

# Design and Implementation of First Order Digital Differentiators at Microwave Frequencies

Srinivasa Rao Sankranti<sup>1,\*</sup>, Tirumala Krishna Battula<sup>2</sup>, Malleswara Rao Veera<sup>1</sup>

<sup>1</sup>Dept. of ECE, GITAM University, Rushikonda, Visakhapatnam, India

<sup>2</sup>Dept. of ECE, UCEV, JNT University Kakinada, Vizianagaram, India

**Abstract** Conventional digital differentiators works efficiently up to Low frequency region only. The main purpose of the paper is to design and implement first order Al-Alaoui differentiator at microwave frequencies. Necessary derivations are carried out. The differentiator is implemented using transmission line configurations such as micro strip lines. The simulations are carried out using MATLAB and Advanced design software (ADS) environment.

**Keywords** Microwave Filter, Digital Differentiator, Stub, Micro strip, ADS (Advanced Design Software)

## 1. Introduction

Digital differentiators which are used to find the time-derivative of the incoming signal play vital role in many of the electronic systems. A digital differentiator is defined as,  $H(j\omega) = j\omega$ . The FIR type differentiators find less use in real-time applications. Al-Alaoui have designed a IIR type digital differentiator [1-3] by the interpolation process which is given by,  $H(z) = \frac{8(z-1)}{7T(z+1/7)}$ . The digital differentiators are mainly implemented in circuits for low frequency applications. So, the implementation of differentiators at high frequencies is a problem of practical interest.

The paper is organized as follows. Section 2 deals about design of digital filters at microwave frequencies. Implementation of Al-Alaoui digital differentiator at microwave frequencies is presented in section 3. Finally, Results and conclusions are drawn in Section 4.

## 2. Digital Filters at Microwave Frequencies

The Scattering Matrix which relates incident waves and emergent waves of a two port network is defined as,

$$\begin{pmatrix} b(1) \\ b(2) \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a(1) \\ a(2) \end{pmatrix} \quad (1)$$

The Chain Scattering Matrix [4] of a two port network is defined as,

$$\begin{pmatrix} a(1) \\ b(1) \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} b(2) \\ a(2) \end{pmatrix} \quad (2)$$

The relation between the parameters will be,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11}S_{22}}{S_{21}} \end{pmatrix} \quad (3)$$

The Chain Scattering Matrix for different Transmission Line configurations [5] will be calculated in this Section.

### 2.1. Open Circuited Transmission Line

Consider a Transmission Line with Open Circuited Stub. Let the length of the stub and transmission line be  $l = \lambda_0/4$ , where  $\lambda_0$  is the wavelength at normalized frequency  $\omega_0$ . If the Impedance of the Open Circuited Stub is  $Z_1$ , then the chain scattering matrix is given by,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{o.c} = \begin{pmatrix} 1 + j \frac{Z_0}{2Z_1} \tan(\beta l) & j \frac{Z_0}{2Z_1} \tan(\beta l) \\ -j \frac{Z_0}{2Z_1} \tan(\beta l) & 1 - j \frac{Z_0}{2Z_1} \tan(\beta l) \end{pmatrix} \quad (4)$$

where  $\beta$  is the propagation constant. Let  $\omega$  be the angular frequency and  $\tau$  be the propagation delay caused by the length  $l$ . If  $j \tan(\beta l) = j \tan(\omega \tau)$ , then,

$$j \tan(\omega \tau) = \frac{e^{j\omega\tau} - e^{-j\omega\tau}}{e^{j\omega\tau} + e^{-j\omega\tau}} = \frac{D - D^{-1}}{D + D^{-1}} \quad (5)$$

Then Equation (4) reduces to,

\* Corresponding author:

srinu16ssr@gmail.com (Srinivasa Rao Sankranti)

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$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{o,c} = \frac{1}{1+D^{-2}} \begin{pmatrix} (1+c) + (1-c)D^{-2} & c - cD^{-2} \\ -c + cD^{-2} & (1-c) + (1+c)D^{-2} \end{pmatrix} \quad (6)$$

where  $c = \frac{Z_0}{2Z_1}$  and  $D^{-1} = e^{-j\omega\tau}$ . If we set  $z = D^2$ , then,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{o,c} = \frac{1}{1+z^{-1}} \begin{pmatrix} (1+c) + (1-c)z^{-1} & c - cz^{-1} \\ -c + cz^{-1} & (1-c) + (1+c)z^{-1} \end{pmatrix} \quad (7)$$

The digital transfer function  $H(z)$  or  $S_{21}(z)$  is given by  $\frac{1}{T_{11}(z)}$ . So,

$$H(z) = S_{21}(z) = \frac{1+z^{-1}}{(1+c)+(1-c)z^{-1}} \quad (8)$$

## 2.2. Short Circuited Transmission Line

For the short circuited stub,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{s,c} = \begin{pmatrix} 1 + j\frac{Z_0}{2Z_1} \cot(\beta l) & j\frac{Z_0}{2Z_1} \cot(\beta l) \\ -j\frac{Z_0}{2Z_1} \cot(\beta l) & 1 - j\frac{Z_0}{2Z_1} \cot(\beta l) \end{pmatrix} \quad (9)$$

The above Equation can be expressed in terms of  $c$  and  $D$  as,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{s,c} = \frac{1}{1-D^{-2}} \begin{pmatrix} (1+c) - (1-c)D^{-2} & c + cD^{-2} \\ -c - cD^{-2} & (1-c) - (1+c)D^{-2} \end{pmatrix} \quad (10)$$

The chain scattering matrix in terms of delay  $z^{-1}$  will be,

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{s,c} = \frac{1}{1-z^{-1}} \begin{pmatrix} (1+c) - (1-c)z^{-1} & c + cz^{-1} \\ -c - cz^{-1} & (1-c) - (1+c)z^{-1} \end{pmatrix} \quad (11)$$

The digital transfer function  $H(z)$  or  $S_{21}(z)$  of a short circuited stub is given by,

$$H(z) = S_{21}(z) = \frac{1-z^{-1}}{(1+c)-(1-c)z^{-1}} \quad (12)$$

## 3. Implementation of Digital Differentiators

### 3.1. Implementation of Al-Alaoui Differentiator

First order Al-Alaoui digital differentiator is given by

$$H_1(z) = \frac{8(z-1)}{7T(z+\frac{1}{T})} \quad (13)$$

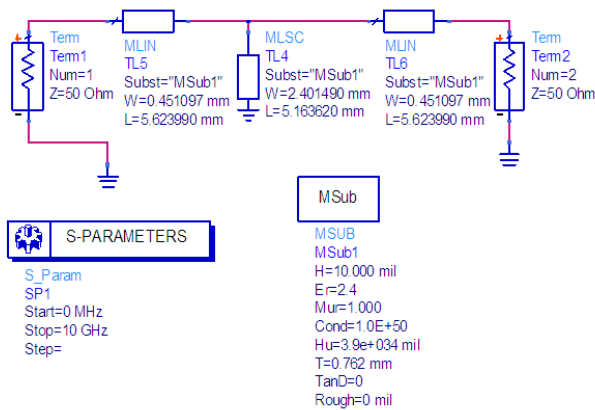


Figure 1. Implementation using ADS Software

Comparing Eqn. (13) with Eqn.(12),  $c = \frac{Z_0}{2Z_1} = \frac{4}{3}$ . If  $Z_0 = 50\Omega$  then  $Z_1 = 18.75\Omega$ . The designed practical filter structure using micro strips is as shown in Figure 1. The experimental setup and the layout generated are as shown in

Fig.5 and 4 respectively. The magnitude and phase responses are shown in Fig.2 and 3.

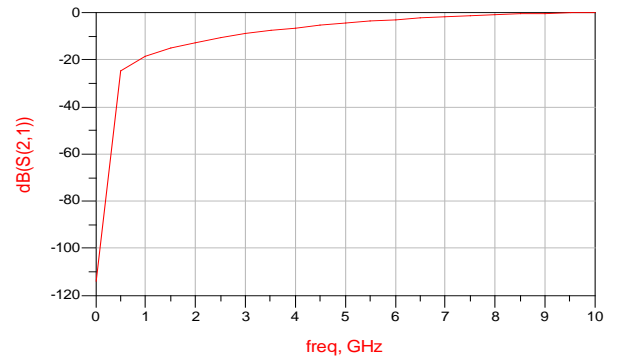


Figure 2. Variation of the gain of  $S_{21}$



Figure 3. Variation of the Magnitude of  $S_{21}$



**Figure 4.** Layout of First order differentiator



**Figure 5.** Experimental setup

## 4. Results and Conclusions

The first order microwave differentiator was constructed by using microstrip to emulate transmission lines. The shunted transmission line having a characteristic impedance of  $18.75\Omega$  and  $50\Omega$  equivalent microstrips are placed both sides symmetrically. The simulation is performed by using Advanced Design System Software (ADS). The magnitude response of  $S_{21}(f)$  of first order differentiator is shown in Fig.2. We generated the physical layout by using ADS. The physical layout was built on FR-4 substrate. By using Network analyzer, the magnitude response of  $S_{21}(f)$  was measured. The simulated and experimental results of a first order differentiator are in good agreement.

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