Applied Mathematics 2014, 4(3): 77-85
DOI: 10.5923/j.am.20140403.02

Unsteady MHD Free Convection Flow Along a Vertical Plate in the Presence of Radiative Heat Flux

Nepal Chandra Roy[1], Bishnu Pada Ghosh[2], Litan Kumar Saha[3,*]

[1]Department of Mathematics, University of Dhaka, Dhaka, Bangladesh
[2]Department of Mathematics, Jagannath University, Dhaka, Bangladesh
[3]Department of Applied Mathematics, University of Dhaka, Dhaka, Bangladesh

Abstract Magnetohydrodynamic (MHD) free convection boundary layer flow of a viscous incompressible fluid along a vertical plate has been studied numerically where the effect of radiation and the surface temperature oscillation is taken into account. The governing boundary layer equations have been solved by employing two distinct methods, namely, the primitive variable method and the Keller box method. A comparison between the primitive variable method and the Keller box method is made that provides a good agreement. The effects of varying the Prandtl number, Pr, the magnetic field parameter, M, and the conduction-radiation parameter, R_d, are discussed in terms of amplitude and phase angle of skin friction for fluid having Prandtl number equals 0.1. Also we have investigated the effects of these parameters on the amplitude of oscillation of transient skin-friction and transient heat transfer and on the streamlines and isolines of temperature.

Keywords Magnetohydrodynamics, Thermal radiation, Boundary layer, Free convection

1. Introduction

The effects of thermal radiation on the free convection flows have been the focus of research interest due to many applications in space technology and in processes involving high temperatures such as nuclear power plants, gas turbines and thermal energy storage. Also the free convection flow in the presence of magnetic field is very important owing to its significant effect on the boundary layer control and on the performance of many engineering devices using electrically conducting fluids. This type of fluid flow finds application in MHD power generation, plasma studies, nuclear reactor using liquid metal coolant and geothermal energy extraction.

The energy equation had lead to a highly nonlinear partial differential equation with the inclusion of radiation effects. Soundalgekar et al. [1] analyzed the radiation effects on free convection flow of a gas past a semi-infinite flat plate. Sakiadis [2] first considered the steady flow on a moving continuous flat surface and developed a numerical solution using a similarity transformation. The effect of magnetic field on the natural convection heat transfer has been studied by Sparrow and Cess [3]. Romig [4] analyzed the effect of electric and magnetic fields on the heat transfer to electrically conducting fluids. The skin friction and heat transfer on a continuous flat surface moving in a parallel free stream was studied numerically by Abdelhafez [5]. Chiam [7] delineates the solutions for steady hydromagnetic flow over a surface stretching with a power-law velocity with the distance along the surface. Hossain and Takhar [8] investigated the effect of radiation on forced and free convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. The effect of radiation on the free convection heat transfer problem in the absent of magnetic field and viscous effects has been analyzed by Hossain et al. [9].

Steady free convection flow in the presence of a magnetic field past a vertical plate has investigated by a number of researchers. The problem of steady, laminar, hydromagnetic forced convection boundary-layer flow over a non-isothermal wedge with permeable surface considering the thermal radiation and heat generation or absorption effects investigated by Chamkha et al. [9]. Afify [11] studied the effects of chemical reaction on free convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching surface in the presence of a magnetic field. Duwairi [12] and Damseh et al. [13] investigated the radiation and magnetic effects on the skin friction and heat transfer for forced convection conditions. The steady nonlinear hydromagnetic flow of an incompressible, viscous and electrically conducting fluid with heat transfer over a surface of variable temperature stretching with a power-law velocity in the presence of variable transverse magnetic field has been analyzed by Devi and Thiyagarajan [14]. Ishak et al. [15] investigated the problem of mixed convection adjacent to a vertical,

* Corresponding author:
lksaha_math@yahoo.com (Litan Kumar Saha)
Published online at http://journal.sapub.org/am

continuously stretching sheet in the presence of a variable magnetic field. Aydın and Kaya [16] presented numerically the effects of thermal radiation taking into account the suction or injection on a steady MHD mixed convective flows of a viscous incompressible fluid about a permeable vertical plate. Kandasamy et al. [17] studied the effects of thermophoresis and chemical reaction on MHD mixed convective heat and mass transfer past a porous wedge with variable viscosity in the presence of suction or injection. Mixed convection heat transfer about a semi-infinite inclined plate with the effects of magneto and thermal radiation analyzed by Aydin and Kaya [18]. Seth et al. [19] investigated the effects of radiation on unsteady hydromagnetic natural convection transient flow near an impulsively moving vertical flat plate with ramped wall temperature in a porous medium. Das [20] studied the effects of thermal radiation on steady two dimensional boundary layer flow of an incompressible electrically conducting fluid over a flat plate considering the partial slip at the surface of the boundary, temperature dependent fluid viscosity, variable thermal conductivity and non-uniform heat generation and absorption. Roy and Hossain [21] examined the natural convection boundary layer flow of a viscous incompressible fluid along a vertical plate taking into account the conduction–radiation interaction and surface temperature oscillations. Samad and Saha [22] investigated the two dimensional heat and mass transfer free convection flow of an MHD Non-Newtonian power-law fluid along a stretching sheet in the presence of magnetic field and assuming a simplified thermal radiation.

Lighthill [23] was the first who analyzed the unsteady forced flow of a viscous incompressible fluid past a flat plate and a circular cylinder with small amplitude oscillation in free stream. The corresponding problem of unsteady free convection flow along a vertical plate with oscillating surface temperature was investigated by Nanda and Sharma [24] and Eshghy et al. [25]. Muhuri and Maiti [26] and Verma [27] have also studied the effect of oscillation of the surface temperature on the unsteady free convection from a horizontal plate.

From the above literature review, we can conclude that most of the works investigated the steady behaviors of the skin friction and heat transfer for a variety of physical parameters. Even though a few works considered the unsteady problem but they used simplified models neglecting the effects of the presence of magnetic field and/or thermal radiation. In this study, we investigate the unsteady free convection flow along a vertical plate taking into account the effects of magnetic field and thermal radiation. The temperature of the plate is assumed to be oscillating about a constant mean temperature θ_w with a small amplitude ε as posed by Kelleher and Yang [6]. We solve the governing boundary layer equations using two distinct methods, namely, the primitive variable method and the Keller box method. Then the effects of the physical parameters such as the Prandtl number, Pr, the magnetic field parameter, M, and the conduction-radiation parameter, R_d, have been presented in terms of amplitude and phase angle of skin friction as well as transient skin-friction and heat transfer.

2. Mathematical Formulation

Let us consider a two-dimensional unsteady laminar free convection boundary layer flow of an optically dense viscous and incompressible fluid along a vertical flat plate in the presence of magnetic field and radiative heat transfer. The surface temperature of the plate is assumed to be oscillating with small amplitude about a constant mean temperature. The plate is maintained at the surface temperature ΔT while the ambient temperature of the fluid is $\overline{T}_\infty$. The coordinate system and the flow configuration are shown in Figure 1.

With a view to elucidate the flow system, let $\overline{x}$ denote the distance along the plate from the leading edge, $\overline{y}$ the normal distance from the plate and $\overline{u}$, $\overline{v}$ the corresponding velocity components respectively, ν the coefficient of viscosity, ρ the density of the fluid, g the acceleration due to gravity, β the coefficient of volumetric expansion, σ the electrical conductivity, B_0 the magnetic field strength, α the thermal diffusivity, κ the thermal conductivity, $\overline{T}$ the temperature of the fluid in the boundary layer, $\overline{T}_\infty$ the ambient temperature of the fluid and q_r represents the radiative heat flux in the $\overline{y}$ direction. Then the conservation equations for the unsteady, laminar, two-dimensional boundary layer flow under the usual Boussinesq approximation are

$$\frac{\partial \overline{u}}{\partial \overline{x}}+\frac{\partial \overline{v}}{\partial \overline{y}}=0 \tag{1}$$

$$\frac{\partial \overline{u}}{\partial t}+\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}}+\overline{v}\frac{\partial \overline{u}}{\partial \overline{y}}=\nu\frac{\partial^2 \overline{u}}{\partial \overline{y}^2}+g\beta\left(\overline{T}-\overline{T}_\infty\right)-\frac{\sigma B_0^2}{\rho}\overline{u} \tag{2}$$

$$\frac{\partial \overline{T}}{\partial t}+\overline{u}\frac{\partial \overline{T}}{\partial \overline{x}}+\overline{v}\frac{\partial \overline{T}}{\partial \overline{y}}=\alpha\left(\frac{\partial^2 \overline{T}}{\partial \overline{y}^2}-\frac{1}{\kappa}\frac{\partial q_r}{\partial \overline{y}}\right) \tag{3}$$

The boundary conditions to be satisfied are:

$$\begin{aligned}&\overline{y}=0:\ \overline{u}=0,\quad \overline{v}=0,\\ &\overline{T}-\overline{T}_\infty=\Delta T\left(1+\varepsilon\exp\left(i\omega t\right)\right)\\ &\overline{y}=\infty:\ \overline{u}\to 0,\ \ \overline{T}\to\overline{T}_\infty\end{aligned} \tag{4}$$

where $\varepsilon << 1$ and ω is the frequency of oscillation and $\Delta T=\overline{T}-\overline{T}_\infty$.

In this study, the radiative heat flux term is simplified by using the Rosseland diffusion approximation which is valid for an optically thick fluid as follows:

$$q_r=-\frac{4\sigma_1}{3\left(a+\sigma_s\right)}\frac{\partial \overline{T}^4}{\partial \overline{y}} \tag{5}$$

where σ_1 is the Stefan-Boltzman constant, a is the Rosseland mean absorption coefficient and σ_s is the scattering coefficient.

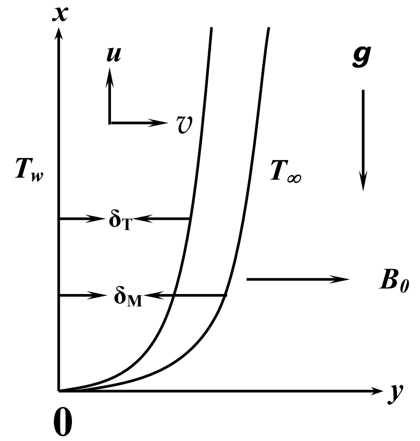

Figure 1. Flow configuration and coordinate system

Now the boundary condition for the surface temperature (4) suggests that the solutions of (1)-(3) can be taken of the form:

$$\bar{u} = \bar{u}_s + \varepsilon \exp(i\omega t)\bar{u}_1$$
$$\bar{v} = \bar{v}_s + \varepsilon \exp(i\omega t)\bar{v}_1 \quad (6)$$
$$\bar{T} - \bar{T}_\infty = \left(\bar{T}_w - \bar{T}_\infty\right)\left(\bar{\theta}_s + \varepsilon \exp(i\omega t)\bar{\theta}_1\right).$$

Substituting the expressions (6) into (1)-(3) and equating the same order of ε, sets of equations are obtained. Equations for $\bar{u}_s$, $\bar{v}_s$ and $\bar{\theta}_s$ are the steady-state equations.
We thus obtain

$$\frac{\partial \bar{u}_s}{\partial \bar{x}} + \frac{\partial \bar{v}_s}{\partial \bar{y}} = 0 \quad (7)$$

$$\bar{u}_s \frac{\partial \bar{u}_s}{\partial \bar{x}} + \bar{v}_s \frac{\partial \bar{u}_s}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}_s}{\partial \bar{y}^2} + g\beta\left(\bar{T}_w - \bar{T}_\infty\right)\bar{\theta}_s - \frac{\sigma B_0^2}{\rho}\bar{u}_s \quad (8)$$

$$\bar{u}_s \frac{\partial \bar{\theta}_s}{\partial \bar{x}} + \bar{v}_s \frac{\partial \bar{\theta}_s}{\partial \bar{y}} = \alpha \frac{\partial^2 \bar{\theta}_s}{\partial \bar{y}^2} + \frac{16\alpha \bar{T}_\infty^3}{3(a+\sigma_s)\kappa}\frac{\partial}{\partial \bar{y}}\left\{\left(1+\Delta\bar{\theta}_s\right)^3 \frac{\partial \bar{\theta}_s}{\partial \bar{y}}\right\} \quad (9)$$

subject to the boundary conditions

$$\bar{u}_s = 0,\ \ \bar{v}_s = 0,\ \ \bar{\theta}_s = 1 \ \text{ at } \ \bar{y} = 0$$
$$\bar{u}_s \to 0,\ \ \bar{\theta}_s \to 0 \ \text{ as } \ \bar{y} \to \infty \quad (10)$$

and $\bar{u}_1$, $\bar{v}_1$ and $\bar{\theta}_1$ are the components of the unsteady flow, which satisfy the equations:

$$\frac{\partial \bar{u}_1}{\partial \bar{x}} + \frac{\partial \bar{v}_1}{\partial \bar{y}} = 0 \quad (11)$$

$$\bar{u}_s \frac{\partial \bar{u}_1}{\partial \bar{x}} + \bar{u}_1 \frac{\partial \bar{u}_s}{\partial \bar{x}} + \bar{v}_s \frac{\partial \bar{u}_1}{\partial \bar{y}} + \bar{v}_1 \frac{\partial \bar{u}_s}{\partial \bar{y}} + i\omega \bar{u}_1 = \nu \frac{\partial^2 \bar{u}_1}{\partial \bar{y}^2} + g\beta\left(\bar{T}_w - \bar{T}_\infty\right)\bar{\theta}_1 - \frac{\sigma B_0^2}{\rho}\bar{u}_1 \quad (12)$$

$$\bar{u}_s \frac{\partial \bar{\theta}_1}{\partial \bar{x}} + \bar{u}_1 \frac{\partial \bar{\theta}_s}{\partial \bar{x}} + \bar{v}_s \frac{\partial \bar{\theta}_1}{\partial \bar{y}} + \bar{v}_1 \frac{\partial \bar{\theta}_s}{\partial \bar{y}} + i\omega \bar{\theta}_1 = \alpha \frac{\partial^2 \bar{\theta}_1}{\partial \bar{y}^2} + \frac{16\alpha \bar{T}_\infty^3}{3(a+\sigma_s)\kappa}\frac{\partial}{\partial \bar{y}}\left\{\left(1+\Delta\bar{\theta}_s\right)^3 \frac{\partial \bar{\theta}_1}{\partial \bar{y}} + 3\Delta\left(1+\Delta\bar{\theta}_s\right)^2 \bar{\theta}_1 \frac{\partial \bar{\theta}_s}{\partial \bar{y}}\right\} \quad (13)$$

with the boundary conditions

$$\bar{u}_1 = 0,\ \ \bar{v}_1 = 0,\ \ \bar{\theta}_1 = 1 \quad \text{at } \bar{y} = 0$$
$$\bar{u}_1 \to 0,\ \ \bar{\theta}_1 \to 0 \quad \text{as} \quad \bar{y} \to \infty \quad (14)$$

We introduce the following group of non-dimensional variables into equations (7)-(9) and (11)-(13):

$$(\bar{u}_s, \bar{u}_1) = \frac{\nu}{L} Gr_L^{\frac{1}{2}} (u_s, u_1), \quad (\bar{v}_s, \bar{v}_1) = \frac{\nu}{L} Gr_L^{\frac{1}{4}} (v_s, v_1),$$
$$x = \frac{\bar{x}}{L}, \quad \frac{\bar{y}}{L} Gr_L^{\frac{1}{4}}, \quad (\bar{\theta}_s, \bar{\theta}_1) = (\theta_s, \theta_1) \quad (15)$$

Thus equations (7)-(9) reduce to

$$\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} = 0 \quad (16)$$

$$u_s \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_s}{\partial y} = \frac{\partial^2 u_s}{\partial y^2} + \theta_s - M u_s \quad (17)$$

$$u_s \frac{\partial \theta_s}{\partial x} + v_s \frac{\partial \theta_s}{\partial y} = \frac{1}{\Pr}\left[\frac{\partial^2 \theta_s}{\partial y^2} + \frac{4}{3} R_d \frac{\partial}{\partial y}\left\{\left(1+\Delta\theta_s\right)^3 \frac{\partial \theta_s}{\partial y}\right\}\right] \quad (18)$$

with boundary conditions

$$u_s = 0,\ \ v_s = 0,\ \ \theta_s = 1 \quad \text{at } y = 0$$
$$u_s \to 0,\ \ \theta_s \to 0 \quad \text{as} \quad y \to \infty \quad (19)$$

And equations (11)-(13) take the form

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \quad (20)$$

$$u_s \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_s}{\partial y} + iSu_1 = \frac{\partial^2 u_1}{\partial y^2} + \theta_1 - M u_1 \quad (21)$$

$$u_s \frac{\partial \theta_1}{\partial x} + u_1 \frac{\partial \theta_s}{\partial x} + v_s \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_s}{\partial y} + iS\theta_1 = \frac{1}{\Pr}\left[\frac{\partial^2 \theta_1}{\partial y^2} + \frac{4}{3}R_d \frac{\partial}{\partial y}\left\{(1+\Delta\theta_s)^3 \frac{\partial \theta_1}{\partial y} + 3(1+\Delta\theta_s)^2 \Delta\theta_1 \frac{\partial \theta_s}{\partial y}\right\}\right] \tag{22}$$

The corresponding boundary conditions are

$$u_1 = 0,\ v_1 = 0,\ \theta_1 = 1 \quad \text{at } y = 0; \qquad u_1 \to 0,\ \theta_1 \to 0 \quad \text{as } y \to \infty \tag{23}$$

Here $\Pr = \nu/\alpha$ is the Prandtl number, $Gr_L = g\beta\Delta TL^3/\nu^2$ is the Grashof number, $S = \omega L^2/\nu Gr_L^{1/2}$ is the Stroughtel number, $M = \sigma B_0^2/\mu Gr_L^{1/2}$ is the magnetic parameter, $R_d = 4\sigma_1 T_\infty^3/\kappa(a+\sigma_s)$ is the Planck constant or the conduction-radiation parameter and $\Delta = (T_w/T_\infty - 1)$ is the surface temperature parameter. Here we consider that the surface is heated for which Δ is positive (i.e. when $T_w/T_\infty > 1$).

Having obtained the solutions of the sets of equations (16)-(19) and (20)-(23), we can readily get the values of the physical quantities, namely, the skin-friction and the heat transfer at the surface of the plate, which are important from the experimental point of view. In this study, the results are illustrated in terms of amplitudes and phases of the skin-friction and the heat transfer rate given by the following relations:

$$A_u = \sqrt{\tau_r^2 + \tau_i^2}, \quad A_t = \sqrt{\chi_r^2 + \chi_i^2}$$

$$\phi_u = \tan^{-1}\frac{\tau_i}{\tau_r}, \quad \phi_t = \tan^{-1}\frac{\chi_i}{\chi_r} \tag{24}$$

where (τ_r, τ_i) and (χ_r, χ_i) are the corresponding real and imaginary parts of the skin friction and the heat transfer, respectively.

3. Methods of Solution

The steady set of equations (16)-(19) have been solved by employing the primitive variable method for different values of the pertinent parameters Pr, R_d and M. The steady-state solutions are then applied in finding the solutions of unsteady equations (20)-(23), which provide the fluctuating parts of the flow and temperature fields.

3.1. Primitive-variable Formulation (PVF)

We introduce the following group of primitive variable transformations to reduce the equations (16)-(19) and (20)-(23) into parabolic form:

$$(u_s, u_1) = x^{1/2}(U_s, U_1),\ (v_s, v_1) = x^{-1/4}(V_s, V_1),$$

$$(\theta_s, \theta_1) = (\Theta, \theta),\ Y = x^{-1/4}y, X = x,\ Sx^{1/2} = \xi.$$

Thus from (16)-(19), we get

$$\frac{\partial V_s}{\partial Y} = -\left[\frac{1}{2}U_s + X\frac{\partial U_s}{\partial X} - \frac{1}{4}Y\frac{\partial U_s}{\partial Y}\right] \tag{25}$$

$$\frac{1}{2}U_s^2 + XU_s\frac{\partial U_s}{\partial X} + \left(V_s - \frac{1}{4}YU_s\right)\frac{\partial U_s}{\partial Y} = \frac{\partial^2 U_s}{\partial^2 Y} + \Theta - MU_s \tag{26}$$

$$XU_s\frac{\partial \Theta}{\partial X} + \left(V_s - \frac{1}{4}YU_s\right)\frac{\partial \Theta}{\partial Y} = \frac{1}{\Pr}\left\{1 + \frac{4}{3}R_d(1+\Delta\Theta)^3\right\}\frac{\partial^2 \Theta}{\partial Y^2} + \frac{4}{\Pr}\Delta R_d(1+\Delta\Theta)^2\left(\frac{\partial \Theta}{\partial Y}\right)^2 \tag{27}$$

and the boundary conditions take the form

$$U_s = V_s = 0,\quad \Theta = 1,\ \text{at}\ Y = 0; \qquad U_s \to 0,\quad \Theta \to 0 \quad \text{as}\ Y \to \infty \tag{28}$$

Again the equations (20)-(23) reduce to

$$\frac{\partial V_1}{\partial Y} = -\left[\frac{1}{2}U_1 + \frac{1}{2}\xi\frac{\partial U_s}{\partial \xi} - \frac{1}{4}Y\frac{\partial U_1}{\partial Y}\right] \tag{29}$$

$$U_sU_1 + \frac{1}{2}\xi U_s\frac{\partial U_1}{\partial \xi} + \left(V_s - \frac{1}{4}YU_s\right)\frac{\partial U_1}{\partial Y} + \left(V_1 - \frac{1}{4}YU_1\right)\frac{\partial U_s}{\partial Y} + i\xi U_1 = \frac{\partial^2 U_1}{\partial^2 Y} + \theta - MU_1 \tag{30}$$

$$\frac{1}{2}\xi U_s\frac{\partial \theta}{\partial \xi} + \left(V_s - \frac{1}{4}YU_s\right)\frac{\partial \theta}{\partial Y} + \left(V_1 - \frac{1}{4}YU_1\right)\frac{\partial \Theta}{\partial Y} + i\xi\theta = \frac{1}{\Pr}\left\{1 + \frac{4}{3}R_d(1+\Delta\Theta)^3\right\}\frac{\partial^2 \theta}{\partial Y^2} + \frac{8}{\Pr}\Delta R_d(1+\Delta\Theta)^2\frac{\partial \Theta}{\partial Y}\frac{\partial \theta}{\partial Y} + \frac{1}{\Pr}\left\{8\Delta^2 R_d(1+\Delta\Theta)\left(\frac{\partial \Theta}{\partial Y}\right)^2 + 4\Delta R_d(1+\Delta\Theta)^2\frac{\partial^2 \Theta}{\partial Y^2}\right\}\theta \tag{31}$$

with the boundary conditions

$$U_1 = V_1 = 0,\quad \Theta = 1,\ \text{at}\ Y = 0; \qquad U_1 \to 0,\quad \Theta \to 0 \quad \text{as}\ Y \to \infty \tag{32}$$

In the present analysis, solutions of the set of parabolic partial differential equations, (25)-(27), for the steady mean flow, and (29)-(31) for the unsteady mean flow, are obtained numerically using the finite difference method.

3.2. Stream-Function Formulation (SFF)

We introduce the following group of well-known transformations to get the similarity equations for the steady state equations (7)-(9):

$$\psi_s = \nu Gr_x^{\frac{1}{4}} F(\eta), \quad \bar{\theta}_s = \Theta(\eta), \quad \eta = \frac{\bar{y}}{\bar{x}} Gr_x^{\frac{1}{4}} \tag{33}$$

where $\psi_s(\bar{x},\bar{y})$ satisfies the equation (7) of continuity for the steady state flow.

Thus we have

$$F''' - \frac{1}{2}F'^2 + \frac{3}{4}FF'' + \Theta - MF' = 0 \tag{34}$$

$$\frac{1}{\Pr}\left\{1 + \frac{4}{3}R_d\left(1+\Delta\Theta\right)^3\right\}\Theta'' + \frac{4\Delta}{\Pr}R_d\left(1+\Delta\Theta\right)^2\left(\Theta'\right)^2 + \frac{3}{4}F\Theta' = 0 \tag{35}$$

The boundary conditions to be satisfied by the above equations are

$$F(0) = F'(0) = 0,\ \Theta(0) = 1, F'(\infty) = 0, \Theta(\infty) = 0 \tag{36}$$

Here primes denote differentiation with respect to η.

The transformation (33) suggests the following group of transformations for the equations (11)-(13), for the fluctuating part of the problem:

$$\psi_1 = \nu Gr_x^{\frac{1}{4}} f(\eta,\xi), \bar{\theta}_1 = \theta(\eta,\xi),\ \xi = \frac{\omega \bar{x}^2}{\nu} Gr_x^{-1/2} \tag{37}$$

where $\psi_1(\bar{x},\bar{y})$ is the oscillating part of the stream function satisfying equation (11). Equation (12) and (13) then reduces to

$$f''' + \frac{3}{4}Ff'' - F'f' + \frac{3}{4}F''f - i\xi f' + \theta - Mf' = \frac{1}{2}\xi\left(F'\frac{\partial f'}{\partial \xi} - F''\frac{\partial f}{\partial \xi}\right) \tag{38}$$

$$\begin{aligned}&\frac{1}{\Pr}\left\{1+\frac{4}{3}R_d\left(1+\Delta\Theta\right)^3\right\}\theta'' + \frac{8}{\Pr}R_d\left(1+\Delta\Theta\right)^2\Delta\Theta'\theta' \\ &+\frac{8\Delta^2}{\Pr}R_d\left(1+\Delta\Theta\right)\left(\Theta'\right)^2\theta + \frac{3}{4}F\theta' \\ &+\frac{3}{4}\Theta'f + \frac{4\Delta}{\Pr}R_d\left(1+\Delta\Theta\right)^2\Theta''\theta \\ &-i\xi\theta = \frac{1}{2}\xi\left(F'\frac{\partial\theta}{\partial\xi} - \Theta'\frac{\partial f}{\partial \xi}\right)\end{aligned} \tag{39}$$

and the boundary conditions become

$$\begin{aligned} f(\xi,0) = f'(\xi,0) = 0, \quad \theta(\xi,0) = 1, \\ f'(\xi,\infty) = \theta(\xi,\infty) = 0. \end{aligned} \tag{40}$$

A very efficient and accurate implicit finite difference method is employed to solve the system of equations (38)-(40).

4. Results and Discussion

The solutions of the equations governing the unsteady natural convection boundary layer flow with a viscous incompressible fluid along a vertical plate are obtained using the finite difference method (primitive variable formulation) and the Keller-box method (the stream-function formulation) for entire region. The results are expressed in terms of amplitude and phase angle of skin friction showing the effects of the physical parameters involved in the flow field, such as, the Prandtl number, Pr, the conduction-radiation parameter, R_d and the magnetic parameter, M while the surface temperature parameter is assumed to be fixed as θ_w = 1.1.

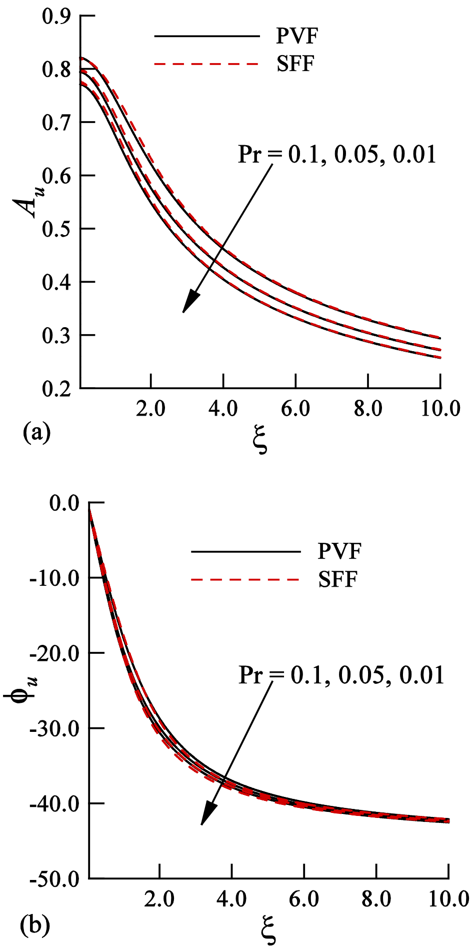

Figure 2. (a) Amplitude and (b) phase angle of skin-friction for different Pr. The solid (black) and dashed (red) curves are due to primitive variable formulation (PVF) and stream-function formulation (SFF), respectively

A comparison between the primitive variable formulation method and the stream-function formulation method is made in Figures 2(a) and (b). The figures show that the primitive variable formulation method gives a good agreement with the stream-function formulation method. As a result, the following results will be presented only by primitive variable formulation method. In addition to, Figures 2(a) and (b) express the effects of the Prandtl number, Pr, on the amplitude and phase angle of skin friction, respectively.

From both figures we observe that the amplitude of skin friction profiles decreases and the phase angle of that increases with the increase of the Prandtl number, Pr. This confirms the finding of Samanta and Guha [28] that the skin friction decreases with increasing the value of the Prandtl number.

Figures 3(a) and (b) exhibit the effects of magnetic field parameter (M) on the amplitude and phase angle of the skin friction, respectively. We see that the amplitude of the skin friction becomes smaller whereas the phase angle is found to be higher when the magnetic field parameter is increased. Since the phase angle of the skin friction for higher value of M slowly decreases along ξ compared to that for smaller M, accordingly the momentum boundary layer thickness increases with the increase of the magnetic field parameter, M. Samanta and Guha also found the similar tendencies of the skin friction and the momentum boundary layer with the variation of the magnetic field parameter.

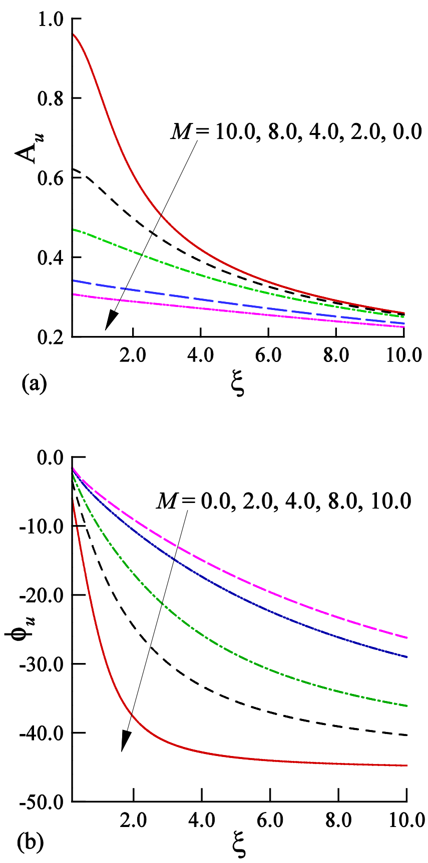

Figure 3. Effect of magnetic field parameter (M) on (a) amplitude and (b) phase angle of skin-friction

The effects of varying the conduction-radiation parameter, R_d, on the amplitude and phase angle of the skin friction have been presented in Figures 4(a) and (b), respectively. When the conduction-radiation parameter (R_d) is increased, the amplitude of the skin friction increases while the phase angle of the skin friction slightly decreases.

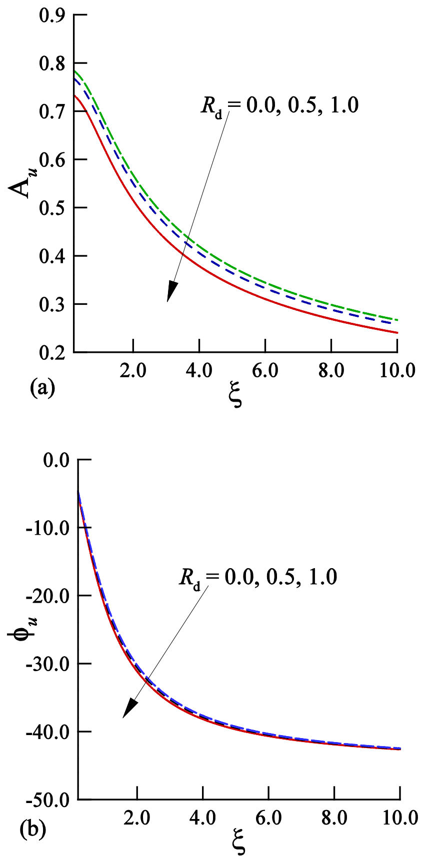

Figure 4. Effect of conduction-radiation parameter (R_d) on (a) amplitude and (b) phase angle of skin-friction

4.1. Effects of Different Physical Parameters on Transient Skin-friction and Heat Transfer

Now we discuss the effects of the physical parameters, such as M, R_d, and ξ on the transient skin-friction coefficient, τ, and heat transfer coefficients, q, are measured from the following relations while Pr = 0.1.

$$\tau = \tau_s + \varepsilon A_u \cos(\omega t + \phi_u)$$

and

$$q = q_s + \varepsilon A_t \cos(\omega t + \phi_t)$$

where τ_s and q_s are respectively the steady mean skin-friction and mean heat transfer.

Figures 5 and 6 show the effects of the variation of the magnetic field parameter, M, on the amplitude of oscillation of the transient skin-friction and heat transfer. From these figures, we observe that the amplitude of oscillation of the transient skin-friction decreases and that of transient heat transfer increases while M increases.

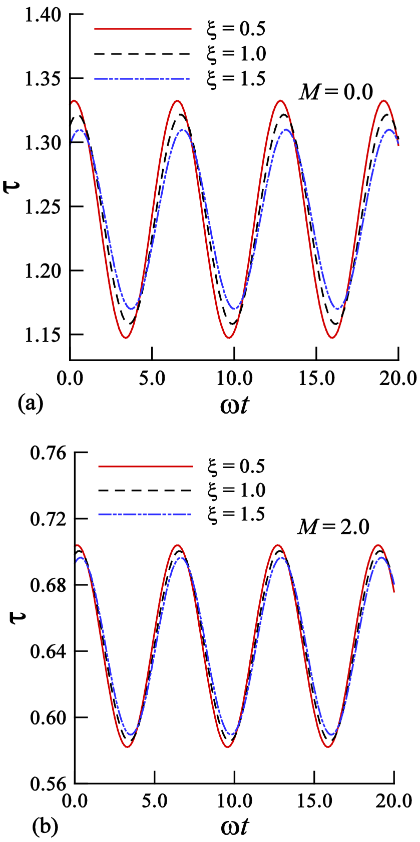

Figure 5. Numerical values of transient skin-friction coefficient against ωt while (a) $M = 0.0$ and (b) $M = 2.0$

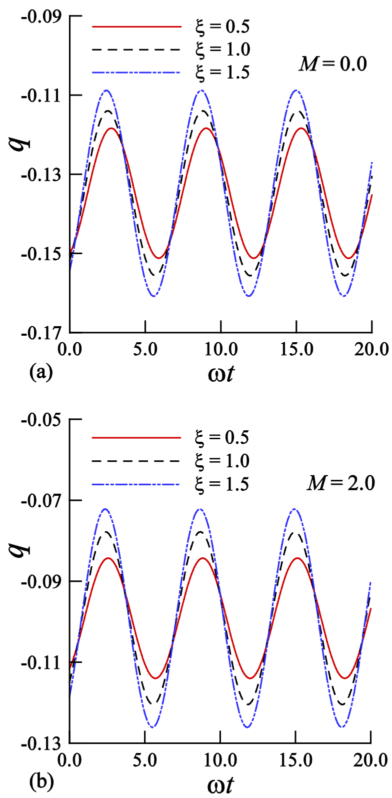

Figure 6. Numerical values of transient heat transfer coefficient against ωt while (a) $M = 0.0$ and (b) $M = 2.0$

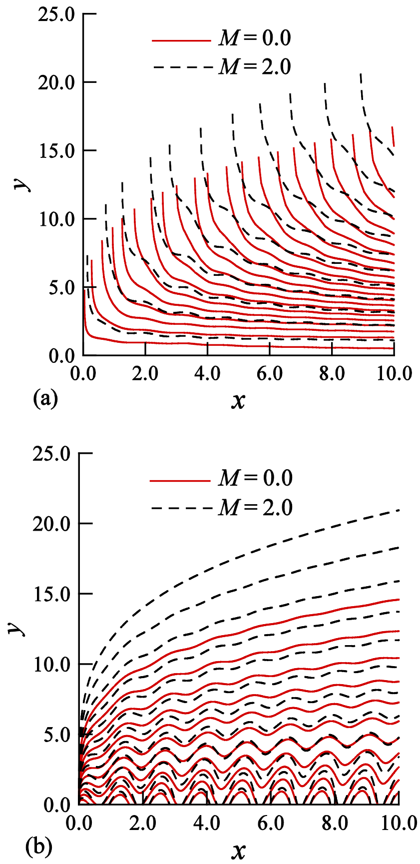

Figure 7. The curves represent (a) streamlines and (b) isolines of temperature in the boundary layer for different values of M when τ is 5 times of x

4.2. Effects of the Physical Parameters on the Streamlines and Isolines of Temperature

Now we discuss the effects of the magnetic field parameter, M, on the streamlines and isolines of temperature in the boundary layer regimes that are shown in Figures 7(a) and (b), respectively. From the figures it is evident that both the momentum boundary layer and thermal boundary layer increase because of the increases of the magnetic field parameter, M. The reason behind these observations is that the magnetic field applied to the flow field accelerates the fluid flow as the fluid is assumed to be electrically conducting.

The effects of the change of the conduction-radiation parameter, R_d, on the streamlines and isolines of temperature are presented in Figures 8(a) and (b), respectively. We observe that the thicknesses of both the thermal boundary layer and momentum boundary layer increase as the value of R_d increases. Now it is clear from Figure 8(a) that the radiative heat transfer accelerates the momentum flux. Consequently, the momentum boundary layer becomes thicker for higher R_d. Figure 8(b) shows that the evolution of temperature increases across the plate owing to the radiative heat transfer. As a result, the thermal boundary layer also increases.

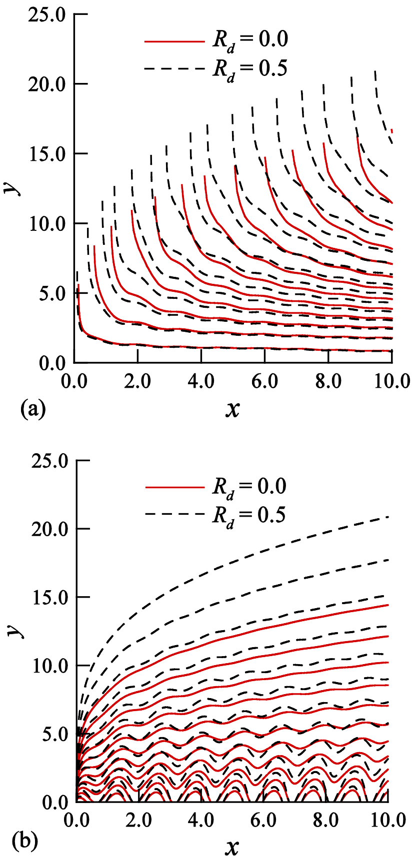

Figure 8. The curves represent (a) streamlines and (b) isolines of temperature in the boundary layer for different values of R_d when τ is 5 times of x

5. Conclusions

Unsteady MHD free convection flow along a vertical plate is studied in the presence of magnetic field with thermal radiation. Two distinct methods, namely, the primitive variable formulation method and the Keller-box method are used. Detailed numerical calculations have been carried out to examine the effects of the Prandtl number, Pr, the magnetic field parameter, M, and the conduction-radiation parameter, R_d, on the amplitude and phase angle of the skin friction, transient skin friction and heat transfer as well as streamlines and isolines of temperature. Solutions obtained by primitive variable formulation method are compared with that of Keller-box method and found to be in excellent agreement.

From the present analysis, it is seen that the amplitude of skin friction decreases and the phase angle of the skin friction increases with the increase of the Prandtl number, Pr and the magnetic field parameter, M. On the other hand, the amplitude of the skin friction become higher and phase angle of the skin friction decreases for lager conduction-radiation parameter. In addition, the amplitude of oscillation of the transient skin-friction decreases but that of transient heat transfer increases when the magnetic field parameter is increased. As the frequency parameter ξ increases, the amplitude of oscillation of the transient skin-friction decreases while the transient heat transfer increases.

It is also found that both the momentum boundary layer and thermal boundary layer increase owing to the increase of the magnetic field parameter, M. The thicknesses of both the thermal boundary layer and momentum boundary layer increase with an increase of the conduction-radiation parameter, R_d.

REFERENCES

[1] Soundalgekar, V. M., Takhar, H. S., and Vighnesam, N. V., 1960, The combined free and forced convection flow past a semi-infinite plate with variable surface temperature, Nuclear Eng. Design, 110, 95–98.

[2] Sakiadis, B. C., 1961, Boundary layer behavior on continuous solid surface, II. The boundary layer on a continuous flat surface, A. I. Ch. E. J., 7, 221–225.

[3] Sparrow, E. M., and Cess, R. D., 1961, Effect of magnetic field on free convection heat transfer, Int. J. Heat Mass Transf., 3, 267–274.

[4] Romig, M., 1964, The influence of electric and magnetic field on heat transfer to electrically conducting fluids, Adv. Heat Transf., 1, 268–352.

[5] Hafez, T. A. A., 1985, Skin friction and heat transfer on a continuous flat surface moving in a parallel free stream, Int. J. Heat Mass Transf., 28, 1234–1237.

[6] Kelleher, M. D., and Yang, K. T., 1968, Heat transfer response of laminar free-convection boundary layers along a vertical heated plate to surface-temperature oscillations, J. Appl. Math. Phys. ZAMP, 19, 31–44.

[7] Chiam, T. C., 1995, Hydromagnetic flow over a surface stretching with a power-law velocity, Int. J. Eng. Sci. 33(3), 429–435.

[8] Hossain, M. A., and Takhar, H. S., 1996, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat Mass Transf., 31, 243–248.

[9] Hossain, M. A., Alim, M. A., and Rees, D. A. S., 1999, The effect of radiation on free convection from a porous vertical plate, Int. J. Heat Mass Transf., 42, 181–191.

[10] Chamkha, J. A., Mujtaba, M., Quadri, A., and Issa, C., 2003, Thermal radiation effects on MHD forced convection flow adjacent to a non-isothermal wedge in the presence of a heat source or sink, Heat and Mass Transf., 39, 305–312.

[11] Afify, A. A., 2004, MHD free convective flow and mass transfer over a stretching sheet with chemical reaction, Heat and Mass Transf., 40, 495–500.

[12] Duwairi, H. M., 2005, Viscous and Joule heating effects on forced convection flow from radiate isothermal porous surfaces, Int. J. Numer Meth Heat Fluid Flow, 15, 429–440.

[13] Damseh, R. A., Duwairi, H. M., and Al-Odat, M., 2006, Similarity analysis of magnetic field and thermal radiation effects on forced convection flow, Turkish J. Eng. Env. Sci., 30, 83–89.

[14] Devi, S. P. A., and Thiyagarajan, M., 2006, Steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature, Heat Mass Transf., 42, 671–677.

[15] Ishak, A., Nazar, R., and Pop, I., 2008, Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet, Heat Mass Transf., 44, 921–927.

[16] Aydin, O., and Kaya, A., 2008, Radiation effect on MHD mixed convection flow about a permeable vertical plate, Heat Mass Transf., 45, 239–246.

[17] Kandasamy, R., Muhaimin, I., and Khamis, A. B., 2009, Thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction, Heat Mass Transf., 45, 703–712.

[18] Aydin, O., and Kaya, A., 2009, MHD mixed convective heat transfer flow about an inclined plate, Heat Mass Transf., 46, 129–136.

[19] Seth, G. S., Ansari, Md. S., and Nandkeolyar, R., 2011, MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature, Heat Mass Transf., 47, 551–561.

[20] Das, K., 2012, Impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties, Heat Mass Transf., 48, 767–778.

[21] Roy, N. C., and Hossain, M. A., 2010, The effect of conduction–radiation on the oscillating natural convection boundary layer flow of viscous incompressible fluid along a vertical plate, Proc. IMechE, Part C: J. Mech. Eng. Sci., 224, 1959-1972.

[22] Samad, M. A., and Saha, K. C., 2014, Radiative heat and mass transfer of an MHD free convective flow of non-Newtonian power-law fluids along a continuously moving stretching sheet with uniform surface heat flux, Dhaka Univ. J. Sci., 62(1), 37- 44.

[23] Lighthill, M. J., 1954, The response of laminar skin-friction and heat transfer to fluctuations in the stream velocity, Proc. Royal Soc. London, Series A, 224, l-23.

[24] Nanda, R. S., and Sharma, V. P., 1963, Free convection laminar boundary layers in oscillatory flow. J. Fluid Mech., 15, 419-428.

[25] Eshghy, S., Arpaci, V. S., and Clark, J. A., 1965, The effect of longitudinal oscillation on free convection from vertical surface. J. Appl. Mech., 32, 183–191.

[26] Muhuri, P. K., and Maiti, M. K., 1967, Free convection oscillatory flow from a horizontal plate. Heat Mass Transf., 10, 717–732.

[27] Verma, R. L., 1982, Free convection fluctuating boundary layer on a horizontal plate, J. Appl. Math. Mech. (ZAMM), 63, 483–487.

[28] Samanta, S., and Guha, A., 2013, Magnetohydrodynamic free convection flow above an isothermal horizontal plate, Commun. Nonlinear Sci. Numer. Simulat., 18, 3407–3422.