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The Rotational Velocity of Barred Spiral Galaxies in the General Relativity Solution

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Abstract This work describes the hypothesis in which the explanation of the rotational velocity of barred spiral galaxies is based on the relativistic solution. Considering a previous relativistic solution for the unbarred spiral galaxies, this solution assumes that the entire spiral galaxy disk would rotate like a rigid (or solid) body. It is considered that both the Solar System and spiral galaxies are behaving like a rigid body in rotation. Then, having both systems the same dynamical behaviour, which can be explained by the same relativistic solution, these two apparently different cases are unified in the same solution. Thus, the stars and gas in the spiral galaxy must be rotating with the system in an almost the same and uniform angular velocity. Nevertheless, according to the Kerr metric for the rotating black hole, at the zone of the bar of barred spiral galaxies, stars and gas must be changing their direction toward the rotation poles of the black hole. On these assumptions, we apply the equation based on the relativistic solution to the barred spiral galaxies. More precisely, we present examples of the rotation curves of unbarred spiral galaxies: NGC 4378 and NGC 4594, and the barred spiral galaxies: Milky Way and, NGC 7541. Comparing our calculations with the observations we find a good approximation.

Keywords Galaxies: spiral, Kinematics and dynamics, General Theory of Relativity

1. Introduction

One of the problems that classical gravitation theory has had is to justify the large velocities observed for stars and gas in spiral galaxies and how they differ from the rotational velocities of planets. Indeed, precise measurements (with a variety of techniques) of the velocities of stars and gases in spiral galaxies [1,2] make clear that the action of the galaxy on its stars and the gas estimated according to the classical gravitation theory cannot account for the measured velocities, if the gas is assumed to be a stable component of the galaxy, as described by the quasi-stationary density wave theory, which characterizes spirals as rigidly rotating, long-lived patterns (i.e. steady spirals) [3]. Thus, we can consider that in principle, stars and gas in spiral galaxies do not follow a rotational motion as described in classical gravitational theory, so they must obey other considerations to satisfy the observations.

The issue is that the classical Newtonian law of gravitation only considers two terms, which are the Newtonian gravity force and the centrifugal force to define the classical total force [4]. Then, it has some limitations proper of this classical theory since it does not consider any other energy or force involved in the rotating systems. This is the reason why is required to add some matter or forces to adjust the calculations to the rotational velocities observed in spiral galaxies, such as the dark matter concept [5], which has not been detected to date, or even develop some adjustments like the proposed in the Modified Newtonian Dynamics (MOND) theory [6,7].

On the other hand, the net or absolute total force in the relativistic solution, derived from the General Theory of Relativity (GTR) [8] considers a third term, which includes the force related to the Coriolis force in a rotating system, which is inverse of the distance to the fourth power. This is the main reason why the relativistic solution can be considered to provide an exact solution which does not require any additional matter and does not require any adjustment to fit to the rotational velocities observed in spiral galaxies.

Then, based on the relativistic solution to calculate the rotational velocities of unbarred spiral galaxies and their spiral geometry [9], we consider that the rotational velocity of barred spiral galaxies can also be calculated from this relativistic solution. Furthermore, considering the Kerr metric for a rotating black hole, we describe the dynamics and geometry to be applied to the barred spiral galaxies, specifically the dynamic which indicates that gas and matter are carried towards the axial poles defined by the rotating axially-symmetric.

A barred spiral galaxy, type as "SB" (spiral, barred) in the Hubble sequence, is a spiral galaxy with a central

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bar-shaped structure composed of stars and gas. For instance, the Milky Way galaxy, where the Solar System is located, is classified as a barred spiral galaxy [10].

In this solution, the dynamics of the stars in a spiral galaxy is not exclusively due to the combined gravitational field of all the stars, gas, or any other massive object in the galaxy, but rather considers that the entire galaxy disk moves and rotates as a rigid (or solid) body.

Previous works [9,11] show the possibility that both the Solar System and spiral galaxies are behaving like a rigid (or solid) body in rotation. Thus, having both systems the same dynamical behavior, which can be explained by the same relativistic solution, these two apparently different cases are unified in the same solution.

This work aims to describe the hypothesis that the explanation of the rotational velocity of some barred spiral galaxies is based on the solution of general relativity. Thus, we revisit the equation that describes the rotational velocity of stars in unbarred spiral galaxies, based on the relativistic solution to be applied to the barred spiral galaxies. In particular, we show examples of the calculated rotation curves of unbarred spiral galaxies NGC 4378 and NGC 4594, and barred spiral galaxies Milky Way and, NGC 7541 (with very different masses and sizes), comparing our calculations with the observations, finding a good approximation.

2. Kerr Metric in the Barred Spiral Galaxies

The Kerr metric [12] is an exact solution of the Einstein field equations of general relativity that generalizes to a rotating uncharged black hole of the Schwarzschild metric [13]. This metric describes the geometry of empty spacetime around a rotating axially-symmetric black hole of massive resting mass $M_\bullet$ with a quasi-spherical event horizon. According to the Kerr metric, a rotating black hole with angular velocity $\Omega_\bullet$ difference of zero should exhibit frame-dragging [14]. Roughly speaking, this effect predicts that objects coming close to a rotating mass will be entrained to participate in its rotation, because of the swirling curvature of spacetime itself associated with rotating bodies. In the case of a rotating black hole, at close enough distances, all objects, even light, must rotate with the black hole. This region is called the ergosphere. Rotating black holes have surfaces where the metric seems to have apparent singularities; the size and shape of these surfaces would depend on the black hole's mass and angular momentum. The outer surface encloses the ergosphere and has a shape like a flattened sphere. The inner surface marks the event horizon [15] (Figure 1).

The Kerr metric has two physically relevant surfaces on which it appears to be singular. The inner surface corresponds to an event horizon like that observed in the Schwarzschild metric; this occurs where the purely radial component g_{rr} of the metric goes to infinity. Solving the quadratic equation $1/g_{rr} = 0$ yields the solution:

$$r_H^{\pm} = \frac{r_S \pm \sqrt{r_S^2 - 4a^2}}{2}, \tag{1}$$

where r_H is the radius of the inner surface of the singularity, r_s is the Schwarzschild radius, and where for brevity, the length-scales $a = J_\bullet/M_\bullet c$ is the ratio between the angular momentum of rotation $J_\bullet$ and the mass $M_\bullet$ of the black hole. Another apparent singularity occurs where the purely temporal component g_{tt} of the metric changes sign from positive to negative. Again, solving a quadratic equation $g_{tt} = 0$ yields the solution:

$$r_E^{\pm} = \frac{r_S \pm \sqrt{r_S^2 - 4a^2 \cos^2\theta}}{2}, \tag{2}$$

where r_E is the radius of the purely temporal component of the singularity. Due to the $\cos^2\theta$ term in the square root, this outer surface resembles a flattened sphere that touches the inner surface at the poles of the rotation axis, where the colatitude θ equals 0 or π; the space between these two surfaces is called the ergosphere (Figure 1). Within this volume, the purely temporal component g_{tt} is negative, i.e., acts like a purely spatial metric component. Consequently, particles within this ergosphere must co-rotate with the inner mass, if they are to retain their time-like character.

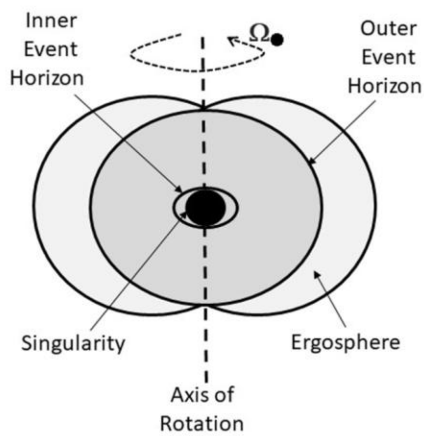

Figure 1. Kerr metric of a rotating black hole

On the other hand, the barred spiral galaxy is a grouping of astronomical objects, stars and gas that keeps rotating due to the force of gravity of a rotating system. Its shape is spiral with a central bar that starts from diametrically opposite points of the galactic nucleus (Figure 2). In fact, it is observed that each of the two main arms of an unbarred spiral also start mainly from diametrically opposite points in the galactic nucleus, but do not present a bar. It is currently believed that in a barred galaxy, the bar funnels interstellar gas from the arms of the spiral galaxy into its interior.

Taking into account the Kerr metric, we can consider that stars and gas that are moving at the outer zones with the spiral arms, when reaching the region of one of the two rotation poles of the black hole, where the outer event horizon is near of the surface, this strongly suggests (but does not yet confirmed) that matter and gas clouds interact at the edges of the bar losing angular momentum and thus facilitating the creation of a flow of matter and gas that is diverted following a straight path from the outside region towards the axial poles, then forming the bar (Figure 2).

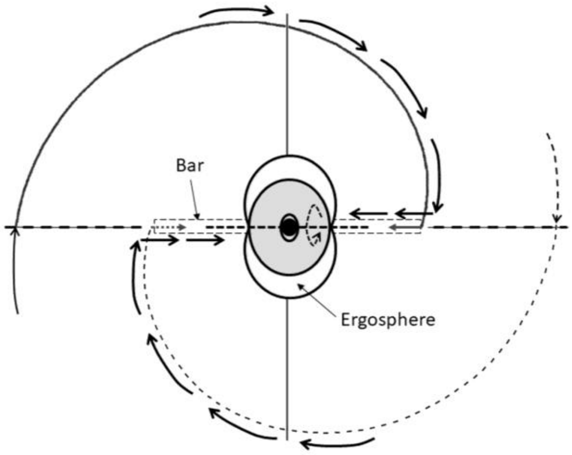

Figure 2. The dynamics around the Kerr metric of a rotating black hole, and the forming of the bar of a barred spiral galaxy

3. General Theory of Relativity Solution for the Total Force

A relativistic solution for the angular movement of the celestial bodies in a rotating system can be determined by a study of the solutions of Einstein's equations following the standard procedure [16-18]. Thus, the total force for a rotating system is given as

$$|F| = \frac{GMm}{r^2} - \frac{L^2}{mr^3} + \frac{3GML^2}{mc^2r^4}, \tag{3}$$

where r is a position vector and L is the orbital angular momentum of the rest mass m, given by

$$\vec{L} = mr\vec{v} = mr^2\vec{\omega}. \tag{4}$$

The first term in Eq. (3) represents the Newtonian gravitational force, which is described by the inverse-square law. This is the case of Newton equation [4] that describes the orbital motion of planets around the Sun. The second term represents the centrifugal force in circular motion. The third term is related to the Coriolis force, which includes the inverse of the distance to the fourth power, which is derived from the GTR.

Furthermore, a previous work [11] shows that it is possible that both the Solar System and spiral galaxies are behaving like a rigid (or solid) body in rotation. Thus, having both systems the same dynamical behavior, which can be explained by the same relativistic solution, these two apparently different cases are unified in the same solution. In that solution, the total angular frequency Ω_T of the system can be obtained from Eq. (3) by adding the apsidal precession for a rotating and orbiting body, giving

$$\Omega_T = \Omega_v + \omega_\varphi = \left(\frac{GMv^2}{6c^2r^3}\right)^{\frac{1}{2}} + \frac{6\pi GM}{a\left(1-e^2\right)c^2t_P}, \tag{5}$$

where Ω_v is the angular velocity of the circular motion of the Solar System, Ω_φ is the angular velocity of the apsidal precession, a is the semi-major axis of the elliptical orbit, e is the eccentricity and t_P is the time in seconds of a sidereal year. Then, for one revolution (2π radians), the total period is given by

$$T_T = \frac{2\pi}{\Omega_T} = \frac{2\pi}{\Omega_v + \omega_\varphi},$$
$$T_T = \frac{2\pi}{\left(\frac{GMv^2}{6c^2r^3}\right)^{\frac{1}{2}} + \frac{6\pi GM}{a\left(1-e^2\right)c^2t_P}}. \tag{6}$$

Substituting the Earth-Sun system data [12] into Eq. (6) for the maximum distance r_{Max} (during the aphelion), and the related minimum orbital velocity v, total angular frequency equals 7.72554 × 10^{-12} radians per second (50.2881" arc per year, or 0.0139689° per year). For one revolution (2π radians), total period in the case of the Earth equals 8.133 × 10^{11} seconds (25,771.5 years), which is according to the observed period of Earth's axial precession.

4. Rotational Velocity for Unbarred Spiral Galaxies

According to this relativistic solution [19], at the stage where the entire spiral galaxy disk rotates like a rigid (or solid) body, the motion of the stars and gas that forms the spiral galaxy should be perceived from Earth as an almost uniform rotation of the galaxy. Considering the third term in Eq. (3), we can reduce common terms, giving

$$F_c = \frac{3GM\left(mr^2\Omega\right)^2}{mc^2r^4} = \frac{3GMm\Omega^2}{c^2}. \tag{7}$$

where Ω is the angular velocity of the rotating system.

In the same way as with the equation for classical mechanics scenario, we equate the centrifugal force with Eq. (7), obtaining

$$\frac{3GMm\Omega^2}{c^2} = m\frac{v^2}{r},$$
$$v = \left(\frac{3GMr\Omega^2}{c^2}\right)^{\frac{1}{2}}, \tag{8}$$

with M in this case being the resting mass $M_\bullet$ of the galactic nucleus. In a rigid body, the angular velocity Ω of the system remains uniform. To calculate the rotational velocity, we can use as datum the angular velocity of the system, if known, or its equivalence with angular momentum J, which increases almost linearly with respect to any distance from the nucleus. Then, according to Eq. (4), we can write the angular momentum as

$$J = M_g r^2 \Omega, \tag{9}$$

where M_g is the mass of the system (in this case, mass of the spiral galaxy). The angular momentum of each galaxy can be calculated from the specific angular momentum $j = J/M_g$ [20], as a function of distance in the spiral Sa galaxies. Then

$$v = \left(\frac{3GM_\bullet}{c^2 r^3} \frac{J^2}{M_g^2} \right)^{\frac{1}{2}} = \left(\frac{3GM_\bullet j^2}{c^2 r^3} \right)^{\frac{1}{2}}. \tag{10}$$

One can verify the spiral behaviour from Eq. (8), having that $\theta = \Omega t$ and, for the sake of describing the geometry of this equation, considering the Schwarzschild radius r_s [13]. Therefore, we write this in polar coordinates as

$$\frac{2v}{3}\left(\frac{r}{t}\right) = \frac{2GM_\bullet r\Omega}{c^2}\left(\frac{v}{r}\right),$$

$$r = \pm\frac{3}{2} r_s \theta, \tag{11}$$

$$\frac{1}{\theta r} = \pm\frac{3}{2} r_s u^2,$$

which is the equation of a spiral. The distance r can be described by the reciprocal $u = 1/r$ as a function of θ according to the Binet equation in the solution of the relativistic equation derived for Schwarzschild metric [21,22].

We know from the observations that spiral galaxies normally mainly have two arms that start at the poles axially-symmetric from the centre of the galactic nucleus. The arms can be geometrically represented in Eq. (11) by considering the opposite sign of this equation for the second arm, so we include the (±) to consider both arms of the spiral galaxy.

In addition, Eqs. (10) and (11) can also be applied for unbarred spiral galaxies Sb and Sc, since these other types of galaxies follow the same geometry and dynamics, only by changing the number of laps performed before reaching the centre.

5. Spiral Geometry in Barred Spiral Galaxies

Considering the Kerr metric for a rotating black hole, specifically the dynamic which indicates that gas and matter are carried from the outside towards the axial poles, following a linear path and forming the bar, we find the geometry of the barred spiral galaxies.

According to the Kerr metric, geometry of empty spacetime around a rotating axially-symmetric black hole would form the bar from the spiral towards the axial poles. Thus, from Eq. (11), we define the equation for the geometry of barred spiral galaxies, giving

$$r = \pm\frac{3}{2} r_s{}^{\theta}, \tag{12}$$

which is the equation of a spiral with two centres. In the same way, we also include the (±) in Eq. (12) to consider both arms of the barred spiral galaxy (Figure 2).

The length of the bar will depend on how far the effect of the horizon event at the rotation poles reaches to transport matter and gas from the outer zone towards the singularity. According to the Kerr metric, it depends on the mass of the black hole, the rotation speed, and the closeness that the outer horizon event reaches to the external environment at the rotation poles.

6. Comparison between Calculated and Observed Rotational Velocities in Spiral Galaxies

Our purpose is to calculate the rotational velocity of some unbarred and barred spiral galaxies based on the relativistic solution and assuming their behaviour is like a rigid (or solid) body, plotting the rotation curves with the calculated values and compare them with the observed rotation curves. In particular, we focus on the rotation curves of two spiral Sa and two spiral SB galaxies. More precisely, we take as an example the known rotation curves of the spiral Sa galaxies: NGC 4378 and NGC 4594, and spiral SB galaxies: Milky Way and, NGC 7541, assuming that the only relevant contribution to the movement of their different regions is due to their behaviour as a rigid (or solid) body in rotation and considering only the movements found on the galactic plane.

Then, calculation the rotational velocity is given by Eq. (10), considering the known values for each of these four spiral galaxies [23-29]. From this information, it is observed that the nucleus masses of barred spiral galaxies are larger than the nucleus masses of unbarred spiral galaxies. The results are tabulated in Table 1 (Rotational velocities) for each galaxy.

Rotation curves are plotted in Figure 3, which shows the calculated results for these galaxies.

To make a comparison between the calculus given by Eq. (10) and the observed rotation curves of these galaxies [30,31], we superimpose the calculated rotation curves on the known curves.

In Figure 1, the solid lines show the rotation curves obtained from the observations. The rotation curves calculated with Eq. (10) are shown as dashed lines. We can see a good approximation between the calculated rotation curves and the known rotation curves of these spiral galaxies. The calculated rotation curves from Eq. (10) can also be applied to other known spiral galaxies to compare them with the observations.

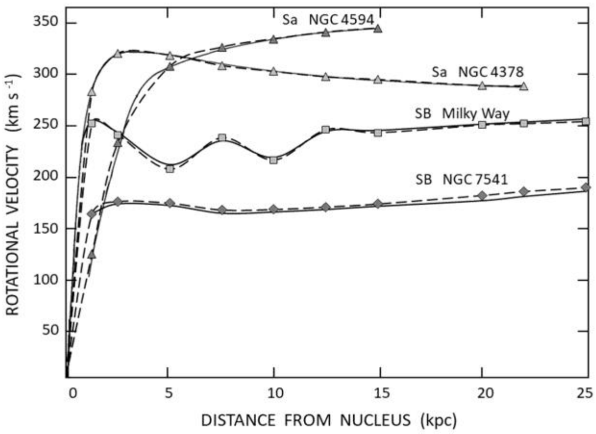

Figure 3. Plot of respective rotational velocities versus distance from nucleus for the unbarred spiral galaxies: NGC 4378 and NGC 4594, and the barred spiral galaxies: Milky Way and, NGC 7541. The dashed lines are calculated from Eqs. (10), and the solid lines come from the observations

In Figure 4, the respective angular momentum along the different distances from the nucleus for these spiral galaxies are plotted. In this way, it is shown the same almost linear pattern of angular momentum for all these spiral galaxies.

On the other hand, the Oort constants [32] have not been considered in this work, since such constants are empirically derived parameters that particularly characterize the local rotational properties of the Milky Way.

7. Conclusions

The aim of this work is to describe the hypothesis in which the explanation of the rotational velocity of some unbarred and barred spiral galaxies is based on the general relativity solution. Furthermore, it is considered that both the Solar System and spiral galaxies are behaving like a rigid (or solid) body in rotation. Thus, having both systems the same dynamical behaviour, which can be explained by the same relativistic solution, these two apparently different cases are unified in the same solution.

In this work, the relativistic solution of the rotational curves for the unbarred spiral galaxies and the Kerr metric for the rotating black hole are related to the observed bar of the barred spiral galaxies.

Then, this work shows the use of Eq. (3) derived from the general relativity solution, applied here to calculate the rotational velocity of some spiral galaxies.

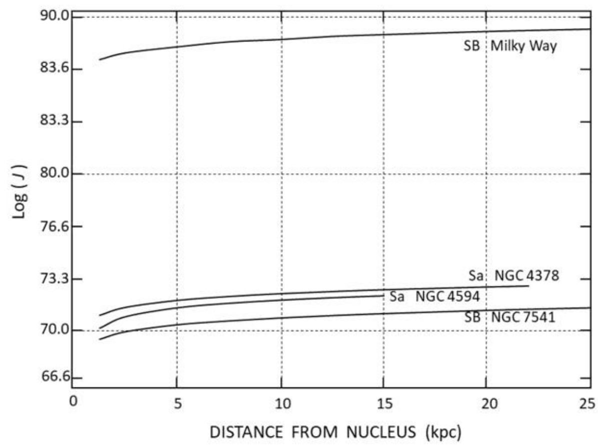

Figure 4. Plot of respective angular momentum for the unbarred spiral galaxies: NGC 4378 and NGC 4594, and the barred spiral galaxies: Milky Way and, NGC 7541. The curves show the same almost linear pattern for all these spiral galaxies along the different distances from their respective nucleus

Table 1. Rotational velocities: Tabulation of respective rotational velocities calculated from Eq. (10) for the unbarred spiral galaxies: NGC 4378 and NGC 4594, and the barred spiral galaxies: Milky Way and, NGC 7541

Galaxy	Sa NGC 4378			Sa NGC 4594			SB Milky way			SB NGC 7541		
Central mass ($M_\bullet$)	7.9×10^{7} $M_\odot$			6.6×10^{8} $M_\odot$			2×10^{10} $M_\odot$			4.9×10^{10} $M_\odot$		
Total mass (M_g)	3.65×10^{11} $M_\odot$			3.66×10^{11} $M_\odot$			1.3×10^{12} $M_\odot$			4.6×10^{11} $M_\odot$		
Distance from nucleus (kps)	Log(J)	J (kg m^2 s^{-1})	Rotational velocity (km s^{-1})	Log(J)	J (kg m^2 s^{-1})	Rotational velocity (km s^{-1})	Log(J)	J (kg m^2 s^{-1})	Rotational velocity (km s^{-1})	Log(J)	J (kg m^2 s^{-1})	Rotational velocity (km s^{-1})
0	0	0	0	0	0	0	0	0	0	0	0	0
1.25	70.91	8.21×10^{70}	280	70.09	1.22×10^{70}	120	87.31	2.03×10^{87}	249	69.38	2.38×10^{69}	160
2.5	71.42	2.64×10^{71}	318	70.82	6.63×10^{70}	230	87.74	5.47×10^{87}	238	69.86	7.24×10^{69}	172
5	71.87	7.41×10^{71}	316	71.40	2.49×10^{71}	305	88.12	1.33×10^{88}	204	70.31	2.03×10^{70}	170
7.5	72.12	1.32×10^{72}	306	71.69	4.85×10^{71}	324	88.45	2.81×10^{88}	235	70.55	3.58×10^{70}	164
10	72.30	1.99×10^{72}	300	71.88	7.66×10^{71}	332	88.59	3.92×10^{88}	213	70.74	5.53×10^{70}	164
12.5	72.44	2.73×10^{72}	295	72.04	1.09×10^{72}	339	88.80	6.25×10^{88}	243	70.89	7.84×10^{70}	167
15	72.55	3.56×10^{72}	292	72.16	1.45×10^{72}	343	88.91	8.11×10^{88}	240	71.02	1.05×10^{71}	170
20	72.73	5.37×10^{72}	286	-	-	-	89.11	1.29×10^{89}	247	71.23	1.69×10^{71}	178
22	72.79	6.19×10^{72}	285	-	-	-	89.17	1.49×10^{89}	249	71.30	2.00×10^{71}	182
25	-	-	-	-	-	-	89.26	1.82×10^{89}	251	71.39	2.47×10^{71}	186

Thus, we apply Eq. (10) to describe the rotational velocity of stars and gas in some spiral galaxies based on this solution. We present preliminarily examples of the rotation curves of unbarred spiral galaxies: NGC 4378 and NGC 4594, and barred spiral galaxies: Milky Way and, NGC 7541.

Comparing our calculations with the observations we find a good approximation. It is shown that the respective angular momentum for these spiral galaxies has the same almost linear pattern along the different distances from their respective nucleus. One of the significances of this result based on the general relativity solution is that it is possible to fit the known rotation curves of the spiral galaxies without any need of introducing dark matter at all. The next step in proving the galaxy dynamics on the general relativity solution is to make more detailed observations of their angular momentum to confirm if the way in which spiral galaxies rotate is mainly according to rigidly rotating and long-lived patterns, as steady spirals.

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