International Journal of Composite Materials 2014, 4(2): 130-134
DOI: 10.5923/j.cmaterials.20140402.12

# Bending of a Fiber-Reinforced Composite Beam with Debonded Fibers

**Yahya Berrehili**

Equipe de Modélisation et Simulation Numérique, Université Mohamed 1er, Ecole Nationale des Sciences Appliquées, Oujda, 60000, Maroc

**Abstract** The paper is devoted to the illustration of effective behavior on a classical bending problem of a fibred-reinforced composite beam with debonded fibers (but in contact with the matrix). We show that when the period of the microstructure is small enough, the displacement field solution is approximated by an explicit displacement field which depends not only of loading, volume fraction of fibers in the matrix and the homogenized stiffness tensor but also new homogenized coefficients ignored in the literature, characterizing the composite structure with debonded fibers (see [Berrehili, Y. and J.-J. Marigo, 2013, The homogenized behavior of unidirectional fiber-reinforced composite materials in the case of debonded fibers, Mathematics and Mechanics of Complex Systems, sous presse.])

**Keywords** Homogenization, Composite beam, Debonded fibers, Modeling, Behavior, Bending problem

## 1. Introduction

The microstructure of the composite materials in the aeronautical sector is so small that it is very difficult or even impossible to study directly, by finite element, this type of materials: since it requires discretizing fine enough its various components, mainly in the contact area (fibers-matrix) for sensing the great stress gradients present in this area. This may cause then a exceeding of the capacity of a computer (memory and time required for the calculation). To overcome these difficulties, multi-scale methods (including homogenization method in particular) were introduced to approach the real problem by a limit problem, in tending to zero the parameter characterizing the fineness of the microstructure [1]. The results are well established on the effective behavior of composite materials with perfectly bonded fibers (see [2] [3] [4] [5] [6] [7] and [8] [9] [10] [11] [12]), but not many studies have been made in the case where the constituents are debonded (see [13] [14] [15] [16] and [17] [18] [19]). The objective of this work is to illustrate the effective behavior on a classical elastostatic problem of bending of a fiber-reinforced composite beam with debonded fibers but in contact with the matrix. Although simple, the bending problem, one of the three famous classical problems (traction/compression, bending and torsion), is of paramount importance for practical applications. For solving this probllem, we will try, with intuitive remarks, to guess the analytical form of a part of the solution which will serve as a starting assumption. We then calculate the full solution using the equilibrium equations and the boundary conditions, i.e. to determine all the constants in a unique way. The uniqueness of these constants ensures, thanks to the Lax-Milgram theorem, the uniqueness of the sought solution of the bending problem considered.

Specifically, the paper is organized as follows. The next section is devoted to the setting of the problem which consists to study the bending problem of a fiber-reinforced composite beam with debonded fibers but in contact with the matrix. The third section is devoted to a brief general recall of homogenization results obtained in [20] and adaptation of these results to our bending problem of a "debonded" composite beam by formulating the homogenized problem associated with our bending problem which consists to solve an equilibrium equations system (coupled not classic). We solve therefore this system in the fourth section. And we end with a conclusion in the fifth section.

## 2. Position of the Real Problem

We consider a cylindrical fibrous composite beam with circular cross section which occupies, in its natural reference configuration, the solid cylinder $\mathbf{\Omega}=\mathbf{D}\times]0, L[$ of $IR^3$ where $\mathbf{D}$ is the disk with center $\mathbf{O}$ and radius R. We denote by $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ the canonical basis of $IR^3$ and $(x_1,x_2,x_3)$ the coordinates of a point $\mathbf{x}$ of $\mathbf{\Omega}$. The beam is assumed of axis $\mathbf{O}x_3$ and with two circular straight sections $\Sigma_0$ and $\Sigma_L$ located respectively in the planes $x_3=0$ and $x_3=L$. The two sections $\Sigma_0$ and $\Sigma_L$ are submitted respectively to a density of surface forces

* Corresponding author:
yahyaberrehili@yahoo.fr (Yahya Berrehili)
Published online at http://journal.sapub.org/cmaterials

$\mathbf{F}=-\beta x_1\mathbf{e}_3$ and $-\mathbf{F}=\beta x_1\mathbf{e}_3$ (see Figure 1.), β being a given arbitrary real constant. With this type of loading, one says that the cylinder considered is subjected to simple bending with axis $\mathbf{O}x_3$. The lateral surface force applied on $\Sigma_{LAT}$ is supposed null and the density of the volume forces applied on **Ω** is assumed negligible.

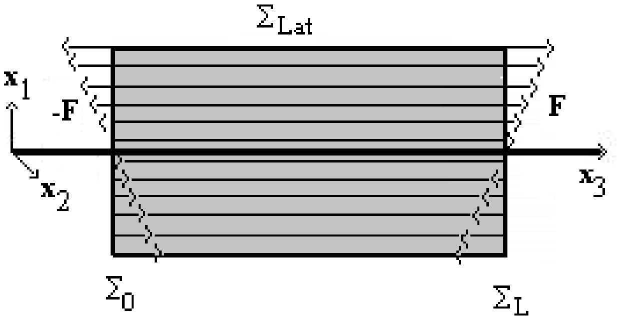


**Figure 1.** The cylindrical composite beam

The fibers of the composite beam are supposed all debonded from the matrix. We assume further that during the deformation, the fibers remain in contact with the matrix and can sliding without friction. This express that the normal displacement field is continuous and the shear is null on the interfaces fibers-matrix $\mathbf{I}^\varepsilon$(ε denotes the period of the microstructure destined to tending to zero[21][22]).

The problem consists in seeking for the displacement and the associated stress fields, denoted respectively by $\mathbf{u}^\varepsilon$ and $\boldsymbol{\sigma}^\varepsilon$ so that the dependence in ε be explicit, solutions to the following bending problem:

$$\text{Div}\ \boldsymbol{\sigma}^\varepsilon = \mathbf{0} \text{ in } \boldsymbol{\Omega}\backslash\mathbf{I}^\varepsilon, \quad (1)$$

$$\boldsymbol{\sigma}^\varepsilon = \mathbf{A}^\varepsilon\boldsymbol{\varepsilon}(\mathbf{u}^\varepsilon) \text{ in } \boldsymbol{\Omega}\backslash\mathbf{I}^\varepsilon, \quad (2)$$

$$\boldsymbol{\varepsilon}(\mathbf{u}^\varepsilon) = \frac{1}{2}(\nabla\mathbf{u}^\varepsilon + \nabla^T\mathbf{u}^\varepsilon) \text{ in } \boldsymbol{\Omega}\backslash\mathbf{I}^\varepsilon, \quad (3)$$

$$\boldsymbol{\sigma}^\varepsilon\mathbf{n}=-\beta x_1\mathbf{e}_3 \text{ on } \Sigma_0, \quad (4)$$

$$\boldsymbol{\sigma}^\varepsilon\mathbf{n}=\beta x_1\mathbf{e}_3 \text{ on } \Sigma_L, \quad (5)$$

$$\boldsymbol{\sigma}^\varepsilon\mathbf{n}=\mathbf{0} \text{ on } \Sigma_{Lat}, \quad (6)$$

$$[\mathbf{u}^\varepsilon]\bullet\mathbf{n}=0,\ [\boldsymbol{\sigma}^\varepsilon]\mathbf{n} = \mathbf{0},\ \boldsymbol{\sigma}^\varepsilon\mathbf{n}\wedge\mathbf{n} = \mathbf{0} \text{ sur } \mathbf{I}^\varepsilon, \quad (7)$$

where $\mathbf{A}^\varepsilon$ denote the elasticity tensor of the composite beam, $\boldsymbol{\varepsilon}(\mathbf{u}^\varepsilon)$ the strain field associated with the displacement field $\mathbf{u}^\varepsilon$, **n** the outer normal of $\Sigma_0(\mathbf{n}=-\mathbf{e}_3)$, $\Sigma_L(\mathbf{n}=\mathbf{e}_3)$ or $\Sigma_{Lat}(\mathbf{n}=n_1\mathbf{e}_1+n_2\mathbf{e}_2$ with $n_1^2+n_2^2=1)$ and $[\mathbf{u}^\varepsilon]$ the jump of the displacement field $\mathbf{u}^\varepsilon$ across the interface $\mathbf{I}^\varepsilon$. The three relations of the last line reflect the continuity of normal displacement field, the continuity of the stress vector and the nullity of the shear on the debonded interfaces $\mathbf{I}^\varepsilon$.

It is assumed that the two materials, fibers and matrix, constituting the composite structure, are elastic homogeneous and isotropic, of Lamé coefficients respective $(\lambda_f,\mu_f)$ and $(\lambda_m,\mu_m)$. The coefficients of the elasticity tensor of the composite structure, $A^\varepsilon_{ijkl}$, $1\leq i,j,k,l\leq 3$, are given therefore by:

$$A^\varepsilon(\mathbf{x})_{ijkl} = \lambda\left(\frac{\mathbf{x}}{\varepsilon}\right)\delta_{ij}\delta_{kl}+\mu\left(\frac{\mathbf{x}}{\varepsilon}\right)(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}), \quad (8)$$

where we conventionally denote by $\delta_{ij}$ the Kronecker symbol equal to 1 if i=j and 0 if i≠j. λ and μ are the Lamé coefficients, defined in the periodic cell $\mathbf{V}=\mathbf{V}_f\cup\mathbf{V}_m$ (reunion of the fiber part $\mathbf{V}_f$ and the matrix part $\mathbf{V}_m$) by $(\lambda(\mathbf{y}),\ \mu(\mathbf{y}))=(\lambda_f,\mu_f)$ if $\mathbf{y}\in\mathbf{V}_f$ and $(\lambda(\mathbf{y}),\ \mu(\mathbf{y}))= (\lambda_m,\mu_m)$ if $\mathbf{y}\in\mathbf{V}_m$ (see Figure 2).

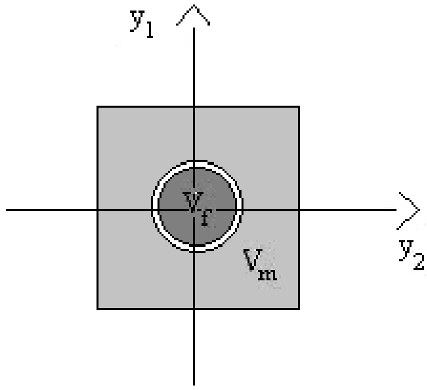


**Figure 2.** The unit periodic cell $\mathbf{V}=\mathbf{V}_f\cup\mathbf{V}_m$

# 3. Homogenized Macroscopic Problem

Following the classical two-scale procedure in homogenization theory of periodic media [1][23][20], we assume that $\mathbf{u}^\varepsilon$ can be expanded as follows:

$$\mathbf{u}^\varepsilon(\mathbf{x})=\mathbf{u}^0(\mathbf{x},y_1,y_2)+\varepsilon\mathbf{u}^1(\mathbf{x},y_1,y_2)+\varepsilon^2\mathbf{u}^2(\mathbf{x},y_1,y_2)+\ldots \quad (9)$$

where **x** is the macroscopic variable($\mathbf{x}\in\boldsymbol{\Omega}$) and, $y_1=x_1/\varepsilon$ and $y_2=x_2/\varepsilon$ are the coordinates of the microscopic variable **y** describing the unit cell $\mathbf{V}\backslash\boldsymbol{\Gamma}$, with $\mathbf{V}=]-1/2,+1/2[\text{x}]-1/2,+1/2[$ and $\boldsymbol{\Gamma}=\{(y_1,y_2\in\mathbf{V}\ /\ y_1^2+y_2^2=\alpha^2\}$ where $0<\alpha<1/2$ ($V_f=\pi\alpha^2$ denotes the volume fraction of fibers in the matrix). And the $\mathbf{u}^i=(u^i_1,u^i_2,u^i_3)$, $i\geq 0$, are the $\mathbf{V}\backslash\boldsymbol{\Gamma}$-periodic fields with respect to the variable microscopic **y**. We have shown in [20] that the result obtained differs in general from the usual property of the homogenization theory. Indeed, because of debonding of the fibers from the matrix, the leading term $\mathbf{u}^0$ of the asymptotic displacement field expansion depends in general on the microscopic coordinates **y**. Moreover a new macroscopic field enters in the effective kinematics of the composite structure. Specifically, we obtain a classical vector field representing the macroscopic displacement of the matrix **u** and additional macroscopic scalar fields δ and ω interpreted respectively as the sliding and the rotation of the fibers with respect to the matrix. Consequently we obtain a generalized homogenized problem (not classic). It contains in additional of the homogenized stiffness tensor (of the debonded composite structure) $\mathbf{A}^{hom}$, new homogenized tensors **K**, **T** and **Σ**. **K** and **T** are interpreted as the effective rigidity tensors respectively to the extension and to the torsion (of the debonded fibers), and **Σ** as the effective stress tensor. These three tensors are obtained by solving new cell problems [20] ignored in the existent literature. It appears in the equilibrium equations of the effective problem as coupling coefficients of these three macroscopic fields, **u**, δ and ω.

The leading term $\mathbf{u}^0(\mathbf{x},\mathbf{y})$ of the asymptotic displacement field expansion $\mathbf{u}^\varepsilon(\mathbf{x})$ can be written as in [20]:

$$\mathbf{u}^0(\mathbf{x},\mathbf{y}) = \mathbf{u}(\mathbf{x}) + \chi_f(\mathbf{y})(\delta(\mathbf{x})\mathbf{e}_3 + \omega(\mathbf{x})\mathbf{e}_3\wedge\mathbf{y}) \quad (10)$$

where $\chi_f(\mathbf{y})$, with $\mathbf{y}\in\mathbf{V}$, is the characteristic function of $\mathbf{V}_f$ (equal to 1 on $\mathbf{V}_f$ and 0 on $\mathbf{V}_m$).

But the forces applied are not working in rotation (since it is a problem of simple bending), we will obtain therefore simply $\omega = 0$ and $\mathbf{T} = \mathbf{0}$. $\mathbf{u}^0$ is written then:

$$\mathbf{u}^0(\mathbf{x},\mathbf{y}) = \mathbf{u}(\mathbf{x}) + \chi_f(\mathbf{y})\delta(\mathbf{x})\mathbf{e}_3, \quad (11)$$

where the couple of macroscopic fields $(\mathbf{u},\delta)$ is a solution of the following homogenized problem(posed on the same domain $\boldsymbol{\Omega}$, but which one has replaced the composite structure by an equivalent homogeneous medium(see Figure 3.), characterized by the tensors $\mathbf{A}^{hom}$, $\mathbf{K}$ and $\boldsymbol{\Sigma}$(see [20]):

$$\mathrm{Div}\left(\boldsymbol{\sigma} + \frac{\partial\delta}{\partial x_3}\boldsymbol{\Sigma}\right) = \mathbf{0} \quad \text{in } \boldsymbol{\Omega} \quad (12)$$

$$\frac{\partial}{\partial x_3}\left(\boldsymbol{\Sigma}.\boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K}\frac{\partial\delta}{\partial x_3}\right) = 0 \quad \text{in } \boldsymbol{\Omega} \quad (13)$$

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{A}^{hom}\boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x}) \text{ in } \boldsymbol{\Omega} \quad (14)$$

$$\boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla^T\mathbf{u})(x) \text{ in } \boldsymbol{\Omega} \quad (15)$$

$$\left(\boldsymbol{\sigma} + \frac{\partial\delta}{\partial x_3}\boldsymbol{\Sigma}\right)\mathbf{n}(\mathbf{x}) = \beta x_1\mathbf{e}_3 \text{ on } \Sigma_0 \quad (16)$$

$$\left(\boldsymbol{\sigma} + \frac{\partial\delta}{\partial x_3}\boldsymbol{\Sigma}\right)\mathbf{n}(\mathbf{x}) = \beta x_1\mathbf{e}_3 \text{ on } \Sigma_L \quad (17)$$

$$\left(\boldsymbol{\sigma} + \frac{\partial\delta}{\partial x_3}\boldsymbol{\Sigma}\right)\mathbf{n}(\mathbf{x}) = \mathbf{0} \text{ on } \Sigma_{Lat} \quad (18)$$

$$\left(\boldsymbol{\Sigma}.\boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K}\frac{\partial\delta}{\partial x_3}\right)(\mathbf{x})n_3(\mathbf{x}) = V_f\beta x_1 \text{ on } \Sigma_0 \quad (19)$$

$$\left(\boldsymbol{\Sigma}.\boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K}\frac{\partial\delta}{\partial x_3}\right)(\mathbf{x})n_3(\mathbf{x}) = V_f\beta x_1 \text{ on } \Sigma_L \quad (20)$$

$$\left(\boldsymbol{\Sigma}.\boldsymbol{\varepsilon}(\mathbf{u}) + \mathbf{K}\frac{\partial\delta}{\partial x_3}\right)(\mathbf{x})n_3(\mathbf{x}) = 0 \text{ on } \Sigma_{Lat} \quad (21)$$

The first equation is a three-dimensional equilibrium. The second one is a family of scalar equations, identified by the indices $(x_1,x_2)$. It is the equations of beam-type ($\mathbf{K}\frac{\partial\delta}{\partial x_3}$ represents the normal effort). We see that, in the first equation, the term $\frac{\partial\delta}{\partial x_3}\boldsymbol{\Sigma}$ plays the role of a pre-stressed field whereas in the second one, the term $\boldsymbol{\Sigma}\boldsymbol{\varepsilon}(\mathbf{u})$ plays the role of a pre-tensioning field of the beam composite. This system of equations is completed by boundary conditions (16)-(21).

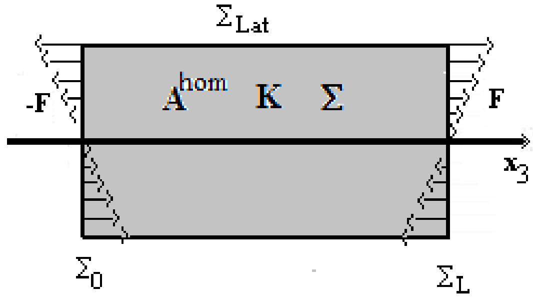


**Figure 3.** The homogenized composite beam

# 4. Resolution

It should be noted that the problem is symmetrical: we have a geometrical and loading symmetries with respect to the plans $x_2$=0 and $x_3$=L/2. The couple solution $(\mathbf{u},\delta)$, with $\mathbf{u}=(u_1,u_2,u_3)$, can be then searched under the following form:

$$\mathbf{u}(\mathbf{x}) = (a_1(x_1^2 - x_2^2) + a_2x_3^2)\mathbf{e}_1 + 2a_1x_1x_2\mathbf{e}_2 - 2a_2x_1x_3\mathbf{e}_3 \quad (22)$$

and

$$\delta(\mathbf{x}) = a_3x_1. \quad (23)$$

i.e.

$$u_1(\mathbf{x}) = a_1(x_1^2 - x_2^2) + a_2x_3^2 \quad (24)$$

$$u_2(\mathbf{x}) = 2a_1x_1x_2 \quad (25)$$

$$u_3(\mathbf{x}) = -2a_2x_1x_3 \quad (26)$$

$$\delta(\mathbf{x}) = a_3x_1 \quad (27)$$

where $a_1$, $a_2$ and $a_3$ are real numbers to seeking for.

The strain field associated with the displacement field $\mathbf{u}$ is given by,

$$\boldsymbol{\varepsilon}(\mathbf{u})(\mathbf{x}) = \begin{pmatrix} 2a_1\mathbf{x}_1 & 0 & 0 \\ 0 & 2a_1\mathbf{x}_1 & 0 \\ 0 & 0 & -2a_2\mathbf{x}_1 \end{pmatrix}. \quad (28)$$

The form of the new stress tensor $\boldsymbol{\Sigma}$, which we have obtained in [20], is given as follows:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{11} & 0 \\ 0 & 0 & \Sigma_{33} \end{pmatrix}. \quad (29)$$

In the case of a disposition of fibers at the vertices of a square lattice(as in our case), the homogenized stiffness tensor $\mathbf{A}^{hom}$ is given, in the axis system $(\mathbf{O};\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$, by the following matrix[8]:

$$A^{hom} = \begin{bmatrix} \mathbf{A}^{hom}_{1111} & \mathbf{A}^{hom}_{1122} & \mathbf{A}^{hom}_{1133} & 0 & 0 & 0 & 0 \\ & \mathbf{A}^{hom}_{1111} & \mathbf{A}^{hom}_{1133} & 0 & 0 & 0 & 0 \\ & & \mathbf{A}^{hom}_{3333} & 0 & 0 & 0 \\ & & & 2\mathbf{A}^{hom}_{3131} & 0 & 0 \\ & -\mathbf{SYM}- & & & 2\mathbf{A}^{hom}_{3131} & 0 \\ & & & & & 2\mathbf{A}^{hom}_{1212} \end{bmatrix} \quad (30)$$

So, by calculating the stress tensor $\boldsymbol{\sigma}$, given by the constitutive law (14), we obtain:

$$\boldsymbol{\sigma}(\mathbf{x})=\begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix}. \quad (31)$$

with

$$\sigma_{11} = \sigma_{22} = 2a_1x_1(\mathbf{A}_{1111}^{hom} - \mathbf{A}_{1122}^{hom})2a_2x_1\,\mathbf{A}_{1133}^{hom} \quad (32)$$

and

$$\sigma_{33} = 2a_1x_1\,\mathbf{A}_{1133}^{hom}\;\; 2a_2x_1\,\mathbf{A}_{3333}^{hom}. \quad (33)$$

● We verify without difficulty, taking account of (28) and (29), that the equilibrium equations (13) and the condition imposed on $\Sigma_{Lat}$ (21) are satisfied.

● The equilibrium equation (12) gives by cons a relation linking the three constants, $a_1$, $a_2$ and $a_3$, to determine:

$$2a_1(\mathbf{A}_{1111}^{hom} + \mathbf{A}_{1122}^{hom})\; 2a_2\,\mathbf{A}_{1133}^{hom} = a_3\Sigma_{11} \quad (34)$$

● In expliciting now the boundary conditions (16), (17) and (18) defined respectively on $\Sigma_0$, $\Sigma_L$ et $\Sigma_{Lat}$, we obtain:

$$4a_1\,\mathbf{A}_{1133}^{hom}\,2a_2\,\mathbf{A}_{3333}^{hom} = -\beta a_3\Sigma_{33} \quad (35)$$

● Similarly as above, for the boundary conditions (19) and (20) defined respectively on the $\Sigma_0$ et $\Sigma_L$, we deduce the following condition:

$$4a_1\Sigma_{11}+a_2\,\Sigma_{33}= \beta V_f+Ka_3 \quad (36)$$

In summary, one has then to solve the following system of three equations with three unknowns, giving the constants $a_1$, $a_2$ and $a_3$:

$$\begin{cases} -4a_1\Sigma_{11} + a_2\Sigma_{33} - Ka_2 = \beta V_f \\ 4a_1A_{1133}^{hom} - 2a_2A_{3333}^{hom} + a_3\Sigma_{33} = -\beta \\ 2a_1(A_{1111}^{hom} + A_{1122}^{hom}) - 2a_2A_{1133}^{hom} + a_{33}\Sigma_{11} = 0 \end{cases} \quad (37)$$

which admits a unique solution:

$$a_1 = \frac{1}{4C\Sigma_{11}}[(-\beta CV_f + G\Sigma_{33}) - \frac{AB-CD}{AE-CF}(E\Sigma_{33}+KC)] \quad (38)$$

$$a_2 = \frac{G}{C} - \frac{E(AB-CD)}{C(AE-CF)}, \quad (39)$$

$$a_3 = \frac{AB-CD}{AE-CF}, \quad (40)$$

where we have posed:

$$A=8(\mathbf{A}_{1133}^{hom})^2\,4\mathbf{A}_{3333}^{hom}(\mathbf{A}_{1111}^{hom} + \mathbf{A}_{1122}^{hom}) \quad (41)$$

$$B=4C(V_f\,\mathbf{A}_{1133}^{hom}\,\Sigma_{11}) \quad (42)$$

$$C=4(\Sigma_{33}\,\mathbf{A}_{1133}^{hom}\,2\Sigma_{11}\,\mathbf{A}_{3333}^{hom}) \quad (43)$$

$$D=(\mathbf{A}_{1111}^{hom} + \mathbf{A}_{1122}^{hom}) \quad (44)$$

$$E=4(K\,\mathbf{A}_{1133}^{hom} + \Sigma_{11}\Sigma_{33}) \quad (45)$$

$$F=2\Sigma_{33}(\mathbf{A}_{1111}^{hom} + \mathbf{A}_{1122}^{hom})\,2\Sigma_{11}\,\mathbf{A}_{1133}^{hom} \quad (46)$$

$$G=4C(V_f\,\mathbf{A}_{1133}^{hom}\,\Sigma_{11}) \quad (47)$$

The displacement field, defined explicitly by,

$$\mathbf{u}^0(\mathbf{x},\mathbf{y}) = \mathbf{u}(\mathbf{x}) + \chi_f(\mathbf{y})\delta(\mathbf{x})\mathbf{e}_3= (a_1(x_1^2-x_2^2) +a_2x_3^2)\mathbf{e}_1+2a_1x_1x_2\mathbf{e}_2+(\chi_f(\mathbf{y})a_3x_1-2a_2x_1x_3)\mathbf{e}_3 \quad (48)$$

(where $a_1$, $a_2$ and $a_3$ are given by (38)-(40)), is then an approached solution, in first order, of the bending problem (1)-(7). Any other solution is obtained by adding to the expression (48) a rigid displacement, without consequences for the mechanical problem considered.

# 5. Conclusions

We have shown in this paper, by using the results obtained in [20] and homogenization theory of periodic media that the bending problem of a fiber-reinforced composite beam with debonded fibers (but in contact with the matrix) admits a unique solution approached, in first order, by an explicit displacement field, i.e. leading term $\mathbf{u}^0$ of the asymptotic expansion of $\mathbf{u}^\varepsilon$ given by the expression (48). This is because the problem considered is symmetrical. This field depends on two displacement fields, the vector field $\mathbf{u}$ representing the macroscopic displacement of the matrix and the scalar field $\delta$ representing the longitudinal sliding of the fibers with respect to the matrix. These two fields, given explicitly by the expressions (22) and (23), are solutions of the coupled homogenized problem (12)-(21). Moreover, unlike that found in the literature, this field depends on the fineness of the microstructure, i.e. on the small parameter $\varepsilon$ related to the size of the microstructure (presence of microscopic variable $\mathbf{y}$, with $y_1=x_1/\varepsilon$ and $y_2=x_2/\varepsilon$, in its expression (48)). It also depends on the density of surface forces $\mathbf{F}$, the volume fraction of fibers $V_f$, the classical homogenized tensor $\mathbf{A}^{hom}$ and the new homogenized tensors $\mathbf{K}$ and $\boldsymbol{\Sigma}$(calculated once and for all in [20]) characterizing the "debonded" composite.

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