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Improved Ratio-Cum-Product Estimators of Population Mean Using Known Population Parameters of Auxiliary Variables

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Abstract In the present paper, a ratio-cum-product type estimator of finite population mean using known coefficient of kurtosis and median of auxiliary variable has been proposed. The explicit expressions for bias and mean squared error of the proposed estimator with large sample approximation are derived up to the first order of approximation. A comparison has been made with the existing estimators of population mean using auxiliary variable under simple random sampling scheme. An empirical study is also carried out to demonstrate the performance of the suggested estimator along with the existing estimators of population mean under simple random sampling. It has been shown through the empirical study that the proposed estimator has minimum mean squared error among all existing estimators of population mean. It is the best estimator of population mean among all existing estimators.

Keywords Ratio-cum-product estimator, Median, Coefficient of kurtosis, Bias, Mean squared error

1. Introduction

Use of auxiliary information has been in practice to increase the efficiency of the estimators. When population parameters of the auxiliary variable are known, several estimators for population mean of study variable have been discussed in the literature. When the variable under study and the auxiliary variables are highly and positively correlated and the line of regression passes through origin, ratio method of estimation is preferred to use. On the other hand, if they are highly and negatively correlated, product method of estimation is suggested to use.

Let the finite population $U=(U_1,U_2,...,U_N)$ consists of N units. Suppose two auxiliary variables x_1 and x_2 are observed on U_i $(i=1,2,...,N)$, where x_1 is positively and x_2 is negatively correlated with the study variable y. A simple random sample of size n with $n < N$, is drawn using simple random sampling without replacement (SRSWOR) from the population U to estimate the population mean ($\bar{Y}$) of study character y, when the population means $\bar{X}_1=\frac{1}{N}\sum_{i=1}^{N}x_{1i}$ and $\bar{X}_2=\frac{1}{N}\sum_{i=1}^{N}x_{2i}$ of x_1 and x_2 respectively are known.

Usual ratio and product estimators given by Cochran [1] and Robson [12] respectively for estimating the population mean $\bar{Y}$ respectively are defined as,

$$\bar{y}_R=\bar{y}\frac{\bar{X}_1}{\bar{x}_1} \tag{1.1}$$

$$\bar{y}_P=\bar{y}\frac{\bar{x}_2}{\bar{X}_2} \tag{1.2}$$

Upadhyaya and Singh [18] suggested ratio and product estimators utilizing coefficient of variation and coefficient of kurtosis of auxiliary variables as

$$t_1=\bar{y}\left(\frac{\bar{X}_1C_{x_1}+\beta_2(x_1)}{\bar{x}_1C_{x_1}+\beta_2(x_1)}\right) \tag{1.3}$$

$$t_2=\bar{y}\left(\frac{\bar{x}_2C_{x_2}+\beta_2(x_2)}{\bar{X}_2C_{x_2}+\beta_2(x_2)}\right) \tag{1.4}$$

$$t_3=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+C_{x_1}}{\bar{x}_1\beta_2(x_1)+C_{x_1}}\right) \tag{1.5}$$

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$$t_4 = \bar{y}\left(\frac{\bar{x}_2\beta_2(x_2)+C_{x_2}}{\bar{X}_2\beta_2(x_2)+C_{x_2}}\right) \quad (1.6)$$

To estimate $\bar{Y}$, Singh [16] suggested a ratio-cum-product estimator as

$$t_5 = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right) \quad (1.7)$$

Singh and Tailor [15] suggested a ratio-cum-product estimator of $\bar{Y}$ utilizing the correlation coefficient between auxiliary variables as

$$t_6 = \bar{y}\left(\frac{\bar{X}_1+\rho_{x_1x_2}}{\bar{x}_1+\rho_{x_1x_2}}\right)\left(\frac{\bar{x}_2+\rho_{x_1x_2}}{\bar{X}_2+\rho_{x_1x_2}}\right) \quad (1.8)$$

Tailor et.al [17] suggested two estimators of population mean using coefficients of variation and coefficients of kurtosis of auxiliary variables as,

$$t_7 = \bar{y}\left(\frac{\bar{X}_1C_{x_1}+\beta_2(x_1)}{\bar{x}_1C_{x_1}+\beta_2(x_1)}\right)\left(\frac{\bar{x}_2C_{x_2}+\beta_2(x_2)}{\bar{X}_2C_{x_2}+\beta_2(x_2)}\right) \quad (1.9)$$

$$t_8 = \bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+C_{x_1}}{\bar{x}_1\beta_2(x_1)+C_{x_1}}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+C_{x_2}}{\bar{X}_2\beta_2(x_2)+C_{x_2}}\right) \quad (1.10)$$

To the first degree of approximation the mean squared error (MSE) of the estimators $\bar{y}_R$, $\bar{y}_P$, t_1, t_2, t_3, t_4, t_5, t_6, t_7 and t_8 respectively are

$$MSE(\bar{y}_R) = \theta\bar{Y}^2[C_y^2 + C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}] \quad (1.11)$$

$$MSE(\bar{y}_P) = \theta\bar{Y}^2[C_y^2 + C_{x_2}^2 + 2\rho_{yx_2}C_yC_{x_2}] \quad (1.12)$$

$$MSE(t_1) = \theta\bar{Y}^2[C_y^2 + \lambda_1^2C_{x_1}^2 - 2\rho_{yx_1}\lambda_1C_yC_{x_1}] \quad (1.13)$$

$$MSE(t_2) = \theta\bar{Y}^2[C_y^2 + \lambda_2^2C_{x_2}^2 + 2\rho_{yx_2}\lambda_2C_yC_{x_2}] \quad (1.14)$$

$$MSE(t_3) = \theta\bar{Y}^2[C_y^2 + \gamma_1^2C_{x_1}^2 - 2\rho_{yx_1}\lambda_1C_yC_{x_1}] \quad (1.15)$$

$$MSE(t_4) = \theta\bar{Y}^2[C_y^2 + \gamma_2^2C_{x_2}^2 + 2\rho_{yx_2}\gamma_2C_yC_{x_2}] \quad (1.16)$$

$$MSE(t_5) = \theta\bar{Y}^2[C_y^2 + C_{x_1}^2(1-2\kappa_{yx_1}) + C_{x_2}^2\{1+2(\kappa_{yx_2}-\kappa_{x_1x_2})\}] \quad (1.17)$$

$$MSE(t_6) = \theta\bar{Y}^2[C_y^2 + \mu_1C_{x_1}^2(\mu_1-2\kappa_{yx_1}) + \mu_2C_{x_2}^2\{\mu_2+2(\kappa_{yx_2}-\mu_1\kappa_{x_1x_2})\}] \quad (1.18)$$

$$MSE(t_7) = \theta\bar{Y}^2[C_y^2 + \lambda_1C_{x_1}^2(\lambda_1-2\kappa_{yx_1}) + \lambda_2C_{x_2}^2\{\lambda_2+2(\kappa_{yx_2}-\lambda_1\kappa_{x_1x_2})\}] \quad (1.19)$$

$$MSE(t_8) = \theta\bar{Y}^2[C_y^2 + \gamma_1C_{x_1}^2(\gamma_1-2\kappa_{yx_1}) + \gamma_2C_{x_2}^2\{\gamma_2+2(\kappa_{yx_2}-\gamma_1\kappa_{x_1x_2})\}] \quad (1.20)$$

where

$$\kappa_{yx_1} = \rho_{yx_1}\left(\frac{C_y}{C_{x_1}}\right),\quad \kappa_{yx_2} = \rho_{yx_2}\left(\frac{C_y}{C_{x_2}}\right),\quad \kappa_{x_1x_2} = \rho_{x_1x_2}\left(\frac{C_{x_1}}{C_{x_2}}\right),\quad C_y = \frac{S_y}{\bar{Y}},\quad \theta = \left(\frac{1}{n}-\frac{1}{N}\right),$$

$$\lambda_i = \frac{\bar{X}_iC_{x_i}}{\bar{X}_iC_{x_i}+\beta_2(x_i)},\quad \gamma_i = \frac{\bar{X}_i\beta_2(x_i)}{\bar{X}_i\beta_2(x_i)+C_{x_i}},\quad \mu_i = \frac{\bar{X}_i}{\bar{X}_i+\rho_{x_1x_2}},\quad C_{x_i} = \frac{S_{x_i}}{\bar{X}_i},\quad \rho_{yx_i} = \frac{S_{yx_i}}{S_yS_{x_i}},$$

$$S_y^2 = \frac{\sum_{j=1}^{N}(y_j-\bar{Y})^2}{(N-1)},\quad S_{x_i}^2 = \frac{\sum_{j=1}^{N}(x_{ij}-\bar{X}_i)^2}{(N-1)},\quad S_{yx_i}^2 = \frac{\sum_{j=1}^{N}(y_j-\bar{Y})(x_{ij}-\bar{X}_i)}{(N-1)},\quad i=1,2$$

Many authors, such as Kadilar and Cingi ([2], [3]), Shabbir and Gupta [13], Singh and Vishwakarma [14], Koyuncu and Kadilar ([4], [5], [6], [7]), Sanaullah *et al*. [8], Mouhamed et al. [9], Parmar et al. [11], Tailor et al [17], Yadav et al. ([19],

[20]), Onyeka et al. [10] have improved the ratio and product estimators as given in (2.1) and (2.3 for the population mean of the study variable in the stratified random sampling.

2. Proposed Estimator

Making the use of auxiliary information more appropriately, assuming that the information on co-efficient of kurtosis and median of auxiliary variables x_1 and x_2 are known, we propose the estimator as,

$$t=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+M_d(x_1)}{\bar{x}_1\beta_2(x_1)+M_d(x_1)}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+M_d(x_2)}{\bar{X}_2\beta_2(x_2)+M_d(x_2)}\right) \tag{2.1}$$

To obtain the bias and mean square error of the proposed estimator, we assume that

$\bar{y}=\bar{Y}(1+e_0)$, $\bar{x}_1=\bar{X}_1(1+e_1)$, $\bar{x}_2=\bar{X}_2(1+e_2)$ such that $E(e_0)=E(e_1)=E(e_2)=0$ and $E(e_0^2)=\theta C_y^2$, $E(e_1^2)=\theta C_{x_1}^2$, $E(e_2^2)=\theta C_{x_2}^2$, $E(e_0e_1)=\theta\rho_{yx_1}C_yC_{x_1}$, $E(e_0e_2)=\theta\rho_{yx_2}C_yC_{x_2}$, $E(e_1e_2)=\theta\rho_{x_1x_2}C_{x_1}C_{x_2}$.

Expressing the proposed estimator t in terms of e_i's, we get

$$t=\bar{Y}(1+e_0)\left(\frac{\bar{X}_1\beta_2(x_1)+M_d(x_1)}{\bar{X}_1(1+e_1)\beta_2(x_1)+M_d(x_1)}\right)\left(\frac{\bar{X}_2(1+e_2)\beta_2(x_2)+M_d(x_2)}{\bar{X}_2\beta_2(x_2)+M_d(x_2)}\right)$$

$$=\bar{Y}(1+e_0)\left(\frac{\bar{X}_1\beta_2(x_1)+M_d(x_1)}{\bar{X}_1\beta_2(x_1)+M_d(x_1)+\bar{X}_1\beta_2(x_1)e_1}\right)\left(\frac{\bar{X}_2\beta_2(x_2)+M_d(x_2)+\bar{X}_2\beta_2(x_2)e_2}{\bar{X}_2\beta_2(x_2)+M_d(x_2)}\right)$$

$$=\bar{Y}(1+e_0)(1+\eta_1e_1)^{-1}(1+\eta_2e_2)$$

$$=\bar{Y}(1+e_0)(1-\eta_1e_1+\eta_1^2e_1^2)(1+\eta_2e_2)$$

$$=\bar{Y}(1+e_0)(1-\eta_1e_1+\eta_1^2e_1^2-\eta_1\eta_2e_1e_2+\eta_2e_2)$$

$$=\bar{Y}(1+e_0-\eta_1e_1+\eta_1^2e_1^2-\eta_1\eta_2e_1e_2+\eta_2e_2-\eta_1e_0e_1+\eta_2e_0e_2)$$

or $$t-\bar{Y}=\bar{Y}(e_0-\eta_1e_1+\eta_1^2e_1^2-\eta_1\eta_2e_1e_2+\eta_2e_2-\eta_1e_0e_1+\eta_2e_0e_2), \tag{2.2}$$

where $\eta_i=\dfrac{\bar{X}_i\beta_2(x_i)}{\bar{X}_i\beta_2(x_i)+M_d(x_i)}$ $\quad i=1,2$

Taking expectation on both sides of (2.2), we get

$$E(t-\bar{Y})=\bar{Y}E(e_0-\eta_1e_1+\eta_1^2e_1^2-\eta_1\eta_2e_1e_2+\eta_2e_2-\eta_1e_0e_1+\eta_2e_0e_2)$$

$$=\bar{Y}[E(e_0)-\eta_1E(e_1)+\eta_1^2E(e_1^2)-\eta_1\eta_2E(e_1e_2)+\eta_2E(e_2)-\eta_1E(e_0e_1)+\eta_2E(e_0e_2)]$$

Substituting the values of $E(e_0)$, $E(e_1)$, $E(e_2)$, $E(e_1^2)$, $E(e_1e_2)$, $E(e_0e_1)$ and $E(e_0e_2)$ we get the bias of t as

$$B(t)=\theta\bar{Y}[\eta_1C_{x_1}^2(\eta_1-\kappa_{yx_1})+\eta_2C_{x_2}^2(\eta_2\kappa_{yx_2}-\eta_1\kappa_{x_1x_2})] \tag{2.3}$$

To find the mean squared error of the suggested estimator t up to first degree of approximation square and take expectation on both sides of (2.2). That is,

$$MSE(t)=E(t-\bar{Y})^2=\bar{Y}^2E(e_0-\eta_1e_1+\eta_2e_2)^2$$

$$=\bar{Y}^2E(e_0^2+\eta_1^2e_1^2+\eta_2^2e_2^2-2\eta_1e_0e_1+2\eta_2e_0e_2-2\eta_1\eta_2e_1e_2)$$

After substituting the values of $E(e_0^2)$, $E(e_1^2)$, $E(e_2^2)$, $E(e_0e_1)$, $E(e_0e_2)$ and $E(e_1e_2)$, we get the mean squared error of t as

$$MSE(t)=\theta\bar{Y}^2[C_y^2+\eta_1C_{x_1}^2(\eta_1-2\kappa_{yx_1})+\eta_2C_{x_2}^2\{\eta_2+2(\kappa_{yx_2}-\eta_1\kappa_{x_1x_2})\}] \tag{2.4}$$

3. Efficiency Comparison

We know that the variance of sample mean $\bar{y}$ in simple random sampling without replacement (SRSWOR) is,

$$V(\bar{y})=\theta S_y^2=\theta\bar{Y}^2C_y^2 \tag{3.1}$$

From (1.11) – (1.20), (2.4) and (3.1) we have

(i) $MSE(t)<V(\bar{y})$ if $\kappa_{yx_1}>\frac{\eta_1}{2}$ and $\kappa_{yx_2}>(\eta_1\kappa_{x_1x_2}-\frac{\eta_2}{2})$ (3.2)

(ii) $MSE(t)<MSE(\bar{y}_R)$ if $\kappa_{yx_1}<\left(\frac{1+\eta_1}{2}\right)$ and $\kappa_{yx_2}<(\eta_1\kappa_{x_1x_2}-\frac{\eta_2}{2})$ (3.3)

(iii) $MSE(t)<MSE(\bar{y}_P)$ if $\kappa_{yx_1}>\left(\frac{\eta_1}{2}-\eta_2\kappa_{x_1x_2}\right)$ and $\kappa_{yx_2}>-(\frac{1+\eta_2}{2})$ (3.4)

(iv) $MSE(t)<MSE(t_1)$ if $\kappa_{yx_2}<(\eta_1\kappa_{x_1x_2}-\frac{\eta_2}{2})$ (3.5)

(v) $MSE(t)<MSE(t_2)$ if $\kappa_{yx_2}>-(\eta_1\kappa_{x_1x_2}-\frac{\eta_2}{2})$ (3.6)

(vi) $MSE(t)<MSE(t_3)$ if either $\kappa_{yx_1}>\left(\frac{\gamma_1+\eta_1}{2}\right)$ if $\gamma_1<\eta_1$ and $\kappa_{yx_2}<(\eta_1\kappa_{x_1x_2}-\frac{\eta_2}{2})$ (3.7)

or $\kappa_{yx_1}<\left(\frac{\gamma_1+\eta_1}{2}\right)$ if $\gamma_1>\eta_1$ and $\kappa_{yx_2}<(\eta_1\kappa_{x_1x_2}-\frac{\eta_2}{2})$ (3.8)

(vii) $MSE(t)<MSE(t_4)$ if either $\kappa_{yx_2}>-\left(\frac{\gamma_2+\eta_2}{2}\right)$ if $\gamma_2>\eta_2$ and $\kappa_{yx_1}<(\frac{\eta_1}{2}-\eta_2\kappa_{x_1x_2})$ (3.9)

or $\kappa_{yx_2}<-\left(\frac{\gamma_2+\eta_2}{2}\right)$ if $\gamma_2<\eta_2$ and $\kappa_{yx_1}<(\frac{\eta_1}{2}-\eta_2\kappa_{x_1x_2})$ (3.10)

(viii) $MSE(t)<MSE(t_5)$ if either $\kappa_{yx_1}>-\left(\frac{1+\eta_2}{2}\right)$ if $\eta_2<1$ and $\kappa_{yx_2}>\left\{\frac{1+\eta_1}{2}-\frac{\kappa_{x_1x_2}(\eta_1\eta_2-1)}{\eta_1-1}\right\}$ (3.11)

or $\kappa_{yx_1}<-\left(\frac{1+\eta_2}{2}\right)$ if $\eta_2>1$ and $\kappa_{yx_2}>\left\{\frac{1+\eta_1}{2}-\frac{\kappa_{x_1x_2}(\eta_1\eta_2-1)}{\eta_1-1}\right\}$ (3.12)

(ix) $MSE(t)<MSE(t_6)$ if one of the following conditions is satisfied

$\kappa_{yx_2}<\left(\frac{\mu_1+\eta_1}{2}\right)$ if $\eta_1<\mu_1$ and $\kappa_{yx_2}>\left\{\frac{\kappa_{x_1x_2}(\eta_1\eta_2-\mu_1\mu_2)}{\eta_2-\mu_2}-\frac{\eta_2+\mu_2}{2}\right\}$ if $\eta_2<\mu_2$ (3.13)

$$\kappa_{yx_2} < \left(\frac{\mu_1+\eta_1}{2}\right) \text{if } \eta_1 < \mu_1 \text{ and } \kappa_{yx_2} < \left\{\frac{\kappa_{x_1x_2}(\eta_1\eta_2-\mu_1\mu_2)}{\eta_2-\mu_2} - \frac{\eta_2+\mu_2}{2}\right\} \text{if } \eta_2 > \mu_2 \quad (3.14)$$

$$\kappa_{yx_2} > \left(\frac{\mu_1+\eta_1}{2}\right) \text{if } \eta_1 > \mu_1 \text{ and } \kappa_{yx_2} > \left\{\frac{\kappa_{x_1x_2}(\eta_1\eta_2-\mu_1\mu_2)}{\eta_2-\mu_2} - \frac{\eta_2+\mu_2}{2}\right\} \text{if } \eta_2 < \mu_2 \quad (3.15)$$

$$\kappa_{yx_2} > \left(\frac{\mu_1+\eta_1}{2}\right) \text{if } \eta_1 > \mu_1 \text{ and } \kappa_{yx_2} < \left\{\frac{\kappa_{x_1x_2}(\eta_1\eta_2-\mu_1\mu_2)}{\eta_2-\mu_2} - \frac{\eta_2+\mu_2}{2}\right\} \text{if } \eta_2 > \mu_2 \quad (3.16)$$

$$\text{(x)} \quad MSE(t) < MSE(t_7) \text{ if } \frac{C_{x_1}^2}{C_{x_2}^2} < \frac{\lambda_2 C_{x_2}^2\{\lambda_2+2(\kappa_{yx_2}-\lambda_1\kappa_{x_1x_2})\} - \eta_2 C_{x_2}^2\{\eta_2+2(\kappa_{yx_2}-\eta_1\kappa_{x_1x_2})\}}{\eta_1-\gamma_1} \quad (3.17)$$

$$\text{(xi)} \quad MSE(t) < MSE(t_8) \text{ if } \frac{C_{x_1}^2}{C_{x_2}^2} < \frac{\gamma_2 C_{x_2}^2\{\gamma_2+2(\kappa_{yx_2}-\gamma_1\kappa_{x_1x_2})\} - \eta_2 C_{x_2}^2\{\eta_2+2(\kappa_{yx_2}-\eta_1\kappa_{x_1x_2})\}}{\eta_1-\gamma_1} \quad (3.18)$$

4. Empirical Study

The performance of the proposed estimator is assessed with that of SRSWOR sample mean, traditional Ratio-cum-Product estimators, and existing modified estimators for a certain hypothetical population of size 30, that is given as below:

Sl. No.	Y	X_1	X_2	Sl. No.	Y	X_1	X_2
1	3.8	56	12.71	16	16.9	40	2.13
2	6.0	58	4.30	17	17.4	49	2.14
3	7.3	21	2.06	18	18.3	48	2.23
4	7.8	63	11.36	19	19.7	49	2.20
5	8.9	19	6.50	20	21.0	29	2.18
6	8.9	26	2.09	21	22.4	30	2.16
7	9.3	24	1.21	22	24.1	31	2.57
8	9.5	69	2.78	23	25.0	32	12.15
9	11.5	15	1.95	24	25.2	125	8.00
10	12.5	18	1.63	25	25.9	95	2.36
11	12.6	19	13.12	26	26.3	96	8.25
12	14.2	20	12.25	27	27.6	97	2.11
13	14.1	45	2.36	28	28.9	98	2.58
14	15.0	38	2.07	29	32.7	31	2.08
15	15.5	39	2.31	30	36.2	34	2.07

The population parameters of the auxiliary variables and the constants computed from the above populations are given below:

$N = 30$	$n = 10$	$\bar{Y} = 17.5$	$\bar{X}_1 = 47.1333$
$\bar{X}_2 = 4.4637$	$\rho_{x_1y} = 0.3637$	$\rho_{x_2y} = -0.1994$	$\rho_{x_1x_2} = 0.0736$
$\beta_2(x_1) = 0.6206$	$\beta_2(x_2) = 0.2296$	$C_y = 0.4758$	$C_{x_1} = 0.6046$
$C_{x_2} = 0.8727$	$\theta = 0.0667$	$M_d(x_1) = 36$	$M_d(x_2) = 2.21$

Table 4.1. The variance of SRSWOR sample mean, mean squared errors of the existing and proposed modified estimators

$V(\bar{y})$	4.6129
$MSE(\bar{y}_R)$	7.7989
$MSE(\bar{y}_p)$	16.7563
$MSE(t_1)$	7.5756
$MSE(t_2)$	15.2646
$MSE(t_3)$	7.5865
$MSE(t_4)$	7.3175
$MSE(t_5)$	18.3606
$MSE(t_6)$	17.9278
$MSE(t_7)$	16.7655
$MSE(t_8)$	9.4541
$MSE(t)$*	4.4003

*Proposed Estimator

To see the performance of the proposed estimator $\mathbf{t}$ in comparison to $\bar{y}$, $\bar{y}_R, \bar{y}_p, t_1, t_2, t_3, t_4, t_5, t_6, t_7$ and t_8, we calculated the percent relative efficiency of proposed estimator with respect to $\bar{y}, \bar{y}_R, \bar{y}_p, t_1, t_2, t_3, t_4, t_5, t_6, t_7$ and t_8. The Percent relative efficiency (%) of the proposed estimator t with respect to the existing estimators (e) has been computed as $\mathbf{PRE(t)} = \left\{\frac{\mathbf{MSE(e)}}{\mathbf{MSE(t)}}\right\}\mathbf{X100}$ and is presented in Table 4.2.

Table 4.2. Percent relative efficiencies of proposed estimator over other estimators

Estimator	$\bar{y}$	$\bar{y}_R$	$\bar{y}_p$	t_1	t_2	t_3	t_4
PRE	104.8320	177.2350	380.7982	172.1610	346.8995	172.4094	166.2948
Estimator	t_5	t_6	t_7	t_8	t		
PRE	417.2575	407.4214	381.0089	214.8515	100.0000		

From the values of Table 4.2, it is observed that the proposed estimator is more efficient than the usual unbiased estimator $\bar{y}$, ratio estimator $\bar{y}_R$, product estimator $\bar{y}_p$ and other existing estimators $t_1, t_2, t_3, t_4, t_5, t_6, t_7,$ and t_8 with considerable gain in efficiency.

5. Conclusions

In this paper, a ratio-cum-product estimator for the estimation of finite population mean with known coefficient of kurtosis and median of auxiliary characters has been proposed. The bias and mean squared error of the proposed estimator are obtained and compared with that of the SRSWOR sample mean, ratio estimator, product estimator, Singh and Tailor [15] and Singh [16], Upadhyaya and Singh [18] and Parmar et.al [11] estimators. Further, we have derived the conditions for which the proposed estimator is more efficient than the existing estimators. We have also assessed the performance of the proposed estimator with that of the existing estimators for a hypothetical population. It is observed that the mean squared error of the proposed estimator is less than the mean squared errors of the existing estimators. Hence, we strongly recommend that the proposed estimator may be preferred over the existing estimators for the use of practical application.

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