

# The Liar is Not a Paradox

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**Abstract** The solution to The Liar is logical and splits into two: Classical and Nonclassical. While in Classical Logic The Liar is not a paradox because the assignment of truth-values is impossible task, in Non-classical Logic it is not a paradox because the assignment of truth-values proves that there is no absurd conclusion. The right logical embedment for the contents of this paradox seems to be the Logic of the Paradox: truth-value indeterminate when it is possible to prove that something is both 'true' and 'false'. This nonclassical logical system means no paradoxes arise, so that this 'paradox' is 'solved' in Nonclassical Logic and, because its contents do not belong to Classical Logic, it doesn't exist there as a paradox either. Logic is either classical or nonclassical. The contents of this paradox do not constitute a paradox in Logic therefore.

**Keywords** Liar paradox, Classical logic, Nonclassical logic, LP, Solution

## 1. Introduction

"The Liar Paradox is an argument that arrives at a contradiction by reasoning about a Liar Sentence. The Classical Liar Sentence is the self-referential sentence:

This sentence is false.

It leads to the same difficulties as the sentence, *I am lying*. Experts in the field of philosophical logic have never agreed on the way out of the trouble despite 2,300 years of attention. Here is the trouble. It is a sketch of the Paradox, the argument that reveals the contradiction:

Let  $L$  be the Classical Liar Sentence. If  $L$  is true, then  $L$  is false. But the converse also can be established, as follows. Assume  $L$  is false. Because the Liar Sentence is just the sentence *L is false*, the Liar Sentence is therefore true, so  $L$  is true. What has now been shown is that  $L$  is true if, and only if, it is false. Since  $L$  must be one or the other, it is both." (Dowden 2022)

'This sentence is false' is the sentence we judge in The Liar, but 'this sentence' is an expression formed of two sigmatoids (Pinheiro 2022, p. 61) that starts with a world reference (Pinheiro 2022, p. 61) that is slots that will be filled with letters and spaces and perhaps even punctuation marks. It is by the way slots that may never end since we can definitely program a machine to type to infinity and find ways of having it powered forever, so that there is always a continuation to the set of sigmatoids already typed. After the sentence is fully typed, 'this sentence' refers to 'this sentence is false', so that we have the figures of the world reference for this expression before the sentence is fully typed, let's

call that TE1, and of the world reference for this expression after the sentence is fully typed, or TE2. While TE2=this sentence is false, TE1 is best represented by a question mark or a place holder, so, were it Mathematics, TE1= $x$  would be suitable. If the sentence is never finished, it is impossible to tell if the sentence is true or false, but the paradox is based on hypotheticals, and the hypotheticals would still hold, so that it would survive that.

From (Pinheiro 2023, pp. 4-5):

"Whoever solves the Russell's Paradox will solve The Liar in its current shape. We claim to have solved The Russell's [13]. The solution is that there is a time issue: when the set of the sets that do not contain or belong to themselves is being formed, it (itself) is not formed yet and therefore cannot be considered as a possible set that does not contain (or belong to) itself. *This assertion is false* = *This assertion* is clearly not true and the assertion is still being formed by the time *This assertion* appears as a subject and therefore cannot appear as a sentence or subject being added a predicate at that moment. There is an error in speech there. *This assertion is false* assumes '*this assertion*' was available for the grabs as a subject by the time it was put there, but it had not yet been formed, so that it actually wasn't. In the same way, the *set of the sets that do not belong or contain themselves* was not formed by the time we are judging sets as for that rule, so that it could not have been picked and therefore there is an error in the question and also in the assertion; a breach. We cannot have '*this assertion*' as a subject in Logic or A as a candidate for inclusion in the *set of the sets that do not contain or belong to themselves* if we call this set A in Mathematics or Logic. In the same way, we say limits do not exist in Mathematics: if the lateral limits are different, then the limit of the function at that point does not exist. *This assertion is false* does not have a truth-value because this sentence breaches a tacit rule of formation of sentences in

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Logic, so that it can at most be said, or left as a question, just like the limit of the function at a point when such a limit does not exist: we leave it indicated as an operation, but we cannot solve it. Does  $A$  belong to  $A$  if  $A$  is the set of the sets that do not belong to themselves? It is a question that cannot be answered because the answer does not exist. In this case, we have not found a problem with CL or Aristotelian Logic either<sup>1</sup> (...)."

These paradoxes are not paradoxes in CL because the conclusion is never arrived at there, but they can be paradoxes in Nonclassical Logic, NCL, where, contrary to CL, the contradiction might be welcome.

In this paper, we first study the meaning of 'this sentence' and determine that it is a special expression, which allows for the world reference to vary from place holder to actual sentence. We then prove once more that assigning truth-values in CL is mission impossible because of how special this expression is and we then conclude that, in CL, this paradox doesn't exist. At this point, we realize that a system allowing for the negation of the law of non-contradiction would also finish with the paradox, so that it was never a paradox anyway (there is only classical or nonclassical logic, so that, if it is not a paradox in both of them, it is not a paradox in Logic).

## 2. Development

'This sentence is in English' sounds immediately true and acceptable. 'This sentence is false' sounds confusing and unacceptable. Yet the structure of these two sentences seems to be the same. 'This sentence' is an expression that changes world references as the sentence changes: before we put the full stop there and finish the quote, 'this sentence' is a question mark, an  $x$  for Mathematics. When we finish the quote, 'this sentence' is 'this sentence is in English' in the first case and 'this sentence is false' in the second case. We then have

- a) this sentence = this sentence is in English
- and
- b) this sentence=this sentence is false.

It is like typing about an unfinished drawing: Someone's hand is still drawing when we type 'your drawing is beautiful'. Yet they may stop going the way they are going all of a sudden and change all into the most horrible drawing of all, undeniably ugly. They may also keep on going in the same way they were going before forever. If they keep on drawing forever, the sentence 'your drawing is beautiful' has indeterminate truth-value even though we could change it into 'your drawing is beautiful (this far)' to make it 'deserve' the truth-value true.

When the subject is used as a subject we don't have enough elements to assert what we are asserting, so that the

sentence cannot have its truth value determined *a priori*<sup>2</sup>. It is only *a posteriori*<sup>3</sup>, once the sentence is concluded, that we can safely assign a truth-value to the sentence. Yet most things have the truth-value determined *a priori*. 'This sentence is false' has an improper subject, of the type that is blurred because if we add a sentence before that one, say 'Tom is handsome', 'this sentence' now has meaning *a priori*. If we say our universe has no other sentence before that one, 'this sentence' will become 'this sentence' from after it is fully typed, so that the subject is a place holder, an  $x$ , and it is fully determined in the first case, of Tom, *a priori*, and it is only determined by the end of it in the second case, *a posteriori*.

We found an expression that accepts both world references: one that points at an empty space, ready to receive a sentence, and one that contains a sentence. The plurality of world references for this expression is what makes its sentence be a proposition: This sentence, while  $x$ , cannot be assigned a truth-value.

We are obliged to establish something similar to the theory of types of Russell (Pinheiro 2016, pp. 110-111) to deal with this. We now must have 'this sentence' *a priori* and 'this sentence' *a posteriori* to fully analyze these assertions. 'This sentence' *a priori* does not hold truth-value and 'this sentence' *a posteriori* holds truth-value. Because it is impossible to tell one from the other, since the shape is identical, 'this sentence', we would have to say something like it gets assigned truth-value and it doesn't or (exclusive) it doesn't accept truth-values. If it is inside of CL, we cannot assign truth-values to it because it cannot be the case that it both accepts those and it doesn't, due to the Law of Non-contradiction, LNC. Indeterminate could be the right name for its truth-value in NCL, though.

Doomen reaches similar conclusion in what regards The Liar. He talks about the world reference being the slots for the subject of the key-sentence (Doomen 2023, p. 8): "The statement 'This statement is not true' must refer to *something* (a state of affairs) in order to be able to determine that it is (not) true, but that complement is lacking." Since that is lacking, we have no way to judge the sentence.

Doomen compares this sentence with "This statement, that the capital of Wyoming is Cheyenne, is not true", which he classifies as a genuine paradox (Doomen 2023, p. 8). He expresses the idea of the slots in the world reference as 'no content' (Doomen 2023, p. 8). He then concludes like this (Doomen 2023, p. 9): "To what has been advanced in this section I can add, for completeness, that it should now be clear that the question of whether the PNC<sup>4</sup> is true is irrelevant for the present inquiry: if no statement is made through the liar paradox, no contradiction manifests itself to begin with. In other words, the liar paradox should not be taken to be symbolically expressible as ' $\neg (p \wedge \neg p)$ ' and not even as ' $\neg p$ '.

The liar paradox, I maintain, is neither true nor not true,

<sup>1</sup> In the same way the inexistent limit is not a problem for the mathematical logic (for more details on mathematical logic please see (Pinheiro 2017)).

<sup>2</sup> *A priori* here means simply beforehand.

<sup>3</sup> *A posteriori* here means after the event or experience.

<sup>4</sup> PNC is Principle of Non-Contradiction.

since there is no paradox to begin with, but if one insists on using the term, the paradox is an underdeveloped one.”

It is worth discussing a bit more: **this sentence=this sentence is in English**. We again don't know the entire sentence is in English until we finish typing it. It could go like this: **this sentence is in English e eu sou bonita**. There is no a priori truth here either.

There is a special way of reading the paradox inside of the universe of the 3-valued Kleene's Logic that will still make it be a paradox: “Let  $L$  be the Classical Liar Sentence (‘This sentence is false’). If  $L$  is true, then  $L$  is false. But the converse also can be established, as follows. Assume  $L$  is false. Because the Liar Sentence is just the sentence  $L$  is false, the Liar Sentence is therefore true, so  $L$  is true. What has now been shown is that  $L$  is true if, and only if, it is false. Since  $L$  must be one or the other, it is both.” (Dowden 2022) If following Tré (2002), we still end up with the paradox because there is nothing saying that if we conclude it is true and false then it is  $\square_{\text{Boolean}}$ . There is nothing there to say that a proposition can be both true and false at the same time, so it can't and we end up with the paradox anyway. We can actually build it using the same steps we use for CL, so that Kleene's Logic, unfortunately, is not how the paradox works yet. We would need to have a logical system that makes it possible that truth-value 0 with truth-value 1 for the same assertion be valued I, using the simplification seen in (Wikipedia 2023) as basis for the notation, to solve the paradox, so that only in this special system the paradox would already be solved. With Priest's Logic of Paradox, LP (Priest 1979), it seems that the paradox is finally not a paradox anymore, making justice to the name: since assuming the sentence is true leads to its falsity and vice-versa, the sentence is ‘indeterminate’/both. Priest then calls the truth-value of this sentence P or paradoxical (Tennant 2019, p.482).

### 3. Conclusions

The Liar is not a paradox in CL because its main sentence cannot be assigned truth-values in it, so that it is not a valid argument against CL. The reason is that its subject has got double world reference and, while existing a priori, it does not admit truth-value True or False, since it consists of empty slots. Very special conditions lead to this and those include no previous sentence, since, if we say something like ‘Tom is ugly’ right before saying ‘This sentence is false’, there is no paradox and the subject is determined a priori in full, not consisting of empty slots anymore. Inside of these very special conditions, though, the Liar Sentence cannot have truth-value in CL because of its very special subject, which does not belong there. It does not belong there because of the LNC: it cannot both admit and not admit truth-values. There is a special logical system, which then has to be Nonclassical,

that proves that the Liar Paradox is not a paradox and it seems that this system might be LP, a system largely defended by Graham Priest. In this special logical system, truth-value 1 (as seen in CL) with truth-value 0 (as seen in CL) returns I for the main sentence. The Liar may survive the 3-valued Kleene's Logic, but will disappear with this special system, so that it is not a paradox in NCL. A possible obstacle, which tells us that LP might still need to be modified to cope with The Liar, is that A false with A true, making  $B=A$ , will return  $A \wedge B$  false (Tennant 2019, p.482). Priest (1979) seems to address that by saying that if a proposition gets to be evaluated as true and false then it is paradoxical and if A and B are paradoxical, or A and A, in our case, we get  $A \wedge B$  (or  $A \wedge A$  in our case) paradoxical. There might be an issue with the co-existence of both rules, though.

### REFERENCES

- [1] Doomen, J. (2023). The Liar Paradox: A Case of Mistaken Truth Attribution. *Global Philosophy*, 33(22), 1–11. <https://doi.org/10.1007/s10516-023-09666-2>.
- [2] Dowden, B. (2022). Liar Paradox. In *Internet Encyclopedia of Philosophy*. <https://iep.utm.edu/par-liar/>.
- [3] Pinheiro, M. (2016). Russell's Paradox, Our Solution, and the Other Solutions. *International Journal of Theoretical and Applied Mathematics*, 2(2), 110–114. <https://www.sciencepublshinggroup.com/journal/paperinfo.aspx?journalid=347&doi=10.11648/j.ijtam.20160202.22>.
- [4] Pinheiro, M. (2017). Explosion Law or Ex Falso Quodlibet: May We Swap, Master? *International Journal of Advances in Philosophy*, 1(1), 10–20. <http://article.sapub.org/10.5923.j.ap.20170101.03.html>.
- [5] Pinheiro, M. R. (2022). Higher Order Vagueness (Corrigendum). *IOSR Journal of Humanities and Social Science*, 27(11), 61–66. <https://doi.org/10.9790/0837-2711026166>.
- [6] Pinheiro, M. R. (2023). The Sorites, The Liar, CL, NCL, Mathematical Induction, Descartes and Aristotle. *International Journal of Advances in Philosophy*, 5(1), 1–6. <https://doi.org/10.5923/j.ap.20230501.01>.
- [7] Priest, G. (1979). The Logic of Paradox. *Journal of Philosophical Logic*, 8(1), 219–241. <https://www.jstor.org/stable/30227165>.
- [8] Tennant, N. (2019). GP's Lp. In: C. Başkent and T. Macaulay Ferguson, eds., *Graham Priest on Dialetheism and Paraconsistency*. Switzerland: Springer Nature, pp. 481–506.
- [9] Tré, G. (2002). Extended Possibilistic Truth Values. *International Journal of Intelligent Systems*, 17(4), 369–446.
- [10] Wikipedia. (2023). Many-valued Logic. In *Wikipedia*. [https://en.wikipedia.org/wiki/Many-valued\\_logic](https://en.wikipedia.org/wiki/Many-valued_logic).