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Mathematical Modeling of Age Specific Adult Literacy Rates in Bangladesh

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Abstract The purpose of this study is to estimate age specific adult literacy rates (ASALRs) for male, female and both sexes population in Bangladesh as well as to fit some mathematical models to these estimated ASALRs and their forward cumulative distribution. For this study, the data is taken from Population Census 2001. Quasi Newton Method is employed to fit these models. Moreover, t-test, F-test and cross validity prediction power (CVPP) are used to check the accuracy as well as validation of the model. In this study, it is found that ASALRs for male, female, both sexes population and forward cumulative distribution for female population follow third degree polynomial model. But, forward cumulative distribution of ASALRs for male and both sexes population follow quadratic polynomial model. These models are well fitted in accordance with t-test, F-test and CVPP. The proportion of variance of these models are more than 94%, 99%, 98%, 99%, 99% and 99% respectively.

Keywords Age specific adult literacy rates (ASALRs), Polynomial model, Cross validity prediction power (CVPP), t-test and F-test

1. Introduction

Bangladesh is a populated country of an area of 147,570 square kilometer. It has a population of about 142,319 thousands with a density of 964 per square kilometer (BBS, 2011). Literacy is defined as the ability of a person to both read and write, with understanding, a short simple statement on his everyday life. Adult literacy rate is the percentage of people ages 15 and above who can, with understanding, read and write a short, simple statement on their everyday life. It is observed that poverty and adult literacy are related inversely. If, adult literacy becomes high, then poverty will be low. Higher adult literacy rates indicate the higher level of development of any country. So, data of adult literacy rate and its level are very important for policy makers, researchers and planners for socio-economic and educational planning of a country. Adult literacy rates of Bangladesh in 1974, 1981, 1991 and 2001 are 25.8, 29.2, 35.3 and 60.5 percent respectively which show a gradually upward trend. Policymakers of Bangladesh feel to change our population into valuable manpower through proper education for overall development. The literacy rate among male population age fifteen and over is 54.0 percent where as for female is 41.4 percent (BBS, 2012). The numbers of female literacy rates are quite low in comparison with male literacy rates. The main reasons of slower rate of progress of female development in Bangladesh are misunderstanding about religious concept, socio-economic structure, lack of parental education and less expectation of education for female. It is observed that female drop outs after age 15 are high. To take various steps to increase the level of adult literacy rates, there is no alternative way to estimate literacy rates by age and sex wise specially age 15 and over. ASALRs of Bangladesh are available from some reliable sources. But, there is a gap that the data which are collected from different sources are not unique. The main cause of variation is the variation of definition of literacy. Moreover, Islam and Hossain (2013a) reported that ASALRs for male, female and both sexes population of rural area follow third degree polynomial model. And their forward cumulative distribution follow second degree polynomial model. Islam and Hossain (2014) investigated that ASALRs for male, female and both sexes of urban area in Bangladesh follow cubic polynomial model.

Therefore, the main objectives of this study are addressed below:

i) to estimate ASALRs for male, female and both sexes population in Bangladesh in 2001,

ii) to study the level and pattern of ASALRs for male, female and both sexes population in Bangladesh in 2001, and

iii) to fit some mathematical models to ASALRs and forward cumulative distribution of ASALRs for male, female and both sexes population in Bangladesh in 2001.

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2. Data and Data Source

A secondary data on age specific adult literacy rates (ASALRs) for male, female and both sexes population in Bangladesh have been taken from Population Census 2001 (BBS, 2003) which is shown in Table 1 and their forward cumulative distribution are demonstrated in Table 2.

Table 1. Age Specific Adult Literacy Rates by Sex, 2001

Age Group	Male			Female			Both Sexes		
	Total Population	Literacy		Total Population	Literacy		Total Population	Literacy	
		Number	Rate		Number	Rate		Number	Rate
15-19	6291600	4309220	68.49	5671660	3864900	68.14	11963260	8174120	68.33
20-24	4858780	3179800	65.44	6056920	3202300	52.87	10915700	6382100	58.47
25-29	4894500	2716420	55.50	5865140	2491080	42.47	10759640	5207500	48.40
30-34	4312760	2163420	50.16	4435700	1631180	36.77	8748460	3794600	43.37
35-39	4203620	2031220	48.32	3794700	1289340	33.98	7998320	3320560	41.52
40-44	3425820	1649380	48.15	2774280	827080	29.81	6200100	2476460	39.94
45-49	2610480	1312320	50.27	1990680	568960	28.58	4601160	1881280	40.89
50-54	2175300	1000440	45.99	1826300	410480	22.48	4001600	1410920	35.26
55-59	1309380	625320	47.76	1047060	231680	22.13	2356440	857000	36.37
60+	4208180	1649500	39.20	3382160	524200	15.50	7590340	2173700	28.64

Table 2. Forward Cumulative Distribution of Age Specific Adult Literacy Rates by Sex, 2001

Age Group	FCD of Male	FCD of Female	FCD of Both Sexes
15-19	68.49	68.14	68.33
20-24	133.93	121.01	126.8
25-29	189.43	163.48	175.2
30-34	239.59	200.25	218.57
35-39	287.91	234.23	260.09
40-44	336.06	264.04	300.03
45-49	386.33	292.62	340.92
50-54	432.32	315.1	376.18
55-59	480.08	337.23	412.55
60+	519.28	352.73	441.19

Note: FCD: Forward Cumulative Distribution

3. Methods and Methodological Issues

3.1. Estimation of ASALRs

To estimate ASALRs for male, female and both sexes population, the following formula is used

$$ASALRs = \frac{l_{15+a}}{p_{15+a}} X 100$$ (Shryock et al., 1976)

where l_{15+a} is the number of literates in age group 15 and over and p_{15+a} is the population in age group 15 and over.

3.2. Model Fitting

a) Using the scattered plot (Fig. 1-Fig.3) of ASALRs by age group in years for male, female and both sexes population in Bangladesh, it seems that ASALRs for male, female and both sexes in Bangladesh can be fitted by polynomial model with respect to different ages in year. Therefore, an nth degree polynomial model is considered and the form of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \; x^{\,i} + u \;;$$

where, x is the mean value of the age group; y is ASALRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. Here, a suitable n is found out for which the error sum of square is minimum.

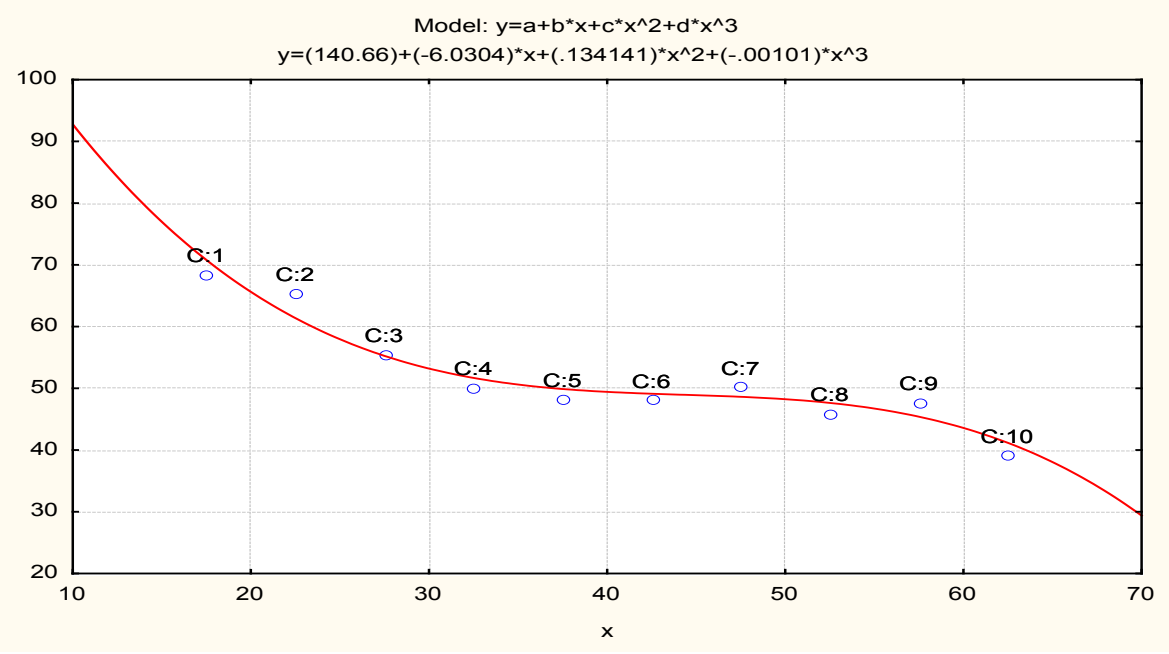

Figure 1. Observed and Fitted ASALRs for Male in 2001. X: Age in Years and Y: ASALRs for Male

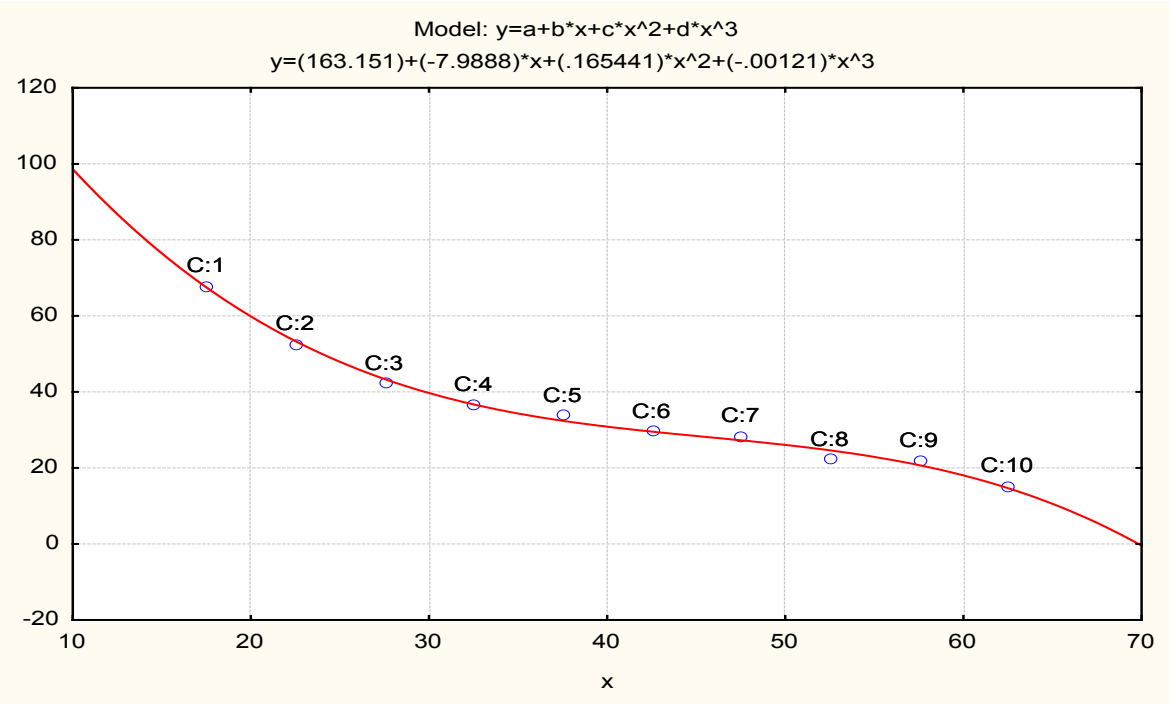

Figure 2. Observed and Fitted ASALRs for Female in 2001. X: Age in Years and Y: ASALRs for Female

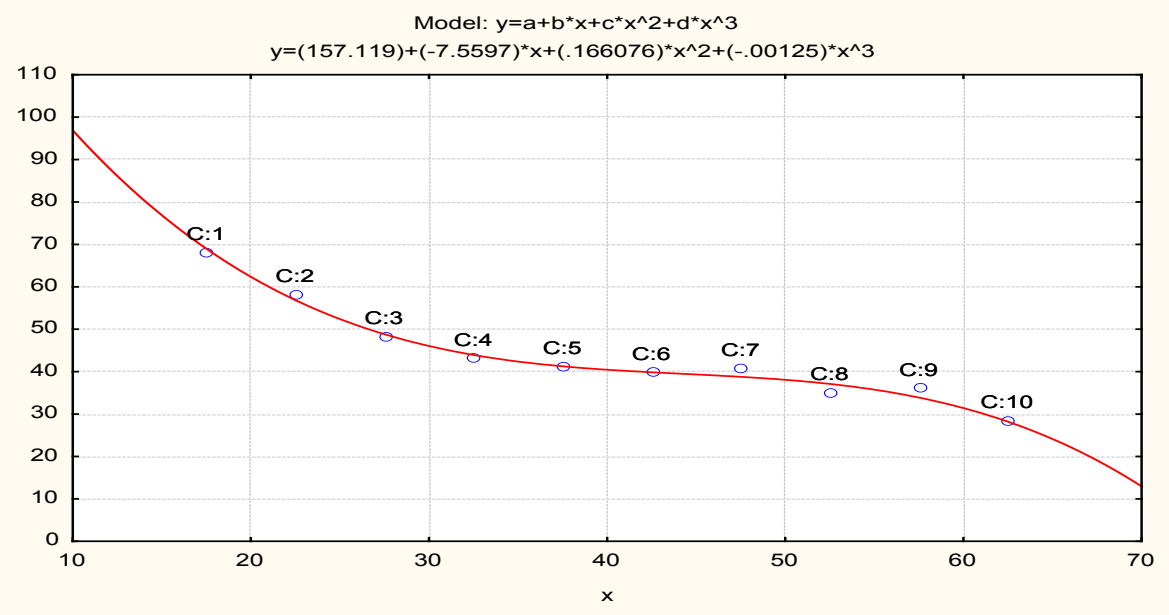

Figure 3. Observed and Fitted ASALRs for Both Sexes in 2001. X: Age in Years and Y: ASALRs for Both Sexes

b) Using the dotted plot (Fig. 4-Fig.6) of forward cumulative ASALRs for male, female and both sexes population in Bangladesh by age groups, it is observed that it follows an nth degree polynomial model with respect to ages. Therefore, the structure of the model is

$$y = a_0 + \sum_{i=1}^{n} a_i \; x^{\,i} + u \;,$$

where, x is the average value of the age group; y is forward cumulative ASALRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. A suitable n is selected such that the error sum of square is lowest.

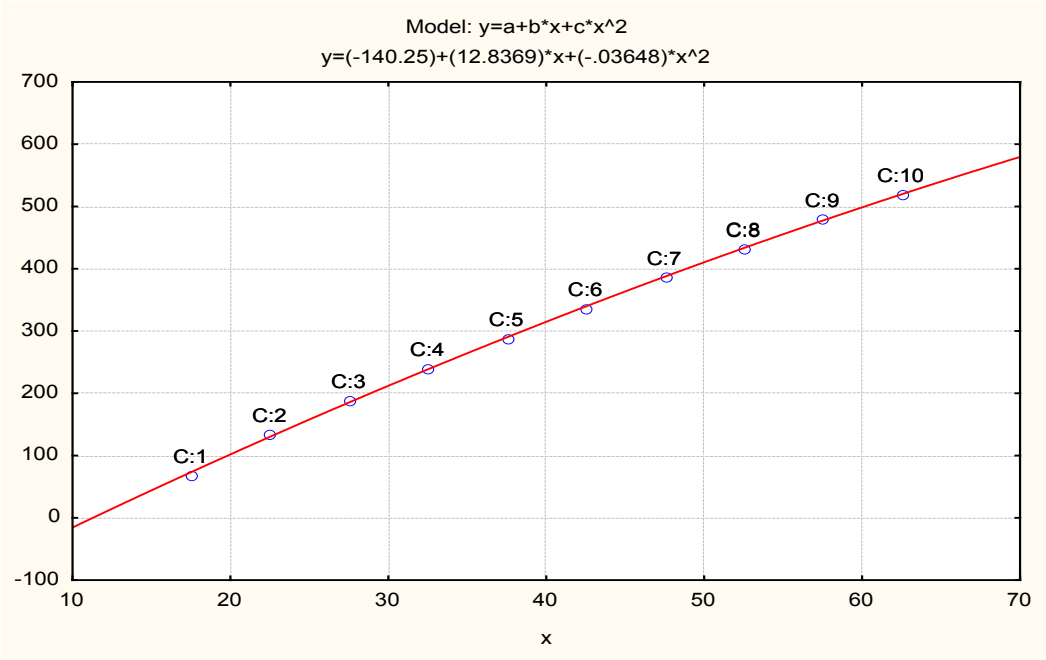

Figure 4. Observed and Fitted FCD of ASALRs for Male in 2001. X: Age in Years and Y: FCD of ASALRs for Male

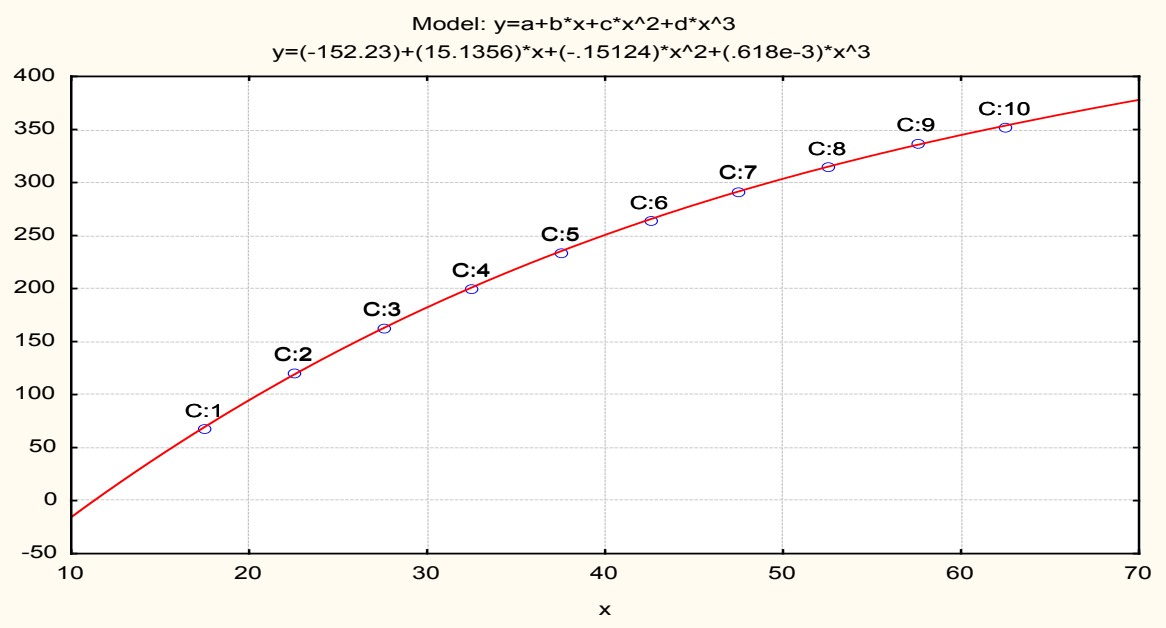

Figure 5. Observed and Fitted FCD of ASALRs for Female in 2001. X: Age in Years and Y: FCD of ASALRs for Female

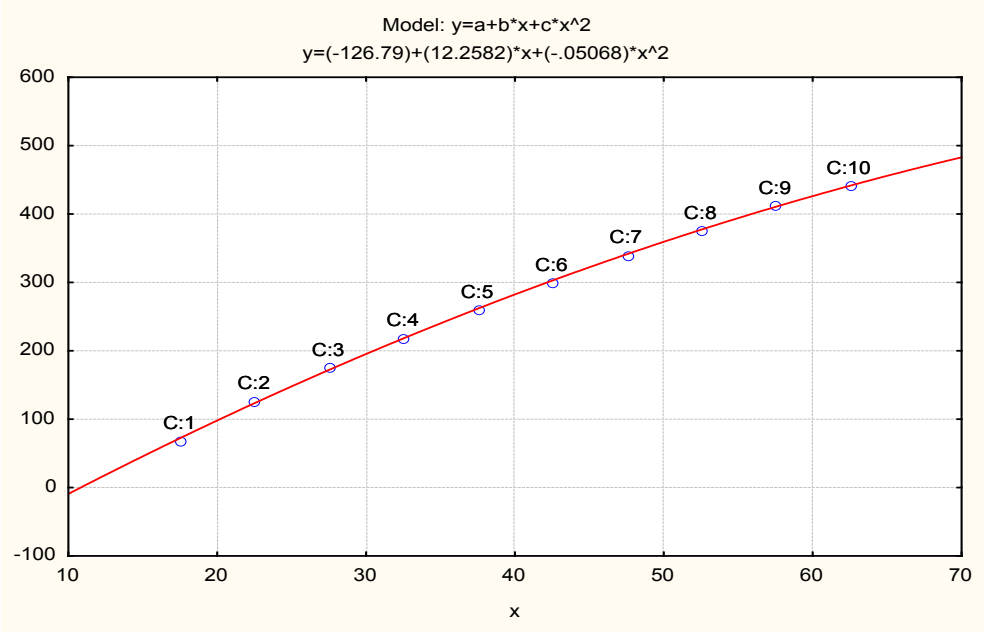

Figure 6. Observed and Fitted FCD of ASALRs for Both Sexes in 2001. X: Age in Years and Y: FCD of ASALRs for Both Sexes

The software STATISTICA was used to fit these mathematical models.

3.3. Model Validation Technique

To check how much these models are stable over the population, the cross validity prediction power (CVPP), ρ^2_{cv}, is applied. Here

$$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2);$$

where, n is the number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable (Stevens, 1996). The shrinkage coefficient of the model is the positive value of (ρ^2_{cv} - R^2); where ρ^2_{cv} is CVPP and R^2 is the coefficient of determination of the model. Here, 1-shrinkage is the stability of R^2 of the model. The information on model fittings and estimated CVPP has been demonstrated in Table 3. It is noted that CVPP was also employed as model validation by Islam et al. (2004a and 2013), and Islam (2004b, 2004c, 2005a, 2007a, 2011, 2012a and 2013).

3.4. F-test

To verify the measure of the overall significance of the

model as well as the significance of R^2, the F-test is employed here. The formula for F-test is given below:

$$F = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)}$$

where k = the number of parameters is to be estimated, n = the number of cases and R^2 is the coefficient of determination in the model (Gujarati, 1998). It is noted that Islam (2005b, 2007b, 2008 and 2012b), Hossain and Islam (2013), Islam and Hossain (2013b) used F-test as overall significance of a model.

4. Results and Discussion

The ASALRs for male, female and both sexes population of Bangladesh in 2001 have been estimated and presented in the Table 1. To see the level and pattern of ASALRs for male, female and both sexes population of Bangladesh, the data have been plotted in graph paper in Fig. 7. It is found that ASALRs for male, female and both sexes population are showing gradually decreasing curve. From the figure, it is also observed that the correlation between age and ASALRs for female is inversely related. Adult literacy rates at the age group15-19 for both male and female is 68 percent. But, at the age group 55-59 and 60+ for male are 47 and 39 percent respectively where as for female are 22 and 15 percent respectively. From the above observation and Fig. 7, it is found that adult literacy rates of female is very low in all age group compared to male population in Bangladesh. Table 2 is prepared for forward cumulative distribution of ASALRs male, female and both sexes population of Bangladesh and which shows the total ASALRs in every age group for clear understanding.

The polynomial model is constructed for ASALRs for male population of Bangladesh in 2001 and the fitted equation is as follows:

$$y = 140.66 - 6.0304x + 0.134141x^2 - 0.00101x^3 \quad (1)$$

t-stat (7.0306) (-3.4900) (2.9227) (-2.6592)
P-value (0.0004) (0.0129) (0.0265) (0.0375)

Again, another polynomial model is fitted to ASALRs for female population of Bangladesh in 2001 and the fitted equation is

$$y = 163.151 - 7.9888x + 0.165441x^2 - 0.00121x^3 \quad (2)$$

t-stat (15.2606) (-8.6485) (6.7428) (-5.9349)

P-value (0.0000) (0.0001) (0.0005) (0.0010)

Again another polynomial model is constructed for ASALRs for both sex population of Bangladesh in 2001 and the fitted equation is as follows:

$$y = 157.119 - 7.5597x + 0.166076x^2 - 0.00125x^3 \quad (3)$$

t-stat (12.8622) (-7.1625) (5.9239) (-5.3606)

P-value (0.0000) (0.0003) (0.0010) (0.0017)

Moreover, polynomial model is fitted to forward cumulative distribution of ASALRs for male population of Bangladesh in 2001 and the fitted equation is given by

$$y = -140.25 + 12.8369x - 0.03648x^2 \quad (4)$$

t-stat (-15.8617) (26.777) (-6.1627)
P-value (0.0000) (0.0000) (0.0004)

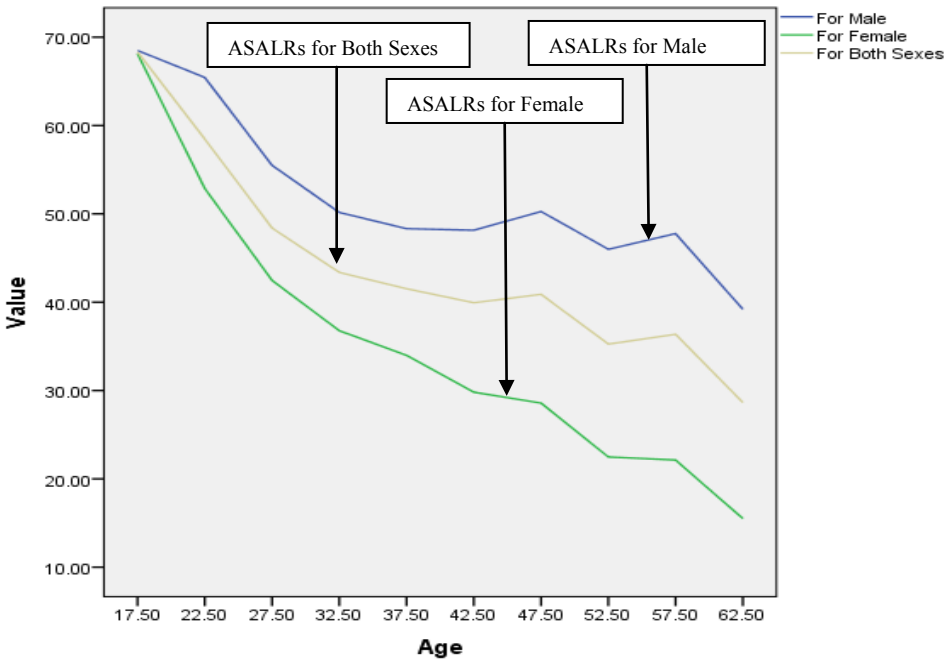

Figure 7. ASALRs for male, female and both sexes population of Bangladesh in 2001

Again, another polynomial model is constructed for forward cumulative distribution of ASALRs for female population of Bangladesh in 2001 and the fitted equation is as follows:

$$y = -152.23 + 15.1356x - 0.15124x^2 + 0.000618x^3 \quad (5)$$

t-stat (-12.0904) (13.9131) (-5.2340) (2.5803)
P-value (0.0000) (0.0000) (0.0019) (0.0417)

Finally, another polynomial model is constructed for forward cumulative distribution of ASALRs for both sexes population of Bangladesh in 2001 and the fitted equation is given below:

$$y = -126.79 + 12.2582x - 0.05058x^2 \quad (6)$$

t-stat (-17.9363) (31.9825) (-10.7104)
P-value (0.0000) (0.0000) (0.000014)

The estimated CVPP, ρ_{cv}^2 corresponding to their R^2 are shown in Table 3. The observed and fitted values are depicted in Fig. 1 to Fig. 6. In this table, all fitted models from equation (1) to equation (6) are highly cross validated and their shrinkage's are very small. Moreover, it is observed that all the parameters of the fitted models are statistically significant with large proportion of variation explained. The stability of these models are more than 84%, 98%, 96%, 99%, 99% and 99% respectively and their corresponding shrinkage coefficients are in the 6th column of Table 3.Moreover, the stability of R^2 of these models are more than 90%, 99%, 97%, 99%, 99% and 99% respectively.

Table 3. Information on Model Fittings and Estimated CVPP of the Equations of ASALRs and its Forward Cumulative Distribution of Bangladesh in 2001

Models	n	k	R^2	ρ_{cv}^2	Shrinkage	Variance explained (%)
Equation 1	10	3	0.94155	0.845692	0.09585800	94.155%
Equation 2	10	3	0.99467	0.985929	0.00874120	99.467%
Equation 3	10	3	0.98722	0.966261	0.02095920	98.722%
Equation 4	10	2	0.99960	0.999246	0.00035429	99.960%
Equation 5	10	3	0.99980	0.999472	0.00032800	99.980%
Equation 6	10	2	0.99963	0.999302	0.00032771	99.963%

The calculated values of F-statistic of the models (1) - (6) are 988.74 with (3, 6) degrees of freedom (d.f.), 1837.103 with (3, 6) degrees of freedom (d.f.), 1981.967 with (3, 6) degrees of freedom (d.f.), 33096.98 with (2, 7) degrees of freedom (d.f.), 57190.32 with (3, 6) degrees of freedom (d.f.), 39667.52 with (2, 7) degrees of freedom (d.f.) respectively where as the corresponding tabulated values are only 9.78, 9.78, 9.78, 9.55, 9.78 and 9.55 at 1% level of significance, respectively. Therefore, from these statistics it is also seen that all these constructed models are highly statistically significant. Hence, the fits of all these models are well.

Table 4. Information of F-statistics of the Equations of ASALRs and its Forward Cumulative Distribution of Bangladesh in 2001

Models	n	k	Cal. F	Tab.F (at 1% level)
Equation 1	10	4	988.74	9.78 with (3,6) d.f.
Equation 2	10	4	1837.103	9.78 with (3,6) d.f.
Equation 3	10	4	1981.967	9.78 with (3,6) d.f.
Equation 4	10	3	33096.98	9.55 with (2,7) d.f.
Equation 5	10	4	57190.32	9.78 with (3,6) d.f.
Equation 6	10	3	39667.52	9.55 with (2,7) d.f.

5. Conclusions

In this study, ASALRs for male and female population of Bangladesh are showing more or less same pattern. The ASALRs for female is less than that of male at each age. Moreover, ASALRs for female is downward pattern with increasing of ages but ASALRs for male is downward pattern due to ages excepting the ages 45-49 and 55-59. The ASALRs and its forward cumulative distribution of ASALRs for female follow cubic polynomial models but forward cumulative distribution of ASALRs for male and both sexes population follow quadratic polynomial model.

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