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A New Hybrid Fuzzy Time Series Forecasting Model Combined the Time -Variant Fuzzy Logical Relationship Groups with Particle Swam Optimization

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Abstract Fuzzy time series forecasting models are used to overcome traditional time series methods when the historical data of traditional time series approaches contain uncertainty or need to be represented by linguistic values. Besides, fuzzy time series forecasting methods do not require any assumption valid. Generally, fuzzy time series forecasting methods consist of three major stages such as fuzzification, determination of fuzzy logic relationships or fuzzy relationship matrix, and defuzzification. All these stages of fuzzy time series are very important on the forecasting performance of the model. In this paper, a new hybrid fuzzy time series forecasting model is proposed based on three computational approaches such as: the new concept of time-variant fuzzy relationship group is used to establish time-variant fuzzy relationship group in the determination of fuzzy logical relationships stage, named called the time - variant fuzzy logical relationship groups (TV-FLRGs), the proposed forecasting rules is applied to calculate the forecasting value for the TV-FLRGs and particle swarm optimization technique (PSO) is aggregated with TV-FLRGs to adjust interval lengths and find proper intervals in the universe of discourse with the objective of increasing forecasting accuracy. To verify the effectiveness of the proposed model, two numerical data sets are selected to illustrate the proposed method and compare the forecasting accuracy with existing methods. The results show that the proposed model gets a higher average forecasting accuracy rate to forecast the Taiwan futures exchange (TAIFEX) and enrolments of the University of Alabama than the existing methods based on the first – order and high-order fuzzy time series.

Keywords Enrolments, TAIFEX, Forecasting, Fuzzy time series (FTS), Time – variant fuzzy logical relationship groups, High-order fuzzy time series, Particle swarm optimization

1. Introduction

In recent years, fuzzy time series has been widely applied to many fields such as forecasting enrolments, crop productions, stock index, temperature, etc.,. Based on the fuzzy set theory, Song and Chissom [1] first proposed the concept of fuzzy time series. They developed two fuzzy time series forecasting models: the time-invariant model [1] and the time-variant model [2] which use the max–min operations to forecast the enrolments of the University of Alabama. Compared with traditional time series models, these fuzzy time series models can deal with the forecasting problems in which the historical data are represented by linguistic values rather than traditional time series model. Unfortunately, their methods had many drawbacks such as huge computation when the fuzzy rule matrix is large and lack of persuasiveness in determining the universe of discourse and the length of intervals. Therefore, in order to overcome those disadvantages, Chen [3] proposed the first-order fuzzy time series model by using simple arithmetic calculations to replace max-min composition operations [1, 2] in the process of mining fuzzy logical relationships and performing prediction for better forecasting accuracy. After that, the fuzzy time series methods have received increasing attention in many forecasting applications. To achieve better forecasting accuracy, in [4] presented an effective approach which can properly adjust the lengths of intervals. Subsequently, in order to further enhance forecasting accuracy of model, Chen [5] extended his previous work [3] to a high-order time-invariant fuzzy time series model to forecast the enrolments of the University of Alabama. Yu showed models of refinement relation [6] and weighting scheme [7] for improving forecasting accuracy. Singh [8] presented a simplified and robust computational method for the forecasting rules based on one and various parameters as fuzzy relationships. In addition, in [9] exploited neural

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networks to construct FTS model. The model was used to forecast stock index and obtained better forecasting results. In the recent years, many researchers started applying nature inspired computation techniques for optimization purpose in FTS forecasting. Chen et al. [10, 11]. improved the first-order and high-order fuzzy time series model by introducing genetic algorithm. Lee et al. [12] presented method for forecasting the temperature and the TAIFEX based on the high – order fuzzy logical relationship groups and genetic algorithm. They also used simulated annealing techniques [13] to adjust the length of each interval in the universe of discourse for increasing the forecasting accuracy rate. Particle swarm optimization technique has been successfully applied in many applications as can be found in [14-21]. From Chen's model [3], Kuo et al. [14] introduced a new hybrid forecasting model which combined fuzzy time series with PSO algorithm to find the proper length of each interval. Then, by improving method [14]. Kuo et al. [15] presented a new hybrid forecast method to solve the TAIFEX forecasting problem based on FTS and PSO algorithm. In addition, in [16] proposed a new method for the temperature prediction and the TAIFEX forecasting, based on two-factor high-order fuzzy logical relationships and particle swarm optimization. Singh and Borah [17] also utilized PSO algorithm to construct unequal-sized intervals for developing Type-2 fuzzy model of stock time series on basis of the scheme of supervised learning. Dieu N.C et al. [18, 19] introduced the concept of time-variant fuzzy logical relationship group and used it in the determining of fuzzy logical relationship stage. Huang et al. [20] proposed a new forecasting model based on FTS and PSO by using the global information of fuzzy logical relationships is aggregated with the local information of latest fuzzy fluctuation to find the forecasting value in FTS. Moreover, a novel method of partitioning the universe of discourse of time series based on interval information granules is proposed in [21] for improving forecasting accuracy. Some other techniques for determining best intervals and interval lengths based on clustering techniques such as: the automatic clustering techniques are found [22], and the fuzzy c-means clustering in [23]. Other approaches as, a high-order algorithm for Multi-Variable FTS [24] and a vector autoregressive model for Multi-Variable FTS [25] based on fuzzy clustering are presented to deal various forecasting problems such as: enrolments forecasting, Gas forecasting, Rice produce prediction and Handy-max and Panamax data of the chartering rates of a group of dry bulk cargo ships, respectively.

The above-mentioned researches showed that the lengths of intervals, fuzzy logical relationships and fuzzy defuzzificated techique are three critical factors for forecasting accuracy. Therefore, the objective of the present research is to develop a new model for forecasting in fuzzy time series models which combined the TV-FLRGs is proposed in [18] and PSO algorithm. Firstly, the proposed method fuzzifies the historical data into fuzzy sets to create high-order TV-FLRGs. Secondly, the novel defuzzication rules of forecasting are proposed to calculate the forecasting value for these TV-FLRGs. Finally, a new hybrid forecasting model based on aggregated the high – order TV-FLRGs and PSO algorithm for the optimized lengths of intervals is developed to adjust the lengths of intervals in the universe of discourse with an aim to increase the forecasting accuracy. The empirical study on the enrolments data at the University of Alabama and the stock market dataset of TAIFEX show that the performance of proposed model is better than those of any existing models.

This paper is organized as follows. In Sec. 2, a brief review of the basic concepts of FTS and algorithms are introduced. In Sec. 3, an improved forecasting model based on the high – order TV-FLRGs and PSO algorithm is presented. Section 4 evaluates the forecasting performance of the proposed method with the existing methods on the enrolment data of the University of Alabama and the TAIFEX data. Finally, conclusion remarks are given in Sec. 5.

2. Basic Concepts of Fuzzy Time Series and Algorithms

2.1. Basic Concepts of Fuzzy Time Series

This section briefly reviews the basic fuzzy time series concepts. The main difference between the fuzzy time series and traditional time series is that the values of the fuzzy time series are represented by fuzzy sets rather than real value. Let $\mathbf{U} = \{\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_n}\}$ be an universal set; a fuzzy set A_i of U is defined as $\boldsymbol{A_i} = \{\mu_{\mathbf{Ai}}(\mathbf{u_1})/\mathbf{u_1} +, \mu_{\mathbf{Ai}}(\mathbf{u_2})/\mathbf{u_2} \ldots + \mu_{\mathbf{Ai}}(\mathbf{u_n})/\mathbf{u_n}\}$, where $\mu_{\mathbf{Ai}}$ is a membership function of a given set A, such that $\mu_{\mathbf{Ai}}$:U→[0,1], $\mu_{\mathbf{Ai}}(\mathbf{u_i})$ indicates the grade of membership of u_i in the fuzzy set A, such that $\mu_{\mathbf{Ai}}(\mathbf{u_i}) \in [\mathbf{0}, \mathbf{1}]$, and $1 \le i \le n$ .. General definitions of FTS are given as follows:

Definition 1: Fuzzy time series [1, 2]

Let $Y(t)(t = .., 0, 1, 2 ..)$, a subset of R, be the universe of discourse on which fuzzy sets $f_i(t)$ $(i = 1,2 \ldots)$ are defined and if $F(t)$ be a collection of $f_1(t), f_2(t), \ldots$, then $F(t)$ is called a FTS on $Y(t)(t \ldots, 0, 1,2 \ldots)$. With the help of the following example, the notions of FTS can be explained:

Example: The common observations of daily weather condition for certain area can be described using the daily common words "hot", "very hot", "cold", "very cold", "good", "very good", etc. All these words can be represented by fuzzy sets.

Definition 2: Fuzzy logic relationships (FLRs) [1, 3]

The relationship between F(t) and F(t-1) can be denoted by $F(t-1) \rightarrow F(t)$. Let $A_i = F(t)$ and $A_j = F(t-1)$, the relationship between F(t) and F(t -1) is denoted by fuzzy logical relationship $A_i \rightarrow A_j$ where A_i and A_j refer to the current state or the left - hand side and the next state or the right-hand side of fuzzy time series.

Definition 3: The high- order fuzzy logical relations [5]

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1), F(t-2), \ldots, F(t-m+1)\ F(t-m)$ then this fuzzy relationship is represented by by $F(t-m), \ldots, F(t-2), F(t-1) \rightarrow F(t)$ and is called an m- order fuzzy time series.

Definition 4: Fuzzy logic relationship groups (FLRGs) [3]

Fuzzy logical relationships with the same fuzzy set in the left-hand side of the fuzzy relationships can be grouped into a fuzzy logic relationship group. Suppose there are exists fuzzy logic relationships as follows: $A_i \rightarrow A_{k1}$, $A_i \rightarrow A_{k2}$,.., $A_i \rightarrow A_{km}$; these fuzzy logic relationship can be grouped into the same FLRG as: $A_i \rightarrow A_{k1}, A_{k2}, \ldots, A_{km}$.

The same fuzzy set appear more than once time on the right hand side, according to Chen model [3], it can be only counted one time but Yu model [6], the recurrence of fuzzy set can be admitted.

Definition 5: The concept of Time-variant fuzzy logical relationship groups [18].

The fuzzy logical relationship is determined by the relationship of F(t-1)→ F(t). If, $F(t) = A_i(t)$ and $F(t-1= Aj(t-1)$, The relationship F(t-1)→F(t) is replaced by $A_j(t-1) \rightarrow A_i(t)$. The same way, at the time t, we will have the following fuzzy logical relationship $A_j(t-1) \rightarrow A_i(t); (t1-1) \rightarrow A_{i1}(t1), \ldots$ and $A_j(tp-1) \rightarrow A_{ip}(t)$ with $t1, t2, .., tp \leq t$. It is noted that $A_i(t_1)$ and $A_i(t_2)$ has the same linguistic value as A_i, but appear at different times t_1 and t_2, respectively. It means that if the fuzzy logical relationship took place before $A_j(t-1) \rightarrow A_i(t)$, the fuzzy logical relationships can be grouped into the same FLRG as $A_j(t-1) \rightarrow A_{i1}(t1), A_{i2}(t2), A_{ip}(tp), A_i(t)$ and it is called first – order time-variant fuzzy logical relationship group.

Definition 6: The m – order time-variant fuzzy logical relationship groups [19].

If there are the m - order fuzzy logical relationships having the same left-hand side, shown as follows:

$A_{i1}(t1-m), \ldots, A_{im}(t1-1) \rightarrow A_{k1}(t1)$

……………………………………………

$A_{i1}(tp-m), \ldots, A_{im}(tp-1) \rightarrow A_{kp}(tp)$

The notation $A_{i1}(t1)$, $A_{i2}(t2)$, …, $A_{im}(tp)$ indicate the fuzzy set A_{i1}, A_{i2} …, A_{im} which appear in the m- order fuzzy relationships at time t1, t2,…,tp, respectively.

It can be eliminated the time variable on the left-hand side of the fuzzy logical relationships as follows:

$A_{i1}, A_{i2}, \ldots, A_{im} \rightarrow A_{k1}$

………………………

$A_{i1}, A_{i2}, \ldots, A_{im} \rightarrow A_{kp}$

with t1< t2<…<tp, then these fuzzy logical relationships at the time tp can be grouped into a TV- FLRG, shown as follows:

$A_{i1}, A_{i2}, \ldots, A_{im} \rightarrow A_{k1}, \ldots, A_{kp}$

2.2. Particle Swarm Optimization Algorithm

PSO was first introduced by Eberhart and Kannedy in 1995, is a random searching algorithm based on group cooperation and is inspired by simulating the social behaviour of animals, such as fish schooling, birds flocking and the swarm theory. It is particle swarm optimization initializes each particle randomly, and then finds the optimal solution through iteration. At each step of optimization, the particles update themselves by tracking their own best position and the best particle [14]. To get the optimal solution, the particles update their own speed and positions according to the following formulas:

$$V_{id}^{k+1} = \omega^k * V_{id}^k + C_1 * Rand() * (P_{best\_id} - X_{id}^k) + C_2 * Rand() * (G_{best} - X_{id}^k) \quad (1)$$

$$X_{id}^{k+1} = X_{id}^k + X_{id}^{k+1} \quad (2)$$

$$\omega^k = \omega_{max} - \frac{k*(\omega_{max} - \omega_{min})}{iter\_max} \quad (3)$$

where, X_{id}^k is the current position of a particle id in k-th iteration;

- ✓ V_{id}^k is the velocity of the particle id in k-th iteration, and is limited to $[-V_{max}, V_{max}]$, where V_{max} is a constant pre-defined by user.
- ✓ P_{best_id} is the position of the particle id that experiences the best fitness value.
- ✓ G_{best} is the best one of all personal best positions of all particles within the swarm.
- ✓ Rand() is the function can generate a random real number between 0 and 1 under normal distribution.
- ✓ C_1 and C_2 are acceleration values which represent the selfcondence coefficient and the social coefficient, respectively.
- ✓ ω is the inertia weight factor accoding to Eq. (3).

A briefly description of the standard PSO is summarized in the following algorithm 1.

Algorithm 1: Standard PSO algorithm

1. initialize positions X_{id} and velocities V_{id} of all P_n particles (P_n is the number of particles).

*2. **while** the stop condition (the maximal moving steps are reached) is not satisfied **do***

*2.1 **.for** particle id, (1≤ $i \leq P_n$) **do***

✓ calculate the fitness value of particle id

✓ update the personal best position of particle id according to the fitness value

*end **for***

2.2. update the global best position of all particles according to the fitness value.

*2.3. **for** particle id, (1≤ $i \leq P_n$) **do***

✓ move particle id to another position according to (1) and (2)

*end **for** end **while***

2.3. Time – Variant Fuzzy Logical Relationship Groups Algorithm

Suppose there are fuzzy time series F(t), t =1, 2 ,…, q which it is presented by fuzzy sets as follows:

$$A_{i1},\ A_{i2},\ \ldots,\ A_{iq}.$$

Based on the Definition 5 and 6 of the time - variant fuzzy logical relationship groups, an algorithm for TV-FLRGs is proposed as follows:

Algorithm 2: The m - order TV- FLRGs algorithm

1: initializethe m-order TV–FLRGs t=m;F(1),F(2),..,F(m-1) → F(m) or $A_{j2},\ldots,A_{jm} \rightarrow A_{k1}(m)$
*2: **for**t: = m to q **do***
for** h: = m **down to** 1 **do
Establish all m- order FLRs $A_{j2}(t\text{-}m),\ldots,A_{jm}(t\text{-}1) \rightarrow A_{k1}(t)$
end for
*3: **for** v: = 1 to t-1 **do***
for** h = 1 to v **do
if there are fuzzy logical relationship $A_{j2},\ldots,A_{jm} \rightarrow A_{k2}(h)$ at the same left - hand side Then add A_{k2} into fuzzy logical relationship groups as follows: $A_{j2},\ldots,A_{jm} \rightarrow A_{k1}, A_{k2}$
*end **for** end **for***

3. A New Forecasting Model Based on the TV-FLRGs and PSO Algorithm

In this section, a new forecasting model which combined the high – order TV-FLRGs and PSO algorithm is introduced. In the proposed model, three key aspects have been applied to approach the lengths of intervals and fuzzy logical relationships on time series data to increase forecasting accuracy. Firstly, original historical data are used instead of the variations of historical data in our forecasting model. Second, the high – order TV-FLRGs are derived from the concept of time-variant fuzzy relationship group and calculate the forecasting output based on the fuzzy sets on the right-hand side of the high – order TV-FLRGs. Third, the lengths of optimal intervals are obtained by employing PSO algorithm. The detail of the proposed model is presented as follows:

3.1. A New Forecasting Model Based on the TV-FLRGs

To verify the effectiveness of the proposed model, all historical enrolments data [14] from 1971s to 1992s are used to illustrate the high - order fuzzy time series forecasting process. The step-wise procedure of the proposed model is detailed as follows:

Step 1: Define the universe of discourse U

Assume Y(t) be the historical data of enrolments at year t ($1971 \leq t \leq 1992$). The universe of discourse is defined as $U = [D_{min},\ D_{max}]$. In order to ensure the forecasting values bounded in the universe of discourse U, we set $D_{min} = I_{min} - N_1$ and $D_{max} = I_{max} + N_2$; where I_{min}, I_{max} are the minimum and maximum data of Y(t); N_1 and N_2 are two proper positive integers to tune the lower bound and upper bound of the U. From the historical data [14], we obtain $I_{min} = 13055$ và $I_{max} = 19337$. Thus, the universe of discourse is defined as U= $[I_{min} - N_1, I_{max} + N_2]$= [13000, 20000] with $N_1 = 55$ and $N_2 = 663$.

Step 2: Partition U into equal length intervals

Divide U into equal length intervals. Compared to the previous models in [3] and [14], we cut U into seven intervals, $u_1, u_2, \ldots, u_7$, respectively. The length of each interval is $L = \frac{D_{max} - D_{min}}{7} = \frac{20000-13000}{7} = 1000$. Thus, the seven intervals are defined as follows:

u_i = [13000 +(i-1)*L, 13000 + i *L), with $(1 \leq i \leq 7)$ gets seven intervals as:

u_1 = [13000, 14000), u_2 = [14000,15000), …, u_6 = [18000,19000), u_7 = [19000, 20000).

Step 3: Define the fuzzy sets for observations

Each of interval in Step 2 represents a linguistic variable of "enrolments" in [3]. For seven intervals, there are seven linguistic values which are A_1= "not many", A_2="not too many", A_3="many", A_4= "many many", A_5= "very many", A_6= "too many", and A_7="too many many" to represent different areas in the universe of discourse on U, respectively. Each linguistic variable represents a fuzzy set A_i and its definitions is described in (4) and (5) as follows.

$$A_i = {a_{i1}}/{u_1} + {a_{i2}}/{u_2} + \ldots + {a_{ij}}/{u_j} + \cdots + {a_{i7}}/{u_7} \quad (4)$$

$$a_{ij} = \begin{cases} 1 & j = i \\ 0.5 & j = i-1 \ or\ j = i+1 \\ 0 & otherwise \end{cases} \quad (5)$$

Here, the symbol '+' denotes the set union operator, $a_{ij} \in [0,1]$ $(1 \leq i \leq 7, 1 \leq j \leq 7)$, u_j is the j-th interval of U. The value of a_{ij} indicates the grade of membership of u_j in the fuzzy set A_i. For simplicity, the different membership values of fuzzy set A_i are selected by according to Eq. (5). According to Eq. (4) and (5), a fuzzy set contains 7 intervals. Contrarily, an interval belongs to all fuzzy sets with different membership degrees. For example, u_1 belongs to A_1 and A_2 with membership degrees of 1 and 0.5 respectively, and other fuzzy sets with membership degree is 0.

Step 4: Fuzzy all historical enrolments data

In order to fuzzify all historical data, it's necessary to assign a corresponding linguistic value to each interval first. The simplest way is to assign the linguistic value with respect to the corresponding fuzzy set that each interval belongs to with the highest membership degree. For example, the historical enrolment of year 1972 is 13563, and it belongs to interval u_1 because 13563 is within [13000, 14000). So, we then assign the linguistic value ''not many" (eg. the fuzzy set A_1) corresponding to interval u_1 to it. Consider two time serials data $Y(t)$ and $F(t)$ at year t, where $Y(t)$ is actual data and $F(t)$ is the fuzzy set of $Y(t)$. According to Eq. (4), the fuzzy set A_1 has the maximum membership value at the interval u_1. Therefore, the historical data time series on date Y(1972) is fuzzified to A_1. The completed fuzzified results of the enrolments are listed in Table 1.

Table 1. The results of fuzzification according to enrolments data

Year	Actual data	Fuzzy sets	Membership degree
1971	13055	A1	[1 0.5 0 0 0 0 0]
1972	13563	A1	[**1** 0.5 0 0 0 0 0]
1973	13867	A1	[**1** 0.5 0 0 0 0 0]
1974	14696	A2	[0.5 **1** 0.5 0 0 0 0]
1975	15460	A3	[0 0.5 **1** 0.5 0 0 0]
1976	15311	A3	[0 0.5 **1** 0.5 0 0 0]
1977	15603	A3	[0 0.5 **1** 0.5 0 0 0]
1978	15861	A3	[0 0.5 **1** 0.5 0 0 0]
1979	16807	A4	[0 0 0.5 **1** 0.5 0 0]
1980	16919	A4	[0 0 0.5 **1** 0.5 0 0]
1981	16388	A4	[0 0 0.5 **1** 0.5 0 0]
1982	15433	A3	[0 0.5 **1** 0.5 0 0 0]
1983	15497	A3	[0 0.5 **1** 0.5 0 0 0]
1984	15145	A3	[0 0.5 **1** 0.5 0 0 0]
1985	15163	A3	[0 0.5 **1** 0.5 0 0 0]
1986	15984	A3	[0 0.5 **1** 0.5 0 0 0]
1987	16859	A4	[0 0 0.5 **1** 0.5 0 0]
1988	18150	A6	[0 0 0 0 0.5 **1** 0.5]
1989	18970	A6	[0 0 0 0 0.5 **1** 0.5]
1990	19328	A7	[0 0 0 0 0 0.5 **1**]
1991	19337	A7	[0 0 0 0 0 0.5 **1**]
1992	18876	A6	[0 0 0 0 0 0 **1** 0.5]

Step 5. Create all m – order fuzzy logic relations ($m \geq 1$).

Based on Definition 2 and 3, one fuzzy relationship is built by two or more consecutive fuzzy sets in time series. To establish a fuzzy logical relationship with various orders, we should find out any relationship which has the type $\mathrm{F}(\mathrm{t}-m), \mathrm{F}(\mathrm{t}-m+1), \ldots, \mathrm{F}(\mathrm{t}-1) \rightarrow F(t)$, where $\mathrm{F}(\mathrm{t}-m), \mathrm{F}(\mathrm{t}-m+1), \ldots, \mathrm{F}(\mathrm{t}-1)$ and $F(t)$ are called the current state and the next state, respectively. Then a m - order fuzzy logical relationship is got by replacing the corresponding linguistic values as follows: $\mathrm{A}_{\mathrm{im}}, \mathrm{A}_{\mathrm{i(m-1)}}, \ldots, \mathrm{A}_{\mathrm{i2}}, \mathrm{A}_{\mathrm{i1}} \rightarrow \mathrm{A}_{\mathrm{k}}$. Two examples for first-order and three-order are illustrated as follows.

In the case of m = 1, two consecutive fuzzy sets are used to form a first – order fuzzy logical relationship. For example, based on Table 1, one fuzzy relationship $\mathrm{A}_1 \rightarrow \mathrm{A}_2$ is created by replacing the historical data of F(1973) and F(1974) with linguistic values of A_1 and A_2, respectively. All first-order fuzzy relationships from year 1972 to 1992 are shown in column 3 of Table 2.

Similarly, in the case of m = 3, four consecutive fuzzy sets are used to form a three – order fuzzy logical relationship. For example, based on Table 1, a fuzzy relationship $\mathrm{A}_1, \mathrm{A}_1, \mathrm{A}_1 \rightarrow \mathrm{A}_2$ is got as $\mathrm{F}(1971), \mathrm{F}(1972), \mathrm{F}(1973) \rightarrow \mathrm{F}(1974)$, respectively. All three-order fuzzy logical relationships from year 1974 to 1992 are shown in column 4 of Table 2. If the linguistic value of the next state does not exist in the historical data, the symbol '#' is used to denote the unknown linguistic value. The fuzzy logical relationship with unknown linguistic value of the next state is used for testing. For example, a first-order fuzzy logical relationship is F(1992) → F(1993) where the linguistic value of F(1993) is unknown. Therefore, the fuzzy relationship is expressed as $\mathrm{A}_6 \rightarrow \#$.

Table 2. The complete the first - order and three – order fuzzy logical relationships

Years	Fuzzy set	First –order FLRs	Three –order FLRs
1971	A1	---	---
1972	A1	A1 → A1	
1973	A1	A1 → A1	
1974	A2	A1 → A2	A1, A1, A1 → A2
1975	A3	A2 → A3	A1, A1, A2 → A3
1976	A3	A3 → A3	A1, A2, A3 → A3
1977	A3	A3 → A3	A2, A3, A3 → A3
1978	A3	A3 → A3	A3, A3, A3 → A3
1979	A4	A3 → A4	A3, A3, A3 → A4
1980	A4	A4 → A4	A3, A3, A4 → A4
1981	A4	A4 → A4	A3, A4, A4 → A4
1982	A3	A4 → A3	A4, A4, A4 → A3
1983	A3	A3 → A3	A4, A4, A3 → A3
1984	A3	A3 → A3	A4, A3, A3 → A3
1985	A3	A3 → A3	A3, A3, A3 → A3
1986	A3	A3 → A3	A3, A3, A3 → A3
1987	A4	A3 → A4	A3, A3, A3 → A4
1988	A6	A4 → A6	A3, A3, A4 → A6
1989	A6	A6 → A6	A3, A4, A6 → A6
1990	A7	A6 → A7	A4, A6, A6 → A7
1991	A7	A7 → A7	A6, A6, A7 → A7
1992	A6	A7 → A6	A6, A7, A7 → A6
1993	N/A	A6 → #	A7, A7, A6 → #

Step 6**:** Establish all m- order time – variant fuzzy logical relationship groups

In this step, a new method is different from the previous approaches in the way where the fuzzy logical relationship groups are created. In previous studies [3, 14] all the fuzzy logical relationships having the same fuzzy set on the left-hand side or the same current state can be grouped into a same fuzzy logical relationship group. But, according to the Definition 5, 6 and algorithm 2 in Subsection 2.3, the appearance history of the fuzzy sets on the right-hand side of fuzzy logical relationships with the same current state is need to more consider. That is, only the element on the right - hand side appearing before the element on the left-hand side of the fuzzy logical relationship at forecasting time is put together to form fuzzy logical relationship group. For example, suppose that there two first – order fuzzy logical relationships with the left – hand side as follows: $\mathrm{A}_\mathrm{i}, \rightarrow \mathrm{A}_\mathrm{j}$; $\mathrm{A}_\mathrm{i} \rightarrow \mathrm{A}_\mathrm{k}$. These fuzzy logical relationships can be grouped together into two group G1 and G2 in chronological order are listed as follows: G1: $\mathrm{A}_\mathrm{i}, \rightarrow \mathrm{A}_\mathrm{j}$; G2: $\mathrm{A}_\mathrm{i} \rightarrow \mathrm{A}_\mathrm{j}, \mathrm{A}_\mathrm{k}$. From this viewpoint and based on Table 2, we can obtain 22 the first -order TV- FLRGs are shown in column 2 of Table 3. Where, the first 21 groups of the first – order fuzzy logical relationship groups are called the trained patterns (or in training phase), and the last one is called the untrained pattern (or in testing phase). Similarly, we can establish m – order time – variant FLRGs based on Definition 6. For example, assume m=3 and there two 3rd – order fuzzy logical relationships with the left – hand side as follows: A_i, A_j, $A_k \rightarrow A_p$; A_i, A_j, $A_k \rightarrow A_q$. These fuzzy logical relationships can be grouped together into two group G1 and G2 in chronological order are listed as follows: G1: A_i, A_j, $A_k \rightarrow A_m$; G2: A_i, A_j, $A_k \rightarrow A_p$, A_q. From column 4 of Table 2 and based on Definition 6, all the three-order time – variant FLRGs are shown in column 3 of Table 3.

Table 3. The complete the first - order and three – order fuzzy logical relationship groups

No group	First –order TV-FLRGs	Three –order TV-FLRGs
1	A1 → A1	
2	A1→ A1, A1	
3	A1 → A1, A1, A2	A1, A1, A1 → A2
4	A2 → A3	A1, A1, A2 → A3
5	A3 → A3	A1, A2, A3 → A3
6	A3 → A3, A3	A2, A3, A3 → A3
7	A3 → A3, A3, A3	A3, A3, A3 → A3
8	A3 → A3, A3, A3, A4	A3, A3, A3 → A3, A4
9	A4 → A4	A3, A3, A4 → A4
10	A4 → A4, A4	A3, A4, A4 → A4
11	A4 → A4, A4, A3	A4, A4, A4 → A3
12	A3 → A3, A3, A3, A4, A3	A4, A4, A3 → A3
---	--------------------------	---------------------
18	A6 → A6	A3, A4, A6→ A6
19	A6 → A6, A7	A4, A6, A6 →A7
20	A7 → A7	A6, A6, A7 → A7
21	A7 → A7, A6	A6, A7, A7 → A6
22	A6 → #	A7, A7, A6 → #

***Step** 7*. Defuzzify and calculate the forecasting values for all the TV- FLRGs

To defuzzify the fuzzified data and calculate the forecasted values for all first – order and high – order TV - FLRGs, the new defuzzification techniques are developed to calculate the forecasted values for all TV-FLRGs with different orders in training phase. Then we also use defuzzification rule is proposed in [14] for the TV-FLRGs in testing phase. The forecasted values are calculated by the following rules:

Rule 1: In the case of first – order TV-FLRGs

To calculate the forecasted values for all first - order fuzzy logical relationship groups. We consider the appearance of fuzzy sets on the right - hand side in the same group and assign different weights for each fuzzy set in chronological order. Assume that there is the first – order fuzzy logical relationship group whose current state is A_j, shown as follows:

$$A_j(t-1) \rightarrow A_{i1}(t1), A_{i2}(t2), \ldots, A_{ik}(tk)\ldots$$

then the forecasted value of year t is calculated as follows:

$$forecasted = \frac{1 * m_{i1} + 2 * m_{i2} + \cdots + k * m_{ik} + \cdots + p * m_{ip}}{1 + 2 + \cdots + k + \cdots + p}$$

where, - m_{i1}, m_{i2}, m_{ik} are the middle values of the intervals u_{i1}, u_{i2} and u_{ik} respectively, and the maximum membership values of A_{i1}, A_{i2}, ... ,A_{ik} occur at intervals u_{i1}, u_{i2}, u_{ik}, respectively.

k $(1 \leq k \leq p)$ is chronologically determined weights.

For example, the forecasted enrolments of the years 1974 is calculated as follows: From column 2 of Table 2, we can see that the fuzzified enrolments of year 1973 is A1. From column 2 Table 3, we can see that there is a fuzzy logical relationship group A1 → A1, A1, A2 that receives from three fuzzy logical relationships ‘‘A1 → A1, A1 → A1, A1 → A2” in chronological order are 1972, 1973 and 1974, respectively. Then, we can assign different weights for each FLR incrementally, say 1, 2, and 3 (the recent FLR is assigned the highest weight of 3). Therefore, the forecasted enrolments of year 1974 is calculated as follows:

$$\text{forecasted} = \frac{1 * m_1 + 2 * m_1 + 3 * m_2}{1 + 2 + 3} = 14000$$

Where, $m_1 = 13500$, $m_2 = 14500$ are the middle values of the intervals u_1, u_2 respectively. Following the above example, the complete forecasted values for all the first - order FLRGs in column 2 of Table 3 are listed in Table 4.

Rule 2: In the case of high – order TV-FLRGs

In order to estimate all forecasting values for all high – order TV-FLRGs, we consider more information within all next states or fuzzy sets on the right-hand side of all fuzzy relationships in the same group.

The viewpoint of this principle is presented as following. For each group in column 3 of Table 3, we divide each corresponding interval of each next state into p sub-intervals with equal size, and calculate a forecasted value for each group according to Eq. (6).

$$\text{forecasted}_{\text{output}} = \frac{1}{n}\sum_{j=1}^{n} subm_{kj} \tag{6}$$

where, ($1 \leq j \leq n$, $1 \leq k \leq p$)

- ✓ n is the total number of next states or the total number of fuzzy sets on the right-hand side within the same group.
- ✓ $subm_{kj}$ is the midpoint of one of p sub-intervals (*means the midpoint of j-th sub-interval)* corresponding to j-th fuzzy set on the right-hand side where the highest level of A_{kj} occur in this interval.

For example, in column 3 of Table 3, Group 1 of three – order FLRs has only one fuzzy set on the right-hand side as $A1, A1, A1 \rightarrow A2$ where the highest membership level of A2 belongs to interval $u_2 = [14000, 15000)$. In this study, we divide the interval u_2 into four sub-intervals which are

$u_{2,1} = [14000, 14250)$, $u_{2.2} = [14250, 14500)$, $u_{2.3} = [14500, 14750)$, $u_{2.4} = [14750, 15000)$. In Table 3, the three-order fuzzy logical relationship group $A1, A1, A1 \rightarrow A2$ is got as F(1971), F(1972), F(1973) → F(1974); where the historical data of year 1974 is 14696 and it is within sub-interval $u_{2,3} = [14500, 14750)$ and then the midpoint $subm_{2.3}$ of sub-interval $u_{2.3}$ is 14625. The finally, forecasted value for Group 1 according to Eq. (6) is 14625. Forecasted value of all remaining three – order TV- FLRGs are calculated in a similar manner and shown in Table 5.

Rule 3: In the case of FLRGs is empty (*called the untrained pattern*)

To estimate the forecasted value for the untrained pattern in testing phase, we use defuzzification rule is proposed in [14] whose name as mater voting (MV) scheme. For FLRG which contains the unknown linguistic value of the next, the MV scheme gives the highest votes (weights) to the latest past and one vote to other past linguistic values in the current state respectively, and calculates a forecasted value based on Eq. (7) as follows:

$$Forecated_{for\,\#} = \frac{(M_{t1} * w_h) + M_{t2} + \cdots + M_{ti} + \cdots + M_{tm}}{w_h + (\text{m}-1)} \quad (7)$$

Where; the symbol w_h means the highest votes predefined by user, the symbol m is the order of the fuzzy logical relationship, the symbols M_{t1} and M_{ti} denote the midpoints of the corresponding intervals of the latest past and other past linguistic values in the current state. From column 3 of Table 3, it can be shown that last group has the three - order fuzzy logical relationship A7, A7, A6 → # as it is created by the fuzzy relationship F(1990), F(1991), F(1992) → F(1993); since the linguistic value of F(1993) is unknown within the historical data, and this unknown next state is denoted by the symbol '#'. Then, calculating value for "#" based on the current state of this group is computed by Eq. (7). The result of group with unknown next state under w_h of 15 is shown in Table 5.

Table 4. The complete forecasted value for all first –order TV-FLRGs

No group	First –order TV- FLRGs	Forecasted value
1	A1 → A1	13500
2	A1→ A1, A1	13500
3	A1 → A1, A1, A2	14000
4	A2 → A3	15500
5	A3 → A3	15500
6	A3 → A3, A3	15500
7	A3 → A3, A3, A3	15500
8	A3 → A3, A3, A3, A4	15900
9	A4 → A4	16500
10	A4 → A4, A4	16500
11	A4 → A4, A4, A3	16000
12	A3 → A3, A3, A3, A4, A3	15766.7
13	A3 → A3, A3, A3, A4, A3, A3	15690.5
14	A3 → A3, A3, A3, A4, A3, A3, A3	15642.9
15	A3 → A3, A3, A3, A4, A3, A3, A3, A3	15611.1
16	A3 → A3, A3, A3, A4, A3, A3, A3, A3, A4	15788.9
17	A4 → A4, A4, A3, A6	17000
18	A6 → A6	18500
19	A6 → A6, A7	19166.7
20	A7 → A7	19500
21	A7 → A7, A6	18833.3

Table 5. The complete forecasted value for all three –order TV-FLRGs

No group	Three –order TV- FLRGs	Forecasted value
1	A1, A1, A1 →A2	14625
2	A1, A1, A2 → A3	15375
3	A1, A2, A3 → A3	15375
4	A2, A3, A3 → A3	15625
5	A3, A3, A3 → A3	15875
-----	--------------	-----
16	A3, A4, A6 → A6	18875
17	A4, A6, A6 → A7	19375
18	A6, A6, A7 → A7	19375
19	A6, A7, A7 → A6	18875
20	A7, A7, A6 → #	18667

Table 6. The complete forecasted results based on fuzzy time series model with first order and 3rd – order TV-FLRGs under seven intervals

Year	Actual data	Fuzzy set	Forecasted value	
			First order	Third-order
1971	13055	A1	---	---
1972	13563	A1	13500	---
1973	13867	A1	13500	---
1974	14696	A2	14000	14625
1975	15460	A3	15500	15375
1976	15311	A3	15500	15375
1977	15603	A3	15500	15625
1978	15861	A3	15500	15875
1979	16807	A4	15900	16375
1980	16919	A4	16500	16875
1981	16388	A4	16500	16375
1982	15433	A3	16000	15375
1983	15497	A3	15767	15375
1984	15145	A3	15691	15125
1985	15163	A3	15643	15958
1986	15984	A3	15611	15938
1987	16859	A4	15789	16125
1988	18150	A6	17000	17500
1989	18970	A6	18500	18875
1990	19328	A7	19167	19375
1991	19337	A7	19500	19375
1992	18876	A6	18833	18875
1993	N/A	N/A	18500	18667

Based on results of Table 4, Table 5 and Table 1, we complete forecasted output results for the enrolments of University of Alabama the period from 1971 to 1992 based on first – order and third-order fuzzy time series model with seven intervals are listed in Table 6.

To evaluate the performance of the proposed model, the mean square error (MSE) is employed as an evaluation criterion to represent the forecasted accuracy. The MSE value is calculated as follows:

$$MSE = \frac{1}{n}\sum_{i=m}^{n}(F_i - R_i)^2 \quad (8)$$

where, R_i denotes actual data at year i, F_i is forecasted value at year i, n is number of the forecasted data, m is order of the fuzzy logical relationships.

3.2. Forecasting Model Combining the TV-FLRGs and PSO Algorithm

To improve forecasted accuracy of the proposed model, the effective lengths of intervals, TV- FLRGs and defuzzification techniques which are three main issues presented in this paper. A novel hybrid method for forecasting enrolments is developed to adjust the length each of intervals in the universe of discourse without increasing the number of intervals by minimizing the MSE value (8).

In our model, each particle exploits the intervals in the universe of discourse of historical data Y(t). Let the number of the intervals be n, the lower bound and the upper bound of the universe of discourse U on historical data Y(t) be b_0 and b_n, respectively. Each particle is a vector consisting of n-1 elements b_i where $1 \le i \le n-1$ and $b_i \le b_{i+1}$. Based on these n-1 elements, define the n intervals as $u_1 = [b_0, b_1]$, $u_2 = [b_1, b_2]$,,..., $u_i = [b_{i-1}, b_i]$,... and $u_n = [b_{n-1}, b_n]$, respectively. When a particle moves to a new position, the elements of the corresponding new vector need to be sorted to ensure that each element bi arranges in an ascending order. The complete steps of the proposed method are presented in Algorithm 3.

Algorithm 3

initialize positions X_{id} and velocities V_{id} of all P_n particles

2. **while** the stop condition (maximum iterations or minimum MSE criteria) is not satisfied **do**

2.1**.** **for** particle id, ($1 \le i \le P_n$) **do**

- ✓ Define linguistic terms based on the current position of particle id
- ✓ Fuzzify all historical data by Step 4 in Subsection 3.1
- ✓ Create all m − order fuzzy logical relationships by Step 5 in Subsection 3.1
- ✓ Construct all m − order time -variant fuzzy relationship groups by Step 6 in Subsection 3.1
- ✓ Calculate and defuzzification forecasting outputs by Step 7 in Subsection 3.1
- ✓ Compute the MSE values for particle id based on Eq. (8)
- ✓ Update the personal best position of particle id according to the MSE values mentioned above.

end for

2.2. Update the global best position of all particles according to the MSE values mentioned above.

3. **for** particle id, ($1 \le i \le p_n$) **do**

✓ move particle id to another position according to (1) and (2)

end for

✓update ω according to Eq. (3)

end while

4. Experimental Results

4.1. Prepared Data

In this paper, the proposed method is applied to forecast the enrolments of the University of Alabama with the entire historical data from 1971 to 1992 [14] and compared to other forecasting models with the same set of training data, training and testing phases. In addition, the proposed method is also tested in other forecasting problems such as the stock market index of TAIFEX [13] with the historical data from 8/3/1998 to 9/30/1998. Without loss of generality and for simplicity of comparison, the necessary parameters of the proposed model are set the same as HPSO model [14] for forecasting enrolments and NPSO model [15] for forecasting TAIFEX. The detail parameters for each of data set are listed and explained in Table 7.

Table 7. Parameters used for forecasting enrolments and TAIFEX

The essential parameters	Value of enrolments	value of TAIFEX
Number of particles: ***N*** =	30	30
Maximum number of iterations N_{max}	100	100
The value of inertial weigh ω	1.4 to 0.4	1.4 to 0.4
The coefficient $C_1 = C_2$	2	2
The velocity be limited to: ***V*** =	[-100,100]	[-50,50]
The position be limited to: ***X*** =	[13000,20000]	[6200,7600]

Where, - V is the velocity of each particle which is bounded within an appropriate range after updating.

- X is the position of each particle which is bounded within an appropriate range (searching space) after updating.
- The value of inertial weight factor ω is linearly decreased from 1.4 to 0.4 during the updating the velocity of the particles.

4.2. Experimental Results for Forecasting Enrolments

To evaluate the effectiveness of the proposed model with the number of intervals is 14 and the first - order fuzzy logical relationships, six forecasting models i.e., the **SCI** model [2], **C96** model [3], **H01** model [4], **CC06a** model [10], **HPSO** model [14], **Wei Lu'**s et al. model [21] and **AFPSO** model [20] are selected for comparison. It is noted that the entire forecasting models employ the first-order fuzzy logical relationships with different number of intervals. The forecasted accuracy of the proposed model is estimated using the MSE function (8). According to the parameter values in column 2 of Table 7, the proposed model is executed 20 runs. In twenty optimal runs, our forecasting model can be get the 20 different MSE values. The smallest MSE value is chosen as the solution with an acceptable forecasting accuracy. A comparison of the forecasted results between the proposed and existing methods is listed in Table 8.

Table 8. A comparison of the forecasted results of the proposed model with the existing models with first-order FLRs under various number of intervals

Year	Actual data	SCI [2]	C96 [3]	H01 [4]	CC06a [10]	HPSO [14]	Wei Lu [21], h_{max} =22	AFPSO [20]	Proposed model
1971	13055	--	--	--	--	--	--	--	--
1972	13563	14000	14000	14000	13714	13555	13512	13579	13433
1973	13867	14000	14000	14000	13714	13994	13998	13812	13856
1974	14696	14000	14000	14000	14880	14711	14601	14565	14712
1975	15460	15500	15500	15500	15467	15344	15462	15422	15381
1976	15311	16000	16000	15500	15172	15411	15305	15307	15381
1977	15603	16000	16000	16000	15467	15411	15641	15618	15600
1978	15861	16000	16000	16000	15861	15411	15827	15660	15933
1979	16807	16000	16000	16000	16831	16816	16715	16794	16823
1980	16919	16813	16833	17500	17106	17140	17212	17032	16957
1981	16388	16813	16833	16000	16380	16464	16392	16390	16388
1982	15433	16789	16833	16000	15464	15505	15409	15504	15381
1983	15497	16000	16000	16000	15172	15411	15514	15431	15490
1984	15145	16000	16000	15500	15172	15411	15037	15077	15179
1985	15163	16000	16000	16000	15467	15344	15195	15297	15242
1986	15984	16000	16000	16000	15467	16018	16054	15848	15933
1987	16859	16000	16000	16000	16831	16816	16861	16835	16823
1988	18150	16813	16833	17500	18055	18060	18024	18145	18021
1989	18970	19000	19000	19000	18998	19014	19036	18880	18970
1990	19328	19000	19000	19000	19300	19340	19241	19418	19486
1991	19337	19000	19000	19500	19149	19340	19666	19260	19486
1992	18876	19000	19000	19149	19014	19014	18718	19031	18864
MSE		**423027**	**407507**	**226611**	**35324**	**22965**	**14534**	**8224**	**5396**

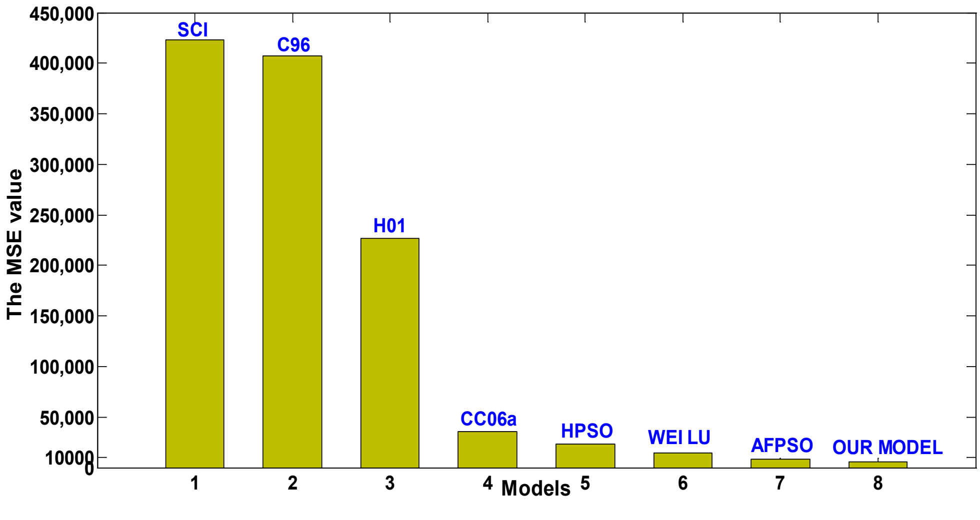

Figure 1. A comparison of the MSE value between proposed model and the existing models based on first – order FTS

As shown in Table 8, the proposed model shows better forecasted accuracy than previous forecasting ones with the value MSE is 5396 which is smallest among all the compared models, where the **HPSO** model [14], **AFPSO** model [20] and our model all use number of intervals is 14. For **Wei Lu'**s model [21] and proposed model, both of them use the PSO algorithm, but our proposed model obtains smaller MSE value in forecasting. The major difference between the Wei Lu's et al model and our model is in the way where the fuzzy logical relationship groups and forecasted rule are created, while Wei Lu's model [21] use the interval information granules to partition the universe of discourse in fuzzification stage that aim to decompose complex problems into simple problems. The above demonstrations showed that the proposed model is more superior than the existing models with first- order fuzzy time series model under different number of intervals in forecasting enrolments of University of Alabama. To be clearly visualized, Fig.1 depicts the trends for forecasting accuracy of the proposed model with existing models by the MSE value.

In addition, our proposed method is also compared with previous models based on the first – order fuzzy logical relationships with different number of intervals, such as: CC06a model [10], HPSO model [14], AFPSO model [20]. The simulation results according to the intervals of each model are presented in Table 9, where the proposed method shows better forecasting accuracy than CC06a model using GA algorithm based the first – order FTS for all intervals. For the HPSO model and the AFPSO model, both of them use the PSO algorithm as the same our model but our model gets the lower MSE values which are 20322, 15472, 12588, 7078 and 5396 for number of intervals is 10, 11, 12, 13,14, respectively. The major difference among three models is in the way where the fuzzy logical relationship groups and forecasted rule they used. The two former models establish all fuzzy logical relationship groups based on Chen's method [3], but our model build fuzzy logical relationship groups from the concept of time-variant fuzzy logical relationship group. This finding suggests that the creation of fuzzy logical relationship groups and consideration of more information within all fuzzy sets on the right-hand of fuzzy logical relationship groups to calculate the forecasting results is an important factor in forecasting model.

Table 9. A comparison of the forecasted accuracy among the existing models and the proposed model based on first – order FTS with different number of intervals

Method	**Number of intervals**						
	8	**9**	**10**	**11**	**12**	**13**	**14**
CC06a	132963	96244	85486	55742	54248	42497	35324
HPSO	119962	90527	60722	49257	34709	24687	22965
AFPSO	27435	24860	19698	19040	16995	11589	8224
Our model	**33983**	**25841**	**20322**	**15472**	**12588**	**7078**	**5396**

Table 10. A comparison of the forecasted results of the proposed method with the existing models based on high – order of the fuzzy time series under different number of intervals

Years	**Actual data**	**S07s**	**C02**	**CC06b**	**HPSO**	**AFPSO**	**Our model**
1971	13055	N/A	N/A	N/A	N/A	N/A	N/A
1972	13563	N/A	N/A	N/A	N/A	N/A	N/A
1973	13867	N/A	N/A	N/A	N/A	N/A	N/A
1974	14696	N/A	N/A	N/A	N/A	N/A	N/A
1975	15460	15500	N/A	N/A	N/A	N/A	N/A
1976	15311	15468	15500	N/A	N/A	N/A	N/A
1977	15603	15512	15500	N/A	N/A	N/A	N/A
1978	15861	15582	15500	N/A	N/A	N/A	N/A
1979	16807	16500	16500	16846	N/A	N/A	N/A
1980	16919	16361	16500	16846	16890	16920	16919
1981	16388	16362	16500	16420	16395	16388	16390
1982	15433	15744	15500	15462	15434	15467	15435
-----	-----	------	-----	-----	------	------	------
1990	19328	19382	19500	19334	19337	19338	19334
1991	19337	19487	19500	19334	19337	19335	19334
1992	18876	18744	18500	18910	18882	18882	18872
MSE		**133700**	**86694**	**1101**	**234**	**173**	**9.23**

In order to verify the forecasting effectiveness of the proposed model under different number of intervals and different high - order FLRGs , five FTS models, named C02 in [5], CC06b in [11], S07s in[8], HPSO in [14] and the AFPSO in [20], are examined and compared. The forecasted accuracy of the proposed model is estimated using the MSE function (8). From the parameters are expressed in column 2 of Table 7. Our proposed model is simulated 20 runs. At the end of the process run, twenty different MSE values were obtained. The smallest MSE value is taken as the optimal solution to be compared. A comparison of the forecasting accuracy with various orders and different number of intervals among the C02, CC06b, S07s, HPSO, AFPSO models and the proposed model, are listed in Table 10, where three models; the HPSO model, the AFPSO model and our model use 9th-order fuzzy relationships and 14 intervals to forecast the enrolments of University of Alabama.

From Table 10, it is obvious that our model has a MSE value is **9.23** which is the lowest among all forecasting models compared. The main difference among all the compared models is the fuzzy logical relationship group algorithms used to forecast. Five fuzzy forecasting models in C02 [5], CC06b [11], S07s [8], HPSO [14] and the AFPSO [20] used the time – invariant fuzzy relation groups algorithm to defuzzify forecasting output, while this study has proposed a method that benefits from the time - variant fuzzy logical relationship groups algorithm. As shown in Table 10, three of models; the HPSO, AFPSO and our model all use the PSO algorithm, but our proposed model gets smaller MSE values in forecasting. The MSE value is calculated according to Eq. (8) as follows:

$$MSE = \frac{1}{13}((16919 - 16919)^2 + \cdots + (18872 - 18876)^2) = 9.23$$

Furthermore, we also perform 10 more runs in each order to be compared with various high-order forecasting models under seven intervals such as **C02** model in [5]. **CC06b** model in [11], **HPSO** model in [14] and **AFPSO** model in [20]. The detail of comparison is shown in Table 11. The trend in forecasting of enrolments based on the high - order FTS under various orders by MSE value can be visualized in Fig.2.

During the simulation, the number of intervals is kept (the number of intervals = 7) with different high – order FTS for the existing models and our model. A comparing of MSE value is listed in Table 10. From Table 11, it can be seen that the accuracy of the proposed model is improved significantly. Particularly, our model gets the lowest MSE value of **358.43** with 8th-order fuzzy relations and the average MSE value of the proposed model is **1657.2**, which is smallest among five forecasting models compared.

4.3. Experimental Results for Forecasting TAIFEX

The historical data of the TAIFEX [13] are used to perform comparative study in the training and testing phases. In order to verify forecasting effectiveness, the proposed model is compared with those of corresponding models for various orders and different intervals. The forecasted accuracy of the proposed method is estimated using the function MSE (8).

Table 11. A comparison of the MSE value between our model and C02 model, CC06b model, HPSO model, AFPSO model under different number of orders and the number of interval is **7**

Models	Number of orders								Average
	2	3	4	5	6	7	8	9	
C02	89093	86694	89376	94539	98215	104056	102179	102789	**95868**
CC06b	67834	31123	32009	24984	26980	26969	22387	18734	**31373**
HPSO	67123	31644	23271	23534	23671	20651	17106	17971	**28121**
AFPSO	19594	31189	20155	20366	22276	18482	14778	15251	**20261**
Our model	**8836.2**	**822.47**	**686.39**	**658.18**	**659.14**	**618.9**	**358.43**	**617.8**	**1657.2**

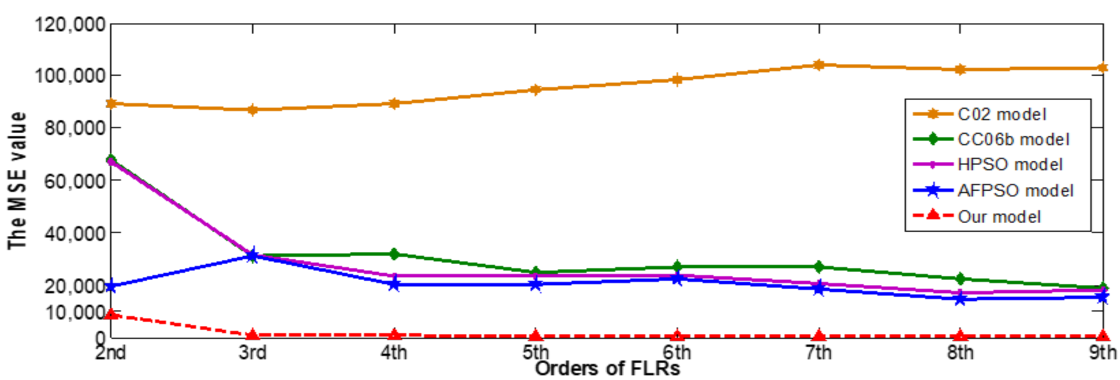

Figure 2. A comparison of the MSE values for 7 intervals with various high-order FLRGs

4.3.1. Experimental Results in the Training Phase

In this subsection, the proposed method is applied to forecast the TAIFEX from 8/3/1998 to 9/30/1998. To verify the superiority of the proposed model under various high-order FLRGs and different numbers of intervals, existing forecasting model, viz., **C96** model [3], **H01b** model [4], **L06** model [12], **L08** model [13], **HPSO** model [14], **NPSO** model [15] and **MTPSO** model [16] and are selected for comparison. During simulation with parameters are expressed in column 3 of Table 7, the number of intervals is kept fix (number of intervals =16) for the existing model and the proposed model. A comparison of the forecasted results using MSE criteria in (8) is shown in Table 12.

As shown in Table 12, the proposed model gets the smallest forecasting error rate by the MSE value among eight forecasting models, when applied to the TAIFEX datasets. More detail comparison, at the same intervals of 16 and different orders FLRs, our model gets the lowest MSE value of **7.98** among four models using the PSO technique such as: HPSO [14], MTPSO [16], NPSO [15] models. Although these models use also PSO technique to catch the proper length of each interval, but the main difference among all the compared models is in the way where the fuzzy logical relationship groups and forecasted rule are created. In addition, it can be seen that the proposed model has smaller MSE value than the L06 and L08 models with different number of orders. The main difference between our model and L06 model [12], L08 model [13] is that the former uses the PSO algorithm but the two models latter use the genetic algorithm (GA) to get the appropriate intervals, respectively. From forecasting results in Table 12, it is clear that the PSO algorithm is more efficiently than the GA algorithm in achieving the appropriate interval lengths. Furthermore in this paper, we also rebuilt three HPSO [14], NPSO [15], AFPSO [20] models are considered to be quite effective in recent years and compare the forecasting accuracy of these models with the proposed model on the same historical data of the TAIFEX with different number of samples as 15, 20, 25, 30, 35, 40, 45 and 47. The results of comparison is presented in Table 13 and Fig.3.

Table 12. A comparison of the forecasting results of the proposed method with the existing models based on the high – order FTS under number of intervals = 16

Date	Actual data	C96	H01b	L06	L08	HPSO	MTPSO	NPSO	Our model
8/3/1998	7552	-	-	-	-	-	-		-
8/4/1998	7560	7450	7450	-	-	-	-		-
8/5/1998	7487	7450	7450	-	-	-	-		-
8/6/1998	7462	7500	7500	7450	-	-	-	7452.54	7463
8/7/1998	7515	7500	7500	7550	-	-	-	7331.62	7514
8/10/1998	7365	7450	7450	7350	-	-	-	7285.63	7362
8/11/1998	7360	7300	7300	7350	-	-	-	7331.62	7362
8/12/1998	7330	7300	7300	7350	7329	7289.56	7325.28	7291.67	7331
8/13/1998	7291	7300	7300	7250	7289.5	7320.77	7287.48	7217.15	7289
8/14/1998	7320	7183.33	7188.33	7350	7329	7289.56	7325.28	7217.15	7316
8/15/1998	7300	7300	7300	7350	7289.5	7222.19	7287.48	7285.63	7302
8/17/1998	7219	7300	7300	7250	7215	7222.19	7221.26	7279.59	7221
8/18/1998	7220	7183.33	7100	7250	7215	7289.56	7221.26	7217.15	7221
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9/25/1998	6871	6850	6850	6850	6848	6850.12	6864.96	6784.34	6872
9/28/1998	6840	6850	6750	6850	6848	6800.07	6842.05	7452.54	6837
9/29/1998	6806	6850	6850	6850	6796	6800.07	6781.01	7331.62	6807
9/30/1998	6787	6850	6750	6750	6796	7289.56	6781.01	7285.63	6789
MSE		**9668.94**	**5437.58**	**1364.56**	**105.02**	**103.61**	**92.17**	**35.86**	**7.98**

Table 13. The comparison between the proposed model and HPSO, NPSO model on the same historical dataset but the different in the numbers of sampling values

Models	Numbers of sampling values							
	15	20	25	30	35	40	45	47
AFPSO	2.98	19.14	69.86	138.38	138.8	185.88	211.04	232.44
HPSO	2.53	7.79	18.04	35.42	40.45	81.73	102.4	108.87
NPSO	0.47	3.75	9.01	13.4	18.6	25.47	32.38	36
Our model	**0.28**	**2.16**	**3.84**	**5.63**	**6.05**	**6.46**	**7.23**	**7.98**

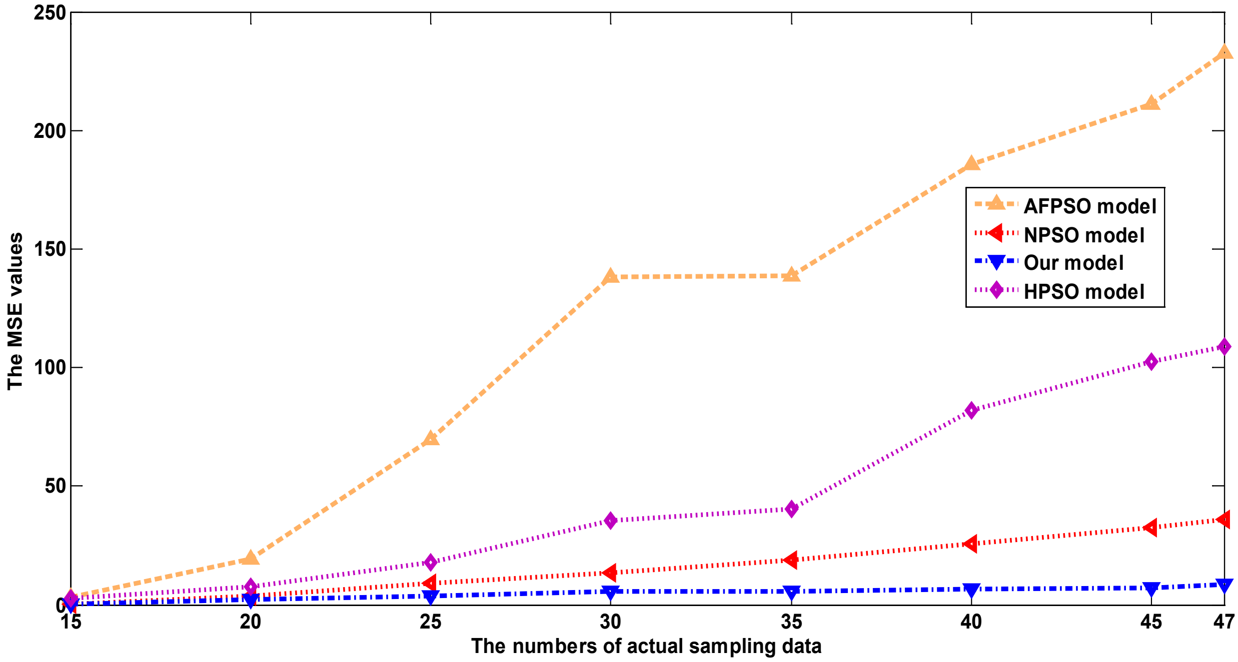

Figure 3. A comparison of the MSE value between our model and the previous methods: AFPSO, HPSO, NPSO models based on the 3rd-order FTS under number of intervals of 16 with different number of samples

From Table 13, it can be seen that our model gives remarkably better forecasting accuracy compared to AFPSO, HPSO, NPSO models with the different number of samples based on the high – order FTS under number of interval is 16. Particularly, at the same intervals of 16 and number of orders is 3, the proposed method obtains the MSE values are 0.28, 2.16, 3.84, 5.63, 6.05, 6.46, 7.23 and 7.98 which are smallest among all forecasting models compared at all. The main difference between the AFPSO model, HPSO model, NPSO model and our model is in the establishing fuzzy logical relationship groups stage created. The three former models use time - invariant fuzzy logical relationship groups, while the proposed model creates fuzzy logical relationship groups by using concept of time - variant fuzzy logical relationship group. From Fig. 3, the graphical comparison clearly shows that the forecasting accuracy according the MSE value of the proposed model is more precise than those of existing models with the all different number of sample data.

4.3.2. Experimental Results in the Testing Phase

To verify the forecasting accuracy for future TAIFEX, the historical data of the TAIFEX index are separated two parts for independent testing. The first part is used as training data set and the second part is used as the testing data set. Based on the historical data for the past days, we can forecast the new TAIFEX index for the next day only. In this study, the historical data of the TAIFEX from 8/3/1998 to 9/23/1998 is used as the training data set and the historical data of the TAIFEX index from 8/24/1998 to 9/30/1998 is used as the testing data set. For example, to forecast the new data of date 9/24/1998, the data under days 8/3/1998 ~ 9/23/1998 are used as the training data set. Similarly, a new data of date 9/25/1998 can be forecasted based on the data under dates 8/3/1998 ~ 9/24/1998. Table 14 shows a comparison for actual data and the forecasted results of the L08 model in [13], HPSO model in [14], MPTSO model in [16] and the proposed model which use 16 intervals with the 3rd - order FLRGs.

Table 14. A comparison of the MSE value for testing phase based on 3rd-order FTS under 16 intervals and which use $w_h = 10$

Date	Actual data	L08 [11]	HPSO [12]	MTPSO [14]	Our model
9/24/1998	6890	6959.07	6861.0	6916.62	6879.25
9/25/1998	6871	6833.52	6897.8	6886.0	6876.5
9/28/1998	6840	6896.95	6912.8	6892.4	6884.0
9/29/1998	6806	6863.76	6858.4	6871.54	6849.16
9/30/1998	6787	6823.38	6800.5	6859.12	6779.2
MSE		**10216.53**	**1955.9**	**6385.86**	**801.08**

The results of comparison in Table 14 indicate that the proposed model is more precise than among all compared models for 3rd – order fuzzy time series model and also gets the smallest **MSE** of **801.08** for testing phase.

5. Conclusions

In this paper, we have presented a hybrid forecasting model in academic enrolments forecasting and the TAIFEX prediction based on two advanced methods, the high – order TV-FLRGs and PSO algorithm. In order to improve the forecasting accuracy of two models; the HPSO model and NPSO model, we consider the appearance history of the fuzzy sets on the right-hand side of the same fuzzy relation to generate high –order TV-FLRGs. Furthermore, we also consider more information within all next states of all fuzzy logical relationships to calculate the forecasting output for them by proposed defuzzification rule. Then, a novel hybrid forecasting model based on aggregated high – order TV-FLRGs and PSO is developed to adjust the length of each interval in the universe of discourse with aim to increase forecasting accuracy. The empirical results show that the proposed model not only obtains higher forecasting accuracy for forecasting the enrolments of University of Alabama than the existing methods, but also supplies an effective tool for forecasting stock markets based on high - order FTS model for both the training and testing phases. The detail of comparison was presented in Table 6-14, Fig. 1, Fig. 2 and Fig 3.

The main contributions of this paper are illustrated in the following. Firstly, we propose fuzzy logical relationships generation method is different from models compared and also show the forecasted accuracy is affected by calculating the forecasting rules from these time-variant fuzzy logical relationship groups. Secondly, the computational results show that the proposed model gets highest forecasted accuracy for the 9th - order FTS model under number of intervals =14 when applied to the enrolment data and also obtains the highest forecasted accuracy for TAIFEX datasets based on the 3^{rd} - order FTS model. Actually, as listed in Table 10 and Table 12 have the MSE value for the proposed model is **9.23** and **7.98** which are the lowest forecast error among the models are compared, respectively.

As a result of implementation, it can be seen that the superior forecasting capability compared with existing forecasting models, but the proposed model is a new forecasting model and only tested by the enrolments dataset and the TAIFEX dataset. Hence, the performance of the proposed method can be changed for every different data sets. To continue assessing the effectiveness of the forecasting model, there are two suggestions for future research: The first, we can apply proposed model to deal with more complicated real-world problems for decision-making such as weather forecast, crop production, road traffic accident forecast and so on. The second, we can use type-2 fuzzy time series or hegde algebras combining with more intelligent algorithm to deal with forecasting problems which has more factors. That will be the future work of this research.

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