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Diagnostics of SGR Magnetospheres Using Coronal Seismology

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Abstract We develop a method of diagnostics of magnetospheres of soft gamma-ray repeaters (SGRs) using high-frequency (20-2400 Hz) quasi-periodic pulsations (QPOs)[14]. The trapped fireball is represented as a set of current-carrying loops, equivalent RLC-circuits, rooted into a neutron star surface. The model explains the observed periods of QPOs and their high quality factor $Q \approx 10^4$-10^5. The parameters of the source of the pulsations at the «ringing tail» stage for three well-known giant SGR flares are determined: the electric current $I \approx (2\text{-}8)\times 10^{19}$ A, the magnetic field $B \approx (0.6\text{-}2.7)\times 10^{13}$ G < B_Q, and the electron number density $n \approx (1.3\text{-}6.0)\times 10^{16}$ cm^{-3}. We also show that high-frequency QPOs can be self-excited for an electric current smaller than the maximum current in the giant pulse of the flare, and/or due to the parametric resonance. The result is consistent with the conclusion made by Rea et al.[11] that a high surface magnetic field is not necessarily required for the magnetar activity.

Keywords Magnetars, Quasi-Periodic Oscillations, Coronal Loops, Magnetic Field

1. Introduction

The strongest cosmic magnets, magnetars, are historically divided into two classes of neutron stars: soft gamma-ray repeaters (SGRs) detected in the hard X-ray/soft-gamma ray band, and anomalous X-ray pulsars (AXPs) first detected in soft X-rays (< 10 keV). Their activity is powered by the decay of ultrastrong magnetic fields and lasts about 10^3-10^5 years (see, e.g., a review[1]). The current information on 23 magnetars is held in McGill SGR/AXP Online Catalog[2].

In the last decade, the primary consideration has been received by SGRs related to very high energy release in the flares, up to 5×10^{46} erg. The peak luminosity of SGR giant flares $L_x \approx 2\times 10^{44}$-$5\times 10^{47}$erg s^{-1} substantially exceeds any previous transient event observed in our Galaxy. The radiation flux from giant SGR flares is so huge that it is reflected by the ionisation level in the bulk of the Earth's ionosphere by several orders of magnitude and the SGR rotating period modulates the ionisation despite the fact that the source is several tenths thousands light years away. For example, the giant SGR 1806-20 flare on December 27th 2004 created disturbances in the daytime lower ionosphere corresponding to the increase in electron density by 2-3 orders of magnitude[3]. Moreover, the lunar echo from SGR 1806-20 flare clearly affected the night dark polar ionosphere[4].

Currently, it is suggested that SGRs are isolated rotating neutron stars with the radius ~ 10 km, the mass ~ 1.5 M_{Sun}, and the spin period 2 to 10 s. SGRs have external magnetic fields of the order of 10^{14}-10^{15} G, two-to-three orders higher than those in classical radio pulsars, and internal fields that can reach 10^{16} G[5]. Three giant flares of magnetars (Table 1), which occurred on 1979 March 5th (SGR 0526-66), on 1998 August 27th (SGR 1900+14), and on 2004 December 27th (SGR 1806-20), with an energy release of $(2\text{-}500)\times 10^{44}$ erg, were accompanied by the "ringing tails" of high-frequency (tens to thousands Hz) quasi-periodic oscillations (QPOs) of the X-ray emission[6],[7],[8].

Table 1. Three giant flares of Soft Gamma Repeaters

	SGR 0526–66 March 5, 1979	SGR 1900+14 August 27, 1998	SGR 1806–20 December 27, 2004
Assumed distance, kpc	50	15	8.7
Energy in initial spike, erg	1.6×10^{44}	$>1.5\times 10^{44}$	$(1.6\text{-}5)\times 10^{46}$
Initial spike duration, s	~ 0.25	~ 0.35	~ 0.5
Main pulse period, s	8.1	5.15	7.56
Energy in pulsating tail, erg	3.6×10^{44}	1.2×10^{44}	1.3×10^{44}

According to current concepts, giant flares in SGRs appear due to sudden perturbations in the metal crust of the

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star: the motion of tectonic plates[9], starquakes, which trigger catastrophic reconfiguration of electric currents and magnetic fields in the magnetosphere, accompanied by enormous energy release. The forming fireball of hot (~ 1 MeV) electron-positron plasma and high-energy photons is the source of the main pulse of the flare, which lasts for several fractions of a second[10]. The fireball escapes from the surface of the star with the relativistic speed. The remaining «residual» - the trapped fireball, i.e. plasma captured by the magnetic field frozen into the crust of the star, decays for several minutes and provides the «tail» of the flare releasing a total energy up to ~ 10^{44} erg, in which, along with spin second oscillations, high-frequency quasi-periodic pulsations (QPOs) are clearly pronounced.

One of the challenges for astrophysicists is the origin of the magnetic field in neutron stars, in particular, in soft gamma repeaters. The values of internal and surface magnetic field in SGR are currently a matter of discussion. Current views consider magnetars as neutron stars with magnetic fields exceeding the electron quantum field $B_Q = m_e^2 c^3 / e\hbar \approx 4.4\times10^{13}$ G at which the nonrelativistic Landau energy $\hbar eB / m_e c$ becomes equal to the electron rest energy $m_e c^2$. However, there are evidences for the existence of SGRs with low magnetic fields. Rea et al.[11] described SGR 0418+5729 with the dipolar magnetic field $B < 7.5\times10^{12}$ G. X-ray observations of the outburst of the magnetar J1822.3-1606 made with *Swift, Rossi X-Ray Timing Explorer* (*RXTE*), *Chandra,* and *X-ray Multi-Mirror Mission-Newton (XMM-Newton)* yielded the surface magnetic field $B \approx 2.7\times10^{13}$ G < B_Q[12]. Moreover, Malov[13] concluded that even a magnetic field of 10^{16} G in the stellar interior cannot explain the giant outbursts of SGRs and that the existence of low-magnetic-field SGRs indicates that the main attribute of a magnetar ($B > B_Q$) may not be inherent in all SGRs/AXPs.

To estimate the surface magnetic field strength, several methods are used, based on various physical phenomena: the spin-down rate, the ion cyclotron resonance, peculiarities of QPOs. Recently, we proposed an independent diagnostic method based on coronal seismology, using the parameters of trapped fireball plasma[14]. The Alfvén and Carlqist's[15] concept of a flare loop as an equivalent electric circuit is the basement of this method. Our approach considers a trapped fireball, the source of high-frequency QPOs, as a set of current-carrying loops, which can be represented as equivalent RLC-circuits. Using the period and the Q-factor of QPOs we can estimate the electric current, magnetic field, and electron density in QPO sources.

This paper is devoted to the further applications of our diagnostic method of SGR coronae based on coronal seismology. Section 2 presents a brief description of the existing methods of the evaluation of magnetic fields. Exsisting QPO models are discussed in Section 3. In Section 4 we use QPO characteristics to diagnose physical parameters of magnetospheres for three well-known giant SGR flares. Excitation mechanisms of QPOs will be analyzed in Section 5. Section 6 presents discussion and conclusions.

2. Magnetic Field Diagnostic Methods

The most popular method of estimation of the surface dipolar field is the rate of kinetic energy loss via magnetic-dipole radiation:

$$B = \left(3Mc^3\dot{P}P / 8\pi^2 R^3_{ns}\right)^{1/2} \approx 3.2\times10^{19}(P\dot{P})^{1/2}G \quad (1)$$

Here, P is the spin period in seconds, $\dot{P}$ is the spin down rate; it is assumed that $R_{ns} \approx 10^6$ cm and $M \approx 10^{45}$ g cm^2 for the star radius and momentum of inertia. Most of the sources with magnetar-like activity have rotational periods 2--12 s and period derivatives 10^{-13} – 10^{-10} ss^{-1}. Therefore, the dipolar field spans 5×10^{13} - 2×10^{14} G[1]. These values exceed the average field in radio pulsars by one to three orders of magnitude; they also exceed the electron quantum field $B_Q \approx 4.4\times10^{13}$ G. Besides, magnetars have a relatively small age $t_c = P / 2\dot{P} \approx 0.2kyr - 0.2My$ High surface field strength ~ 10^{15} G cannot result in a powerful energy release ~ 5×10^{46} erg; it could only be possible either if the mechanism of the flare was extremely efficient or if the interior field was substantially stronger and reached 10^{16} G[16].

Recent observations of SGR 0418+5729 yield $P \approx 9.1$ s, the upper limit $\dot{P} < 6\times10^{-15}$ ss^{-1}, and the corresponding limit on the surface magnetic field $B < 7.5\times10^{12}$ G[11]. The upper limit for $\dot{P}$ implies a characteristic age $t_c > 24$ Myr. With these parameters, the internal toroidal field for SGR 0418+5729 can be estimated as $B^2_{tor} \approx 6L_x t_c / R^3_{NS}$ [5]. Assuming the source distance of 2 Kpc and $L_x \approx 6.2\times10^{31}$ erg s^{-1}, one can obtain $B_{tor} \approx 5\times10^{14}$ G. SGR 0418+5729 may represent the population of low-dipolar-field magnetars that are dissipating the last bit of their internal energy[11]. Another example for a low-magnetic-field magnetar is presented by SGR 1822-1606, with $P \approx 8.44$ s, $\dot{P} \approx 8.3\times10^{-14}$ ss^{-1}, which yields $B \approx 2.7\times10^{13}$ G < B_Q and $t_c \approx 1.6$ Myr[12].

Some models of X-ray spectra suggest a strong absorption line of the proton cyclotron resonance; hence $B \approx 1.6(1+z)E_c(\text{keV})\times10^{14}$G, where $z \approx 0.3$ is the gravitational redshift. For $E_c = 5$ keV, absorption features in SGR 1806-20 bursts give $B \approx 10^{15}$ G[17]. On the other hand, the evidence for the emission line at 6.4 keV obtained during the bursts of SGR 1990+14 with *Rossi X-Ray Timing Explorer* (*RXTE*) implies the surface field strength $B \approx (1.3\text{-}2.6)\times10^{15}$ G, depending on the proton or He^4 cyclotron resonance[18].

An independent evidence for the superstrong surface magnetic field in SGR 1806-20 was obtained by Vietri et al.[19]. They pointed out the largest luminosity variation $\Delta L / \Delta t \approx 6\times10^{43}$ erg s^{-2} in the fastest (625 and 1840 Hz) QPOs in the ringing tail of the 2004 December 27 event in SGR 1806-20, which exceeded the common Cavallo-Fabian-Rees luminosity variability limit $(\Delta L / \Delta t)_{CFR} < 2\times10^{42}$ erg s^{-2} (for a matter-to-radiation conversion efficiency of 100%) by more than an order of magnitude. According to[19], such high $\Delta L / \Delta t$ may be due to the vacuum polarization effect, which reduces the scattering cross-section with respect to the Thompson's because of the presence of a strong magnetic field $B \geq 6.6\times10^{13}$ G in the 30 km size QPO source. Hence the magnetic field on the star surface is $B \geq 10^{15}$ G[19].

3. Existing Models of QPOs

Three giant flares of SGRs were accompanied by high-frequency (tens to thousands Hz) quasi-periodic X-ray pulsations (Table 2). Such pulsations were observed not only in flare tails, 100 to 300 s after the main pulse, which lasted ≤ 1 s, but also at the growth phase of the main pulse[6],[16]. The greatest variety of the pulsations, from 18 to 2384 Hz (Figure 1), were detected by *RXTE* and *Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI)* space missions in the «ringing tail» of the flare of SGR 1806-20[20].

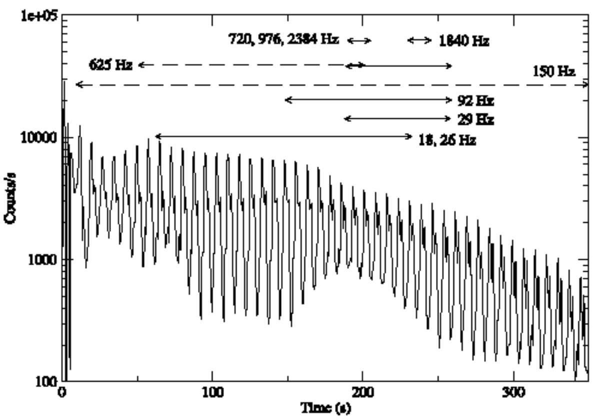

Figure 1. Time periods for various fast QPOs (18 to 2384 Hz) detectable by *RXTE* and *RHESSI* in the 'ringing tail' of SGR 1806-20. The spin period is 7.56 s[20]

A model of the high-frequency pulsations should explain not only their periods and the excitation mechanism, but also their high quality factor $Q \geq 10^4$-10^5. For example, in the flare of SGR 1806-20 on 2004 December 27 pulsations with the frequency 1840 Hz were observed during 50 s, whereas pulsations with the frequency 625 Hz lasted 200 s[19]. Current models are unable to explain the total set of the observed characteristics of pulsations with frequencies of 18 to 2400 Hz.

The most widespread models for high-frequency QPOs are based on global seismic oscillations of the magnetar (see, e.g.[8],[21]). The popularity of these models is caused by intense development of asteroseismology, which provides new insight into the inner structure and crust of a neutron star. In early models of neutron stars, starquakes resulted in excitation of torsion oscillations of the crust with the shear. Motions of the crust provide modulation of super-strong magnetic fields and electric currents in the magnetosphere of a neutron star, due to which the X-ray flux varies. This explains oscillations with periods from 30 Hz (the mode $n = 0$, $l = 2$) to 1840 Hz ($n = 3$). However, the observed pulsations with the frequency ≤ 20 Hz cannot be explained by the torsion oscillations[8]. Seismic models also fail to consider the reason for the high quality factor of the oscillations. In addition, Levin[22] pointed out that the crustal torsion modes decay very rapidly, for the time of the order of 10 oscillations, due to the transfer of their energy to Alfvén waves, which effectively decay in the fluid core of the magnetar. Therefore, Levin suggests that either QPOs should be of a magnetospheric origin, or the magnetic field of the core should display a special configuration before a flare. Currently, seismic models of QPOs take into account the role of the fine structure of the crust and peculiarities in the configuration of the magnetic field of neutron stars[23].

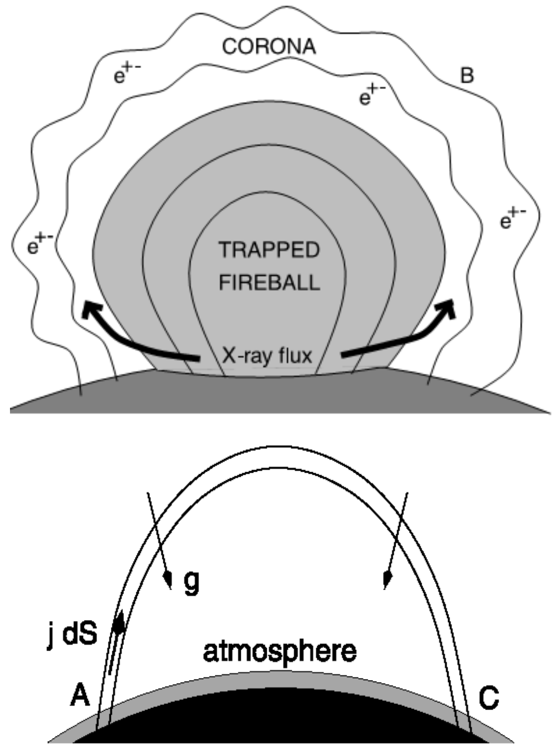

Figure 2. Schematic views of a trapped fireball (upper) and a single current-carrying coronal loop frozen into the neutron star surface (below). The electric current is initiated by starquakes, which leadto a twisting of one or two loop footpoints (A and/or C). The current flows along the loop and is closed in the metal crust of the magnetar[24]. Our approach suggests a multi-loop trapped fireball

Beloborodov and Thompson[24] proposed an alternative non-seismic mechanism of oscillations, when the QPO source is located in the magnetosphere; in this case, the formation of the magnetar's corona, which consists of a set of magnetic loops (Figure 2), is accompanied by non-linear oscillations of the electron-positron pair-creation process. The parallel electric field that arises in a magnetic flux tube as a result of its twisting, and the coronal plasma adjust each other self-consistently: the electric field should be sufficient for the creation of electron-positron pairs. A cyclic behaviour is possible, in which the coronal plasma is periodically saturated and the electric field is reduced by screening. A numerical simulation showed that such oscillations of the electric current in a coronal loop may display sufficiently high frequency ~ 10 кHz[24]. The electron density in a magnetar corona was estimated as $n \sim 10^{17}$ cm^{-3}[24].

Ma et al.[25] interpreted QPOs on the basis of MHD-oscillations of coronal magnetic loops in the corona of a magnetar. These authors suggest that oscillations of magnetic loops are excited by the turbulence at the footpoints located in the metallic crust of the star. The observed QPO frequencies in SGR 19001+14 and SGR 1806-20 (18, 26, 30, 92, 150 Hz) are explained in this study in terms of standing slow sausage-mode oscillations of coronal magnetic loops, which modulate SGR radiation. However, the proposed mechanism is unable to explain high-frequency pulsations with the frequency 625 and 1840 Hz[25]. It is important to note that Eq.(5) suggests Alfvén velocity $V_A = c$ under the magnetar conditions. Due to this circumstance, the expression for kink speed $c_k \cong \sqrt{2}V_A$ derived in[25] has no physical meaning. Moreover, instead of the general formula for the Alfvén velocity (5), the expression $V_A = B/\sqrt{4\pi m_i n}$ was used in[25]. This resulted in $V_A >> c$ even for $B \approx 10^{14}$ G, and the plasma number density $n \approx 10^{28}$ cm^{-3}.

4. The Suggested Approach: An Equivalent Electric (RLC) Circuit

Our model for high-frequency pulsations in magnetars is based on the ideas of coronal seismology, which studies oscillations and waves in stellar coronae[26]. Coronal seismology appeared to be very efficient in diagnostics of the parameters of the coronae and flare plasma not only in the Sun, but also in late-type stars[27]. Currently, two approaches are developed in coronal seismology. The first one studies MHD-oscillations and waves in the basic structures of stellar coronae – coronal magnetic loops. In these formations, flare energy release occurs. The other approach is based on the Alfvén and Carlqvist's[15] idea, according to which a flare loop is considered as an equivalent electric circuit. The basic concepts of this approach are presented in the review of Khodachenko et al.[28] and in the book[26].

The corona of a magnetar and the trapped fireball, to which the ringing tail is generally related, may be presented as a set of current-carrying magnetic loops of various sizes (Figure 2), whose eigen-frequencies and quality factors are given by well-known expressions

$$\nu = \left(2\pi\sqrt{LC}\right)^{-1}, \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{2}$$

Here, R and C are the resistance and capacitance of a coronal loop, L – the inductance specified by the geometry of the loop. The latter value may be expressed by the formula for a thin wire of the length that greatly exceeds the radius $l >> r$:

$$L = 2l\left(\ln\frac{4l}{\pi r} - \frac{7}{4}\right) \tag{3}$$

Given the energy $E = LI^2/2$ that has been released in the flare tail, one may determine the electric current I in the coronal loops and hence the coronal plasma density and the φ-component of the magnetic field. The observed energy release rate $W = RI^2$ makes it possible to find the resistance R of a coronal current-carrying loop, while from the frequency of the oscillations, the loop's capacity C, that is, the quality factor Q of the oscillations, may be estimated. Below, we will illustrate the efficiency of our model by the examples of the most powerful well-known SGR events with developed QPOs.

4.1. The Flare of SGR 1806-20 on December 27th 2004

The ringing tail of the SGR 1806-20 giant flare on December 27th 2004 reveals the largest number of pulsation frequencies in the interval from 18 to 2384 Hz. The total energy released in this flare was of the order of 5×10^{46} erg, while the stored magnetic energy ~ 10^{47} erg[16]. The energy of the pulsating tail was of the order of 10^{44} erg[19]. Taking into account the great variety of QPO frequencies (Table 2), we will suggest that the energy stored in an 'average' loop in the course of the 'ringing tail' is roughly 10^{43} erg. Supposing the length and radius of a loop $l = 3\times10^6$ cm and $r = 3\times10^5$ cm, respectively, we can use Eqs. (3) to find its inductance $L \approx 5\times10^6$ cm = 5×10^{-3} Henry. Assuming that the stored energy of an 'average' loop, $E \approx 10^{43}$ ergs = 10^{36} J, has been released, we obtain the current $I = (2E/L)^{1/2} \approx 2\times10^{19}$ A, and from Biot-Savart law we estimate the φ-component of the magnetic field in the loop $B_\varphi \approx I/cr \approx 6\times10^{12}$ G. The density of electron-positron pairs n in the source of the tail will be obtained from the electric current $I = encS$ and the cross section of the coronal loop S with the radius $r = 3\times10^5$ cm. For $I = 2\times10^{19}$ A, $n = 1.3\times10^{16}$ cm^{-3}, i.e., the Langmuir frequency $\nu_p = \omega_p/2\pi \approx 1$ THz corresponds to sub-millimeter wavelengths.

The power of the energy release in the tail of a flare with the duration ≈ 200 s is of the order of ≈ 10^{41} erg/s[19] i.e. for an «average» loop, the power is $W = RI^2 \approx 10^{40}$ erg/s = 10^{33} W. The resistance of a loop in the ringing tail of the flare of SGR 1806-20 is $R = W/I^2 \approx 2\times10^{-6}$ Ω. This resistance value may be due to anomalous conductivity that emerges in the course of excitation of small-scale plasma waves. The effective (turbulent) resistance may be presented in the form

$$R = R_{eff} = \frac{l}{\sigma_{eff} S} = \frac{l}{S}\frac{4\pi\nu_{eff}}{\omega_p^2}, \tag{4}$$

where the anomalous (turbulent) conductivity $\sigma_{eff} = e^2 n/m\nu_{eff}$. For $l \approx 3\times10^6$ cm, $r \approx 3\times10^5$ cm, $\omega_p \approx 10^{13}$ s^{-1}, and $R_{eff} \approx 10^{-6}$ Ω ≈ 10^{-18} s cm^{-1}, from Eq.(4) we obtain $\nu_{eff} = (W_p/nk_BT)\omega_p \approx 10^{-1}\,\omega_p$. Thereby, the level of small-scale plasma turbulence W_p/nk_BT in the QPO source should be appreciable. One of the possible reasons for this level of plasma waves may be the instability of comparatively dense (n_b/n ~ 0.1) beams of high-energy electrons accelerated in electric fields of the magnetar's corona[24]. Here, n_b is the beam density. The other possible reason for the anomalous resistance in the magnetar's corona is related to the instability of ion-sound waves[24], which emerges when the relative velocity of ions and electrons exceeds the speed of sound, $v_d > c_s = (5k_B T/3m_i)^{1/2}$. Note that, according to current concepts, the corona of a magnetar, in addition to electron-positron pairs, contains around 10% ions. For the temperature of the cooling trapped fireball $T = 3\times10^9$ K, $c_s \approx 6\times10^8$ cm/s << c.

The minimum (ν_1 = 18 Hz) and maximum (ν_2 = 2384 Hz) frequencies of the ringing tail of SGR 1806-20 make it possible to estimate the capacity of current-carrying magnetic loops – equivalent RLC–circuits, using relations (2) for the frequency: $C_1 \approx 1{,}5\times10^{-2}$ F, $C_2 \approx 8\times10^{-7}$ F. On the other hand, the capacity of a coronal magnetic loop may be approximately presented as follows[27]: $C \approx \varepsilon_A S / l$, where $\varepsilon_A = (k/\omega)_A^{\,2} c^2$ is the dielectric permittivity for Alfvén waves. If the displacement current is included in the analysis, the dispersion relation for Alfvén waves is[29]

$$\left(\frac{\omega}{k}\right)_A = \frac{c}{\sqrt{1+\frac{4\pi\rho c^2}{B^2}}} \quad (5)$$

Since in the magnetar corona $B^2 >> 4\pi\rho c^2$ the Alfvén velocity is roughly equal to c. Therefore, $\varepsilon_A \approx 1$, and for the assumed cross-section of the loop $S = \pi r^2 \approx 3\times10^{11}$ cm^2 and its length $l = 3\times10^6$ cm, we obtain $C \approx 10^5$ cm $=10^{-7}$ F, which is by the factor of a few lower than that (C_2) calculated from the formula (2). Note that the sizes of coronal loops in the trapped fireball may differ by several orders of magnitude. We can see that with an increase in the cross-section S and a decrease in the loop length l (a «thick» loop), the coincidence between the calculated capacity C and both C_2 and C_1 may be reached. Using the second relation from the formula (2), we can find the quality factors for the minimum and maximum frequency: $Q_1 \approx 3\times10^5$ and $Q_2 \approx 10^7$, which exceed the observed quality factors of the QPO by one or two orders of magnitude. This discrepancy may be due to both an insufficient sensitivity of the detectors, and the «cooling» of the trapped fireball.

Our model suggests that oscillations of electric current should be in-phase in all points of a loop. On the other hand, variations of the current propagate along the loop with the Alfvén velocity. Therefore, for the condition of phase coincidence, the Alfvén time should be substantially smaller than the period of oscillations. For SGRs considered here, the in-phase condition is satisfied, $\nu_{RLC} \approx$ 20-2500 Hz $< \nu_A \approx c/l \geq 10^4$ Hz, because in magnetar coronae $V_A \approx c$.

The same method was applied to the determination of magnetosphere parameters of the giant flares in SGR 0526–66 (March 5th 1979) and SGR 1900+14 (August 27th 1998). Table 2 presents the data of observations of "ringing tails" with QPOs and the results of calculations of magnetosphere parameters. One can see that our diagnostic method yields the magnetic field in the SGR magnetospheres, which does not exceed the Schwinger critical value $B \approx (6\text{-}27)\times10^{12}$ G $< B_Q$.

Table 2. Pulsating tail properties in giant flares of SGRs and magnetosphere parameters

	SGR 0526–66 March 5, 1979	SGR 1900+14 August 27, 1998	SGR 1806–20 December 27, 2004
Duration, s	~ 200	~ 400	~ 380
Energy, erg	3.6×10^{44}	1.2×10^{44}	1.3×10^{44}
Main pulse period, s	8.1	5.15	7.56
QPO frequencies, Hz	43	28,54,84,155	18,26,30,93,150,625, 720,976,1840,2384
Q-factor	~ 10^4	~ $10^4 - 10^5$	~ $10^4 - 5\times10^5$
Calculated parameters			
Electric current, A	8×10^{19}	3×10^{19}	2×10^{19}
Magnetic field, G	2.7×10^{13}	10^{13}	6×10^{12}
Electron density, cm^{-3}	6×10^{16}	2×10^{16}	1.3×10^{16}

5. Excitation of High-frequency Oscillations of the Current in Coronal Loops

For minor deviations of the electric current $|\tilde{I}| << I$, the equation that describes oscillations of the electric current in a loop may be presented in the form[28]:

$$L\frac{\partial^2 \tilde{I}}{\partial t^2} + \alpha\left(I^2 - I^2_{\max}\right)\frac{\partial \tilde{I}}{\partial t} + \frac{\tilde{I}}{C} = 0 \quad (6)$$

The equation (6) takes into account that, since the anomalous resistance $R_{eff} \sim \nu_{eff}$ is proportional to the power of energy release $W \sim I^2$, then from dimensional relations the effective resistance may be presented as $R_{eff} \sim \alpha I^2$, where α is a factor. Equation (4) gives us the effective collisional frequency $\nu_{eff} / \omega_p \approx 0.1$. This value was used before to obtain the level of small-scale turbulence in a QPO source which determines the effective resistance of "average" loop.

Equation (6) indicates that oscillations will be excited for a current smaller than the maximum current in the giant pulse of the flare, $I < I_{\max}$, that is, not only at the descending, but also at the ascending stage of the flare. Recall that pulsations with the frequency 43 Hz in SGR 0526-66[6] and 50 Hz in SGR 1806-20[16] were also observed at the ascending stage of the pulse phase.

Consider another possible way of generation of oscillations in coronal magnetic loops – excitation due to parametric resonance[30]. As a result of parametric interaction with the coronal loop, the oscillations of the electric current due to perturbations in the crust of the magnetar with the pumping frequency ν may trigger ocsillations in the loop, with the frequency ν, at the sub-harmonics ν/2, and at the first upper frequency of the parametric resonance 3ν/2. A similar effect is observed in the optical and microwave radiation of solar flares[30].

Variations of the parameters of a coronal loop may be described with the equation

$$\frac{d^2 y}{dt^2} + \nu_0{}^2 (1 + q\cos \nu t) y = 0 \qquad (7)$$

Here, ν_0 is the frequency of the eigen-oscillations of the coronal loop. The parameter q specifies the width of the zone around the parametric resonance frequency $\nu_n = n\nu/2$, $n = 1, 2, 3,\ldots$, namely $q\nu_0/2 < \nu/2 - \nu_0 < q\nu_0/2$[31]. The excitation occurs when the frequency of eigen-oscillations of the loop ν_0 falls on the first zone of instability, i.e., when it is close to $\nu/2$. This means that, for parametrical excitation of a coronal loop, the latter should display an appropriate size, density, temperature, and magnetic field. It is not excluded that in the SGR 1901+14 27 flare on 1998 August 27 the QPOs were excited due to parametric resonance: ν = 53 Hz, $\nu/2$ = 26.5 Hz (at the observed frequency 28 Hz), $3\nu/2$ = 79.5 Hz (at the observed frequency 84 Hz). Note that for the pumping frequency ν equal to 56 Hz rather than 53 Hz, with a high accuracy we obtain the observed frequencies $\nu/2$ = 28 Hz and $3\nu/2$ = 84 Hz. Inductive interaction between current-carrying coronal loops also may increase the number of observed QPO frequencies[28].

6. Discussion and Conclusions

QPO models based both on global seismic ocsillations and on MHD-oscillations in coronal magnetic loops face difficulties in explanation for both the observed periods of oscillations and their high quality factors. Firstly, seismic oscillations do not explain pulsations with the frequencies 18 and 26 Hz[8]. Secondly, there are problems with MHD-oscillation model for QPOs, mentioned in Section 3[25]. In addition to that, the existing models do not explain the excitation of the oscillations at the positive slope of the main pulse. Our model provides the explanation from a single point of fiew, and for the total set of the observed frequencies of pulsations, 20 to 2400 Hz, for the excitation of the oscillation both in the «tail» of the flare and at the beginning of the main pulse, and, which is particularly important, for the high quality factor of the pulsations $Q \geq 10^4$. High-frequency variations of the current in coronal loops result in periodic variations of the magnetic field, which modulates the radiation of the magnetar. Although the suggested approach -- the description of coronal loops of a magnetar as equivalent RLC-circuits -- is largely phenomenological, it nonetheless makes it possible to estimate the parameters of the magnetosphere of a neutron star in an independent way.

Currently, numerous techniques for the determination of the magnetic field of magnetars exist. From the deceleration of the rotation of magnetars, the field $B \approx 10^{14} - 10^{15}$ G is derived. From the proton cyclotron resonance, the value $B \approx 10^{15}$ G is obtained. From the model of QPO as Alfvén torsion oscillations, the magnetic field of SGR 1806-20 within the interval $(3\text{-}7)\times10^{15}$ G was found[32]. From the study of variations of the luminosity of pulsations with frequencies 625 and 1840 Hz in the ringing tail of SGR 1806-20, it was found that in the QPO source (a magnetar's corona) the magnetic field $B \approx 6.6\times10^{13}$ G, while on the surface of the star the dipole approximation yields $B \approx 2\times10^{15}$ G. Our estimation for the values of the magnetic field in the QPO sources in SGR flares based on the electric current $I = (2E/L)^{1/2} \approx (2\text{-}8)\times10^{19}$ A gives $B \approx (6\text{-}27)\times10^{12}$ G $< B_Q$. These values are consistent with the recent observations of low-magnetic field SGRs[11],[12], and with the idea that a high surface magnetic field is not necessary for the magnetar-like activity[11],[13]. It also means that the physical processes in magnetar magnetospheres at the "ringing tail" phase can be studied within the non-quantum electrodynamics approach. Within the RLC-model, the current values make it possible to estimate also the electron number density in SGR magnetospheres $n \approx (1.3\text{-}6)\times10^{16}$ cm^{-3}.

We have to stress, however, that our estimates of the electric current and φ-component of the magnetic field in the ringing tail are based on the energy relation $E = LI^2/2$. The energy of the ringing tail of the flare ~ 10^{44} erg is by more than two orders of magnitude less than the total flare energy. Hence, to answer the question about the origin of the magnetar-like activity of neutron stars, more multi-wavelength observations are required.

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