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Conceptual Study of the Effect of Radiation on Free Convective Flow of Mass and Heat Transfer over a Vertical Plate

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Abstract The effects of mass and radiative heat transfer on free convective flow of a viscous incompressible optically thick fluid towards a vertical surface has been investigated. The nonlinear non-dimensional, similarity-transformed boundary-layer equations governing the problem are solved using an efficient numerical method based on the fourth-order Runge-Kutta integration scheme and shooting iteration technique. Numerical calculations were carried out for different values of the various non-dimensional quantities governing the flow regime. The analysis shows that as the radiation parameter, N, increases, the temperature decreases; an increase in the prandtl number leads to a decrease in the temperature profile; a rise in the thermal Grasshof and the mass transfer number leads to increase in the velocity profile and a rise in the Schmidt number Sc leads to a decrease in the concentration profile.

Keywords Radiation, Free Convective Flow, Mass and Heat Transfer, Vertical Plate

1. Introduction

The phenomenon of free or natural convection arises in fluids when temperature changes cause density variations leading to buoyancy forces acting on the fluid particles. Such flows which are driven by temperature differences abound in nature and have been studied extensively because of its applications in engineering, geophysical and astrophysical environments.

When technological processes take place at higher temperatures thermal radiation heat transfer has become very important and its effects cannot be neglected (Siegel & Howel, 2001). The effect of radiation on MHD flow, heat and mass transfer become more important industrially. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes a very important for the design of the pertinent equipment. The quality of the final product depends to a great extent on the heat controlling factors, and the knowledge of radiative heat transfer in the system can lead to a desired product with sought qualities. Different researches have been forwarded to analyze the effects of thermal radiation on different flows (Shateyi and Motsa, 2009).

Several people have shown considerable interest in studying the problem of free convection boundary layer flow and heat transfer as it is being affected by radiation. For example Aziz A.(2009), Vleggaar J (1977), Sivaiah M., Nagarajan A.S., Reddy P. Sreehari (2010), B. Vasu, V. Ramachandra Prasad and N. Bhaskar Reddy (2011), A. Raptis (2001), A. Raptis and C. Perdikis (2004) V.R. Prasad and N.B. Reddy (2008), M.A. Hossain, M.A. Alim, D. Rees (1999) and R.C. Bataller (2008).

Hence, in the present study, we investigate the effects of mass and radiative heat transfer on free convective flow of a viscous incompressible optically thick fluid towards a vertical surface. Such a flow problem occurs in many industrial and technological applications which include the aerodynamic extrusion of plastic sheets cooling of metallic plates in a cooling bath, cooling of nuclear reactors, drawing, annealing and tinning of copper wires. Ishak, Nazar and Pop(2008) The nonlinear nondimensional, similarity-transformed boundary-layer equations governing the problem are solved using an efficient numerical method based on the fourth-order Runge-Kutta integration scheme and shooting iteration technique. Numerical calculations were carried out for different values of the various non-dimensional quantities governing the flow regime.

2. Mathematical Analysis

An unsteady one-dimensional laminar boundary layer flow of a viscous, incompressible, radiating fluid along a semi-infinite vertical plate in the presence of thermal and

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concentration buoyancy effects is considered, by taking the effect of viscous dissipation into account. The x′-axis is taken along the vertical infinite plate in the upward direction and the y-axis normal to the plate. Under these assumptions the Boussinesq's approximation, for the flow field is governed by the following equations:

$$\frac{\partial v}{\partial y}=0 \quad (1.0)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty) \quad (1.1)$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q}{\partial y} \quad (1.2)$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (1.3)$$

With the following the Boundary conditions,

$$u=U, T=T_w, C=C_w, \text{ at } y=0 \quad (1.4)$$

$u \to 0, T \to T_\infty, C \to C_\infty$, as $y \to \infty$ for $t>0$

Nomenclature

$(u,v)=$ Velocity components, $(x,y)=$ Coordinates, $t=$ Time, $T=$ Temperature of flow,
$T_w=$ Plate surface temperature, $T_\infty=$ Temperature outside the flow, $\upsilon=$ Kinematic viscosity,
$g=$ Gravity, $\beta_T=$ Heat transfer coefficient, $\beta_C=$ Mass transfer coefficient
$C=$ Concentration of the fluid, $C_w=$ Concentration at the plate surface, $C_\infty=$ Concentration outside the flow
$\rho=$ Density, $\alpha=$ Thermal diffusion and $D=$ Coefficient of mass diffusion.

In order to write the governing equations and the boundary condition in dimension less form, the following non-dimensional quantities are introduced

$$\frac{u}{u_\infty}=f(\eta), \text{ where } \eta=\frac{y}{2\sqrt{\upsilon t}}, \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

$$\frac{4tg}{u}\beta_T(T_w-T_\infty)=G_T, \ \frac{4tg}{u}\beta_C(C_w-C_\infty)=G_C$$

$$\frac{16T^3{}_\infty\sigma_f}{3\upsilon k_f}=\frac{1}{N}, \ \Pr=\frac{\upsilon}{\alpha}, \ -c=V_o\sqrt{\frac{t}{\upsilon}}, \ Sc=\frac{\upsilon}{D}$$

$q=-\dfrac{4\sigma_f}{3k_f}\dfrac{\partial T^4}{\partial y}$ this is Rosseland approximation. (Brewster 1992)

Where σ_f is the Stefan-Boltzman constant and k_f is the absorption coefficient.

It should be noted that by using the Rosseland approximation the present analysis is limited to optically thick fluids.

In view of the above dimensionless quantities, the governing equations is reduce to the following dimensionless form,

$$\frac{d^2 f(\eta)}{d\eta^2}+2(\eta+c)\frac{df(\eta)}{d\eta}+G_T\theta(\eta)+G_C\phi(\eta)=0 \quad (1.5)$$

$$\frac{d^2\theta(\eta)}{d\eta^2}+\left[\frac{2N\Pr}{N+\Pr}\right](\eta+c)\frac{d\theta(\eta)}{d\eta}=0 \quad (1.6)$$

$$\frac{d^2\phi(\eta)}{d\eta^2}+2Sc(\eta+c)\frac{d\phi(\eta)}{d\eta}=0 \quad (1.7)$$

$$f(0)=\theta(0)=\phi(0)=1 \quad (1.8)$$

$$f(\infty)=\theta(\infty)=\phi(\infty)=0 \quad (1.9)$$

Where,

c = Unsteadiness parameter, $G_T=$ Thermal Grashof number
$G_C=$ Concentration Grashof number $\Pr=$ Prandlt number,
$Sc=$ Schmindt number and N = Radiation Parameter.

Numerical Solution

The numerical solutions were obtained using an efficient numerical method based on the fourth-order Runge-Kutta integration scheme and shooting iteration technique as embedded in Maple 16. The values of the skin-friction coefficient, the local Nusselt number and the local Sherwood number are presented in table 1 and depicted graphically in Figures 1 to 8 to illustrate the influence of the various studied physical parameters.

3. Discussion of the Results

The numerical simulation depicted by the table 1 and Figures in 1 - 8 shows that Skin-friction decreases with increasing thermal Grashof number when the Nusselt and Sherwood numbers are kept constant. The same phenomenon is observed with variation in the Solutal Grashof number. Increase in the magnitude of the Prandtl Schmidt and the radiation numbers results in corresponding increase Skin friction.

In Figures 1 and 2, it is also observed that velocity varies directly with both thermal and mass Grashof number. Figure 3 demonstrates the effect of the radiation parameter on the temperature profile showing an inverse relation. The effect of the unsteadiness parameter on the velocity, temperature and concentration profile is demonstrated in Figures 4 to 6 which vary inversely with the parameter. Figure 7 shows that the temperature decreases with increase in Prandtl number. Finally Figure 8 shows that concentration varies inversely with the Schmidt number.

Table 1. Numerical computations showing the Skin-friction $f'(0)$, the Nusselt number $\theta'(0)$ and the Sherwood number $\phi'(0)$ for various embedded flow parameters

G_T	G_C	C	Pr	N	Sc	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.1	0.72	0.1	0.24	1.1017225079	0.3456276073	0.366483153
0.5	0.1	0.1	0.72	0.1	0.24	0.7603393511	0.3456276073	0.366483153
1.0	0.1	0.1	0.72	0.1	0.24	0.3336104051	0.3456276073	0.366483153
0.1	0.5	0.1	0.72	0.1	0.24	0.8151565808	0.3456276073	0.366483153
0.1	1.0	0.1	0.72	0.1	0.24	0.4569491718	0.3456276073	0.366483153
0.1	0.1	0.5	0.72	0.1	0.24	1.2357556660	0.3570428659	0.402490401
0.1	0.1	1.0	0.72	0.1	0.24	1.3754144122	0.3686115810	0.439142011
0.1	0.1	0.1	3.0	0.1	0.24	1.1039978234	0.3634395174	0.366483153
0.1	0.1	0.1	7.1	0.1	0.24	1.1044346872	0.3669912648	0.366483153
0.1	0.1	0.1	0.72	0.3	0.24	1.1212745534	0.5465161312	0.366483153
0.1	0.1	0.1	0.72	0.5	0.24	1.1299943107	0.6877591023	0.366483153
0.1	0.1	0.1	0.72	0.1	0.62	1.1213656634	0.3456276073	0.624057431
0.1	0.1	0.1	0.72	0.1	2.62	1.1256056135	0.3456276073	0.712703275
0.1	0.1	0.1	0.72	0.1	0.24	1.0869399111	0.3456276073	0.048000000
0.1	0.1	0.1	0.72	0.1	0.24	1.0497691195	0.3456276073	0.606873211

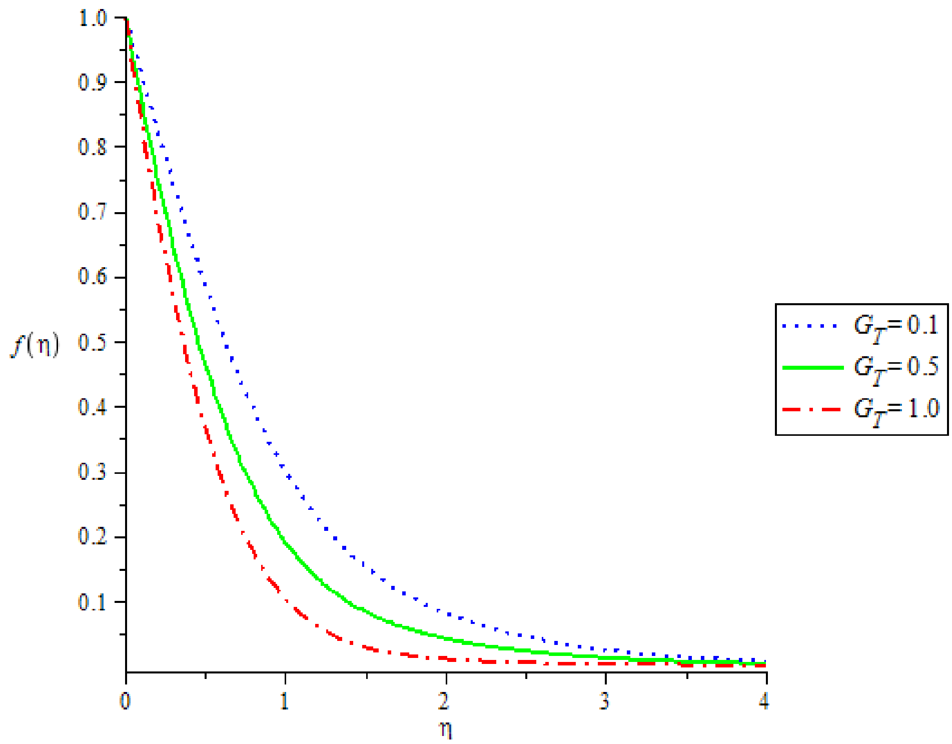

Figure 1. Velocity profiles for various values of G_T when $G_C = 0.1$, N = 0.1, c = 0.5, Pr = 0.72, Sc = 0.24

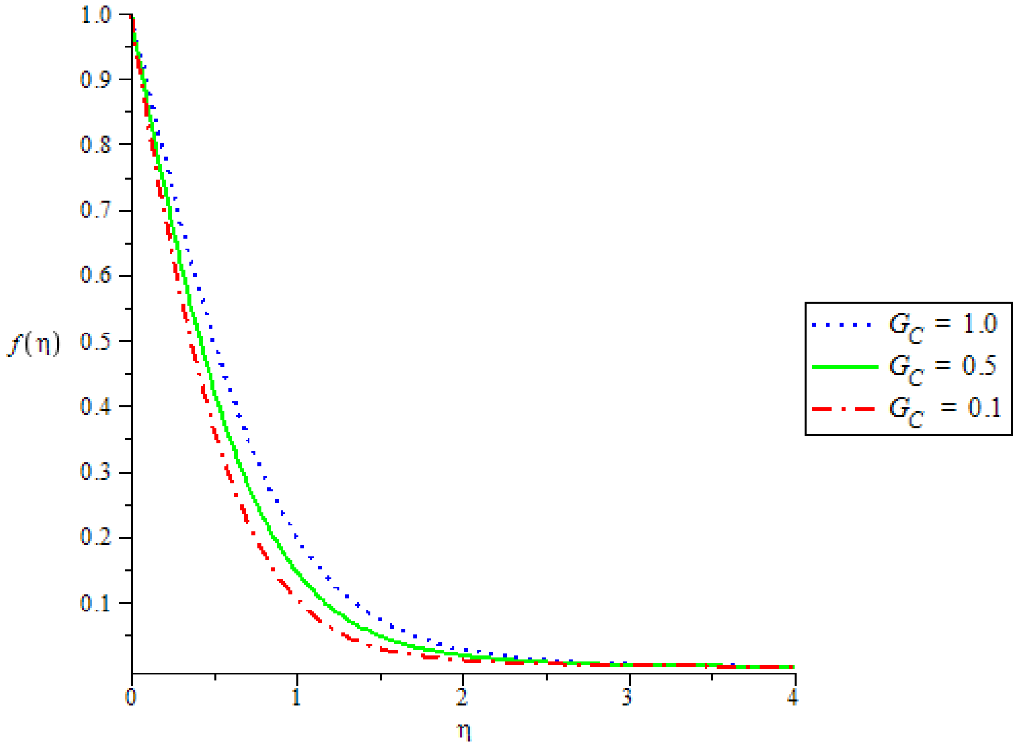

Figure 2. Velocity profiles for various values of G_C when $G_T = 0.1$, $N = 0.1$, $c = 0.5$, $Pr = 0.72$, $Sc = 0.24$

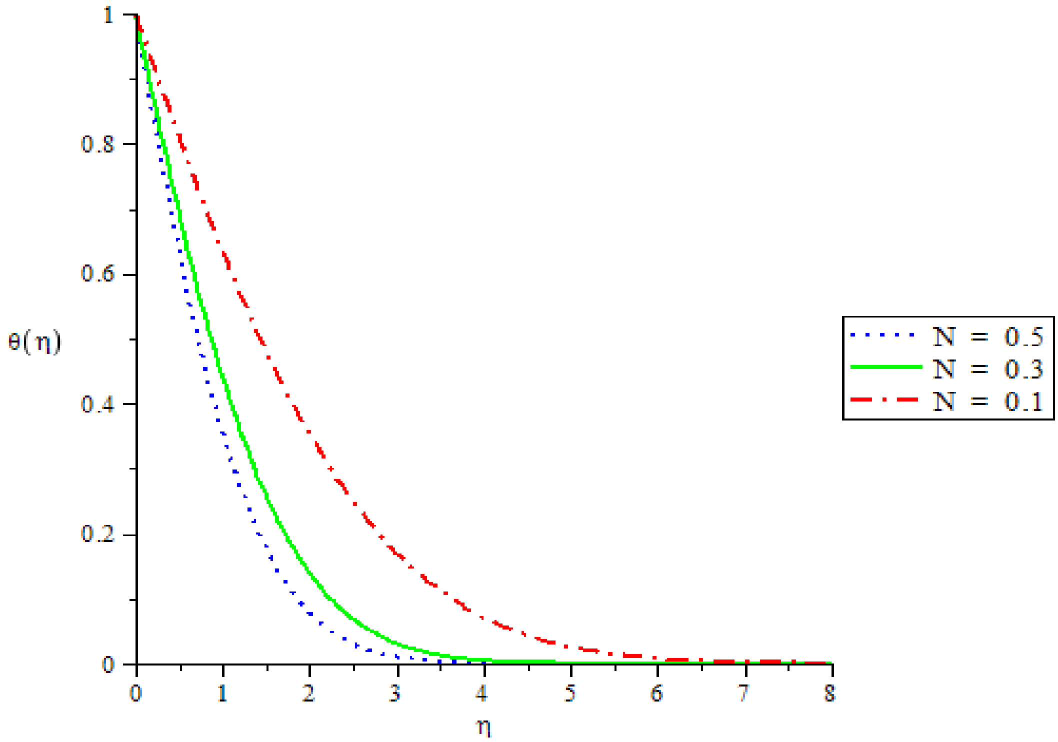

Figure 3. Temperature profiles for various values of radiation parameter N when $G_T = 0.1$, $G_C = 0.1$, $c = 0.5$, $Pr = 0.72$, $Sc = 0.24$

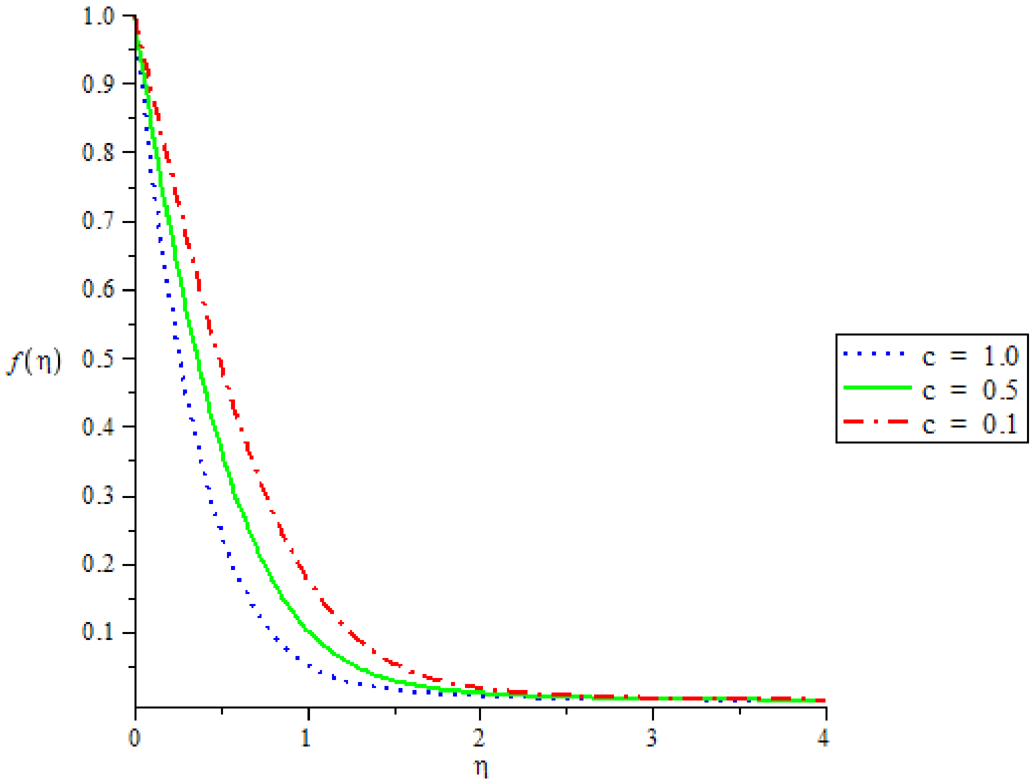

Figure 4. Velocity profiles for various value of c when $G_T = 0.1$, $G_C = 0.1$, N = 0.1, Pr = 0.72, Sc = 0.24

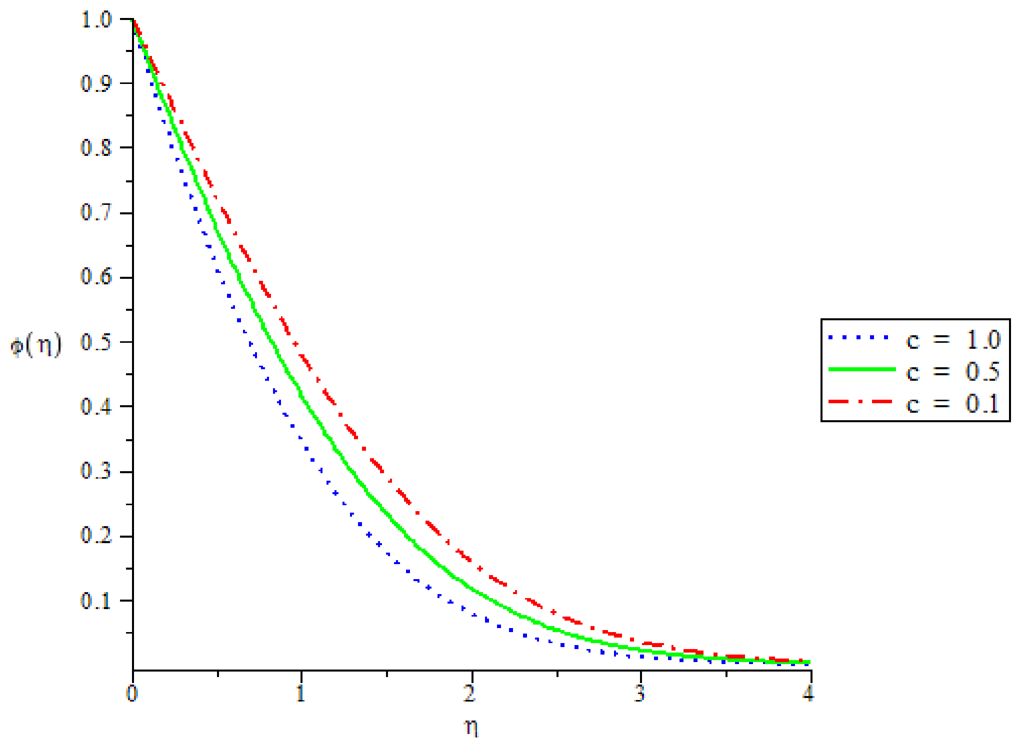

Figure 5. Concentration profiles for various value of c when $G_T = 0.1$, $G_C = 0.1$, N = 0.1, Pr = 0.72, Sc = 0.24

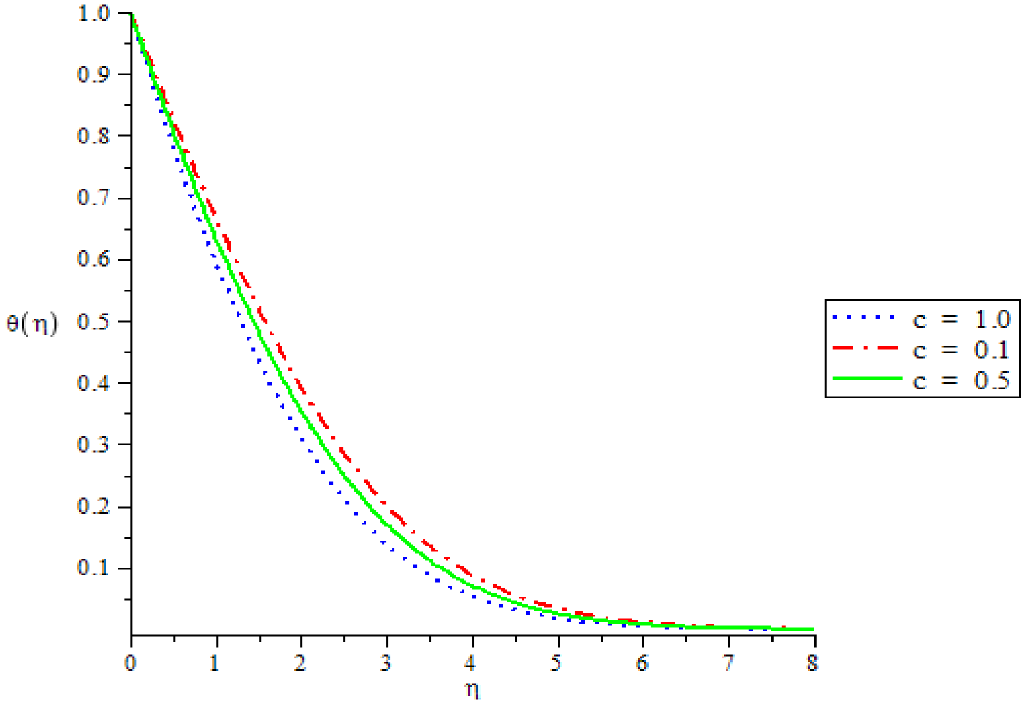

Figure 6. Temperature profiles for various value of c when $G_T = 0.1$, Gc = 0.1, N = 0.1, Pr = 0.72, Sc = 0.24

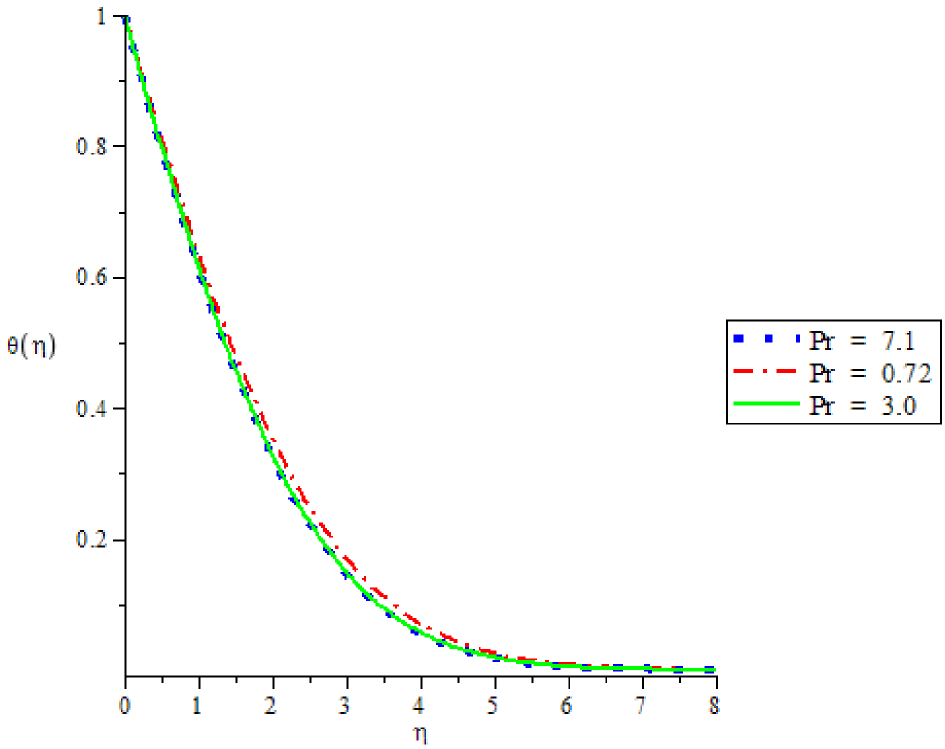

Figure 7. Temperature profiles for various values of Prandtl number Pr when $G_T = 0.1$, $G_C = 0.1$, N = 0.1, c = 0.5, Sc = 0.24

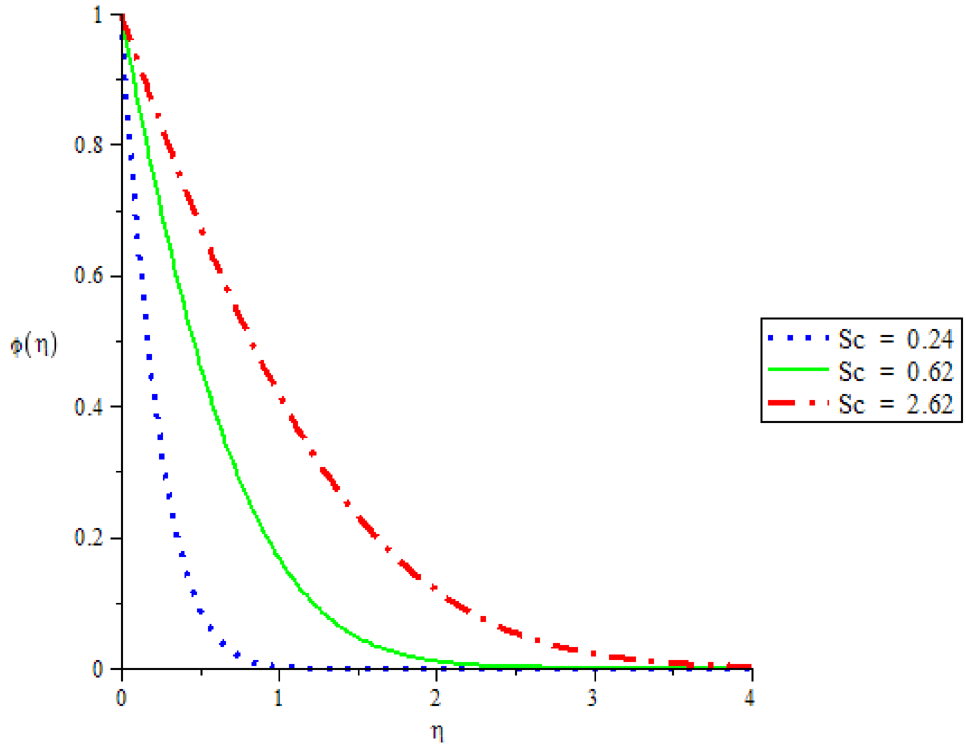

Figure 8. Concentration profiles for various values of Schmidt number Sc when $G_T = 0.1$, $G_C = 0.1$, N = 0.1, c = 0.5, Pr = 0.72

4. Conclusions

In this paper, we have investigated numerically the effects of mass and radiative heat transfer on free convective flow of a viscous incompressible optically thick fluid towards a vertical surface, using the method of similarity transform and the resulting coupled ordinary differential equations solved using the shooting technique embedded in Maple 16.

The particular conclusion drawn from this study reveals the following;

- As the radiation parameter, N, increases the temperature decreases.
- An increase in the Prandtl number leads to a decrease in the temperature profile.
- A rise in the thermal Grashof and the mass transfer number leads to increase in the velocity profile.
- A rise in the Schmidt number Sc leads to a decrease in the concentration profile.

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