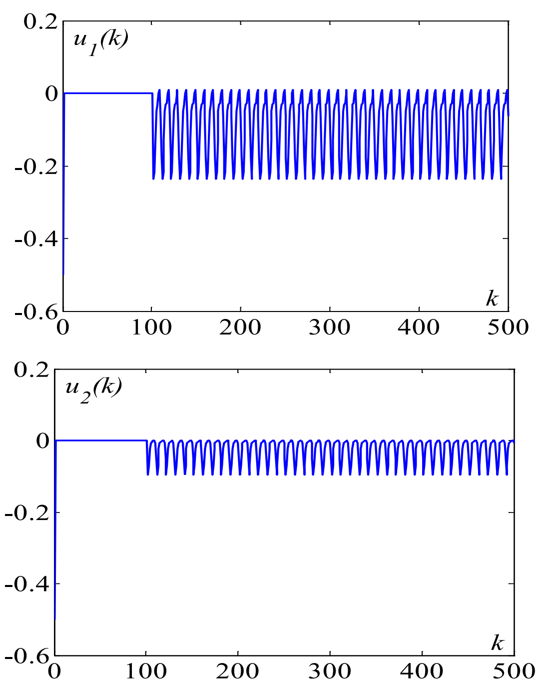

Figure 3. Evolutions of the control signals $u_1(k)$ and $u_2(k)$ (classical sliding mode control)

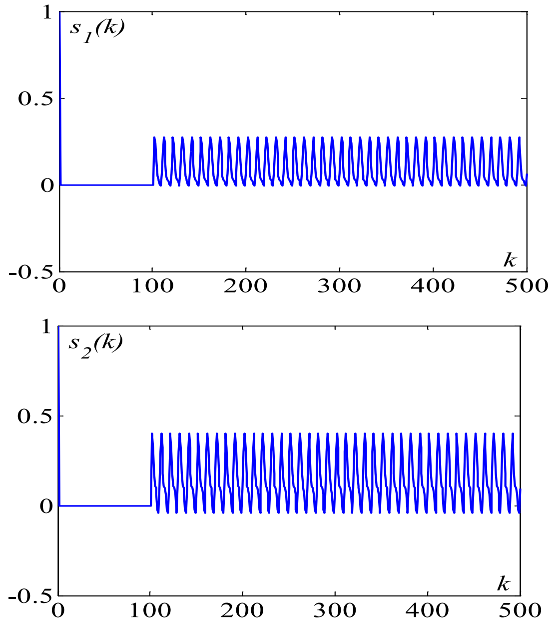

Figure 4. Evolutions of the sliding functions $s_1(k)$ and $s_2(k)$ (classical sliding mode control)

Secondly, to ameliorate the performance in term of the rejection of periodic disturbances, we use the new proposed control law defined by the equation (8).

The simulation results of the system with the considered control law are shown in figures 5, 6 and 7. Figure 5 gives the evolutions of the states $x_1(k)$ and $x_2(k)$. Figure 6 presents the evolutions of the inputs $u_1(k)$ and $u_2(k)$.

The evolution of the sliding functions $s_1(k)$ and $s_2(k)$ is presented in figure 7. These figures prove that relatively satisfactory performances are recorded in terms of rejecting periodic disturbances. A comparison between figure 2 and figure 5 reveals that the using of the new control strategy reduces effectively the periodic disturbances.

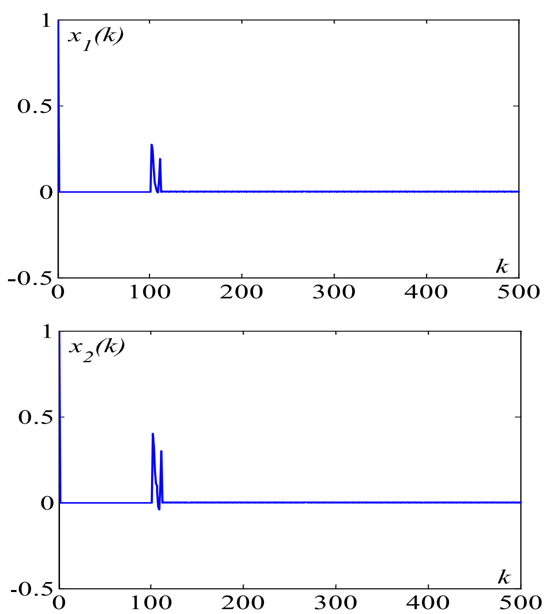

Figure 5. Evolutions of the states $x_1(k)$ and $x_2(k)$ (new sliding mode control (8))

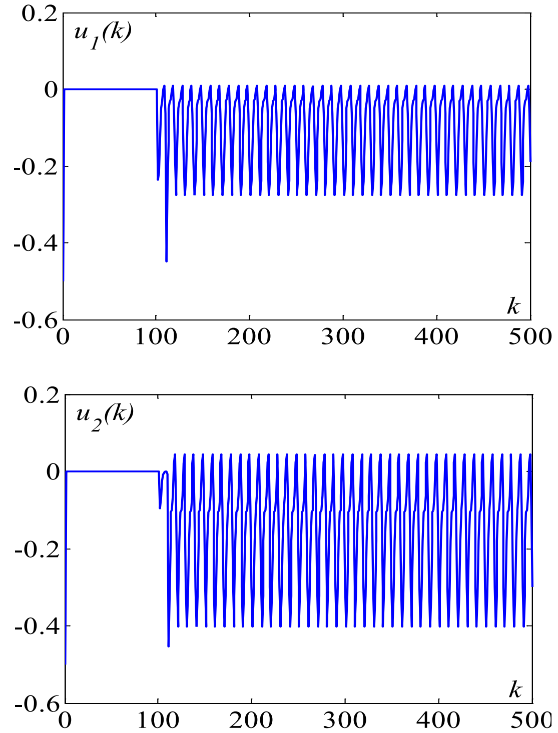

Figure 6. Evolutions of the control signals $u_1(k)$ and $u_2(k)$ (new sliding mode control (8))

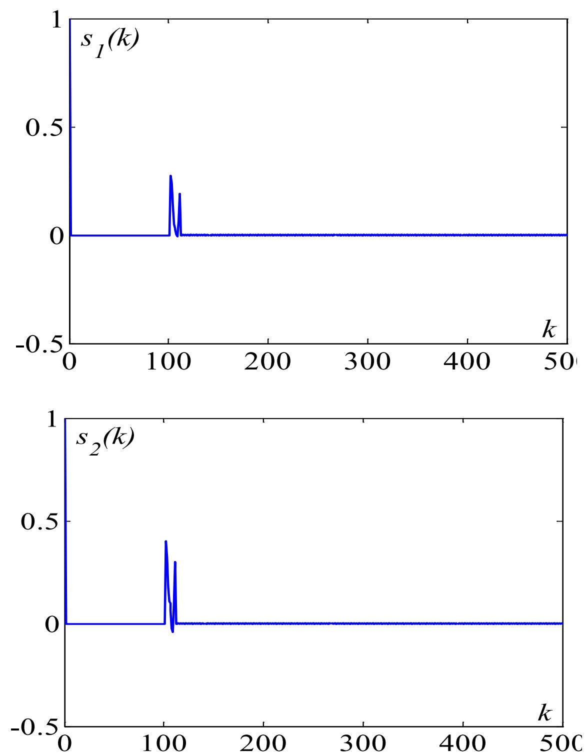

Figure 7. Evolutions of the sliding functions $s_1(k)$ and $s_2(k)$ (new sliding mode control (8))

We suppose that the disturbances change its dynamics and become multi-periodic:

$$\begin{cases} d_1(k) = 0 \\ d_2(k) \end{cases}; \quad k \le 100$$

$$\begin{cases} d_1(k) = 0.1 + 0.1\sin(\frac{2\pi k}{N}) + 0.08\cos(\frac{2\pi k}{N}) + 0.05\sin(\frac{4\pi k}{N}) + 0.01\cos(\frac{4\pi k}{N}) \\ \qquad + 0.1\sin(\frac{2\pi k}{N_1}) + 0.07\cos(\frac{2\pi k}{N_1}) \\ d_2(k) = 0.15 + 0.15\sin(\frac{2\pi k}{N}) + 0.1\cos(\frac{2\pi k}{N}) + 0.07\sin(\frac{4\pi k}{N}) + 0.05\cos(\frac{4\pi k}{N}) \\ \qquad + 0.12\sin(\frac{2\pi k}{N_1}) + 0.08\cos(\frac{2\pi k}{N_1}) \end{cases} ; \; k > 100$$

with N_1=30. The evolution of disturbances is given in figure 8.

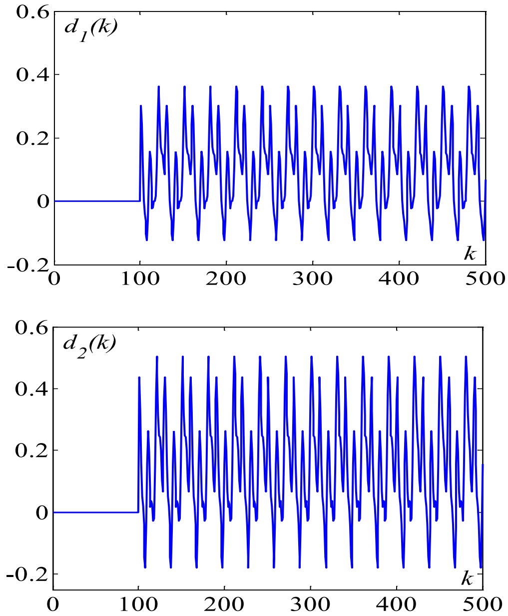

Figure 8. Evolutions of the multi-periodic disturbances $d_1(k)$ and $d_2(k)$

Figure 9 represent the evolutions of the states $x_1(k)$ and $x_2(k)$ when the control law (equation (4)) is applied to the system. It can be seen that this control law is able to reject only the periodic component of period N. The periodic component of period N_1 is not rejected.

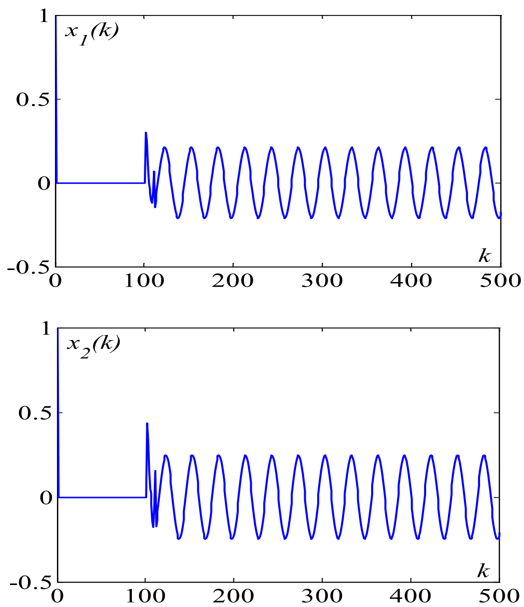

Figure 9. Evolutions of the states $x_1(k)$ and $x_2(k)$ (new sliding mode control (8))

The simulation results of the system with the new strategy control (equation (11)) proposed to reject multi-periodic disturbances are shown in figures 10, 11 and 12. The evolution of the states is given in figure 10. This figure shows that the states $x_1(k)$ and $x_2(k)$ for k >100 move away from zero during few iterations and return to zero. This is due to control signals (see figure 11) which become repetitive after k >100 to eliminate the multi-periodic disturbances. The evolution of sliding functions is presented in figure 12.

The results obtained prove the capability of the proposed control law to reject multi-periodic disturbances.

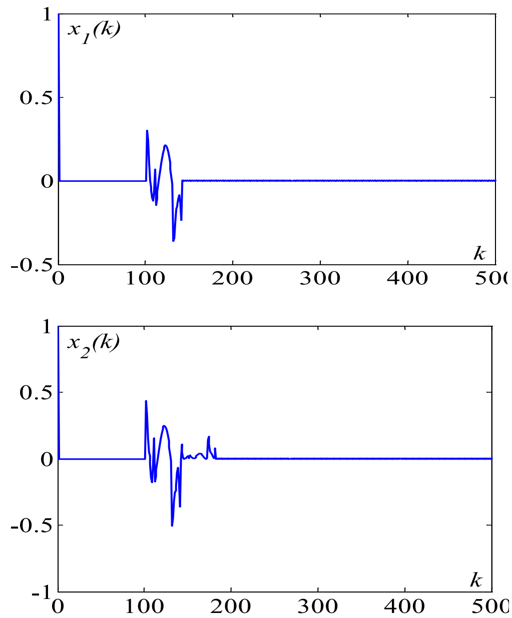

Figure 10. Evolutions of the states $x_1(k)$ and $x_2(k)$ (new sliding mode control (11))

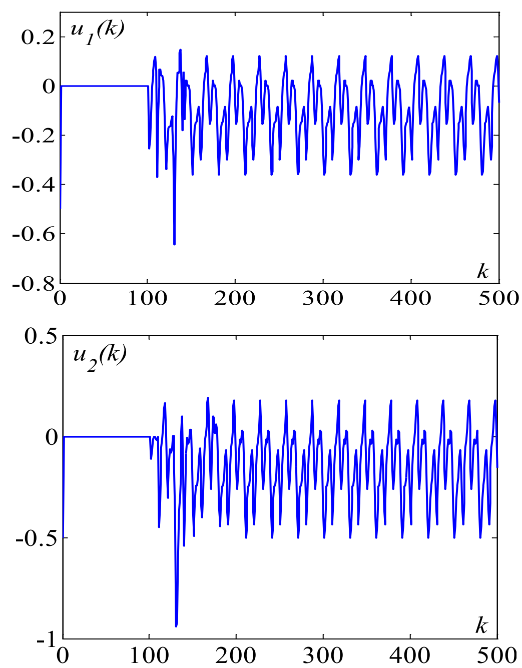

Figure 11. Evolutions of the control signals $u_1(k)$ and $u_2(k)$ (new sliding mode control (11))

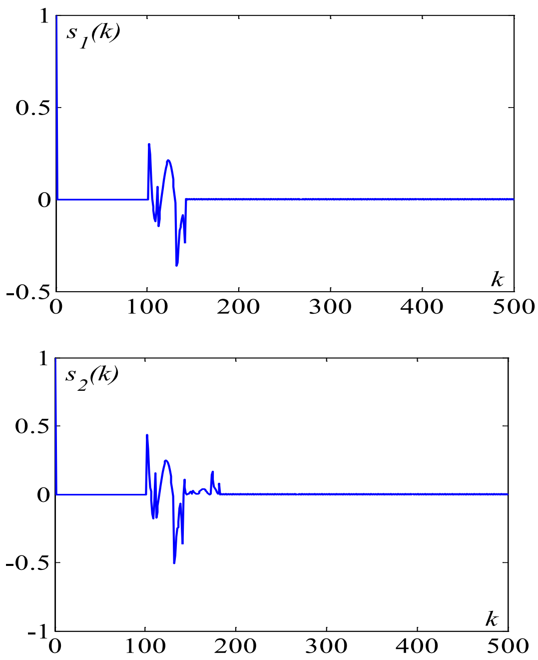

Figure 12. Evolutions of the sliding functions $s_1(k)$ and $s_2(k)$ (new sliding mode control (11)).

5. Conclusions

In this paper, the problem of rejecting periodic and multi-periodic disturbances for nonlinear discrete multivariable system was considered.

To overcome the sensibility of the discrete sliding mode control to periodic and multi-periodic disturbances, a new strategy based on the combination of discrete sliding mode approach and repetitive control was proposed. A stability analysis of the proposed control is studied.

The obtained simulation results using the new discrete repetitive sliding mode control show good performances in term of regulation and periodic and multi-periodic disturbances rejection.

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