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The Sorites, The Liar, CL, NCL, Mathematical Induction, Descartes and Aristotle

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Abstract **Aim:** to study the soundness of The Sorites and The Liar. The soundness of one of these paradoxes may serve as justification for the creation of Nonclassical Logic. **Study design:** The study was designed in a way to allow for those who have a logical background, but no specific formation, to understand all that is involved. **Place and Duration of Study:** This study was conducted in Burwood, Sydney, NSW, Australia, from June of 2022 to February of 2023. **Methodology:** Systematic creation following Bloom's analysis and synthesis of the results of the scholastic application of logical tools to relevant data sets. **Results:** With The Sorites, induction reasoning leads to no conflict, since the conclusion does not eventuate, and non-induction reasoning leads to no conflict because the premise does not eventuate. Yet, we need to describe the blurred region somehow. With The Liar, *'this sentence is false' = 'this* sentence' or *X=X is false* and *X* is clearly different from *X is false*. Similar reasoning appeared when we discussed the Russell's Paradox and we can see that the solutions to these paradoxes will also be similar. Yet, *~T* and *F* are different things in language, since *~T* may mean mainly false, but partially true, so that we need to find a logic that describes *~T* as it is in language. **Conclusions:** Both The Liar and The Sorites are valid but unsound arguments. *X=X is false* is not an acceptable sentence in Logic. Nonclassical Logicians have created alternative logical systems that may be used to deal with blurred regions and problems requiring more than two truth-values, but these systems are not good enough. Both The Liar and The Russell's are about breaching a tacit rule, which is using only what is available; a time issue.

Keywords Sorites, Liar, Priest, Hyde, Aristotle, Descartes, Logic, Paradox

1. Introduction

"The sorites paradox is the name given to a class of paradoxical arguments, also known as little-by-little arguments, which arise as a result of the indeterminacy surrounding limits of application of the predicates involved." [1]

Soros is Greek for heap [1], so that The Sorites would be arguments involving things that are similar or equal to heaps. An example involves heaps of wheat [1]: "since one grain of wheat does not make a heap, it follows that two grains do not; and if two do not, then three do not; and so on. This reasoning leads to the absurd conclusion that no number of grains of wheat make a heap." [2]

"A way to further illustrate the argument involved: if adding one grain of wheat to a nonheap does not make a difference, then let me add one grain per second and tell me what made us end up with an accumulation of wheat that is undeniably a heap by the end of one year.

Yet another way to further illustrate this argument: 1) Adding one grain of wheat does not make any difference [as to the application of the predicate 'is not a heap']; 2) There is an accumulation of *n* grains of wheat, where *n* was attained from adding grains of wheat one by one to the existing accumulation, which had initial amount of grains zero and *n* is large enough to constitute a heap; 3) The accumulation of *n* grains of wheat must be both a heap and a nonheap of wheat." [2]

The human mind syncs with the Principle of Mathematical Induction when thinking of this problem for the first time, so that it imagines one grain of wheat placed in alignment with the previous grain of wheat in a horizontal row, what will indeed mean adding one grain does not make any difference and we will never have a heap [2]. When seeing things with the 'eyes of the mathematical induction', therefore, The Sorites is not a paradox, since the 'absurd conclusion' does not eventuate and there is never an *n* that is large enough to constitute a heap [2]. When the contradiction appears for the first time in the human mind it is as if someone else uttered that *n* grains do form a heap and they presented a vertical accumulation of grains to the observer for the first time when saying that. Now it is clear that one more grain may make a difference, since it may, for instance, be dropped at the top of

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a perfect vertical row of grains, which did not look like a heap, but then make the grains fall and flatten up [2]. Now the premise is not true forever anymore: some of the grains will make a difference even if added on their own to the previous accumulation [2]. Therefore we again have no paradox. Yet, since we don't know which accumulation the problem originally refers to, if horizontal or vertical, we have what seems to be contradictory conclusions, which is *'it is a heap'* and *'it is not a heap'*, both happening at the 'same' time [2]. Notice that the observer would have to have both sets of paradigms in their heads and no graphical image [2] for them to reach both conclusions.

The Sorites was initially taken to be a proof that the Aristotelian Logic does not work, since LNC seemed to be breached and its deductions seemed to be incomplete [2], but, later on, when the mental paradigms were split into vertical and horizontal rows and two other syllogisms which were considered to be acceptable were also not found in the description of the Aristotelian Logic we had, the Aristotelian Logic was told to probably be OK once more (the key was the sigmatoid 'respect' in Aristotle's writings and the other two syllogisms not being described by the symbols from the source we consulted). The Principle of Mathematical Induction was told to be OK thanks to the finding involving perspectives: now we had horizontal rows and single-follicle pony tails [2].

We now want to prove that a similar process happens with the Liar Paradox. In this case, we are initially seeing evidence that Classical Logic (CL) needs to be replaced with a better Logic.

"The Liar Paradox is an argument that arrives at a contradiction by reasoning about a Liar Sentence. The Classical Liar Sentence is the self-referential sentence:

This sentence is false.

It leads to the same difficulties as the sentence, I am lying. Experts in the field of philosophical logic have never agreed on the way out of the trouble despite 2,300 years of attention. Here is the trouble. It is a sketch of the Paradox, the argument that reveals the contradiction:

Let *L* be the Classical Liar Sentence. If *L* is true, then *L* is false. But the converse also can be established, as follows. Assume *L* is false. Because the Liar Sentence is just the sentence L is false, the Liar Sentence is therefore true, so *L* is true. What has now been shown is that *L* is true if and only if, it is false. Since *L* must be one or the other, it is both." [3]

We have discussed the shape of this problem in [4]. We however did not talk about *'this sentence is false'*. That is what we do here instead. *'I always lie'* was our choice in [4], but *'I always lie*' involves the sigmatoids [5] *'always'* and *'lie'*, which allow for very different reasoning from that of *'sentence'* and *'false'*: while *'always'* and *'lie'* are clearly relative sigmatoids, for their use is deeply connected to private logic [6], *'sentence'* and *'false'* are clearly absolute sigmatoids, for their use is deeply connected to Classical Logic, which is almost the same as Mathematical Logic [7].

2. Material and Methods

The material used is scientific articles, academic encyclopedias, one newspaper's blog, one academic blog and one academic conference.

The methods used are systematic creation following Bloom's analysis and synthesis of the results of the scholastic application of logical tools to relevant data sets.

3. Results and Discussion

'This sentence is false' is false means *'This sentence is not false'*, not *'This sentence is true'*. It is false or true in Classical Logic (CL) only [7]. When this problem is solved using the tools of CL, the paradox is actual, what proves that there is something wrong with applying CL to non-binary situations, which is quite obvious, since CL is binary in all. This sentence is not false means 'This sentence might be true, or partially true' instead. Perhaps it could be 60% true and 40% false, therefore not quite 'our false' from CL, which is absolute or 100% false.

Even though this is clear enough, things will get confused again when *'this sentence is false'* is translated into symbols.

"*T* `*L*´ if and only if *L*.

Here T is the truth predicate (informally it is the predicate __ is a true sentence) and *L* is the Liar Sentence, namely ~T `*L*´. Substituting the latter for *L* on the right of the above biconditional yields the contradiction:

T`*L*´ if and only if ~*T*`*L*´." [3]

The Liar Sentence is *'this sentence is false'*, as seen in the Introduction. *T`L'* is then *'this sentence is false' is a true sentence*. *~T`L'* is *'this sentences is false' is not a true sentence*.

The departure sentence was *'this sentence is false' is a true sentence iff 'this sentence is false'*. Here the problem is that there is no explanation for the next move. Who said *L* equates *~T`L'* in first place? However, notice that if *L* is true, *'this sentence is false' is true*, but what it says is that this sentence is false and *this sentence* is *'this sentence is false'*, so that *L* is *false* or *not true* or *~T* in Classical Logic. This is to prove that *~T* is not false or *F* at least sometimes in real life, so that it might be false by a percentage, in case we could use Fuzzy Logic [8], it might be both true and false, in case we could use the Logic of the Paradox, LP [9], and so on. We could also decide that Logic does not apply to real life and simply use a couple of sigmatoids to indicate what we mean, so say *partially true* or *mostly false*.

The Sorites was there to almost prove that the Aristotelian Logic was inadequate to deal with real-life problems involving logical reasoning and The Liar is here to almost prove that CL is inadequate to deal with those. Were The Sorites and The Liar evidence of failure of CL or Aristotelian Logic, we would need to find a logical system that solved the problems Aristotelian Logic and CL solved plus these paradoxes or we would need to prove that not all that is

logical can be dealt with by logical systems. We can however prove that CL is not under threat if The Liar is the tool used to point at its problems.

It is pretty obvious that nobody would be able to say, with 100% of confidence, that their 'heap' is 80% accurate. It might be 80.9%, for instance, instead, or perhaps 80.99999%. In the same way, 'true' and 'false' are connected to a private logic, which can only be used, but not described in full by an individual. Individuals might point at nothing and say heap, everyone laughs and knows what they mean, a joke they once told… . They are not insane, they are logical speakers, yet the heap they used escapes, by far, logic. It is all logical in that interaction even so. These are sigmatoids that could never be used in Science, the 'soritical' sigmatoids, let's say. In the same way goes 'false', 'true' and 'sentence'. They need to be defined, as in CL, so we need to use other elements of language to explain them before we can apply them in logical reasoning. False means that, for the instance (x_1, y_1, z_1), where (x, y, z) are the only variables of a sentence, the assertion is not verified. True means, in opposition, that for all instances of (x, y, z) the assertion is verified.

What solves The Sorites is realizing that in induction reasoning there is no paradoxical conclusion and, outside of it, there is no paradoxical premise.

Induction Reasoning (IR): If the main mental paradigm is 'horizontal and linear accumulation', adding one grain of wheat does not make any difference to the application of the predicate 'is not a heap' and the conclusion of the argument is that an accumulation of *n* grains of wheat, *n* being any natural number, 'is not a heap'.

Non-Induction Reasoning (NIR): If the main mental paradigm is 'vertical and linear accumulation', adding one grain of wheat does make a difference to the application of the predicate 'is not a heap', since the pile will break and flatten up at a certain stage and, with that happening a few times, the conclusion of the argument is that an accumulation of *n* grains of wheat, n being a certain natural number, 'is not a nonheap'.

With IR, *step 1* of the syllogism is true: adding one grain of wheat does not make any difference. *Step 2* of the syllogism is false, since *n* grains that accumulated horizontally will not make a heap. *Step 3* is false because, for n grains, we have a nonheap, therefore we have a valid and unsound argument.

The definition of a 'valid/sound argument' comes from [10]: "An argument is valid iff* it is impossible for the premises of the argument to be true while the conclusion is false. Otherwise, an argument is invalid. An argument is sound iff it is valid and its premises are true. Otherwise, an argument is unsound."

With NIR, *step 1* of the syllogism is not true sometimes: adding one grain does make a difference sometimes (as to the predicate 'is not a heap'). *Step 2* is true, since there is an accumulation with *n* grains that will be a heap. *Step 3* is not true because *n* grains will be a heap but not a nonheap. The argument is valid if when assuming *step 1* and *step 2* are true we get *step 3* true. Before *n*, step 1 is true, but, on and after *n*, it is false. Step 2 is true for *n*. The conclusion happens on *n* and there it is impossible for the premises to be true while the conclusion is false, so that NIR is valid and unsound.

There are moments that precede the heap moment and generate the perception that adding a grain does not make any difference and, during those moments, it will still be true that if *n* is large enough we have a heap. The issue is that that will be a nonheap, not a heap, clearly the case. Yet, the amount of grains will be inferior to the value in *Step 2*, so that it is not a heap and a nonheap AT THE SAME TIME: the times are different. The argument also did not breach LNC because the times were different, so there was a 'different respect' [2]. Here we may be told to be using Supervaluationism as presented by Hyde at the conference Ancient Philosophers in the University of Newcastle, 2000, to translate elements of speech into logical entries of the type *nonheap* and *heap*, so that only the Supertrue heap would be a heap.

Both conclusions (*'it is a heap'* and *'it is not a heap'*) are still possible in the universe where the observer has not yet found out the *n* involved and the moment of confusion can then be expressed by means of a logic that allows for both conclusions to be true at the same time. Decisions need to be made ALSO during that time, so that the appearance of a new logical system is justified by the urges of human life.

We then possibly have an argument against the validity of CL and Aristotelian Logic, but notice that, in both of them, we would need the same mental paradigms to have inconsistency, but the mental paradigms are different: when we say we have a nonheap, an image of a nonheap is in our mind. When we say we have a heap, the image of a heap is in it instead. They are different world references [5]. It is as if we had two arguments to vary, two variables, this at the same time, but both Aristotelian and CL judge only one variable at a time. See an example from Cartesian Logic, which also judges only one variable at a time:

"(1) I think. (2) If I think, I exist. (3) Therefore, I exist." [10]

We are judging *step 1* of the syllogism and *'think'* has got one set of paradigms. We are judging *step 2* of the syllogism and *'exist'* has got one set of paradigms. Not two each. Now let's introduce one more set of paradigms: when we think of *'nonexist'*, we don't have a machine translating thinking to a public of non-zero people. Now *I exist* is true and so is *I nonexist at least some of the time*. We join the alternative paradigms and we then have to modify (2) to accommodate: (1) We think. (2) If we think is having the mind working, these subjects all exist. If we think is having a machine translating thinking to a non-zero audience, these subjects nonexist. (3) Therefore we exist. The problem is that the argument is invalid only if what makes the premises true makes the conclusion false and we then have to satisfy both sets of paradigms, so come up with a Stephen Hawking [12], to have (2) true and, in that case, (3) will happen because Stephen Hawking does exist. If we accommodate The Sorites in the same manner, the syllogism will not work in the same way: (from [2]) (1) Adding one grain of wheat does

not make any difference [application of the predicate 'is not a heap']. (2) (If the accumulation is vertical) There is an accumulation of *n* grains of wheat, where *n* was attained from adding grains of wheat one by one to the existing accumulation, which had initial amount of grains zero and *n* is large enough to constitute a heap. (If the accumulation is horizontal, *n* might be as large as possible, but the accumulation will always be a nonheap) (3) Therefore, the accumulation of *n* grains of wheat must be both a heap and a nonheap of wheat. The main problem is that there is no accumulation that satisfies both cases in (2), but there was intersection between the types of thinking. We then have to split the problem into two: vertical and horizontal accumulation.

One could say that logic simply does not apply to the blurred region and saying *'it is a heap'* and *'it is a nonheap'* at the same time is simply being confused or irrational, therefore not logical, but what about The Liar?

'This sentence is false' still means we need an alternative to both Aristotelian Logic and CL, a multivalued logic, to the least. Otherwise we need to come up with an argument that proves that this sentence is not logical. If we call *'this sentence' X*, we are saying *'X is false'*. We then want to call this *X* and have *X=X is false*. Why can't we do that? We can't do that because *X=X*, but *X* is different from *X is false* and it suffices we consider the world references involved to tell they are different things. Yet, in language, *'this sentence is false' is different from 'this sentence'*. This is the same problem we saw in Russell's Paradox [13], the set of the sets that do not belong to themselves. We all know there is an issue with the level of the elements there and a few people have already tried to create theories that would address that. {5} cannot belong to {5}. It can only be contained in {5}. There is an inadequacy that appears as a result of ignoring the rules of application of the relations in Mathematics. In the same way, The Liar is a breach of the rule. We simply cannot have *A is false=A*. Whoever solves the Russell's Paradox will solve The Liar. Yet, if we stick to the previous conclusion, the equality between *~T* and *F* does not hold, so that we cannot swap *L* for *~T`L´* and the conclusion of The Liar is then not true.

"*T `L´* if and only if *L* (1). *T`L´* if and only if *~T`L´* (2)." [3] *T `L´* if and only if *L and ~T`L´* (3) is The Liar. (2) will never be true if we determine that the equality between *~T* and *F* does not hold. The conclusion will be false for the same reason. It is then impossible for the premises ((1) and (2)) to be true while the conclusion ((3)) is false. The argument is then valid and unsound.

With this, we are left with the blurred region and the necessity of having many truth-values, more than two, as reasons to introduce nonclassical logic systems, NCL, but we may simply decide to say logic does not apply to the blurred region. Fuzzy Logic has been used in machine reasoning for air-conditioners [14] with success, so that it allowed machine circuits to pass information and decide what to do in the blurred area. In the past, those circuits used only CL, therefore binary code or 0s and 1s. Brazilians created the paraconsistent robot [15], a robot that does not crash upon receiving conflicting information from the environment. In the past, robots would enter short circuit upon receiving conflicting information from the environment.

4. Conclusions

Both The Liar and The Sorites are valid but unsound arguments. Not all that is logical in real life can be described or addressed by logical systems. Private logic is a logical system, but only the own individual can describe this system and their description is always going to be incomplete or inaccurate, given that the individual changes all the time or doesn't know themselves well enough or does not use a universal language competently or others. Private logic also includes moments of pure irrationality, which the individual can only describe a posteriori.

We were left with the moment of confusion for The Sorites and the translation of 'false' for The Liar as reasons to introduce NCL. The moment of confusion seems to include conclusions coming from different premises and, once they are split, there is no confusion. We definitely need a logical system that accepts at least three truth-values instead of CL, which accepts only two, true or false. Otherwise we are not able to cope with the simplest logical arguments in real life: negating something is not frontally opposing that something at least sometimes, so that saying I LIE is false may imply even I LIE, since I LIE might have been said with irony. If it implies I LIE, then I LIE is false and true at the same time, under different mental paradigms, so that Aristotle's LNC is not breached, but both are acceptable results, so that CL has to be replaced with a three-valued system, which allows for both I LIE is false and I LIE is true being OK, such as LP.

The applications of Fuzzy Logic and paraconsistency resume to machines so far, so that these systems appear as inferior in logical reasoning to CL and Aristotle's Logic, what perhaps points at non-equivalency, so impossibility of applying the NCL systems to the same level of problems we apply CL or Aristotle's Logic. Indeed, if we attribute degrees of pertinence to a predicate, we will eventually think our own assignment is wrong, since it is mechanical, not purely human and that will make evolution in reasoning not possible. If we think that things might be and not be, we will simply be confused all the time and therefore think that anything is OK, what will again not allow for progression in reasoning: the same conclusion we marked as good today might be bad tomorrow, since there is no logical cause for it. As seen in the UNICAMP experiment, the decisions are arbitrarily assigned to the machine at the time of need. If NCL could solve the problem of progression in reasoning, they would have a system that could replace CL and Aristotle's, but, so far, it is not the case.

Whoever solves the Russell's Paradox will solve The Liar

in its current shape. We claim to have solved The Russell's [13]. The solution is that there is a time issue: when the set of the sets that do not contain or belong to themselves is being formed, it (itself) is not formed yet and therefore cannot be considered as a possible set that does not contain (or belong to) itself. *This assertion is false = This assertion* is clearly not true and the assertion is still being formed by the time *This assertion* appears as a subject and therefore cannot appear as a sentence or subject being added a predicate at that moment. There is an error in speech there. *This assertion is false* assumes *'this assertion'* was available for the grabs as a subject by the time it was put there, but it had not yet been formed, so that it actually wasn't. In the same way, the *set of the sets that do not belong or contain themselves* was not formed by the time we are judging sets as for that rule, so that it could not have been picked and therefore there is an error in the question and also in the assertion; a breach. We cannot have *'this assertion'* as a subject in Logic or A as a candidate for inclusion in the *set of the sets that do not contain or belong to themselves* if we call this set A in Mathematics or Logic. In the same way, we say limits do not exist in Mathematics: if the lateral limits are different, then the limit of the function at that point does not exist. *This assertion is false* does not have a truth-value because this sentence breaches a tacit rule of formation of sentences in Logic, so that it can at most be said, or left as a question, just like the limit of the function at a point when such a limit does not exist: we leave it indicated as an operation, but we cannot solve it. Does A belong to A if A is the set of the sets that do not belong to themselves? It is a question that cannot be answered because the answer does not exist. In this case, we have not found a problem with CL or Aristotelian Logic either and at most the blurred region remains as justification for the appearance of NCL, but, as said before, that is simply an area of confusion and, when reasoning is well sorted, the confusion is not there anymore.

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Competing Interests

There are no competing interests.

Definitions, Acronyms, Abbreviations

LNC: Law of Non-Contradiction
LP: Logic of the Paradox
CL: Classical Logic
NCL: Nonclassical Logic

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