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Effect of Change of Origin of Variables on Ratio Estimator of Population Mean

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Abstract In last two decades, a number of modified ratio estimators for population mean have been developed by using the known information of auxiliary variable under different sampling strategies. The functional form of these estimators is, mathematically, not justifiable because these estimators have been obtained by using transformation of auxiliary variable by adding a unit free constant or quantities with different units. But these modified estimators are more efficient than the usual ratio estimator of population mean. So, it is clear that there is some effect of change of origin of auxiliary variable on the usual ratio estimator. In the present paper, we have discussed the effect of change of origin of auxiliary as well as study variable on ratio estimator of population mean and have also obtained the optimum transformations in the cases of over estimation and under estimation. Also, the conditions for validity of approximate results of the ratio estimator have been obtained which can be easily verified in practical situations. Further, we have carried out a simulation study to verify the theoretical results.

Keywords Efficiency, Ratio Estimation, Simple Random Sampling, Transformation

1. Introduction

[1] initiated the use of auxiliary information to estimate population mean in sample survey. He proposed the ratio estimator $\bar{y}_R = \bar{y}\frac{\bar{X}}{\bar{x}}$ to estimate population mean $(\bar{Y})$ which can also be used to estimate population total (Y) where $\bar{y}$ and $\bar{x}$ denote the sample means of study and auxiliary variables respectively and $\bar{X}$ denotes the population mean of auxiliary variable. This ratio estimator is more efficient than mean per unit estimator if $\rho > \frac{1}{2}\frac{C_x}{C_y}$, where ρ is the correlation coefficient between the auxiliary (x) and the study (y) variables; C_x and C_y are coefficients of variation for auxiliary and study variables respectively.

Further, on the same pattern, [2] proposed a product estimator $\bar{y}_P = \bar{y}\frac{\bar{x}}{\bar{X}}$ for $\bar{Y}$. The product estimator is more efficient than mean per unit estimator if $\rho < -\frac{1}{2}\frac{C_x}{C_y}$. Using the power transformation, [3] proposed a new estimator

$$\bar{y}_{pwr} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha},$$

to estimate $\bar{Y}$ and he found the optimum value of α for which the MSE of $\bar{y}_{pwr}$ is minimum. After this, many statisticians proposed different kinds of estimators using auxiliary information. But [4] proposed a modified ratio estimator for $\bar{Y}$ using the additional known information of coefficient of variation of auxiliary variable as:

$$\bar{y}_{sd} = \bar{y}\frac{\bar{X} + C_x}{\bar{x} + C_x},$$

Thereafter, it became a trend to propose modified ratio estimators for $\bar{Y}$ by using additional known information of auxiliary variable. Some related examples are as follows:

[5]

$$\bar{y}_{pd} = \bar{y}\frac{\bar{x} + C_x}{\bar{X} + C_x},$$

[6]

$$\bar{y}_{sk} = \bar{y}\frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)},$$

where $\beta_2(x)$ is coefficient of kurtosis of auxiliary variable.

[7]

$$\bar{y}_{us1} = \bar{y}\frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x} + C_x},$$

$$\bar{y}_{us2} = \bar{y}\frac{C_x\bar{X} + \beta_2(x)}{C_x\bar{x} + \beta_2(x)},$$

[8]

$$\bar{y}_{ST} = \bar{y}\frac{\bar{X} + \rho}{\bar{x} + \rho},$$

[9]

$$\bar{y}_{kho} = \bar{y}\frac{a\bar{X} + b}{\alpha(a\bar{x} + b) + (1-\alpha)(a\bar{X} + b)},$$

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where a and b are real numbers or some known parameters of auxiliary variable.

[10]

$$\bar{y}_{R(\alpha)} = \bar{y}\left(\frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x} + C_x}\right)^{\alpha},$$

where α is some real constant.

[11]

$$\bar{y}_{kk} = k\bar{y}_{kho},$$

where k is some real constant.

[12]

$$\bar{y}_{ts} = \bar{y}\left[\alpha\left(\frac{C_x\bar{X} + \beta_2(x)}{C_x\bar{x} + \beta_2(x)}\right) + (1-\alpha)\left(\frac{C_x\bar{x} + \beta_2(x)}{C_x\bar{X} + \beta_2(x)}\right)\right],$$

[13]

$$\bar{y}_{yt1} = \bar{y}\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1},$$

$$\bar{y}_{yt2} = \bar{y}\frac{\beta_2(x)\bar{X} + \beta_1}{\beta_2(x)\bar{x} + \beta_1},$$

$$\bar{y}_{yt3} = \bar{y}\frac{\beta_1\bar{X} + \beta_2(x)}{\beta_1\bar{x} + \beta_2(x)},$$

$$\bar{y}_{yt4} = \bar{y}\frac{C_x\bar{X} + \beta_1}{C_x\bar{x} + \beta_1},$$

where β_1 is the coefficient of skewness of auxiliary variable.

All the above estimators are more efficient than the classical ratio estimator but in these estimators, researchers have added a pure number in the quantity which possesses a unit which is mathematically not possible. For example, the mean of auxiliary variable $\bar{X}$ is measured in centimeters and the coefficient of variation is always a pure number, still [4] and [5] added these two quantities. The main question which arises here is "Can we add these two quantities?" This question was first raised by [14]. If the answer is in the negative, then the question arises about the authenticity of the estimators proposed by different researchers using above assumption.

The aim of the present paper is to discuss these issues and find out the situation where one can use the classical ratio estimator. In Section-2, the basic conditions for the ratio estimator of population mean have been discussed. The effects of change of origin of variables on ratio estimator of population mean have been presented in Section-3. A detailed discussion about the clarification given by [15] on the transformation of auxiliary variable has been presented in Section-4 and the idea for further study has been proposed in Section-5. In Section-6, a simulation study has been carried out to verify theoretical results. In the last section, the conclusion of the present paper has been drawn.

2. Ratio Estimator

In all the modified ratio estimators, researchers had changed either origin only or both the origin and scale of auxiliary variable. Due to this change, there is some reduction in bias and MSE of modified ratio estimators as compared to those of the classical ratio estimator. It means that there is some effect of change of origin and scale of auxiliary variable. To understand this reduction of bias and MSE, there is a need to understand the assumptions of classical ratio estimator which are as follows:-

1. To obtain the approximate results for ratio estimator, the conditions are

$$\left|\frac{\bar{x}-\bar{X}}{\bar{X}}\right| \ll 1 \quad \& \quad \left|\frac{\bar{y}-\bar{Y}}{\bar{Y}}\right| \ll 1 \tag{1}$$

From the Chebychev's inequality, we have

$$P(|\bar{x} - \bar{X}| < 3\sigma_{\bar{x}}) \geq \frac{8}{9},$$

where $\sigma_{\bar{x}}^2 = V(\bar{x}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_x^2 = (1-f)\frac{S_x^2}{n}$ and $f = \frac{n}{N}$.

The above inequality shows that $3\sigma_{\bar{x}}$ is the upper bound of $|\bar{x} - \bar{X}|$ with probability more than $\frac{8}{9}$. Using this upper bound in (1), we get

$$\frac{3\sigma_{\bar{x}}}{|\bar{X}|} \ll 1,$$

or

$$\sqrt{(1-f)}\frac{3S_x}{\sqrt{n}} \ll |\bar{X}|$$

After ignoring the finite population correction, we get

$$\frac{3S_x}{\sqrt{n}} \ll |\bar{X}| \tag{2}$$

Similarly, one can obtain

$$\frac{3S_y}{\sqrt{n}} \ll |\bar{Y}| \tag{3}$$

Now one can easily verify the conditions given in equation (1) with the help of inequalities (2) and (3).

If the inequalities (2) and (3) are not satisfied then one can use new variables using the transformation $x \to x + \gamma = x'$ and $y \to y + \eta = y'$. Since the left hand side of these inequalities is independent of change of origin, these transformations affect only the right side. These new variables can further be used in ratio estimation of population mean. In the present paper, we assume that these inequalities are satisfied for variables x and y.

2. Classical ratio estimator of population mean is more efficient than mean per unit estimator provided

$$\rho > \frac{1}{2}\frac{C_x}{C_y} \tag{4}$$

In above inequality, left hand side is independent of change of origin of variables whereas right hand side is dependent on change of origin of variables. In the following section, we will first discuss some results of ratio estimator.

3. Effect of Change of Origin of Variables

After assuming the conditions given in (1), one can easily obtain the approximate bias and MSE for classical ratio estimator as

$$Bias(\bar{y}_R) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}\left(C_x^2 - \rho C_y C_x\right) \quad (5)$$

$$MSE(\bar{y}_R) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_y C_x\right) \quad (6)$$

It is known that

$$MSE(\bar{y}_R) = V(\bar{y}_R) + Bias^2(\bar{y}_R)\,, \quad (7)$$

where $V(\bar{y}_R)$ is the minimum value of $MSE(\bar{y}_R)$ if bias of the estimator become zero. $Bias(\bar{y}_R)$ may be positive or negative. Both the cases have been discussed as follows:-

Case-I $Bias(\bar{y}_R) > 0$

We have from equation (5),

$$C_x^2 - \rho C_y C_x > 0$$

or

$$\rho < \frac{C_x}{C_y} \quad (8)$$

If we use the transformation $x \to x + \gamma = x'$, where $\gamma > 0$, there is reduction in C_x, that is

$$C_{x'} < C_x \quad (9)$$

One can choose the optimum value of γ as

$$\gamma_{opt} = \frac{\bar{Y}S_x^2}{S_{yx}} - \bar{X},$$

such that

$$\rho = \frac{C_x{'}}{C_y}. \quad (10)$$

Now we propose a modified ratio estimator of population mean as

$$\bar{y}_{R1} = \bar{y}\frac{\bar{X}'}{\bar{x}'} = \bar{y}\frac{\bar{X}+\gamma_{opt}}{\bar{x}+\gamma_{opt}} \quad (11)$$

Approximate expressions for bias and MSE of $\bar{y}_{R1}$ are as

$$Bias(\bar{y}_{R1}) \cong 0$$

and

$$MSE(\bar{y}_{R1}) = \left(\frac{1}{n} - \frac{1}{N}\right)(1-\rho^2)S_y^2 \cong V(\bar{y}_{R1}) \quad (12)$$

Clearly, the modified ratio estimator $\bar{y}_{R1}$ is an approximately unbiased estimator of population mean $\bar{Y}$ and the estimator $\bar{y}_{R1}$ is always more efficient than mean per unit estimator.

In practical situations, the value of γ_{opt} is generally unknown. So one can use the estimated value of γ_{opt} as

$$\hat{\gamma}_{opt} = \frac{\bar{y}s_x^2}{s_{yx}} - \bar{X}\,.$$

Case-II $Bias(\bar{y}_R) < 0$

From equation (5), we get

$$C_x^2 - \rho C_y C_x < 0$$

or

$$\rho > \frac{C_x}{C_y} \quad (13)$$

If we use the transformation $x \to x - \gamma_1 = x''$, where $\gamma_1 > 0$, there is increase in C_x, that is

$$C_{x''} > C_x \quad (14)$$

But due this transformation, the new variable x'' may violate the condition given in (1). So here, we shall use a new transformation $y \to y + \eta = y'$, where $\eta > 0$. If we compare the coefficients of variation of these variables, we get

$$C_{y'} < C_y\,,$$

By using the optimum value of η as

$$\eta_{opt} = \frac{\bar{X}S_{yx}}{S_x^2} - \bar{Y},$$

we get

$$\rho = \frac{C_x}{C_{y'}} \quad (15)$$

We define the new ratio estimator for $\bar{Y} + \eta_{opt}$ as

$$\bar{y}_R' = \bar{y}'\frac{\bar{X}}{\bar{x}} = (\bar{y} + \eta_{opt})\frac{\bar{X}}{\bar{x}} \quad (16)$$

The ratio estimator defined above is approximately unbiased for $\bar{Y} + \eta_{opt}$ and the approximate variance is same as the variance of $\bar{y}_{R1}$ as given in (12).

Modified ratio estimator for $\bar{Y}$ may be defined as

$$\bar{y}_{R2} = \bar{y}_R' - \eta_{opt} \quad (17)$$

Clearly, $\bar{y}_{R2}$ remains unbiased for $\bar{Y}$. Also the variance of $\bar{y}_{R2}$ remains same as the variance of $\bar{y}_{R1}$ given in (12).

The optimum value of η contains some unknown parameters, so one can use the estimated value of η_{opt} in practical situations as

$$\hat{\eta}_{opt} = \frac{\bar{X}s_{yx}}{s_x^2} - \bar{y}\,.$$

From the above discussion, one concludes that if there is a problem of over estimation during ratio estimation of population mean, then it can be solved by shifting the origin of the auxiliary variable. Similarly, if there is a problem of under estimation, then it can be reduced by shifting the origin of the study variable. Further, one more question arises- "What will happen if we change the origin and scale of the auxiliary variable?" To answer this, we consider the following transformation on auxiliary variable.

$$x \to \kappa x + \eta$$

which implies that

$$\bar{X} \to \kappa\bar{X} + \eta, \qquad \bar{x} \to \kappa\bar{x} + \eta$$

In this case, the modified ratio estimator may be defined as

$$\bar{y}_{R3} = \bar{y}\left(\frac{\kappa\bar{X}+\eta}{\kappa\bar{x}+\eta}\right) = \bar{y}\left(\frac{\bar{X}+\frac{\eta}{\kappa}}{\bar{x}+\frac{\eta}{\kappa}}\right).$$

It shows that in ratio estimation, the transformation $x \to \kappa x + \eta$ is equivalent to the transformation $\to x + \frac{\eta}{\kappa}$.

One can modify the ratio estimator $\bar{y}_R$ by shifting the origin of auxiliary variable or study variable and this modified estimator is always more beneficial than the existing one. Different modified ratio estimators proposed by the researchers by using the additional known information of auxiliary variable are mathematically not correct. But these estimators perform better than classical ratio estimator due to the change of origin of auxiliary variable; not due to additional known information.

4. Clarification by Singh and Kumar

[14] and [16] raised a question regarding transformation of auxiliary variables by adding a unit free constant. But [15] try to clarify the same with the help of following three examples:

1. "Let

$$f(x) = \begin{cases} 1 & if\ a < x < b \\ 0 & otherwise \end{cases} \tag{18}$$

The area under the function will be $(b-a)$ which is in fact the distance between two points a and b on a real line, say $(b-a)cm$. Now, to make $(b-a)cm$ as area, we have to multiply it by $1cm$. The required area will be: $(b-a)cm2$."

2. "If we integrate speed (cm/sec) of an object with respect to time, then we get distance in cm. Now distance is not area, as expected after integrating a function, but it is distance in centimeter."

3. "π is constant and an irrational number, but has radian as units of measurements."

On the basis of these three examples, they asserted that these kinds of adjustments of units of measurements are common in practice.

Our Clarification:

First of all, in the above three examples we need to understand whether an adjustment has really been done or not. Let us recall the definition of integration as follows:

Let $'f'$ be a bounded real-valued function on $[a,b]$. It means that $'f'$ is also bounded on each sub-interval corresponding to each partition $'P'$. Let $M_i,\ m_i$ be the least upper bound and greatest lower bound of $'f'$ in δx_i. Then there are two sums,

$$U(P,f) = \sum_{i=1}^{n} M_i \delta x_i$$

and

$$L(P,f) = \sum_{i=1}^{n} m_i \delta x_i$$

respectively called the upper and lower sums of $'f'$ corresponding to the partition $'P'$.

Further, infimum of the set of upper sums is called the **upper integral** and supremum of the set of lower sums is called the **lower integral** over $[a,b]$. That is

$$\int_a^{\bar{b}} f\,dx = \inf\{U(P,f)\}$$

and

$$\int_{\underline{a}}^{b} f\,dx = \sup\{L(P,f)\}$$

If two integrals are equal, i.e.

$$\int_a^{\bar{b}} f\,dx = \int_{\underline{a}}^{b} f\,dx = \int_a^b f\,dx$$

then $'f'$ is said to be Riemann integrable or simply integrable over $[a,b]$ and the common value of these integrals is called the integral of $'f'$ over $[a,b]$. For more details of the concept of integration, see [17].

From the above definition, it is clear that

The units of integral of $'f'$ = (units of $'f'$)×(units of $'x'$).

In equation (18), if both $'f'$ and $'x'$ have units in $'cm'$ then from the definition, we have

$$\int_a^b f\,dx = (b-a)cm^2.$$

Similarly, when we integrate speed*(cm/sec)* of an object with respect to time*(sec)*, then we get units of the integral in cm. Further, we know that each individual real number is a constant and π is one of them. The number π does not possess any units like radian. For example, if height of a person is 72 inches, it does not mean that 72 has inches as units. The units are possessed by variable height but not by 72.

From above discussion, it is clear that in all the three examples given by [15], there is no adjustment of units.

5. Further Study

In the last two decades, a number of modified estimators have been developed under various sampling strategies using transformations of auxiliary variables by shifting origin by adding units free constants or quantities with different units. This type of modifications are mathematically not justifiable. So there is a need to find out the actual reasons about what happens when we change the origin under those sampling strategies. Some of the work which has been done using this pattern by different researchers is mentioned in [18-36] and many more.

6. Simulation Study

A simulation study has been carried out to verify the theoretical results by using software R. Firstly, results have been obtained on the basis of *1,00,000* samples each of size *n=3, 5* and *7* from each of normal populations $N(\bar{Y}=25,\ \bar{X}=5,\ \sigma_y^2=49,\ \sigma_x^2=49,\rho)$, where $\rho = 0.3, 0.5, 0.7, 0.9$. For these populations, auxiliary variable x does not satisfy the condition

$$\frac{3S_x}{\sqrt{n}} \ll 1.$$

Table 1. When The Conditions are not Satisfying

ρ	0.3	0.5	0.7	0.9
$n = 3$				
Actual Bias ($\bar{y}_R$)	230.722895	-26.12035	1.272922	-189.92528
Approx. Bias ($\bar{y}_R$)	12.08667	11.43333	10.78	10.12667
Actual MSE ($\bar{y}_R$)	21341314455	119444988	5328050	1846187721
Approx.MSE ($\bar{y}_R$)	238.4667	212.3333	186.2	160.0667
Actual Bias ($\bar{y}_R'$)	0.485596	0.355074	0.232909	0.06464257
Approx. Bias ($\bar{y}_R'$)	0.326667	0.196	0.065333	-0.06533
Actual MSE ($\bar{y}_R'$)	26.597502	18.77098	11.165717	3.596584
Approx.MSE ($\bar{y}_R'$)	18.94667	13.72	8.493333	3.266667
$n = 5$				
Actual Bias ($\bar{y}_R$)	196.899968	8.495421	-0.890275	-5.547268
Approx. Bias ($\bar{y}_R$)	7.252	6.86	6.468	6.076
Actual MSE ($\bar{y}_R$)	3306100220	10777495	16048314	22078935
Approx.MSE ($\bar{y}_R$)	143.08	127.4	111.72	96.04
Actual Bias ($\bar{y}_R'$)	0.29869843	0.211394	0.1279243	0.04211807
Approx. Bias($\bar{y}_R'$)	0.326667	0.196	0.065333	-0.06533
Actual MSE ($\bar{y}_R'$)	15.061803	10.560222	6.258956	2.080361
Approx. MSE ($\bar{y}_R'$)	11.368	8.232	5.096	1.96
$n = 7$				
Actual Bias ($\bar{y}_R$)	-106.987649	19.25837	-22.33229	4.280546
Approx. Bias ($\bar{y}_R$)	5.18	4.9	4.62	4.34
Actual MSE ($\bar{y}_R$)	1316222565	7628726	109972370	1356378
Approx.MSE ($\bar{y}_R$)	102.2	91	79.8	68.6
Actual Bias ($\bar{y}_R'$)	0.20451674	0.1420131	0.0750127	0.02826361
Approx. Bias ($\bar{y}_R'$)	0.326667	0.196	0.065333	-0.06533
Actual MSE ($\bar{y}_R'$)	10.431577	7.405863	4.369064	1.451982
Approx. MSE ($\bar{y}_R'$)	8.12	5.88	3.64	1.4

Table 2. Problem of Over Estimation

ρ	Bias ($\bar{y}_R$)	MSE ($\bar{y}_R$)	Bias ($\bar{y}_{R1}$)	MSE ($\bar{y}_{R1}$)	Bias ($\bar{y}_{R1}'$)	MSE ($\bar{y}_{R1}'$)
when $n = 5$						
0.3	1.655770	168.9897	0.00027615	4.5504491	0.024736	6.861163
0.5	1.473163	154.3566	0.00039894	3.7788297	0.0347981	5.627979
0.7	1.421140	141.2586	-0.00459157	2.5601980	0.030450	3.828913
0.9	1.389383	128.6809	-0.00370513	0.9540783	0.044629	1.440474
when $n = 7$						
0.3	1.109625	114.02038	0.00570	3.254664	0.016759	4.066155
0.5	0.9998352	106.3099	-0.00504	2.683296	0.006694	3.367004
0.7	1.0198715	97.28919	-0.000076	1.816818	0.020412	2.267031
0.9	0.9522851	87.51277	0.00378	0.67798	0.032549	0.841154
when $n = 9$						
0.3	0.836387	86.81129	0.002245	2.533021	0.009581	2.9544848
0.5	0.812951	79.88055	0.006425	2.086275	0.017265	2.4451311
0.7	0.767774	73.08879	0.000351	1.414371	0.017391	1.6549719
0.9	0.749307	66.91465	0.001753	0.527241	0.025924	0.6163458

If we use the transformation $x \rightarrow x' = x + 20$, then the new auxiliary variable x' satisfies the above condition. Actual and approximate values for the biases and MSEs of the usual ratio estimator $\bar{y}_R$ and the modified ratio estimator $\bar{y}_R'$ after using transformation $x \rightarrow x'$ are given in Table-1.

Secondly, the problem of over estimation has been considered. For this purpose, 1,00,000 samples each of size $n = 3, 5$ and 7 have been drawn from each of normal population $N(\bar{Y} = 100,\ \bar{X} = 25,\ \sigma_y^2 = 25,\ \sigma_x^2 = 49, \rho)$, where $\rho = 0.3, 0.5, 0.7, 0.9$. The values of biases and MSEs of $\bar{y}_R, \bar{y}_{R1}$ and $\bar{y}_{R1}'$ have been presented in Table-2.

Results Based on Table-1 and Table-2:

1. Table-1 shows that if the study and auxiliary variables satisfy the conditions given in (2) and (3), then the approximate values for biases and MSE's of usual ratio estimator are very close to the actual values.
2. Table-2 shows that if we use the optimum transformation of auxiliary variable, then we can get an almost unbiased ratio estimator for population mean.

7. Conclusions

A number of modified ratio estimators have been proposed by different researchers using the additional known information of auxiliary variable. Basically, they have applied a transformation on auxiliary variable by adding some unit-less quantity or a quantity with different units in auxiliary variable. Such type of transformations are mathematically not correct. But from these modified ratio estimators, one can form an idea that the change of origin of auxiliary variable affect the efficiency of the ratio estimator. In the present paper, it has been shown that the efficiency of ratio estimator also depends upon the origin of the auxiliary and study variables. Also, the efficiencies of the modified estimators have improved due to the change of origin and not due to additional known information of auxiliary variable. Further, the conditions for validity of the approximate results for ratio estimators have been developed in (2) and (3), which can easily be checked in practical situations before using the ratio estimators.

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