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Notched Strength Estimation of Graphite/Epoxy Laminated Composite with Central Crack under Uniaxial Tensile Loading

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Abstract Notched strength of composite material with different type of irregularities has been estimated analytically by many researchers and scholars. Some models are extremely complicated and leads to a cumbersome calculation process and others are of limited use. This intrigued authors to seek an estimation which is simple and accurate. This paper represents statistical approach for prediction of notch strength of composite laminates of different lay-ups containing a central crack using point stress criterion and average stress criterion. For calculation of characteristic lengths for point stress criterion and average stress criterion, different expressions are used which are very simple and accurate. Final equations for notched strength of laminated composites are in the simple polynomial form and can simplify further calculations. Results are compared with experimental data of graphite/epoxy composite and data obtain from improved inherent flaw model. The notched strength estimations are found to be within range of tested and improved inherent flaw model values.

Keywords Laminated composite, Notched strength, Characteristic length, Central crack

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1. Introduction

In the development of fracture mechanics for composite materials many theories and models have been suggested. In which most popular approaches are linear elastic fracture mechanics (LEFM) model suggested by Waddoups, Eisenmann and Kaminski (WEK) [1]. WEK model of estimation of notched strength was based on LEFM which assumes some intense energy region around a hole and known as crack in traverse direction of loading. Later on Whitney and Nuismer [2, 3], suggested two failure criterion called point stress criterion (PSC) and average stress criterion (ASC). According to PSC failure of a composite laminate occur when applied uniaxial tensile stress at some distance (d_0) from discontinuity, perpendicular to loading direction, is equal to or greater than unnotched strength of laminates i.e.

$$\sigma_y(d_0, 0) = \sigma_0 \tag{1}$$

Where σ_y is normal stress, applied along y direction and σ_0 is unnotched strength of laminated composite.

Another model ASC of Whitney-Nuismer [2, 3] which assumes the failure occur when average stress σ_y over some distance a_0 is equal to or more than the unnotched strength σ_0, i.e.

$$\frac{1}{a_0}\int_a^{a+a_0} \sigma_y(x, 0)dx = \sigma_0 \tag{2}$$

Where a is half crack length.

In Whintney-Nuisimer's both models characteristic length d_0 and a_0 are assumed to be material constant but later on experimental results have shown that d_0 and a_0 are not material constants. As the failure process of composite depends not only type of material but also notch shape and type of lay-ups. PSC has been modified by Pipes et al [4] and Kim et al [5] which gave a three parameter expression of characteristic length d_0 for a hole type notch, i.e.

$$d_0 = k^{-1}\left(\frac{2R}{W}\right)^n \tag{3}$$

Where k and n are constants for particular material and related to width (W) and radius (R) of hole. An expression of characteristic lengths in terms of fracture parameters, notched and unnotched strength for a crack notch presented by Potti et al [6] gives accurate and precise results and quite simple in calculation. In this paper fracture strength of graphite/epoxy composite of different lay-ups is calculated with PSC and ASC where characteristic lengths are calculated from expression presented by Potti et at [6]. Values obtained are compared with experimental values from Morries and Hahn [7] and values obtained from inherent flaw model (IFM) presented by Potti et al [8].

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2. Analytical Work

2.1. Point Stress Criterion

Based on the point stress fracture criterion it is assumed that laminate will fail when the stress at some distance d_0 away from the opening edge and on the axis normal to the loading, reaches unnotched strength [2, 3], i.e.

$$\sigma_y(d_0, 0) = \sigma_0 \qquad \text{(From equation 1)}$$

Fracture strength of infinite width plate under uniaxial tensile loading according to PSC is given by

$$\frac{\sigma_N^\infty}{\sigma_0} = \sqrt{1 - \xi^2} \tag{4}$$

Where σ_N^∞ is notched strength of infinite width laminated composite and

$$\xi = \frac{a}{a + d_0} \tag{5}$$

The characteristic dimension d_0 is initially expressed in terms of fracture parameters (k_f and m) as [6]:

$$d_0 = \frac{\gamma}{2\pi} \tag{6}$$

Where

$$\gamma = \left(\frac{k_f}{\sigma_0}\right)^2 \left\{1 - m\left(\frac{\sigma_N^\infty}{\sigma_0}\right)\right\}^2 \tag{7}$$

From equations (5) and (6) we obtain

$$\xi = \left\{1 + \frac{\gamma}{2\pi a}\right\}^{-1} \tag{8}$$

Now from equation (4), (7) and (8) we get a following equation:

$$z^2 = 1 - \frac{1}{(\alpha z^2 + \beta z + \eta)^2} \tag{9}$$

Where $z = \frac{\sigma_N^\infty}{\sigma_0}$, $\alpha = \frac{\left(\frac{k_f m}{\sigma_0}\right)^2}{2\pi a}$, $\beta = -m\frac{\left(\frac{k_f}{\sigma_0}\right)^2}{\pi a}$ and

$$\eta = \frac{\left(\frac{k_f}{\sigma_0}\right)^2}{2\pi a} + 1,$$

Equation (9) can be simplified assuming different constants, and finally we get a polynomial equation of sixth degree.

$$Az^6 + Bz^5 + Cz^4 + Dz^3 + Ez^2 + Fz + G = 0 \tag{10}$$

Where $A = \alpha^2$, $B = 2\alpha\beta$, $C = \beta^2 - \alpha^2 + 2\eta\alpha$, $D = 2\beta(\eta - \alpha)$, $E = \eta^2 - \beta^2 - 2\eta\alpha$, $F = -2\beta\eta$ and $G = 1 - \eta^2$.

Equation (10) can be solved with help of Bisection method for determining roots with initial approximation between $z = 0$ and $z = 1$.

2.2. Average Stress Criterion

Based on the average stress fracture criterion it is assumed that when the normal stress averaged over some distance (a_0), away from the opening edge and on the axis normal to the applied load reaches or greater than the unnotched strength of the laminate, the laminate will fail [2, 3] i.e.

$$\frac{1}{a_0}\int_a^{a+a_0} \sigma_y(x, 0)\,dx = \sigma_0 \qquad \text{(From equation 2)}$$

According to ASC

$$\frac{\sigma_N^\infty}{\sigma_0} = \sqrt{\frac{1-\varphi}{1+\varphi}} \tag{11}$$

Where

$$\varphi = \frac{a}{a + a_0} \tag{12}$$

a is half crack length, a_0 is average characteristic length, which can be expressed as [6]

$$a_0 = \frac{2\gamma}{\pi} \tag{13}$$

Putting $a_0 = \frac{2\gamma}{\pi}$ in equation (12) we get an expression

$$\varphi = \left\{1 + \frac{2\gamma}{\pi a}\right\}^{-1} \tag{14}$$

Where

$$\gamma = \left(\frac{k_f}{\sigma_0}\right)^2 \left\{1 - m\left(\frac{\sigma_N^\infty}{\sigma_0}\right)\right\}^2 \tag{15}$$

From equation (11), (14) and (15) we get

$$\lambda_1 z^4 + \lambda_2 z^3 + \lambda_3 z^2 + \lambda_4 z + \lambda_5 = 0 \tag{16}$$

Where

$$z = \frac{\sigma_N^\infty}{\sigma_0},\ \lambda_1 = \left(\frac{mk_f}{\sigma_0}\right)^2,\ \lambda_2 = -2m\left(\frac{k_f}{\sigma_0}\right)^2,$$

$$\lambda_3 = \left\{\left(\frac{k_f}{\sigma_0}\right)^2 + \pi a - \left(\frac{mk_f}{\sigma_0}\right)^2\right\},\ \lambda_4 = 2m\left(\frac{k_f}{\sigma_0}\right)^2 \text{ and}$$

$$\lambda_5 = -\left(\frac{k_f}{\sigma_0}\right)^2.$$

Equation (16) can be solved with Bisection method with initial approximation between $z = 0$ and $z = 1$.

Table 1. Unnotched strength of laminates with standard deviation and coefficient of variation [7]

Material	Lay-up	Average unnotched strength (σ_0) (MPa)	Standard deviation (MPa)	Coefficient of variation (%)
Thornel300/N5208	$[0/\pm45]_{2s}$	540.7	51.4	9.5
	$[0/\pm45]_s$	540.7	51.4	9.5
	$[0/90/\pm45]_s$	460.1	9.1	2

Table 2. Fracture parameters [8]

Material	Lay-up	$k_f(\mathrm{MPa}\sqrt{\mathrm{m}})$	m
Thornel300/N5208	$[0/\pm45]_{2s}$	47.9	0.559
	$[0/\pm45]_s$	41.7	0.358
	$[0/90/\pm45]_s$	47.1	0.225

Table 3. Notched strength for lay-up $[\pm45/0]_{2s}$

Thickness (t) (mm)	Crack 2a (mm)	Notched strength (σ_N) (Mpa)			
		Experimental[7]	IFM [8]	Present analysis (ASC)	Present analysis (PSC)
1.72	4.91	301.5	304.0	304.54	293.33
1.72	10.08	264.8	243.2	251.02	255.2
1.72	14.99	207.2	207.5	220.09	221.33
1.69	20.15	185.6	178.6	199.38	206.8
1.71	25.15	150.4	154.4	183.29	182.48

Table 4. Notched strength for lay-up $[\pm45/0]_s$

Thickness (t) (mm)	Crack 2a (mm)	Notched strength σ_N(MPa)			
		Experimental [7]	IFM [8]	Present analysis (ASC)	Present analysis (PSC)
0.88	4.95	289.2	302.6	305.80	314.28
0.88	10.16	238.1	236.7	243.64	247.9
0.88	15.11	213.6	199.1	211.88	209.52
0.88	20.19	180.9	170.1	189.84	194.31
0.86	25.27	141.0	145.7	173.78	182.49
0.84	30.10	119.6	124.6	161.88	165.72

Table 5. Notched strength for lay-up $[0/90/\pm45]_s$

Thickness (t) (mm)	Crack 2a (mm)	Notched strength σ_N(MPa)			
		Experimental [7]	IFM [8]	Present analysis (ASC)	Present analysis (PSC)
1.17	5.08	316.3	317.9	319.90	334.26
1.16	10.03	264.0	259.9	267.43	273.8
1.18	15.24	216.6	219.7	232.72	235.8
1.20	20.06	191.1	190.6	211.3	217.79
1.19	25.15	164.2	163.9	194.39	201.29

3. Results and Discussion

The Whitney-Nuismer [2, 3] fracture model (viz., point stress criterion, average stress criterion) are applied to correlate the experimental data [7] of graphite/epoxy, Thornel300/N5208 laminated composite containing centre crack under uniaxial tensile loading. Width of each laminate is 50.8 mm and lay-ups are $[0/\pm45]_{2s}$, $[0/\pm45]_s$ and $[0/90/\pm45]_s$. The properties of Thornel300/N5208 laminated composite are mentioned in Table 1 and 2. The comparative results of PSC and ASC analysis of unnotched strength for lay-up orders $[0/\pm45]_{2s,}$ $[0/\pm45]_s$ and $[0/90/\pm45]_s$ are presented in Table 3, 4 and 5 respectively.

Different laminates of graphite/epoxy composite are considered and good amount of fracture data generated. It is observed that procedure described in the preceding section has increased the effectiveness of the suggested models. Morries and Hans [7] have presented fracture data on Thornel 300/Narmo 5208 epoxy. Total 35 centre cracked tension specimen were tested. Three specimen for each crack length of $[0/\pm45]_{2s}$, two specimen for each crack length of $[0/\pm45]_s$ and two specimen for each crack length of $[0/90/\pm45]_s$ were tested. Width of specimen was 50.8 mm and length 304.8 mm with crack length 5.08 mm to 25.4 mm.

Figure 1, 2 and 3 shows the variation of strength ratio $\left(\frac{\sigma_N^\infty}{\sigma_0}\right)$ with crack size for $[0/\pm45]_{2s,}$ $[0/\pm45]_s$ and $[0/90/\pm45]_s$ respectively.

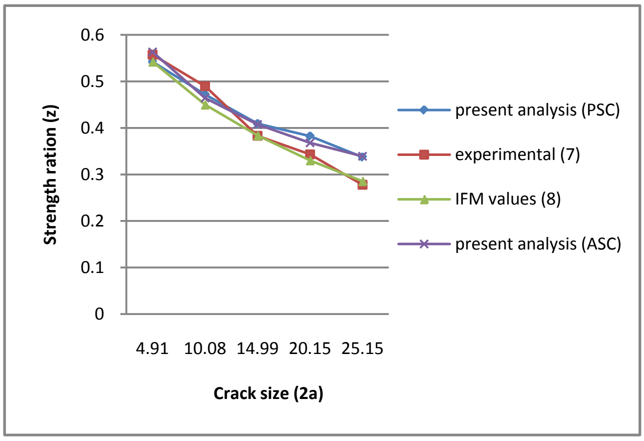

Figure 1. Variation of strength ratio (z) with crack size (2a) for [±45/0]$_{2s}$ laminate

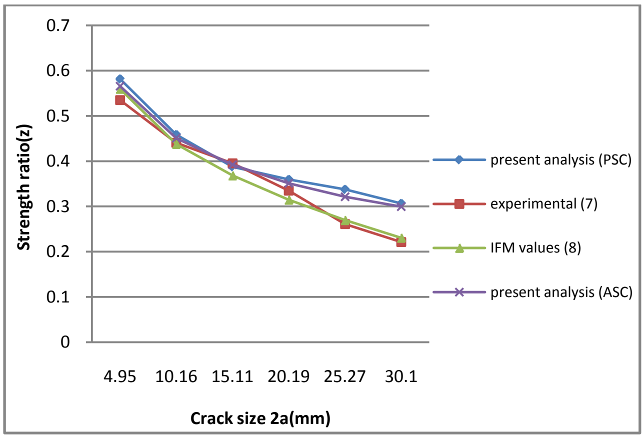

Figure 2. Variation of strength ratio (z) with crack size (2a) for [±45/0]$_s$ laminate

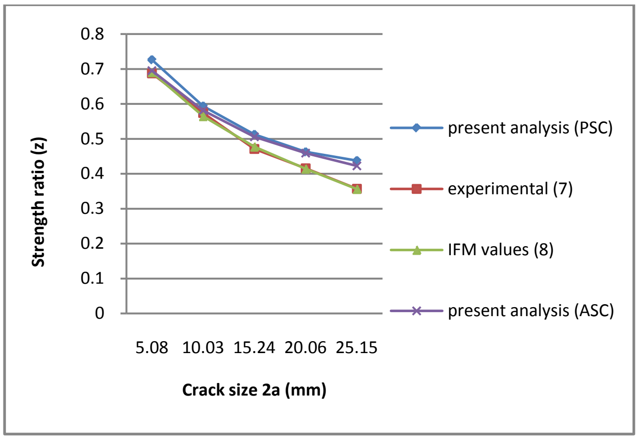

Figure 3. Variation of strength ratio (z) with crack size (2a) for [0/90/±45]$_s$ laminate

All the curves are giving approximately same results. Figure 4 and 5 shows variation of characteristic length with crack size for PSC and ASC respectively. Figure 6 shows the average values of characteristic length for PSC and ASC for different lay-ups.

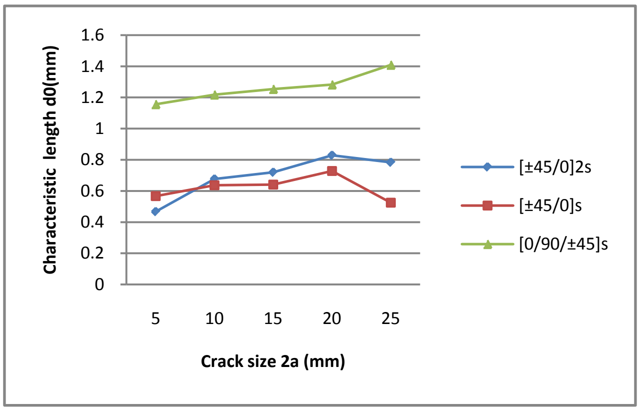

Figure 4. Variation of characteristic length (d_0) with crack size (2a) (point stress criterion)

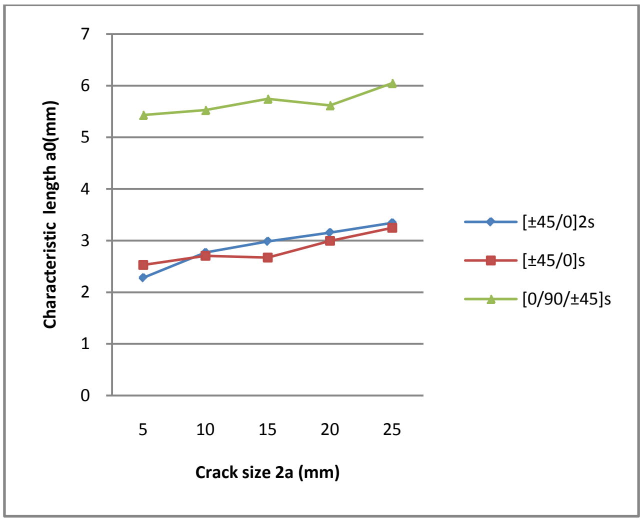

Figure 5. Variation of characteristic length (a_0) with crack size (2a) (average stress criterion)

In general it is seen that the characteristic length estimation based on the PSC and ASC are not very close to each other for all laminates. The average stress criterion values are nearly four times higher than the point stress criterion (see figure 6). This represent that the characteristic length depends on the surface fracture energy. The crack length is generally dominated by the progress of debonding fiber fracture and increasing crack length [7]. However, the results indicate that the strength ratio decreases with increase in crack size due to concentration of stress at crack tip. Figure 6 shows the average values of characteristic length for PSC and ASC for different lay-ups. A finite width correction factor [6] $Y = \sqrt{\sec\frac{\pi a}{W}}$ of a centre crack can be considered to obtain σ_N^∞ by multiplying σ_N. Since here width of plate is large compared to crack length (2a) so $\sigma_N^\infty = \sigma_N$ is assumed.

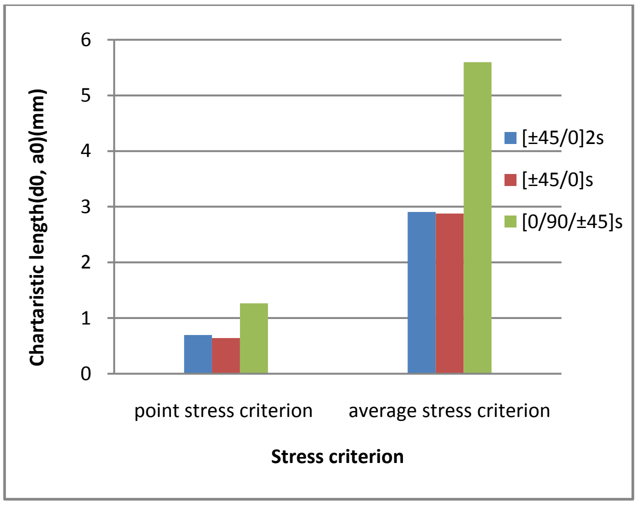

Figure 6. Average values for characteristic lengths (d_0, a_0)

4. Conclusions

All values are found in an acceptable region and as the crack size is increasing the deviation of analytical values from experimental is also increasing. This is because at large crack length the validity of the assumption of infinite width for large crack is ineffective on strength. This can be improved by considering a suitable factor for finite width laminates.

Finally, this analysis has reduced the cumbersome analytical work required for notched strength estimation. The analysis has been improved by considering just a polynomial equation of six and four degree for PSC and ASC respectively.

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