

Use of Non-Conventional Measures of Dispersion for Improved Estimation of Population Mean

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Abstract In this paper, we have proposed some improved modified ratio type estimators for population mean of study variable using auxiliary information in the form of non-conventional measures of dispersion Gini's mean difference, Downton's method and probability weighted moments given by Abid (2016) with linear combination of population coefficient of Skewness and Kurtosis of auxiliary variable. The large sample properties of the proposed estimators have been studied up to the first order of approximation that is the biases and the mean squared errors. A comparison has been made with the existing estimators of population mean. The conditions under which the proposed estimators perform better than the other existing estimators have been mentioned. An empirical study is also carried out to justify the theoretical findings. The theoretical as well as the empirical findings show the improvement of the proposed estimators over other existing estimators for the estimation of population mean.

Keywords Study variable, Auxiliary variable, Bias, Mean Squared Error, Efficiency

1. Introduction

Whenever the population is large, it is very time taking and costly to get the information on each of the population unit. Sampling is a very good alternative to overcome this problem. It is very natural to estimate any parameter of the population by its corresponding statistic. Thus to estimate the population mean, the most appropriate estimator is the sample mean. The sample mean is unbiased, but it seems to have large amount of variation. Now our aim is to seek an estimator which may be biased but it should have lesser mean squared error as compared to sample mean. The use of auxiliary information supplied by the auxiliary variable fulfills our aim to find more and more efficient estimators. The auxiliary variable is the variable about which the experimenter has full information and it is collected with the main variable under study without increasing the cost of the survey. The auxiliary variable may be positively or negatively correlated with the study variable under consideration. When the auxiliary variable has positive relationship with the main variable under study and the line of regression of y on x passes through origin, ratio type estimators are used to estimate population parameters under

consideration. When it has negative relationship with the main variable under study, product type estimators are used otherwise regression type estimators are used for the estimation of parameters under investigation. In the present study, we have considered the positive correlation case only so only ratio type estimators are studied along with the proposed ratio type class of estimators.

Let the finite population of interest consist of N distinct and identifiable units and let (x_i, y_i) , $i = 1, 2, \dots, n$ be a bivariate sample of size n taken from (X, Y) using a *SRSWOR* scheme. Let \bar{X} and \bar{Y} respectively be the population means of the auxiliary and the study variables, and let \bar{x} and \bar{y} be the corresponding sample means which are unbiased estimators of \bar{X} and \bar{Y} respectively. Let ρ be the correlation coefficient between X and Y .

2. Notations

Following notations have been used in this manuscript,

N - Size of the population

n - Size of the sample

Y - Study variable

X - Auxiliary variable

\bar{Y}, \bar{X} - Population means

\bar{y}, \bar{x} - Sample means

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S_y, S_x - Population Standard Deviations

S_{yx} - Population Covariance between Y and X

C_y, C_x - Coefficients of Variation

M_d - Median of the auxiliary variable

ρ - Correlation coefficient between X and Y

$b = \frac{S_{yx}}{S_x^2}$ - Regression coefficient of y on x

$\beta_1 = \frac{N \sum_{i=1}^N (X_i - \bar{X})^3}{(N-1)(N-2)S_x^3}$ - Coefficient of Skewness of

auxiliary variable

$\beta_2 = \frac{N(N+1) \sum_{i=1}^N (X_i - \bar{X})^4}{(N-1)(N-2)(N-3)S_x^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$

Coefficient of Kurtosis of auxiliary variable

$QD = \frac{Q_3 - Q_1}{2}$ - Quartile Deviation

$G = \frac{4}{N-1} \sum_{i=1}^N \left(\frac{2i - N - 1}{2N} \right) X_i$ - Gini's Mean Difference

$D = \frac{2\sqrt{\pi}}{N(N-1)} \sum_{i=1}^N \left(i - \frac{N+1}{2} \right) X_i$ - Downton's Parameters

$S_{pw} = \frac{\sqrt{\pi}}{N^2} \sum_{i=1}^N (2i - N - 1) X_i$ - Probability Weighted

Moments

$B(.)$ - Bias of the estimator

$V(.)$ - Variance of the estimator

$MSE(.)$ - Mean squared error of the estimator

$PRE(t_e, t_p) = \frac{MSE(t_e)}{MSE(t_p)} * 100$ - Percentage relative

efficiency of the estimator t_p over t_e

3. Review of Existing Estimators

As mentioned above the most appropriate estimator for population mean \bar{Y} is the sample mean \bar{y} that is mean per unit estimator given by,

$$t_0 = \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

It is an unbiased estimator of population mean and its variance up to the first order of approximation is,

$$V(t_0) = \frac{1-f}{n} S_y^2 \quad (1)$$

Cochran (1940) keeping in mind that the mean per unit estimator has sufficiently large variance, used the auxiliary variable to estimate the population mean of the main variable and proposed the traditional ratio estimator as,

$$t_R = \bar{y} \frac{\bar{X}}{\bar{x}}$$

It is a biased estimator of population mean and its bias and mean squared error, up to the first order of approximation respectively are,

$$B(t_R) = \frac{1-f}{n} \frac{1}{\bar{X}} [R_1 S_x^2 - \rho S_y S_x]$$

$$MSE(t_R) = \frac{1-f}{n} [S_y^2 + R_1^2 S_x^2 - 2R_1 \rho S_y S_x],$$

$$\text{where } R_1 = \frac{\bar{Y}}{\bar{X}} \quad (2)$$

Kadilar and Cingi (2004) suggested some improved ratio type estimators of population mean as,

$$t_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X}$$

$$t_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)} (\bar{X} + C_x)$$

$$t_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)} (\bar{X} + \beta_2)$$

$$t_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + C_x)} (\bar{X}\beta_2 + C_x)$$

$$t_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \beta_2)} (\bar{X}C_x + \beta_2)$$

The biases and the mean squared errors of above estimators, up to the first order of approximations respectively are,

$$B(t_i) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_i^2, \quad (i=1, 2, \dots, 5)$$

$$MSE(t_i) = \frac{1-f}{n} [R_i^2 S_x^2 + S_y^2 (1 - \rho^2)],$$

$$(i=1, 2, \dots, 5) \quad (3)$$

Where,

$$R_1 = \frac{\bar{Y}}{\bar{X}}, \quad R_2 = \frac{\bar{Y}}{\bar{X} + C_x}, \quad R_3 = \frac{\bar{Y}}{\bar{X} + \beta_2},$$

$$R_4 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + C_x}, \quad R_5 = \frac{\bar{Y}C_x}{\bar{X}C_x + \beta_2}$$

Kadilar and Cingi (2006) suggested some more improved ratio type estimators of population mean as,

$$\begin{aligned}
 t_6 &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \rho)} (\bar{X} + \rho) \\
 t_7 &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \rho)} (\bar{X}C_x + \rho) \\
 t_8 &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x)} (\bar{X}\rho + C_x) \\
 t_9 &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + \rho)} (\bar{X}\beta_2 + \rho) \\
 t_{10} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \beta_2)} (\bar{X}\rho + \beta_2)
 \end{aligned}$$

The biases and the mean squared errors of above estimators, up to the first order of approximations respectively are,

$$\begin{aligned}
 B(t_i) &= \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_i^2, \quad (i = 6, 7, \dots, 10) \\
 MSE(t_i) &= \frac{1-f}{n} [R_i^2 S_x^2 + S_y^2 (1 - \rho^2)], \\
 &\quad (i = 6, 7, \dots, 10)
 \end{aligned} \quad (4)$$

Where,

$$\begin{aligned}
 R_6 &= \frac{\bar{Y}}{\bar{X} + \rho}, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x + \rho}, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho + C_x}, \\
 R_9 &= \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho}, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2}
 \end{aligned}$$

Yan and Tian (2010) suggested two modified ratio type estimators of population mean using information on auxiliary variable in the form of coefficient of skewness and kurtosis respectively as,

$$\begin{aligned}
 t_{11} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1)} (\bar{X} + \beta_1) \\
 t_{12} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + \beta_2)} (\bar{X}\beta_1 + \beta_2)
 \end{aligned}$$

The biases and the mean squared errors of above estimators, up to the first order of approximations respectively are,

$$\begin{aligned}
 B(t_i) &= \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_i^2, \quad (i = 11, 12) \\
 MSE(t_i) &= \frac{1-f}{n} [R_i^2 S_x^2 + S_y^2 (1 - \rho^2)], \quad (i = 11, 12)
 \end{aligned} \quad (5)$$

Where,

$$R_{11} = \frac{\bar{Y}}{\bar{X} + \beta_1}, \quad R_{12} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + \beta_2}$$

Subramani and Kumarpandiyan (2012a, 2012b, 2012c) suggested some modified improved ratio type estimators of population mean using median with coefficient of skewness, kurtosis and coefficient of variation as,

$$\begin{aligned}
 t_{13} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + M_d)} (\bar{X} + M_d) \\
 t_{14} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + M_d)} (\bar{X}C_x + M_d) \\
 t_{15} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + M_d)} (\bar{X}\beta_1 + M_d) \\
 t_{16} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + M_d)} (\bar{X}\beta_2 + M_d)
 \end{aligned}$$

The biases and the mean squared errors of above estimators, up to the first order of approximations respectively are,

$$\begin{aligned}
 B(t_i) &= \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_i^2, \quad (i = 13, 14, \dots, 16) \\
 MSE(t_i) &= \frac{1-f}{n} [R_i^2 S_x^2 + S_y^2 (1 - \rho^2)], \\
 &\quad (i = 13, 14, \dots, 16)
 \end{aligned} \quad (6)$$

Where,

$$\begin{aligned}
 R_{13} &= \frac{\bar{Y}}{\bar{X} + M_d}, \quad R_{14} = \frac{\bar{Y}C_x}{\bar{X}C_x + M_d}, \\
 R_{15} &= \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + M_d}, \quad R_{16} = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + M_d}
 \end{aligned}$$

Jeelani et al. (2013) suggested the following modified ratio type estimator of population mean using coefficient of skewness and quartile difference of auxiliary variable as,

$$t_{17} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + QD)} (\bar{X}\beta_1 + QD)$$

The bias and the mean squared error of above estimator, up to the first order of approximation is given by,

$$\begin{aligned}
 B(t_{17}) &= \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_{17}^2, \\
 MSE(t_{17}) &= \frac{1-f}{n} [R_{17}^2 S_x^2 + S_y^2 (1 - \rho^2)], \quad (7)
 \end{aligned}$$

Where,

$$R_{17} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1 + QD}$$

Abid et al. (2016) introduced some modified improved estimators of population mean using some non-conventional parameters of auxiliary variable as,

$$\begin{aligned}
t_{18} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + G)}(\bar{X} + G) \\
t_{19} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + G)}(\bar{X}\rho + G) \\
t_{20} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + G)}(\bar{X}C_x + G) \\
t_{21} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D)}(\bar{X} + D) \\
t_{22} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + D)}(\bar{X}\rho + D) \\
t_{23} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D)}(\bar{X}C_x + D) \\
t_{24} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + S_{pw})}(\bar{X} + S_{pw}) \\
t_{25} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + S_{pw})}(\bar{X}\rho + S_{pw}) \\
t_{26} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + S_{pw})}(\bar{X}C_x + S_{pw})
\end{aligned}$$

The biases and the mean squared errors of above estimators, up to the first order of approximations respectively are,

$$\begin{aligned}
B(t_i) &= \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_i^2, \quad (i = 18, 19, \dots, 26) \\
MSE(t_i) &= \frac{1-f}{n} [R_i^2 S_x^2 + S_y^2 (1 - \rho^2)], \\
&\quad (i = 18, 19, \dots, 26)
\end{aligned} \tag{8}$$

where,

$$\begin{aligned}
R_{18} &= \frac{\bar{Y}}{\bar{X} + G}, \quad R_{19} = \frac{\bar{Y}\rho}{\bar{X}\rho + G}, \quad R_{20} = \frac{\bar{Y}C_x}{\bar{X}C_x + G}, \\
R_{21} &= \frac{\bar{Y}}{\bar{X} + D}, \quad R_{22} = \frac{\bar{Y}\rho}{\bar{X}\rho + D}, \quad R_{23} = \frac{\bar{Y}C_x}{\bar{X}C_x + D}, \\
R_{24} &= \frac{\bar{Y}}{\bar{X} + S_{pw}}, \quad R_{25} = \frac{\bar{Y}\rho}{\bar{X}\rho + S_{pw}}, \quad R_{26} = \frac{\bar{Y}C_x}{\bar{X}C_x + S_{pw}}.
\end{aligned}$$

4. Proposed Estimators

Motivated by Abid et al. (2016) and Subramani (2013), we

have proposed the following improved modified ratio type estimators of population mean using specific parameter of auxiliary variable along with the non-conventional parameters of auxiliary variable as,

$$\begin{aligned}
t_{p1} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\delta \bar{x} + G)}(\delta \bar{X} + G), \\
t_{p2} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\delta \bar{x} + D)}(\delta \bar{X} + D), \\
t_{p3} &= \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\delta \bar{x} + S_{pw})}(\delta \bar{X} + S_{pw}),
\end{aligned}$$

Where, $\delta = QD/S_x$.

The biases and the mean squared errors of proposed estimators, up to the first order of approximations respectively can be obtained as,

$$\begin{aligned}
B(t_{p_j}) &= \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_{p_j}^2, \quad (j = 1, 2, 3) \\
MSE(t_{p_j}) &= \frac{1-f}{n} [R_{p_j}^2 S_x^2 + S_y^2 (1 - \rho^2)], \\
&\quad (j = 1, 2, 3)
\end{aligned} \tag{9}$$

Where,

$$R_{p1} = \frac{\bar{Y}\delta}{\bar{X}\delta + G}, \quad R_{p2} = \frac{\bar{Y}\delta}{\bar{X}\delta + D}, \quad R_{p3} = \frac{\bar{Y}\delta}{\bar{X}\delta + S_{pw}}$$

5. Efficiency Comparison

In this section, we have compared theoretically the proposed estimator with the existing estimators of population mean and have given the conditions under which the proposed estimator performs better than the other estimators of population mean under simple random sampling without replacement scheme.

5(a). Comparison with mean per unit estimator

From equation (9) and equation (2), we see that the proposed estimators are better than the mean per unit estimator if,

$$\begin{aligned}
MSE(t_{p_j}) - V(\bar{y}) &\leq 0 \quad \text{or,} \\
\frac{1-f}{n} [R_{p_j}^2 S_x^2 - \rho^2 S_y^2] &\leq 0 \quad \text{or,} \\
R_{p_j}^2 &\leq \frac{\rho^2 S_y^2}{S_x^2} \quad \text{or,} \\
R_{p_i} &\leq \pm \frac{\rho S_y}{S_x}, \quad (j = 1, 2, 3)
\end{aligned} \tag{10}$$

5(b). Comparison with usual ratio estimator

The proposed estimators are better than the usual ratio estimator if,

$$MSE(t_{p_j}) - MSE(t_r) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [(R_{p_j}^2 - R_1^2) S_x^2 - \rho^2 S_y^2 + 2R_1 \rho S_y S_x] \leq 0 \quad \text{or,}$$

$$(R_{p_j}^2 - R_1^2) S_x^2 \leq \rho^2 S_y^2 - 2R_1 \rho S_y S_x, \quad (j = 1, 2, 3) \quad (11)$$

5(c). Comparison with Kadilar and Cingi (2004) estimators

From equation (9) and equation (3), the proposed estimators are better than the Kadilar and Cingi (2004) estimators if,

$$MSE(t_{p_j}) - MSE(t_i) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [R_{p_j}^2 S_x^2 - R_i^2 S_x^2] \leq 0 \quad \text{or,}$$

$$R_{p_j} \leq \pm R_i, \quad (j = 1, 2, 3), \quad (i = 1, 2, \dots, 5) \quad (12)$$

5(d). Comparison with Kadilar and Cingi (2006) estimators

From equation (9) and equation (4) it is seen that the proposed estimators are better than the Kadilar and Cingi (2006) estimators under the conditions if,

$$MSE(t_{p_j}) - MSE(t_i) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [R_{p_j}^2 S_x^2 - R_i^2 S_x^2] \leq 0 \quad \text{or,}$$

$$R_{p_j} \leq \pm R_i, \quad (j = 1, 2, 3), \quad (i = 6, 7, \dots, 10) \quad (13)$$

5(e). Comparison with Yan and Tian (2010) estimators

The proposed estimators are better than Yan and Tian (2010) estimators if,

$$MSE(t_{p_j}) - MSE(t_i) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [R_{p_j}^2 S_x^2 - R_i^2 S_x^2] \leq 0 \quad \text{or,}$$

$$R_{p_j} \leq \pm R_i, \quad (j = 1, 2, 3), \quad (i = 11, 12) \quad (14)$$

5(f). Comparison with Subramani and Kumarpandiyan (2012a, 2012b, 2012c) estimators

From equation (9) and equation (6) it is seen that the proposed estimators are better than Subramani and Kumarpandiyan (2012a, 2012b, 2012c) estimators under the conditions if,

$$MSE(t_{p_j}) - MSE(t_i) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [R_{p_j}^2 S_x^2 - R_i^2 S_x^2] \leq 0 \quad \text{or,}$$

$$R_{p_j} \leq \pm R_i, \quad (j = 1, 2, 3), \quad (i = 13, 14, \dots, 16) \quad (15)$$

5(g). Comparison with Jeelani et al. (2013) estimator

The proposed estimators are better than Jeelani et al. (2013) estimator if,

$$MSE(t_{p_j}) - MSE(t_{17}) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [R_{p_j}^2 S_x^2 - R_{17}^2 S_x^2] \leq 0 \quad \text{or,}$$

$$R_{p_j} \leq \pm R_{17}, \quad (j = 1, 2, 3) \quad (16)$$

5(h). Comparison with Abid et al. (2016) estimators

From equation (9) and equation (6) it is seen that the proposed estimators are better than Abid et al. (2016) estimators under the conditions if,

$$MSE(t_{p_j}) - MSE(t_i) \leq 0 \quad \text{or,}$$

$$\frac{1-f}{n} [R_{p_j}^2 S_x^2 - R_i^2 S_x^2] \leq 0 \quad \text{or,}$$

$$R_{p_j} \leq \pm R_i, \quad (j = 1, 2, 3), \quad (i = 18, 19, \dots, 26) \quad (17)$$

6. Numerical Illustration

In this section, we have used the data of Kadilar and Cingi (2004). They have considered the data of only Aegean Region of Turkey under simple random sampling scheme. We have applied our proposed and other ratio type estimators for the data on amount of apple production (as study variable) and number of apple trees (as auxiliary variable) in 106 villages of Aegean Region in 1999. Using these data we have computed MSE of proposed estimators along with the other ratio type estimators mentioned in this manuscript and these estimators are compared with each other with respect to their MSE values.

We observe the statistics about the population. It is worth notable that the correlation between the study and the auxiliary variable is 86% and we have taken a sample of size $n = 40$. It would be important to mention that the sample size has no effect on the efficiency comparisons of the estimators.

The population parameters for the above data are,

$$N = 106 \quad n = 40 \quad \bar{Y} = 2212.59 \quad \bar{X} = 27421.70$$

$$\rho = 0.860 \quad S_y = 11551.53 \quad C_y = 5.22 \quad S_x = 57460.61$$

$$C_x = 2.10 \quad \beta_1 = 2.122 \quad \beta_2 = 34.572 \quad M_d = 7297.50$$

$$QD = 12156.25 \quad G = 40201.69 \quad D = 35634.99$$

$$S_{pw} = 35298.81$$

Table 1. Constants, Biases and MSE of Proposed and other estimators

Estimator	Constant	Bias	Mean Squared error
t_0	Nil	0	2077627.25
t_r	0.0807	171.32	984589.70
t_1	0.0807	151.20	889617.50
t_2	0.0807	151.18	889566.40
t_3	0.0806	150.82	888775.70
t_4	0.0807	151.20	889616.00
t_5	0.0806	151.02	889215.30
t_6	0.0807	151.19	889596.60
t_7	0.0807	151.20	889607.50
t_8	0.0807	151.17	889557.80
t_9	0.0867	151.20	889616.90
t_{10}	0.0806	150.76	888634.40
t_{11}	0.0807	151.14	889492.50
t_{12}	0.0807	151.13	889452.90
t_{13}	0.0637	94.32	763783.60
t_{14}	0.0715	119.04	818477.40
t_{15}	0.0767	136.64	857402.20
t_{16}	0.0801	148.10	884526.80
t_{17}	0.0742	128.08	838466.80
t_{18}	0.0327	24.87	610126.10
t_{19}	0.0475	52.34	670914.00
t_{20}	0.0297	20.59	600579.70
t_{21}	0.0320	23.85	607875.10
t_{22}	0.0498	57.60	682552.70
t_{23}	0.0351	28.59	618381.50
t_{24}	0.0322	24.12	608480.30
t_{25}	0.0500	58.02	683478.00
t_{26}	0.0353	28.90	619061.50
t_{p_1}	0.01018	2.56	546154.92
t_{p_2}	0.01130	3.02	547697.15
t_{p_3}	0.01139	2.79	547683.38

The values of the related constants, biases and the mean squared error of the existing and proposed improved ratio estimators are given in Table-1.

7. Results and Conclusions

In this manuscript we have proposed an improved estimator of population mean using a special parameter of auxiliary variable along with some non-traditional measures of dispersion of auxiliary variable. The expressions for the bias and mean squared error have been obtained up to the first order of approximation. A theoretical as well as numerical comparison of proposed estimators has been made with other existing estimators of population mean. From the above Table-1, it can be observed that the related constants, biases and mean squared errors of the proposed improved ratio type estimators are smaller than the usual ratio estimator and the other existing ratio estimators in literature. Thus the proposed estimators perform better than the usual ratio estimator and the other existing modified ratio estimators in terms of MSE, which indicates that the proposed estimators are more efficient. Therefore it is recommended that the proposed estimators may be used for the efficient estimation of population mean. It can be further noted that among the proposed estimators the estimator t_{p_1} performs better than the other two proposed estimators.

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