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Joint Influence of Double Sampling and Randomized Response Technique on Estimation Method of Mean

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Abstract In this article, the problem of estimation under two-phase random sampling using randomized response technique is considered. In two-phase (double) sampling, the expression of bias and mean square error (MSE) up to the first-order approximations are derived for the proposed estimators. Simulation studies and real data are presented to demonstrate the performance of proposed estimators.

Keywords Double Sampling, Sensitive Variable, Randomized Response Technique

1. Introduction

It is common practice in sample survey we obtain the information about auxiliary variable(s) from a larger sample at first phase and relatively small sample from the second phase by using two-phase sampling procedure. Many authors worked on two-phase random sampling such as: Sukhatme (1962), Singh and Vishwakarma (2007), Sahoo et al. (2010) Noor-ul-amin and Hanif (2012), Sanaullah et al. (2014), etc.

In survey sampling information on sensitive variable would be collected by using randomized response technique introduced by Warner (1965), because direct reliable information on variable of interest is sometime may not possible. Several authors have worked on randomized response techniques on estimation of mean, including Eichhorn and Hayre (1983), Gupta et al. (2002), Chang et al. (2005), Huang (2008). Sousa et al. (2010) introduced ratio estimators by using non-sensitive auxiliary information. Gupta et al. (2014) presented ratio and regression estimator using optional scrambling. Mushtaq et al. (2017) presented a family of estimators of a sensitive variable using auxiliary information in stratified random sampling. Noor-ul-amin et al. (2018) proposed estimation of mean using generalized optional scrambled responses in the presence of non-sensitive auxiliary variable. Saleem I. et al (2019) presented estimation of mean of a sensitive quantitative variable in complex survey: improved estimator and scrambled randomized response model. Partha Parichha et al. (2020) discuss the development of estimation procedure of population mean in Two-Phase Stratified Sampling.

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Encourage the above work, we have suggested a generalized class of estimators in two-phase sampling using randomized response technique. The main purpose is to suggest a strategy of two-phase (double) sampling in randomized response technique and proposed general family of estimators for estimating the finite population mean of a sensitive variable with non-sensitive auxiliary variable based on RRT in two- phase (double) random sampling.

2. Sampling Strategy

2.1. Notations & Scheme of Selection of Sample

We consider the finite population $U=(U_1,U_2,...,U_N)$ in which Y be the sensitive study variable, X be non-sensitive auxiliary variable which is correlated with Y and S be scrambling variable independent of Y and X. The reported response of the respondents is $Z=Y+S$, and n' is the number of units in the first sample whereas n is the number of units in the second sample. Only in the second sample both study and auxiliary variables are observed, in the first sample only auxiliary variable is observed because study variable is expensive.

The two-phase sampling strategy is given below:

1. The first phase, a large sample of a fixed size n' is drawn from N to observe only x or auxiliary variable.
2. The second phase sample, a sub-sample of fixed size n is drawn from n' to observe z and x, so that $(n<n')$.

Let define the following notations:

Let $\bar{z}=\frac{1}{n}\sum_{i\in s} z_i$, $\bar{x}=\frac{1}{n}\sum_{i\in s} x_i$, $\bar{x}'=\frac{1}{n'}\sum_{i\in s'} x_i$.

$e_0 = \dfrac{\bar{z} - \bar{Z}}{\bar{Z}}$, $e_1 = \dfrac{\bar{x} - \bar{X}}{\bar{X}}$ and $e_1' = \dfrac{\bar{x}' - \bar{X}}{\bar{X}}$

$E(e_0) = E(e_1) = E(e_1') = 0$; $E(e_0^2) = \theta C_z^2$,

$E(e_1^2) = \theta C_x^2$, $E(e_1'^2) = \theta' C_x^2$,

$E(e_0 e_1) = \theta C_{zx}$, $E(e_0 e_1') = \theta' C_{zx}$,

$E(e_1 e_1') = \theta' C_x^2 = E(e_1'^2)$.

Where $\theta' = \dfrac{1}{n'} - \dfrac{1}{N}$, $\theta = \dfrac{1}{n} - \dfrac{1}{N}$, $\rho_{zx} = \dfrac{\rho_{yx}}{\sqrt{1 + \dfrac{s_s^2}{s_y^2}}}$,

$C_{zx} = \rho_{zx} C_z C_x$.

2.2. Discussion on Estimators in Double Sampling Strategy in RRT

Firstly, we introduce some existing estimators in double sampling using RRT.

The mean and variance of the usual mean estimator in RRT is given by

$$t_{ys} = \bar{z} \tag{2.1}$$

$$MSE(t_{ys}) = \frac{1-f}{n}\left(S_y^2 + S_s^2\right) \tag{2.2}$$

Where $S_y^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$ and $S_s^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}\left(s_i - \bar{S}\right)^2$.

A ratio estimator in two-phase sampling in RRT is given as:

$$t_{rsd} = \bar{z}\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{2.3}$$

$$MSE\left(t_{rsd}\right) = \bar{Y}^2\left\{\theta C_z^2 + \left(\theta - \theta'\right)\left(C_x^2 - 2C_{zx}\right)\right\} \tag{2.4}$$

And regression estimator in two-phase sampling given as

$$t_{regsd} = \bar{z} + \hat{\beta}_{zx}(\bar{x}' - \bar{x}) \tag{2.5}$$

Where $\hat{\beta}_{zx} = \dfrac{S_{zx}}{S_x^2}$

$$MSE\left(t_{regsd}\right) = \bar{Y}^2 C_z^2\left\{\theta\left(1 - \rho_{zx}^2\right) + \theta'\rho_{zx}^2\right\}. \tag{2.6}$$

3. Proposed Estimators

We propose the following a class of generalized estimators in two-phase sampling:

$$t_{Sid} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\gamma\left(\frac{a\bar{x}' + b}{a\bar{x} + b}\right) + (1-\gamma)\exp\left(\frac{a\left(\bar{x}' - \bar{x}\right)}{a\left(\bar{x}' + \bar{x}\right) + 2b}\right)\right] \tag{3.1}$$

Where k_1 and k_2 are weights whose values are to be determined, γ =0 or 1, a and b are the parameters of the auxiliary variables.

From t_{Sid} for $\gamma = 0$, we obtain the following estimators

$t_{S0d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\exp\left(\dfrac{\left(\bar{x}' - \bar{x}\right)}{\left(\bar{x}' + \bar{x}\right)}\right)\right]$ where $a = 1, b = 0$.

$t_{S1d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\exp\left(\dfrac{\left(\bar{x}' - \bar{x}\right)}{\left(\bar{x}' + \bar{x} + 2C_x\right)}\right)\right]$ where $a = 1, b = C_x$.

$t_{S2d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\exp\left(\dfrac{\left(\bar{x}' - \bar{x}\right)}{\left(\bar{x}' + \bar{x} + 2\beta_2(x)\right)}\right)\right]$ where $a = 1, b = \beta_2(x)$.

$t_{S3d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\exp\left(\dfrac{C_x\left(\bar{x}' - \bar{x}\right)}{C_x\left(\left(\bar{x}' + \bar{x}\right) + 2\beta_2(x)\right)}\right)\right]$ where $a = C_x, b = \beta_2(x)$.

$t_{S4d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\exp\left(\dfrac{\beta_2(x)\left(\bar{x}' - \bar{x}\right)}{\beta_2(x)\left(\left(\bar{x}' + \bar{x}\right) + 2C_x\right)}\right)\right]$ where $a = \beta_2(x), b = C_x$.

From t_{Sid} for $\gamma = 1$, we obtain the following estimators

$t_{S5d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\left(\dfrac{\bar{x}'}{\bar{x}}\right)\right]$ where $a = 1, b = 0$.

$t_{S6d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\left(\dfrac{\bar{x}' + C_x}{\bar{x} + C_x}\right)\right]$ where $a = 1, b = C_x$.

$t_{S7d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\left(\dfrac{\bar{x}' + \beta_2(x)}{\bar{x} + \beta_2(x)}\right)\right]$ where $a = 1, b = \beta_2(x)$.

$t_{S8d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\left(\dfrac{C_x\bar{x}' + \beta_2(x)}{C_x\bar{x} + \beta_2(x)}\right)\right]$ where $a = C_x, b = \beta_2(x)$.

$t_{S9d} = \left[k_1\bar{z} + k_2\left(\bar{x}' - \bar{x}\right)\right]\left[\left(\dfrac{\beta_2(x)\bar{x}' + C_x}{\beta_2(x)\bar{x} + C_x}\right)\right]$ where $a = \beta_2(x), b = C_x$.

Expanding (3.1), we have

$$t_{Sid} \cong \left[k_1\bar{Z}\left(1+e_0\right) + k_2\bar{X}\left(e_1' - e_1\right)\right]\left[\gamma\left(1+ge_1'\right)\left(1+ge_1\right)^{-1} + \left(1-\gamma\right)\exp\left\{\frac{1}{2}g\left(e_1' - e_1\right)\left(1+\frac{1}{2}g\left(e_1' + e_1\right)\right)^{-1}\right\}\right] \quad (3.2)$$

Where $g = \dfrac{a\bar{X}}{a\bar{X} + b}$

$$t_{Sid} - \bar{Y} \cong \left(k_1 - 1\right)\bar{Y} + k_1\bar{Y}\left[e_0 - \frac{1}{2}g\left(1+\gamma\right)\left(e_1 - e_1'\right) + \frac{1}{8}g^2\left(3+5\gamma\right)\left(e_1^2 - e_1'^2\right) - \frac{1}{2}g\left(1+\gamma\right)\left(e_0e_1 - e_0e_1'\right)\right]$$
$$-k_2\bar{X}\left[\left(e_1 - e_1'\right) - \frac{1}{2}g\left(1+\gamma\right)\left(e_1^2 - e_1e_1'\right)\right]. \quad (3.3)$$

Using (3.3), the $Bias$ and MSE of t_{Sid}, are given by

$$Bias\left(t_{sid}\right) \cong \left(k_1 - 1\right)\bar{Y} + k_1\bar{Y}\left(\left(\theta - \theta'\right)\frac{1}{8}g^2\left(3+5\gamma\right)C_x^2 - \frac{1}{2}g\left(1+\gamma\right)\left(\theta - \theta'\right)C_{zx}\right) + k_2\bar{X}\left[\frac{1}{2}g\left(1+\gamma\right)\left(\theta - \theta'\right)C_x^2\right] \quad (3.4)$$

$$MSE\left(t_{Sid}\right) \cong \bar{Y}^2\left[\left(k_1 - 1\right)^2 + k_1^2\left\{\theta C_z^2 + \left(\theta - \theta'\right)\left(\frac{1}{4}g^2C_x^2\left(\gamma^2 + 7\gamma + 4\right) - 2gC_{zx}\left(1+\gamma\right)\right)\right\}\right.$$
$$-2k_1\left(\theta - \theta'\right)\left\{\frac{1}{8}g^2\left(5\gamma + 3\right)C_x^2 - \frac{1}{2}g\left(1+\gamma\right)C_{zx}\right\} + k_2^2\frac{\bar{X}^2}{\bar{Y}^2}\left(\theta - \theta'\right)C_x^2$$
$$\left.-2k_2\frac{\bar{X}}{\bar{Y}}\frac{1}{2}g\left(\theta - \theta'\right)\left(1+\gamma\right)C_x^2 - 2k_1k_2\frac{\bar{X}}{\bar{Y}}\left(\theta - \theta'\right)\left(C_{zx} - g\left(1+\gamma\right)C_x^2\right)\right] \quad (3.5)$$

And optimum values of k_1 and k_2, respectively, are found as,

$$k_{1(opt)} = \frac{1 - \frac{1}{8}\left(\theta - \theta'\right)g^2\left(4\gamma^2 + 3\gamma + 1\right)C_x^2}{1 + \left\{\theta C_z^2\left(1 - \rho_{zx}^2\right) - g^2\frac{1}{4}\left(\gamma + 3\gamma^2\right)\left(\theta - \theta'\right)C_x^2\right\}}$$

$$k_{2(opt)} = \frac{\bar{Y}}{\bar{X}}\left\{\frac{1}{2}g\left(1+\gamma\right) + k_{1(opt)}\left(\frac{C_{zx}}{C_x^2} - g\left(1+\gamma\right)\right)\right\}$$

Substituting these optimum values in (3.5), the minimum MSE of t_{Sid} is given by

$$MSE\left(t_{Sid}\right)_{\min} \cong \bar{Y}^2\left[1-\frac{1}{4}g^2\left(1+\gamma\right)^2\left(\theta-\theta'\right)C_x^2-\frac{\left\{1-\frac{1}{8}g^2\left(4\gamma^2+3\gamma+1\right)\left(\theta-\theta'\right)C_x^2\right\}^2}{\left\{1+\left\{\theta C_z^2\left(1-\rho_{zx}^2\right)-g^2\frac{1}{4}\left(\gamma+3\gamma^2\right)\left(\theta-\theta'\right)C_x^2\right\}\right\}}\right] \tag{3.6}$$

By using (3.6), for different values of a, b and $\gamma=0$ or $\gamma=1$, we can get the minimum MSE_s of t_{Sid} $\left(i=0,1,2,...,9\right)$.

4. Simulation Study & Efficiency Comparison of Proposed Estimators

We use the simulation studies for efficiency comparison by empirically and theoretically. Two populations for simulation studies of size 1000 each from bivariate normal populations for (Y, X), with different covariance matrices are used. The Scrambling variable $S \sim N\left(0, 0.1\sigma_x\right)$ and $Z=Y+S$.

Population 1 Mean of $\left[Y, X\right]$ given as $\mu=\left[2 \quad 2\right]$

$$\Sigma=\begin{bmatrix} 9 & 1.9 \\ 1.9 & 4 \end{bmatrix} \quad \rho_{XY}=0.3209$$

Population 2 Mean of $\left[Y, X\right]$ given as $\mu=\left[2 \quad 2\right]$

$$\Sigma=\begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix} \quad \rho_{XY}=0.8684$$

For all populations, we consider four sample sizes: $n'=100, 200, 300$ and n=25,45, 80 respectively.

The empirical and theoretical MSE's for various sensitive mean estimators are given in Tables 1-2. We estimate the empirical MSE using 10000 samples of different sizes selected from each population.

5. Real Data Set Application of Two-Phase or Double Sampling in RRT

For this analysis, we consider the real population used in Gupta et al. (2012). Let Y be the monthly salaries amount in 2010, X is the number of employees available from business data register and $S \sim N\left(0, 0.1\sigma_x\right)$. For this population, we have:

$N=26980, \rho_{xy}=0.8599$, $\mu_X=113.91, \mu_Y=167.18$ (in thousands of) $\sigma_X=215.8, \sigma_Y=501.4$ and $\sigma_{XY}=93043$, $n'=1000, 5000$, 10000 and $n=300$, 500, 1000 respectively.

Numerical results of empirical and theoretical MSE based on population data is given in Table 3.

The following expression is used to obtain percent relative efficiency (PRE) of different estimators with respect to $\bar{z}$:

$$PRE=\frac{MSE\left(t_{ys}\right)}{MSE\left(t_\alpha\right)}\times 100$$

Where $\alpha=rsd, regsd, S0d, S1d, S2d, S3d, S4d, S5d, S6d, S7d, S8d, S9d$.

In Table 3 we present the empirical and theoretical results of MSE estimates and PRE of the various estimators in the stratified sample.

6. Conclusions

In this study, we suggested the idea of two-phase sampling in randomized response technique. We consider a general class of estimators for mean of sensitive variable based on randomized response technique in two-phase sampling. In Tables 1-3, we present the results of the theoretical and empirical MSE and PRE of the estimators in two-phase sampling using randomized response technique. These results are computed with a simulation studies and using a real data set.

Appendix

Table 1. Empirical and Theoretical MSE, PRE for the estimators relative to RRT mean estimator for Population 1 in Two Phase Sampling

Population 1					*MSE Estimation*		
N	ρ_{XY}	n'	n	*Estimator*	*Empirical*	*Theoretical*	*PRE*
1000	0.3209	100	25	t_{rsd}	0.3575	0.3570	98.74
				t_{regsd}	0.3156	0.3247	108.56
				t_{S0d}	0.2719	0.2809	125.48

Population 1					*MSE Estimation*		
N	ρ_{XY}	n'	n	*Estimator*	*Empirical*	*Theoretical*	*PRE*
				t_{S1d}	0.2701	0.2672	131.92
				t_{S2d}	0.2625	0.2620	134.54
				t_{S3d}	0.2594	0.2578	136.73
				t_{S4d}	0.2589	0.2596	135.78
				t_{S5d}	0.2662	0.2683	131.38
				t_{S6d}	0.2700	0.2690	131.04
				t_{S7d}	0.2752	0.2733	128.97
				t_{S8d}	0.2773	0.2786	126.52
				t_{S9d}	0.2799	0.2782	126.70
		200	45	t_{rsd}	0.1958	0.1944	98.66
				t_{regsd}	0.1778	0.1769	108.42
				t_{S0d}	0.1676	0.1669	114.91
				t_{S1d}	0.1659	0.1643	116.74
				t_{S2d}	0.1621	0.1636	117.24
				t_{S3d}	0.1547	0.1576	121.70
				t_{S4d}	0.1589	0.1595	120.25
				t_{S5d}	0.1614	0.1617	118.61
				t_{S6d}	0.1617	0.1620	118.39
				t_{S7d}	0.1649	0.1651	116.17
				t_{S8d}	0.1627	0.1633	117.45
				t_{S9d}	0.1636	0.1642	116.81
		300	80	t_{rsd}	0.1081	0.1053	98.67
				t_{regsd}	0.0941	0.0955	108.79
				t_{S0d}	0.0905	0.0909	114.30
				t_{S1d}	0.0906	0.0910	114.17
				t_{S2d}	0.0868	0.0898	115.70
				t_{S3d}	0.0869	0.0900	115.44
				t_{S4d}	0.0906	0.0910	114.17
				t_{S5d}	0.0920	0.0918	113.18
				t_{S6d}	0.0922	0.0924	112.44
				t_{S7d}	0.0929	0.0927	112.08
				t_{S8d}	0.0930	0.0932	111.48
				t_{S9d}	0.0931	0.0912	113.92

Table 2. Empirical and Theoretical MSE, PRE for the estimators relative to RRT mean estimator for Population 2 in Two Phase Sampling

Population 1					*MSE Estimation*		
N	ρ_{XY}	n'	n	*Estimator*	*Empirical*	*Theoretical*	*PRE*
1000	0.8684	100	25	t_{rsd}	0.1212	0.1242	187.36
				t_{regsd}	0.1096	0.1091	213.29
				t_{S0d}	0.0943	0.0940	247.55
				t_{S1d}	0.0922	0.0921	252.66
				t_{S2d}	0.0901	0.0903	257.69
				t_{S3d}	0.0900	0.0909	255.99
				t_{S4d}	0.0910	0.0914	254.59
				t_{S5d}	0.0922	0.0924	251.84
				t_{S6d}	0.0929	0.0930	250.21
				t_{S7d}	0.0956	0.0977	238.18
				t_{S8d}	0.0970	0.0970	239.89
				t_{S9d}	0.0984	0.0981	237.21
		200	45	t_{rsd}	0.0579	0.0586	217.92
				t_{regsd}	0.0482	0.0498	256.43
				t_{S0d}	0.0477	0.0472	270.55
				t_{S1d}	0.0476	0.0470	271.70
				t_{S2d}	0.0458	0.0460	277.61
				t_{S3d}	0.0434	0.0440	290.22
				t_{S4d}	0.0443	0.0448	285.04
				t_{S5d}	0.0447	0.0447	285.68
				t_{S6d}	0.0450	0.0489	261.14
				t_{S7d}	0.0456	0.0458	278.82
				t_{S8d}	0.0460	0.0453	281.89
				t_{S9d}	0.0467	0.0460	277.61
		300	80	t_{rsd}	0.0332	0.0324	213.58
				t_{regsd}	0.0301	0.0299	231.43
				t_{S0d}	0.0280	0.0281	246.26
				t_{S1d}	0.0282	0.0281	246.26
				t_{S2d}	0.0278	0.0275	251.63
				t_{S3d}	0.0277	0.0270	256.29
				t_{S4d}	0.0279	0.0280	247.14
				t_{S5d}	0.0281	0.0282	245.39
				t_{S6d}	0.0284	0.0283	244.52
				t_{S7d}	0.0286	0.0288	240.27
				t_{S8d}	0.0288	0.0289	239.44
				t_{S9d}	0.0290	0.0291	237.81

Table 3. Empirical and Theoretical MSE, PRE for the estimators relative to RRT mean estimator for Real data in Two Phase Sampling

Population 1					*MSE Estimation*		
N	ρ_{XY}	n'	n	*Estimator*	*Empirical*	*Theoretical*	*PRE*
26980	0.8599	1000	300	t_{rsd}	436.74	430.82	192.71
				t_{regsd}	394.79	396.80	209.22
				t_{S0d}	390.10	391.23	212.21
				t_{S1d}	389.38	389.00	213.42
				t_{S2d}	380.12	382.32	217.15
				t_{S3d}	378.20	376.09	220.75
				t_{S4d}	379.56	380.77	218.03
				t_{S5d}	382.38	383.01	216.76
				t_{S6d}	384.06	384.88	215.71
				t_{S7d}	385.98	386.04	215.06
				t_{S8d}	388.90	389.00	213.42
				t_{S9d}	390.94	390.82	212.43
		5000	500	t_{rsd}	187.27	183.36	269.63
				t_{regsd}	160.49	161.84	305.48
				t_{S0d}	157.61	157.69	313.52
				t_{S1d}	156.03	155.98	316.96
				t_{S2d}	155.42	155.09	318.78
				t_{S3d}	153.90	153.78	321.49
				t_{S4d}	153.56	153.03	323.07
				t_{S5d}	154.29	154.95	319.07
				t_{S6d}	157.34	157.00	314.90
				t_{S7d}	156.99	157.00	314.90
				t_{S8d}	157.97	157.56	313.78
				t_{S9d}	157.09	157.54	313.82
		10000	1000	t_{rsd}	86.47	87.01	278.74
				t_{regsd}	75.08	75.24	322.34
				t_{S0d}	71.31	71.78	337.87
				t_{S1d}	71.06	71.12	341.01
				t_{S2d}	70.03	70.12	345.87
				t_{S3d}	69.23	68.99	351.54
				t_{S4d}	69.06	69.10	350.98
				t_{S5d}	69.29	69.32	349.82
				t_{S6d}	69.47	69.40	349.46
				t_{S7d}	69.87	69.88	347.06
				t_{S8d}	69.98	69.89	346.32
				t_{S9d}	70.10	70.21	345.43

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