International Journal of Astronomy 2020, 9(2): 27-30
DOI: 10.5923/j.astronomy.20200902.01

The Rotational Velocity of Spiral Sa Galaxies in the General Theory of Relativity Solution

Adrián G. Cornejo

Electronics and Communications Engineering from Universidad Iberoamericana, Santa Rosa, Querétaro, Mexico

Abstract This work describes the hypothesis where the explanation of the rotational velocity of spiral Sa galaxies is based on the General Theory of Relativity solution. In this solution, a spiral Sa galaxy disk would rotate like a solid body, so that the stars into the spiral Sa galaxy must be rotating with the system at a uniform angular velocity. On these assumptions, we define the equation that describes the rotational velocity of stars in a spiral Sa galaxy based on the relativistic solution and present examples of the rotation curves of spiral Sa galaxies NGC 4378 and NGC 4594, comparing our estimates with the observations, finding a good approximation between the estimated rotation curves and the known rotation curves of these spiral Sa galaxies.

Keywords Galaxies: spiral, kinematics and dynamics, General Theory of Relativity

1. Introduction

One of the current problems of gravitation theory is to justify the large velocities observed for stars in spiral galaxies and how they differ from the rotational velocities of planets. Indeed, precise measurements (with a variety of techniques) of the velocities of stars and gases in spiral galaxies [1,2] make it clear that the action of the galaxy on its stars and gas estimated according to classical gravitation theory cannot account for the measured velocities, if the gas is assumed to be a stable component of the galaxy, as described by the quasi-stationary density wave theory, which characterizes spirals as rigidly rotating, long-lived patterns (i.e. steady spirals) [3].

Thus, we can consider that in principle, stars and gas in spiral galaxies do not follow a rotational motion as described in classical gravitational theory, so they must obey other considerations to satisfy the observations.

Currently, there are some plausible theories, but not yet proven and confirmed to date, which attempt to explain the rotational velocities observed in spiral galaxies.

The most popular theory nowadays is the speculation that brings the dark matter concept [4], which has not been detected to date. In that theory, the dynamics of the stars in a spiral galaxy is described by the classical Newtonian theory of gravity, in which other forces are simply added due to the supposed dark matter that is considered part of a galaxy.

* Corresponding author:
adriang.cornejo@gmail.com (Adrián G. Cornejo)
Received: Dec. 6, 2020; Accepted: Dec. 20, 2020; Published: Dec. 26, 2020
Published online at http://journal.sapub.org/astronomy

A second theory, which is currently less popular, is that which considers the influence of gravitational effects of the faraway matter that come from other distant galaxies and massive structures [5]. In that theory, the dynamics of the stars in a galaxy is also described by the classical Newtonian theory of gravity, in which some other forces are added due to the faraway matter outside a spiral galaxy.

Another theory that tries to solve the problem of the rotation of the spiral galaxy is the one that considers a gravito-magnetic field not yet discovered which influences the rotation curves of galaxies [6], among others.

However, these theories do not directly consider the General Theory of Relativity (GTR) [7], which has been shown to be a predictive theory that has been able to coherently describe the dynamics and kinematics behaviour of most celestial bodies in space-time, and which also includes the classic Newtonian theory of gravity.

Based on the General Theory of Relativity solution, we found that the rotational velocity of spiral Sa galaxies can be estimated from the total force equation for a rotational gravitational system solution derived from the General Theory of Relativity, being composed by the sum of the Newtonian force of gravity, the centrifugal force and a force related to the Coriolis force. This solution is different to the classical solution of the Newtonian force in the third term, since it includes the force with the inverse of the distance to the fourth power.

In this solution, the dynamics of the stars in a spiral Sa galaxy is not exclusively due to the combined gravitational field of all the stars, gas and any other massive object in the galaxy, but rather considers that the entire galaxy moves and rotates as a solid body.

The aim of this work is to describe the hypothesis where the explanation of the rotational velocity of some spiral Sa galaxies is based on the General Theory of Relativity solution. Thus, we define the equation that describes the rotational velocity of stars in a spiral Sa galaxy based on the relativistic solution. In particular, we show examples of the estimated rotation curves of spiral Sa galaxies NGC 4378 and NGC 4594 (with very different masses and sizes), comparing our estimates with the observations.

2. General Theory of Relativity Solution for the Total Force

A relativistic solution for the angular movement of the celestial bodies in a rotational system can be determined from a study of the solutions of Einstein's equations following the standard procedure [8,9,10]. Thus, the total force for a rotating system is given as

$$\left|F\right| = \frac{GMm}{r^2} - \frac{L^2}{mr^3} + \frac{3GML^2}{mc^2r^4}, \tag{1}$$

where r is a position vector and L is the orbital angular momentum of the rest mass m, given by

$$\vec{L} = mr\vec{v} = mr^2\vec{\omega}. \tag{2}$$

The first term in Eq. (1) represents the Newtonian gravitational force, which is described by the inverse-square law. This is the case of Newton equation [11] that describes the orbital motion of planets around the Sun. The second term represents the centrifugal force in circular motion. The third term is related to the Coriolis force, which includes the inverse of the distance to the fourth power, which is derived from the GTR.

3. Rotational Velocity in a Spiral Sa Galaxy

According to this relativistic solution, at the stage where a spiral Sa galaxy disk rotates like a solid (or rigid) body, the motion of the stars that form the spiral galaxy should be perceived from Earth as a uniform rotation of the galaxy. Taking into account the third term in Eq. (1), we can reduce common terms, giving

$$F_c = \frac{3GM\left(mr^2\Omega\right)^2}{mc^2r^4} = \frac{3GMm\Omega^2}{c^2}. \tag{3}$$

where Ω is the angular velocity of the rotating system.

In the same way as with the equation for classical mechanics scenario, we equate the centrifugal force with Eq. (3), obtaining

$$\frac{3GMm\Omega^2}{c^2} = m\frac{v^2}{r}, \qquad v = \left(\frac{3GMr\Omega^2}{c^2}\right)^{\frac{1}{2}}, \tag{4}$$

with M in this case being the resting mass $M_\bullet$ of the galactic nucleus. In a solid body, the angular velocity Ω of the system remains uniform. To calculate the rotational velocity, we can use as datum the angular velocity of the system, if known, or its equivalence with angular momentum J, which increases almost linearly with respect to any distance from the nucleus. Then, according to Eq. (2), we can write the angular momentum as

$$J = M_g r^2\Omega, \tag{5}$$

where M_g is the mass of the system (in this case, mass of the spiral galaxy).

The angular momentum of each galaxy can be estimated from the specific angular momentum $j = J/M_g$ [12], as function of distance in the spiral Sa galaxies. Then

$$v = \left(\frac{3GM_\bullet}{c^2r^3}\frac{J^2}{M_g^2}\right)^{\frac{1}{2}} = \left(\frac{3GM_\bullet j^2}{c^2r^3}\right)^{\frac{1}{2}}. \tag{6}$$

One can verify the spiral behaviour from Eq. (4), having that $\theta = \Omega t$ and, for the sake of describing the geometry of this equation, we take into account the Schwarzschild radius r_s [13]. Therefore, we write this in polar coordinates as

$$\frac{2v}{3}\left(\frac{r}{t}\right) = \frac{2GM_\bullet r\Omega}{c^2}\left(\frac{v}{r}\right), \qquad \frac{2}{3}r = r_S\theta, \qquad \frac{1}{\theta r} = \frac{3}{2}r_S u^2, \tag{7}$$

which is the equation of a spiral. The distance r can be described by the reciprocal $u = 1/r$ as function of θ according to the Binet equation in the solution of the relativistic equation derived for Schwarzschild metric [14,15].

4. Comparison between Estimated and Observed Rotational Velocities in Spiral Sa Galaxies

Our purpose is to estimate the rotational velocity of some spiral Sa galaxies based on the relativistic solution and assuming their behaviour is like a solid body, plot the rotation curves with the estimated values and compare them with the observed rotation curves. In particular, we focus on the rotation curves of two spiral Sa galaxies. More precisely, we take as an example the known rotation curves of the spiral galaxies NGC 4378 and NGC 4594, assuming that the only relevant contribution to the movement of their different regions is from their behaviour as a solid body, and considering only the movements found on the galactic plane. Then, an estimation of the rotational velocity is given by Eq. (6), taking into account the known values for each of these two spiral galaxies [16,17,18]. These results are tabulated in

Table 1 (Rotational velocities) for each galaxy.

Table 1. Rotational velocities: Tabulation of rotational velocities estimated from Eq. (6) for spiral Sa galaxies NGC 4378 and NGC 4594

Sa galaxy	NGC 4378		NGC 4594	
Central mass ($M_\bullet$)	7.9×10^7	$M_\odot$	6.6×10^8	$M_\odot$
Total mass (M_g)	3.65×10^{11}	$M_\odot$	3.66×10^{11}	$M_\odot$
Distance from nucleus (kps)	J (kg m^2 s^{-1})	Rotational velocity (km s^{-1})	J (kg m^2 s^{-1})	Rotational velocity (km s^{-1})
0	0	0	0	0
1.25	8.21×10^{70}	280	1.22×10^{70}	120
2.5	2.64×10^{71}	318	6.63×10^{70}	230
5	7.41×10^{71}	316	2.49×10^{71}	305
10	1.99×10^{72}	300	7.66×10^{71}	332
15	3.56×10^{72}	292	1.45×10^{72}	343
20	5.37×10^{72}	286	-	-
22	6.19×10^{72}	285	-	-

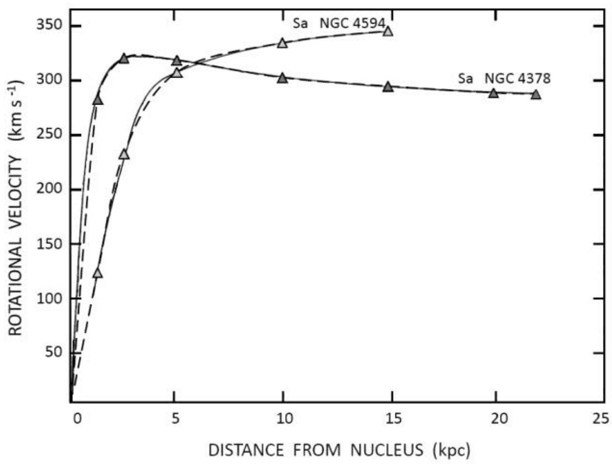

Figure 1. Plots of rotational velocities versus distance from nucleus, for NGC 4378 and NGC 4594 galaxies, respectively. The dashed lines are estimated from Eq. (6) and the solid lines come from the observations [19]

Rotational curves are plotted in Figure 1, which shows the estimated results for both galaxies.

To make a comparison between the estimations given by Eq. (6) and the observed rotational curves of these galaxies [19], we superimpose the estimates on the known curves.

In Figure 1, the solid lines show the rotation curves obtained from the observations. The rotation curves estimated with Eq. (6) are shown as dashed lines. We can see a good approximation between the estimated rotation curves and the known rotation curves of these spiral Sa galaxies. The estimates from Eq. (6) can also be applied to other known spiral Sa galaxies to compare them with the observations.

On the other hand, the Oort constants have not been considered in this work, since such constants are empirically derived parameters that characterize the local rotational properties of the Milky Way [20], which is a spiral SB galaxy (but it's not a spiral Sa galaxy, like those we have focused in this work). Nevertheless, Eq. (6) could be developed in its geometry and kinematics in order to be applied to other types of spiral galaxies, which could be the subject of a further work.

5. Conclusions

The aim of this work is to describe the hypothesis where the explanation of the rotational velocity of some spiral Sa galaxies is based on the General Theory of Relativity solution. Then, this work shows the using of Eq. (1) derived from the General Relativity solution, here applied to estimate the rotational velocity of some spiral Sa galaxies. Then, we define Eq. (6) to describe the rotational velocity of stars in some spiral Sa galaxies based on this solution, and present preliminarily two examples of the rotation curves of spiral Sa galaxies NGC 4378 and NGC 4594, then comparing our estimates with the observations, finding a good approximation between the estimated rotation curves and the known rotation curves of these spiral Sa galaxies. One of the significances of this result based on the General Theory of Relativity solution is that it is hypothetically possible to fit the known rotation curves of the spiral Sa galaxies without any need of introducing dark matter at all. The next step in probing the galaxy dynamics on the General Relativity solution is to make more detailed observations to confirm whether the way in which spiral Sa galaxies rotate is mainly according to rigidly rotating and long-lived patterns, as steady spirals.

ACKNOWLEDGEMENTS

The author would like to thank Professor Sergio S. Cornejo for its review and comments for this work.

REFERENCES

[1] Sofue, Y., 2016, Rotation and mass in the Milky Way and spiral galaxies. Publications of the Astronomical Society of Japan, 69(1). arXiv:1608.08350v1.

[2] Beckman, J. E., Zurita, A., and Vega Beltrán, J. C., 2004, Kinematic measurements of gas and stars in spiral galaxies. Lecture Notes and Essays in Astrophysics, 1, 43-62.

[3] Baba, J., Morokuma-Matsui, K., Miyamoto, Y., Egusa, F., and Kuno, N., 2016, Gas velocity patterns in simulated galaxies: observational diagnostics of spiral structure theories. Monthly Notices of the Royal Astronomical Society, 460(3), 2472-2481.

[4] Sofue, Y., and Rubin, V. C., 2001, Astron. Astrophys. Vol. 39, p.137. arXiv:astro-ph/0010594.

[5] Carati, A., 2011, Gravitational effects of the faraway matter on the rotation curves of spiral galaxies. arXiv:1111.5793

[astro-ph.CO].

[6] De Mees, T., 2007, Deduction of orbital velocities in disk galaxies, Marquis Who'sWho Publications.

[7] Einstein, A., 1955, The Meaning of Relativity, 5th ed. Princeton University Press, Princeton.

[8] Arbab, A. I., 2012, J. Mod. Phys., Vol. 3 No. 29, 1231-1235. arXiv:1105.1911 [physics.gen-ph].

[9] Cheng, T., 2005, Relativity, gravitation, and cosmology, Oxford University Press. pp. 98–99.

[10] Cornejo, A. G., 2014, A lagrangian solution for the precession of Mercury's perihelion, Int. J. Astron. 3, 31–34.

[11] Unzicker, A., 2008, Why do we Still Believe in Newton's Law? Facts, Myths and Methods in Gravitational Physics. arXiv:gr-qc/0702009v8.

[12] Romeo, A. B., and Mogotsi, K. M., 2018, Mon. Not. R. Astron. Soc. 000, 1–5. arXiv:1805.05871 [astro-ph.GA].

[13] Schwarzschild, K., 1916, Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. Sitzungsberichte der Königlichen Preußischen Akademie der Wissenschaften zu Berlin 189–196.

[14] Kren, P., 2007, The source the field or the metric? Gen. Science J., 1246. https://www.gsjournal.net/Science-Journals/Research%20Papers/View/1246.

[15] Fabian, A. C., and Lasenby, A. N., 2019, Astrophysical Black Holes. Publisher: De Gruyter. arXiv:1911.04305v1 [astro-ph.HE].

[16] Akylas, A., and Georgantopoulos, I., 2009, Astron. Astrophys. 500.3: 999–1012. DOI:http://dx.doi.org/10.1051/0004-6361/200811371.

[17] Jardel, J. R. et al., 2011, Orbit-based dynamical model of the sombrero galaxy (NGC 4594). ApJ: 30. arXiv:1107.1238 [astro-ph.CO].

[18] Rubin, V. C., Ford, W. K. Jr., Strom, K. S., Strom, S. E., and Romanishin, W., 1978, ApJ, Vol. 224, p. 782-795.

[19] Rubin, V. C., Ford, W. K. Jr., and Thonnard, N., 1978, ApJ, 225: L107-L111.

[20] Binney, J., and Tremaine, S., 2008, Galactic Dynamics, 2nd ed. Princeton University Press, Princeton.