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Details of Semi-Membrane Shell Theory of Hybrid Anisotropic Materials

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Abstract A new version of partial differential equations and prescribed boundary conditions of already formulated semi-membrane shell theory of anisotropic materials is presented. It is based on the theories been developed by authors previously. The isotropic versions of the theory has been known to be developed nearly a century ago by Vlasov and it has been efficiently utilized for design and analysis. The aerospace vehicle structures and outer space rocket fuel storage tanks designed efficiently however not much for composite materials, while most of recent structural materials are of the combination of anisotropic materials. The semi-membrane cylindrical shell theories are classified as that of very long effective length scale and we adopted here the longest possible longitudinal and short circumferential length scales, respectively, for the formulation. The edge effects due to the prescribed boundary condition penetrate differently depending on material orientation of each layer but all within the limit of length scale $(ah)^{1/2}$ where Donnell-Vlasov bending theory is valid. Demonstrated that beyond the limit of edge effective zone, membrane or pseudo-membrane state dominates, it is traditionally named semi-membrane state. New simplified governing equations of semi-membrane theory of cylindrical shell are formulated and the physical interpretation of the theory is described.

Keywords Semi-membrane theory, Pseudo-membrane theory, Donnel-Vlasov theory, Edge effect

1. Introduction

Shell structures have numerous shapes such as cylindrical, spherical and conical configurations.

Cylindrical shells are efficiently used for space shuttles, rocket fuel storage tanks, aircraft fuselages, above ground fuel storage tanks and deep water submarines.

The advantages of cylindrical shells are not only its functional capacity and aerodynamic features but also a simple coordinate system for the mechanical analysis compared to spherical or conical shapes. For a shell of general shape we will first accept and use the classical assumption of Love then compare our theory with more elaborate theories of Reissner and Donnell-Vlasov as shown in the References [1], [2] and [3].

It is known that Vlasov during the course of simplifying his cylindrical shell equation named Donnell-Vlasov theory he had noticed a proper equation for long longitudinal length, it is interesting that he might have invented the mathematics but never used the terminology "semi-membrane" at all.

Who started to name the terminology and how Vlasov was honored are an interesting issue. When analyzing the cylindrical shells, it is common to utilize known physical parameters such as radius (a) thickness (h) and length (L). Donnell, Vlasov and many other scientists classified when develop the analytical model for circular cylindrical shell as following categories: Short and Intermediate effective length, very long effective Length. Figures 4 and 5 indicated the zoning territory but the border line can be altered by the property of material and thickness of lamination.

Once we built the analysis for the cylindrical shells we could easily convert to the other shape shells as shown in the References, [7], [8], [9], [10], However, the mathematics to describe its behavior and characteristics are complicated and it is more of challenge when the materials are anisotropic and combination of different anisotropic materials, hybrid anisotropic shell structures.

Let us first start with three dimensional coordinate system of shell structures.

The system is of longitudinal (X, z), circumferential (φ, θ) and radial (r) as shown in Figure 1 and 2. The original and non-dimensional coordinates as shown in the figures are used to allow an asymptotic integration process. According to the exact three-dimensional theory of elasticity, a shell element is considered as a volume element. All possible stresses and strains are assumed to exist and no simplifying assumptions are allowed in the formulation of the theory. We

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therefore allow for six stress components, six strain components and three displacements as indicated in the following equation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad i, j = 1,2,3 \quad k,l = 1,2 \tag{1}$$

where σ_{ij} and ε_{kl} are stress and strain tensors respectively and C_{ijkl} are elastic moduli.

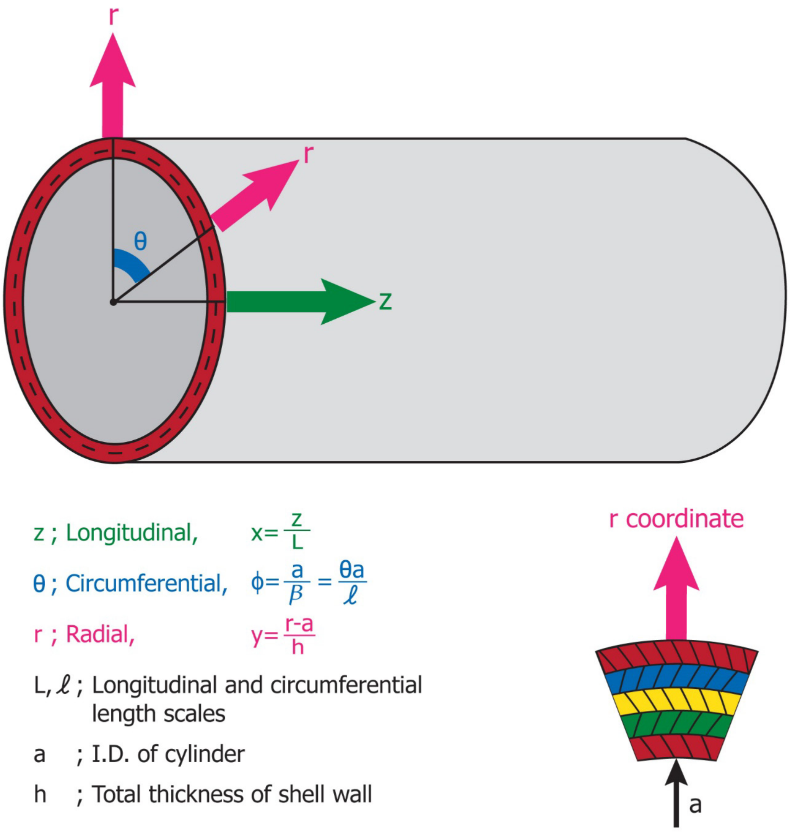

Figure 1. Coordinates of the Cylindrical Shell

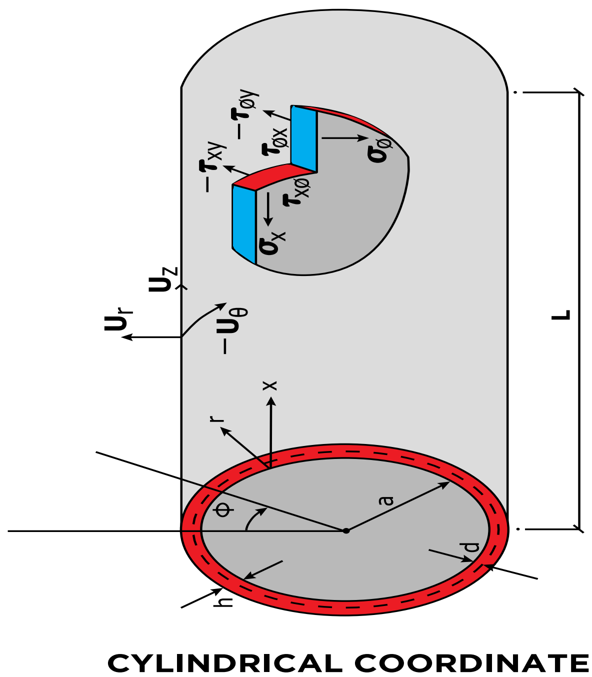

Figure 2. Normal and Shear Stresses

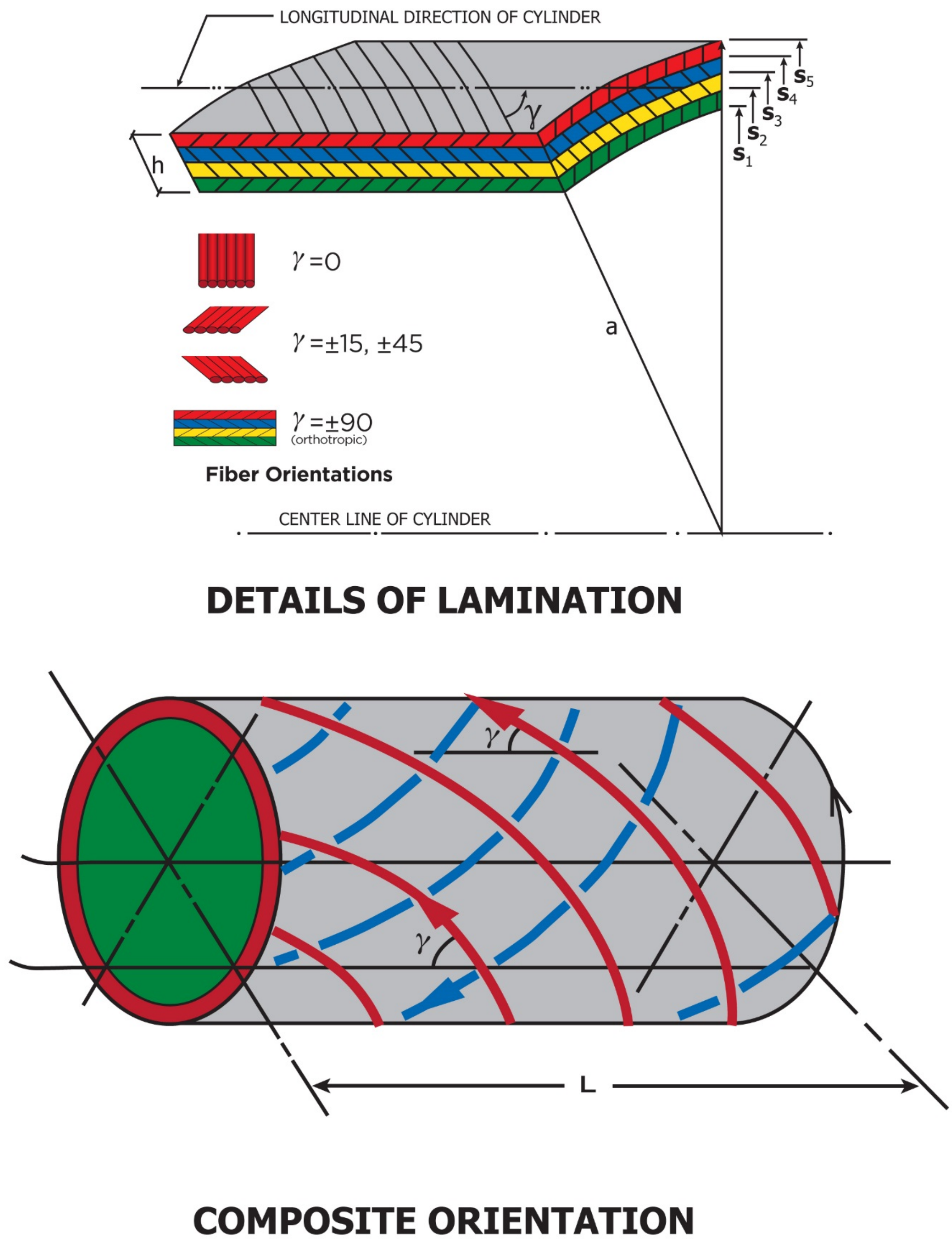

Figure 3. A Laminated Cylindrical Shell, Material Orientation γ

There are thus a total of fifteen unknowns to solve for in a three dimensional elasticity problem. On the other hand, the equilibrium equations and strain displacement equations can be obtained for a volume element and six generalized elasticity equations can be used. A total of fifteen equations can thus be formulated and it is basically possible to set up a solution for a three-dimensional elasticity problem. It is however very complicated to obtain a unique solution which satisfies both the above fifteen equations and the associated boundary conditions. This led to the development of various theories for structures of engineering interest. A detailed description of classical shell theory can be found in various references [1] through [13].

The cylindrical shell theory that we are concerned is among the three classifications of above is very long effective length shell which is well compared to Donnell-Vlasov theory. As shown in Figure 4, the edge effective zone due to the prescribed edge boundary condition represented by curved pattern is limited within close distance from the end and after the zone the deformation is nearly linear with respect to the longitudinal axis, which is very close to membrane analysis. It is more distinct for a cylindrical shell of one end fixed or hinged boundary condition and the other free open. That is semi-membrane status, we will now mathematically formulate the governing equations.

According to Calladine, Vinson and Chung, References [18], [21], [22] respectively, we pick and choose the longitudinal (L) and circumferential (ℓ) length scales as follows:

$$L = a(a/h)^{1/2}, \qquad \ell = a \tag{2}$$

While we keep the circumferential length scale as same as the inner radius (a) of the shell, the longitudinal length scale is the longest we can physically describe. Insert the equation (2) into stress displacement and equilibrium equations and use a small thin shell parameter λ = h/a, where h is the total thickness and a is the inner radius of the shell, we will obtain the following equations.

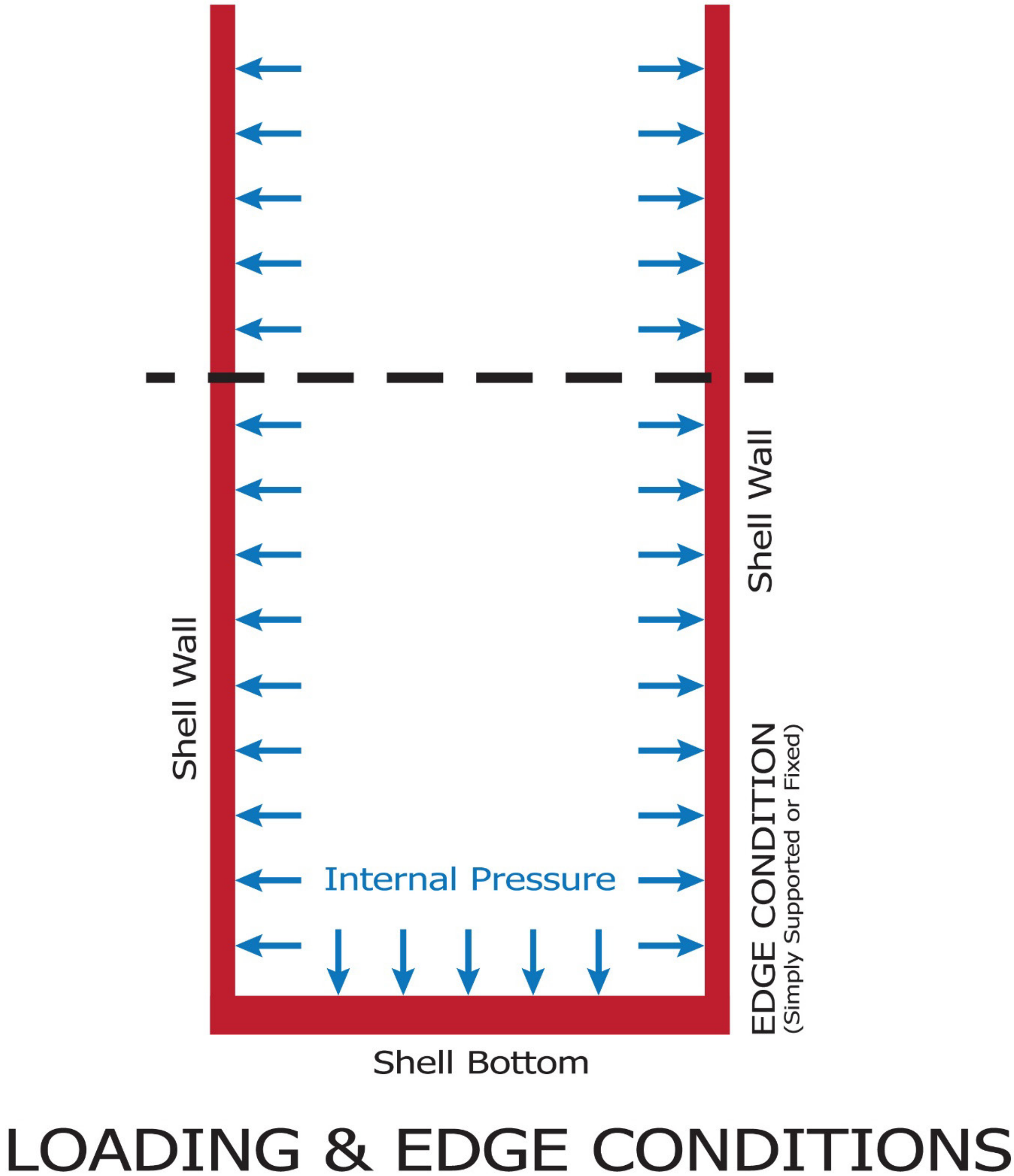

Figure 4. Cylindrical Shell under Internal Pressure

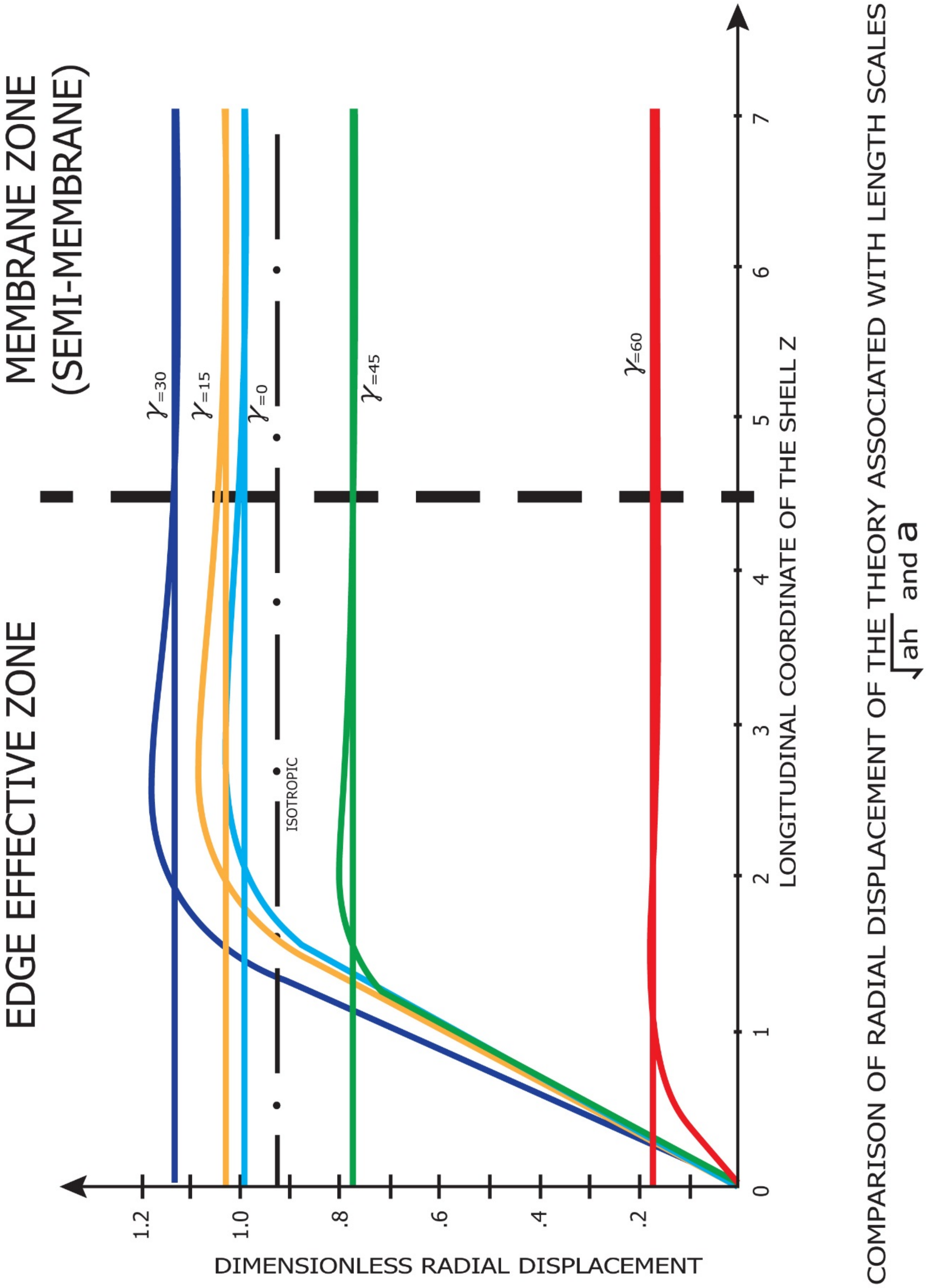

Figure 5. Edge Effective Zone of Bending and Pure Membrane Theories

Stress-displacement relations

$$
\begin{aligned}
&v_{r,y} = \lambda[\bar{S}_{31}t_z + \bar{S}_{32}t_\theta + \bar{S}_{33}t_r + \bar{S}_{34}t_{r\theta} + \bar{S}_{35}t_{rz} + \bar{S}_{36}t_{\theta z}]\\
&v_{z,y} + \lambda^{3/2}v_{r,x} = \lambda[\bar{S}_{51}t_z + \bar{S}_{52}t_\theta + \bar{S}_{53}t_r + \bar{S}_{54}t_{r\theta} + \bar{S}_{55}t_{rz} + \bar{S}_{56}t_{\theta z}]\\
&\lambda v_{r,\phi} + (1+\lambda y)v_{\theta,y} - \lambda v_\theta\\
&\qquad = \lambda(1+\lambda y)[\bar{S}_{41}t_z + \bar{S}_{42}t_\theta + \bar{S}_{43}t_r + \bar{S}_{44}t_{r\theta} + \bar{S}_{45}t_{rz} + \bar{S}_{46}t_{\theta z}]\\
&\lambda^{1/2}v_{z,x} = \bar{S}_{11}t_z + \bar{S}_{12}t_\theta + \bar{S}_{13}t_r + \bar{S}_{14}t_{r\theta} + \bar{S}_{15}t_{rz} + \bar{S}_{16}t_{\theta z}\\
&v_{\theta,\phi} + v_r = (1+\lambda y)[\bar{S}_{21}t_z + \bar{S}_{22}t_\theta + \bar{S}_{23}t_r + \bar{S}_{24}t_{r\theta} + \bar{S}_{25}t_{rz} + \bar{S}_{26}t_{\theta z}]\\
&\lambda^{1/2}(1+\lambda y)v_{\theta,x} + v_{z,\phi}\\
&\qquad = (1+\lambda y)[\bar{S}_{61}t_z + \bar{S}_{62}t_\theta + \bar{S}_{63}t_r + \bar{S}_{64}t_{r\theta} + \bar{S}_{65}t_{rz} + \bar{S}_{66}t_{\theta z}]
\end{aligned}
\tag{3}
$$

Equilibrium equations

$$
\begin{aligned}
&[t_{rz}(1+\lambda y)]_{,y} + \lambda t_{\theta z,\phi} + \lambda^{3/2}(1+\lambda y)t_{z,x} = 0\\
&[t_{r\theta}(1+\lambda y)]_{,y} + \lambda t_{\theta,\phi} + \lambda t_{r\theta} + \lambda^{3/2}(1+\lambda y)t_{\theta z,x} = 0\\
&[t_r(1+\lambda y)]_{,y} + \lambda t_{r\theta,\phi} + \lambda^{3/2}(1+\lambda y)t_{rz,x} - \lambda t_\theta = 0
\end{aligned}
\tag{4}
$$

By going through the first approximation procedure of previous formulation as explained in the References [20], [21], [22], [23], we found the following phenomena.

Transverse Strains; the first three equations of (5) are zero and all displacements shown in (6) independent of thickness coordinate, r, which means it is only membrane state.

In-plane Circumferential and Shear Strains; the longitudinal strain is represented by the combination of longitudinal and circumferential stresses (t_z and t_θ).

In-plane Shear Stresses; t_{rz} does not appear in the first approximation theory.

We now complete the second approximation theory of the asymptotic expansion and integration procedure, which can be shown as follows:

$$
\begin{aligned}
&v^{(2)}_{r,y} = 0\\
&v^{(3)}_{z,y} + v^{(0)}_{r,x} = 0\\
&v^{(2)}_{\theta,y} + v^{(0)}_{r,\phi} - v^{(0)}_{\theta} + y v^{(0)}_{\theta,y} = 0\\
&v^{(3)}_{z,x} = S^{(0)}_{11} t^{(4)}_{z} + S^{(0)}_{12} t^{(4)}_{\theta} + S^{(2)}_{11} t^{(2)}_{z} + S^{(2)}_{12} t^{(2)}_{\theta}\\
&v^{(2)}_{\theta,\phi} + v^{(2)}_{r} = S^{(0)}_{21} t^{(2)}_{z} + S^{(0)}_{22} t^{(2)}_{\theta}\\
&v^{(2)}_{\theta,x} + v^{(3)}_{z,\phi} + y v^{(0)}_{\theta,x} = S^{(0)}_{66} t^{(3)}_{\theta z}\\
&t^{(7)}_{rz,y} + (y t^{(5)}_{rz})_{,y} + t^{(5)}_{\theta z,\phi} + y t^{(2)}_{z,x} + t^{(4)}_{z,x} = 0\\
&t^{(6)}_{r\theta,y} + (y t^{(4)}_{r\theta})_{,y} + t^{(4)}_{\theta,\phi} + t^{(4)}_{r\theta} + t^{(3)}_{\theta z,x} = 0\\
&t^{(6)}_{r,y} + (y t^{(4)}_{r})_{,y} + t^{(4)}_{r\theta,\phi} - t^{(4)}_{\theta} = 0
\end{aligned}
\tag{5}
$$

By integrating the first three equations of the above, we will obtain the following equations:

$$
\begin{aligned}
&v^{(2)}_{r} = V^{(2)}_{r}(x,\phi) \quad , \qquad v^{(3)}_{z} = V^{(3)}_{z}(x,\phi) - V^{(0)}_{r,x} y\\
&v^{(2)}_{\theta} = V^{(2)}_{\theta}(x,\phi) + (V^{(0)}_{\theta} - V^{(0)}_{r,\phi}) y
\end{aligned}
\tag{6}
$$

Inserting the displacements obtained in the equation (5) into the middle three equations of (6) to obtain the following equations:

$$
\begin{aligned}
&V^{(3)}_{z,x} - V^{(0)}_{r,xx} y = S^{(0)}_{11} t^{(4)}_{z} + S^{(0)}_{12} t^{(4)}_{\theta} + S^{(2)}_{11} t^{(2)}_{z} + S^{(2)}_{12} t^{(2)}_{\theta}\\
&S^{(0)}_{21} t^{(2)}_{z} + S^{(0)}_{22} t^{(2)}_{\theta} = V^{(2)}_{\theta,\phi} + (V^{(0)}_{\theta,\phi} - V^{(0)}_{r,\phi\phi}) y + V^{(2)}_{r}\\
&S^{(0)}_{66} t^{(3)}_{\theta z} = V^{(2)}_{\theta,x} + V^{(3)}_{z,\phi} + 2(V^{(0)}_{\theta,x} - V^{(0)}_{r,x\phi}) y
\end{aligned}
\tag{7}
$$

By combining the equation for $V_{z,x}$ in the equation (6) and the last two equations of (7), we can obtain the in-plane stress-strain relations as follows:

$$
\begin{Bmatrix} t^{(2)}_{z} \\ t^{(2)}_{\theta} \\ t^{(3)}_{\theta z} \end{Bmatrix} = [\ C\ ] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{Bmatrix} + [\ C\ ] \begin{Bmatrix} K_1 \\ K_2 \\ K_{12} \end{Bmatrix} y
\tag{8}
$$

where

$$[C] = \begin{bmatrix} S_{11}^{(0)} & S_{12}^{(0)} & 0 \\ S_{21}^{(0)} & S_{22}^{(0)} & 0 \\ 0 & 0 & S_{66}^{(0)} \end{bmatrix}^{-1}$$

$$\begin{aligned} \varepsilon_1 &= V_{z,x}^{(1)} \\ \varepsilon_2 &= V_{\theta,\phi}^{(2)} + V_r^{(2)} \\ \varepsilon_{12} &= V_{\theta,x}^{(2)} + V_{z,\phi}^{(3)} \end{aligned} \tag{9}$$

$$\begin{aligned} K_1 &= 0 \\ K_2 &= V_{\theta,\phi}^{(0)} - V_{r,\phi\phi}^{(0)} \\ K_{12} &= 2(V_{\theta,x}^{(0)} - V_{r,x\phi}^{(0)}) \end{aligned}$$

Substituting t_z, t_θ and $t_{\theta z}$ into the first approximation equations of (5), we will obtain the following transverse stresses:

$$\begin{aligned} t_{rz}^{(5)} &= T_{rz}^{(5)}(x,\phi) - [(V_{\theta,x}^{(2)} + V_{z,\phi\phi}^{(3)})A_{33} + 2(V_{\theta,x\phi}^{(0)} - V_{r,x\phi\phi}^{(0)})B_{33}] \\ &\quad - [V_{z,xx}^{(1)} A_{11} + (V_{\theta,x\phi}^{(2)} + V_{r,x}^{(2)})A_{12} + (V_{\theta,x\phi}^{(0)} - V_{r,x\phi\phi}^{(0)})B_{12}] \\ t_{r\theta}^{(4)} &= T_{r\theta}^{(4)}(x,\phi) - [V_{z,x\phi}^{(1)} A_{12} + (V_{\theta,\phi\phi}^{(2)} + V_{r,\phi}^{(2)})A_{22} + (V_{\theta,\phi\phi}^{(0)} - V_{r,\phi\phi\phi}^{(0)})B_{22}] \\ t_r^{(4)} &= T_r^{(4)}(x,\phi) + [V_{z,x}^{(1)} A_{12} + (V_{\theta,\phi}^{(2)} + V_r^{(2)})A_{22} + (V_{\theta,\phi}^{(0)} - V_{r,\phi\phi}^{(0)})B_{12}] \end{aligned} \tag{10}$$

The boundary conditions at the inner and the outer surface of the shell can be specified as follows:

$$\begin{aligned} & t_{rz}^{(5)} = t_{r\theta}^{(4)} = t_r^{(4)} = 0, && (y=0) \\ & t_{rz}^{(5)} = t_{r\theta}^{(4)} = 0, \quad t_r^{(4)} = p^*, && (y=1) \end{aligned} \tag{11}$$

While the pressure term will be dimensionless as:

$$p^* = p/(\sigma\lambda^2) \tag{12}$$

Note that p* is axi-symmetric and only allowed to vary in longitudinal direction.

$$p^* = p^*(x) \tag{13}$$

After mathematical manipulation and inserting boundary conditions, we can obtain the following final equations:

$$\begin{aligned}
&[\underline{D}_{12} - (\underline{D}_{22}\underline{A}_{12}/\underline{A}_{22})]V_{z,x\phi\phi} + [\underline{A}_{11} - (\underline{A}_{12}^{2}/\underline{A}_{22})]V_{z,xxx} + [\underline{D}_{12} - (\underline{D}_{22}\underline{A}_{12}/\underline{A}_{22})]V_{z,x\phi\phi\phi\phi} \\
&+ [\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{\theta,\phi\phi\phi} + [\underline{B}_{12} - (\underline{A}_{12}\underline{B}_{22}/\underline{A}_{22})]V_{\theta,xx\phi} + \\
&[\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{\theta,\phi\phi\phi\phi\phi} + [-\underline{B}_{12} + (\underline{A}_{12}\underline{B}_{22}/\underline{A}_{22})]V_{r,xx\phi} - \\
&[\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{r,\phi\phi\phi\phi} - [\underline{E}_{22} - (\underline{B}_{22}\underline{D}_{22}/\underline{A}_{22})]V_{r,\phi\phi\phi\phi\phi\phi} + \\
&(\underline{A}_{12}/\underline{A}_{22})p^*_{,xx} = 0
\end{aligned} \tag{14}$$

where

$$\underline{E}_{ij} = \int_0^y E_{ij}\,d\rho \tag{15}$$

The above differential equations are the governing equations and can be solved efficiently as the following simplifications:

$$[\Gamma_1 v_{x,xyy} + \Gamma_2 v_{x,xxx} + \Gamma_3 V_{x,xyyy}] + [\Gamma_4 V_{y,yyy} + \Gamma_5 V_{y,xxy} + \Gamma_6 V_{y,yyyyy}] + \Gamma_7 V_{z,xxy} - \Gamma_8 V_{z,xxy}] + [\Gamma_9 V_{z,yyy} - \Gamma_{10} V_{r,yyyyyy}] + [\Gamma_{11} p_{,xx}] = 0 \tag{16}$$

Where Γ_1 through Γ_{11} are Constants and the deformation of are the functions of longitudinal, circumferential and radial directions, respectively

$$Vx = Vx(x,y,z)$$
$$Vy = Vy(x,y,z)$$
$$Vz = Vz(x,y,z) \tag{17}$$

and the prescribed boundary conditions are simply supported at both ends

$V_{xy} = V_{yx} = V_{zx} = V_{zy}$ (V is any components and x y z are partial differentiations in x y z variables)

when x = 0:

all shear forces ;

$$Vx = 0 \quad Vy = 0 \quad Vz = 0 \tag{18}$$

all bending moments ;

$Vx_{,XX} = 0$ $Vy_{,XX} = 0$ $Vz_{,XX} = 0$
$Vx_{,yy} = 0$ $Vy_{,yy} = 0$ $Vz_{yy} = 0$
$Vx_{,zz} = 0$ $Vy_{,zz} = 0$ $Vz_{,zz} = 0$
$Vx_{,Xy} = 0$ $Vy_{,Xy} = 0$ $Vz_{,Xy} = 0$
$Vx_{,Xz} = 0$ $Vy_{,xz} = 0$ $Vz_{,xz} = 0$

When x = L

all shear forces ;

$$Vx = 0 \quad Vy = 0 \quad Vz = 0 \tag{19}$$

all bending moments ;

$Vx_{,XX} = 0$ $Vy_{,XX} = 0$ $Vz_{,XX} = 0$
$Vx_{,yy} = 0$ $Vy_{,yy} = 0$ $Vz_{yy} = 0$
$Vx_{,zz} = 0$ $Vy_{,zz} = 0$ $Vz_{,zz} = 0$
$Vx_{,Xy} = 0$ $Vy_{,Xy} = 0$ $Vz_{,Xy} = 0$
$Vx_{,Xz} = 0$ $Vy_{,xz} = 0$ $Vz_{,xz} = 0$

2. Conclusions

The theory of circular cylindrical shell is developed and formulated by adopting circumferential length scale as inner radius, a, and longitudinal length scale, $a(a/h)^{1/2}$, which is the longest possible physical length and extended to hybrid anisotropic materials and the governing equations are well compared to classical theories of Vlasov, Calladine and Gould, etc. which are of isotropic material as shown in the references [6], [10] and [11] respectively.

The adopted the circumferential length scale is equal to the inner radius, a, is membrane state circumferentially as shown in the pseudo membrane theory formulated by Chung and Hong as shown in the reference [19]. Longitudinally, a long length scale was adopted, therefore radial deformation and circumferential stresses are of exponentially decaying patterns as explained by Vlasov, Calladine as shown in the references [7], [8] and [20] respectively.

Due to the complexity of hybrid anisotropic material being used, the developed theory is of higher order partial differential equations but it should be considerably simplified considering the axi-symmetric nature of load and deformations and very long longitudinal length scale. The equations and formulations used in this article are clarified by the Reference of [22].

The simplification procedure together with prescribed edge conditions further clarified.

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List of Symbols

a : Inside Radius of Cylindrical Shell
h : Total Thickness of the Shell Wall
S_i : Radius of Each Layer of Wall (i = 1, 2, 3 --- to the number of layer)
L : Longitudinal Length Scale to be defined, Also Actual Length of the Cylindrical Shell
E_i : Young's Moduli in i Direction
G_{ij} : Shear Moduli in i-j Face
S_{ij} : Compliance Matrix of Materials of Each Layer
r : Radial Coordinate
l : Circumferential Length Scale to be defined
γ: Angle of Fiber Orientation
σ : Normal Stresses
ε : Normal Strains
z, θ, r : Generalized Coordinates in Longitudinal, Circumferential and Radial Directions Respectively
τ : Shear Stresses
ε_{ij} : Shear Strains in i-j Face
λ : Shell Thickness / Inside Radius (h/a)
C_{ij} : Elastic Moduli in General
X, φ, Y : Non Dimensional Coordinate System in Longitudinal, Circumferential and Radial Directions Respectively
Γ : Constants used for partial differential equations

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