

Use of Correlation Coefficient and Quartiles of Auxiliary Variable for Improved Estimation of Population Variance

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Abstract The present paper deals with the estimation of population variance using correlation coefficient and quartiles of an auxiliary variable under simple random sampling scheme. Up to the first order of the approximation, the bias and the mean square error of the proposed estimator have been obtained. The optimum value of the characterizing scalar kappa has been obtained and for this optimum value of kappa, the minimum mean square error of the proposed estimator has also been obtained up to the first order of approximation. A comparison of the proposed estimator has been made with existing estimators of population variance under simple random sampling scheme. An empirical study is also carried out to justify the theoretical findings. An improvement over existing estimators has been shown in the sense of having lesser mean square error.

Keywords Ratio estimator, Quartiles, Bias, Mean squared error, Efficiency

1. Introduction

In survey sampling, auxiliary information is used for enhancing the efficiency of the estimate of the parameters of the population for the characteristic under study. Auxiliary information is supplied by the auxiliary variable which is highly positively or negatively correlated with the main variable under study. Ratio type estimators are used when the variables X and Y are positively correlated and the line of regression of y on x passes through origin, while the product type estimators are used when X and Y are negatively correlated to each other otherwise regression estimators are used. In the present study, we are dealing with only the positive correlation.

Let $(x_i, y_i), i = 1, 2, \dots, n$ be the n pair of observations for the auxiliary and study variables, respectively from the population of size N using simple random sampling without replacement. Let \bar{X} and \bar{Y} be the population means of auxiliary and study variables respectively and \bar{x} and \bar{y} be the respective sample means.

Following are the notations which have been used in this manuscript and are already being discussed by Subramani and Kumarpandiyani (2015) as:

N : Size of the population

n : Size of the sample

Y : Study variable

X : Auxiliary variable

ρ : Correlation coefficient between X and Y

\bar{Y}, \bar{X} : Population means

\bar{y}, \bar{x} : Sample means

S_y^2, S_x^2 : Population variances

s_y^2, s_x^2 : Sample variances

C_y, C_x : Coefficient of variations

$$\gamma = \frac{(1-f)}{n}$$

$$f = \frac{n}{N}$$

$$\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2} \mu_{02}^{s/2}}$$

$$\mu_{rs} = \frac{1}{N} \sum_{i=1}^N (y_i - \bar{Y})^r (x_i - \bar{X})^s$$

$$\beta_{1(x)} = \frac{\mu_{03}^2}{\mu_{02}^3} : \text{Skewness of the auxiliary variable}$$

$$\beta_{2(x)} = \frac{\mu_{04}}{\mu_{02}^2} : \text{Kurtosis of the auxiliary variable}$$

Q_1 : First quartile of the auxiliary variable

Q_3 : Third quartile of the auxiliary variable

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Published online at <http://journal.sapub.org/ajor>

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Q_r : Inter quartile range of the auxiliary variable

Q_d : Semi quartile range of the auxiliary variable

Q_a : Semi quartile average of the auxiliary variable

$B(.)$: Bias of the estimator

$V(.)$: Variance of the estimator

$MSE(.)$: Mean squared error of the estimator

$PRE(t_e, t_p) = \frac{MSE(t_e)}{MSE(t_p)} * 100$: Percentage relative efficiency of the estimator t_p over t_e

The appropriate estimator of the population variance is the sample variance defined as

$$t_0 = s_y^2, \quad (1.1)$$

which is an unbiased estimator of the population variance and its variance up to the first order of approximation is given as

$$V(t_0) = \frac{1-f}{n} S_y^4 (\lambda_{40} - 1), \quad (1.2)$$

$$\text{where } \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2} \mu_{02}^{s/2}}, \quad \mu_{rs} = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^r (X_i - \bar{X})^s$$

$$\text{and } f = \frac{n}{N}.$$

Isaki (1983) utilizing the auxiliary information, proposed the following classical ratio estimator for the population variance as

$$t_R = s_y^2 \left(\frac{S_x^2}{s_x^2} \right), \quad (1.3)$$

where

$$s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2,$$

$$S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2, \quad \bar{X} = \frac{1}{N} \sum_{i=1}^N X_i, \quad \bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i,$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i.$$

The expressions for the Bias and Mean Square Error (MSE) of the estimator in (1.3) up to the first order of approximation respectively are as

$$B(t_R) = \frac{(1-f)}{n} S_y^2 [(\lambda_{40} - 1) - (\lambda_{22} - 1)], \quad (1.4)$$

$$MSE(t_R) = \frac{(1-f)}{n} S_y^4 [(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)], \quad (1.5)$$

(Vide Kendall and Stuart (1977))

In the series of improvement, many authors have proposed different estimators of population variance utilizing the parameters of the auxiliary variable. Following table-1 which was also given by Subramani and Kumarpandiyam (2012) represents different estimators along with their bias, mean square error and constant.

Where symbols have their usual meanings. C_x is the coefficient of variation of auxiliary variable, $\beta_{2(x)}$ is the coefficient of kurtosis, Q_i ($i=1, 2, 3$) are the quartiles of the auxiliary variable. Q_r , Q_d and Q_a are the functions of quartiles defined by,

$$Q_r = Q_3 - Q_1, \quad Q_d = (Q_3 - Q_1)/2, \quad Q_a = (Q_3 + Q_1)/2.$$

Thus, in general the mean square errors of the estimators given in above table may be written as,

$$MSE(\hat{S}_i^2) = \lambda S_y^4 [(\beta_{2(y)} - 1) + R_i^2 (\beta_{2(x)} - 1) - 2R_i (\lambda_{22} - 1)] \quad (i = 1, 2, \dots, 10) \quad (1.6)$$

The latest references in the series of improvement can be made of Shukla *et al.* (2015) and Yadav *et al.* (2015a, 2015b, 2016).

Table 1. Bias, MSE and Constants of different estimators

Estimator	Bias	MSE	Constant
$\hat{S}_1^2 = s_y^2 \left[\frac{S_x^2 + C_x}{s_x^2 + C_x} \right]$ Kadilar and Cingi (2006a)	$\lambda S_y^2 R_1 \left[R_1 (\beta_{2(x)} - 1) - (\lambda_{22} - 1) \right]$	$\lambda S_y^4 \left[(\beta_{2(y)} - 1) + R_1^2 (\beta_{2(x)} - 1) - 2R_1 (\lambda_{22} - 1) \right]$	$R_1 = \frac{S_x^2}{S_x^2 + C_x}$
$\hat{S}_2^2 = s_y^2 \left[\frac{S_x^2 + \beta_{2(x)}}{s_x^2 + \beta_{2(x)}} \right]$ Upadhyaya and Singh (1999)	$\lambda S_y^2 R_2 \left[R_2 (\beta_{2(x)} - 1) - (\lambda_{22} - 1) \right]$	$\lambda S_y^4 \left[(\beta_{2(y)} - 1) + R_2^2 (\beta_{2(x)} - 1) - 2R_2 (\lambda_{22} - 1) \right]$	$R_2 = \frac{S_x^2}{S_x^2 + \beta_{2(x)}}$

$\hat{S}_3^2 = s_y^2 \left[\frac{S_x^2 \beta_{2(x)} + C_x}{s_x^2 \beta_{2(x)} + C_x} \right]$ <p>Kadilar and Cingi (2006b)</p>	$\lambda S_y^2 R_3 \begin{bmatrix} R_3 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_3^2 (\beta_{2(x)} - 1) \\ -2R_3 (\lambda_{22} - 1) \end{bmatrix}$	$R_3 = \frac{S_x^2 \beta_{2(x)}}{S_x^2 \beta_{2(x)} + C_x}$
$\hat{S}_4^2 = s_y^2 \left[\frac{S_x^2 C_x + \beta_{2(x)}}{s_x^2 C_x + \beta_{2(x)}} \right]$ <p>Kadilar and Cingi (2006b)</p>	$\lambda S_y^2 R_4 \begin{bmatrix} R_4 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_4^2 (\beta_{2(x)} - 1) \\ -2R_4 (\lambda_{22} - 1) \end{bmatrix}$	$R_4 = \frac{S_x^2 C_x}{S_x^2 C_x + \beta_{2(x)}}$
$\hat{S}_5^2 = s_y^2 \left[\frac{S_x^2 + Q_1}{s_x^2 + Q_1} \right]$ <p>Subramani and Kumarpandiyan (2012)</p>	$\lambda S_y^2 R_5 \begin{bmatrix} R_5 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_5^2 (\beta_{2(x)} - 1) \\ -2R_5 (\lambda_{22} - 1) \end{bmatrix}$	$R_5 = \frac{S_x^2}{S_x^2 + Q_1}$
$\hat{S}_6^2 = s_y^2 \left[\frac{S_x^2 + Q_3}{s_x^2 + Q_3} \right]$ <p>Subramani and Kumarpandiyan (2012)</p>	$\lambda S_y^2 R_6 \begin{bmatrix} R_6 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_6^2 (\beta_{2(x)} - 1) \\ -2R_6 (\lambda_{22} - 1) \end{bmatrix}$	$R_6 = \frac{S_x^2}{S_x^2 + Q_3}$
$\hat{S}_7^2 = s_y^2 \left[\frac{S_x^2 + Q_r}{s_x^2 + Q_r} \right]$ <p>Subramani and Kumarpandiyan (2012)</p>	$\lambda S_y^2 R_7 \begin{bmatrix} R_7 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_7^2 (\beta_{2(x)} - 1) \\ -2R_7 (\lambda_{22} - 1) \end{bmatrix}$	$R_7 = \frac{S_x^2}{S_x^2 + Q_r}$
$\hat{S}_8^2 = s_y^2 \left[\frac{S_x^2 + Q_d}{s_x^2 + Q_d} \right]$ <p>Subramani and Kumarpandiyan (2012)</p>	$\lambda S_y^2 R_8 \begin{bmatrix} R_8 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_8^2 (\beta_{2(x)} - 1) \\ -2R_8 (\lambda_{22} - 1) \end{bmatrix}$	$R_8 = \frac{S_x^2}{S_x^2 + Q_d}$
$\hat{S}_9^2 = s_y^2 \left[\frac{S_x^2 + Q_a}{s_x^2 + Q_a} \right]$ <p>Subramani and Kumarpandiyan (2012)</p>	$\lambda S_y^2 R_9 \begin{bmatrix} R_9 (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_9^2 (\beta_{2(x)} - 1) \\ -2R_9 (\lambda_{22} - 1) \end{bmatrix}$	$R_9 = \frac{S_x^2}{S_x^2 + Q_a}$
$\hat{S}_{10}^2 = s_y^2 \left[\frac{S_x^2 \rho + Q_3}{s_x^2 \rho + Q_3} \right]$ <p>Khan and Shabbir (2013)</p>	$\lambda S_y^2 R_{10} \begin{bmatrix} R_{10} (\beta_{2(x)} - 1) \\ -(\lambda_{22} - 1) \end{bmatrix}$	$\lambda S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + R_{10}^2 (\beta_{2(x)} - 1) \\ -2R_{10} (\lambda_{22} - 1) \end{bmatrix}$	$R_{10} = \frac{S_x^2 \rho}{S_x^2 \rho + Q_3}$

2. Proposed Estimator

Motivated by Prasad (1989) and Khan and Shabbir (2013), we propose an efficient ratio estimator of population variance as

$$\hat{S}_{YM}^2 = \kappa S_y^2 \left[\frac{S_x^2 \rho + Q_3}{S_x^2 \rho + Q_3} \right], \quad (2.1)$$

where κ is a suitable constant to be determined such that the mean square error of \hat{S}_{YM}^2 is minimum.

In order to study the large sample properties of the proposed estimator, \hat{S}_{YM}^2 , we define $S_y^2 = S_y^2(1 + \varepsilon_0)$ and $S_x^2 = S_x^2(1 + \varepsilon_1)$ with $E(\varepsilon_i) = 0$ for $(i = 0, 1)$.

In case of simple random sampling without replacement, ignoring finite population correction term, the following

expectations could be obtained either directly or by the method due to Kendall and Stuart (1977) as

$$E(\varepsilon_0^2) = \frac{1-f}{n}(\lambda_{40} - 1), \quad E(\varepsilon_1^2) = \frac{1-f}{n}(\lambda_{04} - 1),$$

$$E(\varepsilon_0 \varepsilon_1) = \frac{1-f}{n}(\lambda_{22} - 1).$$

Expressing \hat{S}_{YM}^2 in terms of ε_i 's ($i = 0, 1$), we have

$$\hat{S}_{YM}^2 = \kappa S_y^2 (1 + e_0)(1 + R_{10}e_1)^{-1}$$

After simplifying and retaining terms up to the first order of approximation, we have:

$$\hat{S}_{YM}^2 = \kappa S_y^2 (1 + e_0 - R_{10}e_1 - R_{10}e_0e_1 + R_{10}^2e_1^2)$$

Subtracting S_y^2 on both the sides, we obtain,

$$\hat{S}_{YM}^2 - S_y^2 = \kappa S_y^2 (1 + e_0 - R_{10}e_1 - R_{10}e_0e_1 + R_{10}^2e_1^2) - S_y^2 \quad (2.2)$$

Taking expectation on both sides of (2.2), we have the bias of proposed estimator \hat{S}_{YM}^2 as:

$$B(\hat{S}_{YM}^2) = \lambda \kappa S_y^2 [R_{10}^2(\lambda_{04} - 1) - R_{10}(\lambda_{22} - 1)] + S_y^2(\kappa - 1) \quad (2.3)$$

where $\lambda = \frac{(1-f)}{n}$.

The mean squared error of the proposed estimator \hat{S}_{YM}^2 is obtained by squaring both sides of (2.2), simplifying and taking expectation on both sides, up to the first order of approximation as,

$$MSE(\hat{S}_{YM}^2) = S_y^4 [\kappa^2 \lambda (\lambda_{40} - 1) + (3\kappa^2 - 2\kappa) R_{10}^2 \lambda (\lambda_{04} - 1) - 2(2\kappa^2 - \kappa) R_{10} \lambda (\lambda_{22} - 1) + (\kappa - 1)^2] \quad (2.4)$$

$MSE(\hat{S}_{YM}^2)$ is minimum for,

$$\kappa = \frac{A}{B} \quad (2.5)$$

where,

$$A = 1 + R_{10}^2 \lambda (\lambda_{04} - 1) - R_{10} \lambda (\lambda_{22} - 1) \text{ and}$$

$$B = 1 + \lambda (\lambda_{40} - 1) + 3R_{10}^2 \lambda (\lambda_{04} - 1) - 4R_{10} \lambda (\lambda_{22} - 1)$$

The minimum MSE of the estimator \hat{S}_{YM}^2 for this optimum value of κ is:

$$MSE_{\min}(\hat{S}_{YM}^2) = S_y^4 \left[1 - \frac{A^2}{B} \right] \quad (2.6)$$

Here, we pass an important remark that the value of the unknown parameters involved in A and B can be obtained from the previous surveys or the experience gathered in due course of time, for instance, vide Murthy (1967), Reddy (1973, 1974), Srivenkataramana & Tracy (1980), Singh & Vishwakarma (2008), Singh & Kumar (2008) and Singh & Karpe (2010). If the unknown parameters in A and B are replaced by their estimates, then we obtain the same minimum mean squared error as in (2.6).

3. Efficiency Comparison

From (2.6) and (1.2), we have:

$$V(t_0) - MSE_{\min}(\hat{S}_{YM}^2) = S_y^2 \left[1 - \frac{A^2}{B} - \lambda(\lambda_{40} - 1) \right] > 0 \quad \text{if} \quad \frac{A^2}{B} + \lambda(\lambda_{40} - 1) < 1 \quad (3.1)$$

From (2.6) and (1.5), we have:

$$MSE(t_R) - MSE_{\min}(\hat{S}_{YM}^2) = S_y^2 \left[1 - \frac{A^2}{B} - \lambda\{(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)\} \right] > 0$$

$$\text{if} \quad \frac{A^2}{B} + \lambda\{(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1)\} < 1 \quad (3.2)$$

From (2.5) and (1.6), we have:

$$MSE(\hat{S}_i^2) - MSE_{\min}(\hat{S}_{YM}^2) = S_y^2 \left[1 - \frac{A^2}{B} - \lambda\{(\lambda_{40} - 1) + R_i^2(\lambda_{04} - 1) - 2R_i(\lambda_{22} - 1)\} \right] > 0, \quad (i = 1, 2, \dots, 10)$$

$$\text{if} \quad \frac{A^2}{B} + \lambda\{(\lambda_{40} - 1) + R_i^2(\lambda_{04} - 1) - 2R_i(\lambda_{22} - 1)\} < 1, \quad (3.3)$$

4. Numerical Illustration

To justify the theoretical findings of different estimators, we have considered the following real populations.

Population-1: Italian bureau for the environment protection-APAT Waste 2004

Y: Total amount (tons) of recyclable-waste collection in Italy in 2003.

X: Total amount (tons) of recyclable-waste collection in Italy in 2002.

$N = 103$ (Cities), $n = 40$, $\bar{Y} = 626.2123$, $\bar{X} = 557.1909$, $\rho = 0.9936$, $S_y = 913.5498$, $C_y = 1.4588$,

$S_x = 818.1117$, $C_x = 1.4683$, $\lambda_{04} = 37.3216$, $\lambda_{40} = 37.1279$, $\lambda_{22} = 37.2055$, $Q_1 = 142.9950$,

$Q_3 = 665.6250$, $Q_r = 522.6300$, $Q_d = 261.3150$, $Q_a = 404.3100$

Population-2: Italian bureau for the environment protection-APAT Waste 2004

Y: Total amount (tons) of recyclable-waste collection in Italy in 2003.

X: Number of inhabitants in 2003.

$N = 103$, $n = 40$, $\bar{Y} = 62.6212$, $\bar{X} = 556.5541$, $\rho = 0.7298$, $S_y = 91.3549$, $C_y = 1.4588$,

$S_x = 610.1643$, $C_x = 1.0963$, $\lambda_{04} = 17.8738$, $\lambda_{40} = 37.1279$, $\lambda_{22} = 17.2220$, $Q_1 = 259.3830$,

$Q_3 = 628.0235$, $Q_r = 368.6405$, $Q_d = 184.3293$, $Q_a = 443.7033$.

Population-3: Murthy (1967)

Y: Output for 80 factories in a region.

X: Fixed capital.

$N = 80$, $n = 20$, $\bar{Y} = 51.8264$, $\bar{X} = 11.2646$, $\rho = 0.9413$, $S_y = 18.3549$, $C_y = 0.3542$,

$S_x = 8.4563$, $C_x = 0.7507$, $\lambda_{04} = 2.8664$, $\lambda_{40} = 2.2667$, $\lambda_{22} = 2.2209$, $Q_1 = 5.1500$,

$Q_3 = 16.975$, $Q_r = 11.825$, $Q_d = 5.9125$, $Q_a = 11.0625$.

Population-4: Singh and Chaudhary (1986)

$N = 70$, $n = 25$, $\bar{Y} = 96.7000$, $\bar{X} = 175.2671$, $\rho = 0.7293$, $S_y = 60.7140$, $C_y = 0.6254$,

$$S_x = 140.8572, C_x = 0.8037, \lambda_{04} = 7.0952, \lambda_{40} = 4.7596, \lambda_{22} = 4.6038, Q_1 = 80.1500, \\ Q_3 = 225.0250, Q_r = 144.8750, Q_d = 72.4375, Q_a = 152.5875.$$

Table 2. Bias and Mean square error of different estimators

Estimator	Bias				MSE			
Population	I	II	II	IV	I	II	II	IV
\hat{S}_1^2	2420.6810	135.9827	10.4399	364.3702	67038384403	35796605	3850.1552	1415839
\hat{S}_2^2	2379.9609	135.8179	9.2918	363.9722	670169790	35796503	3658.4051	1414994
\hat{S}_3^2	2422.3041	135.9929	10.7222	364.4139	670393032	35796611	3898.5560	1415931
\hat{S}_4^2	2393.4791	135.8334	8.8117	363.8627	670240637	35796512	3580.8342	1414762
\hat{S}_5^2	2259.9938	133.4494	8.1749	359.3822	669558483	35795045	3480.5516	1427990
\hat{S}_6^2	1667.7818	129.8456	3.9142	350.4482	667000531	35792955	2908.6518	1408858
\hat{S}_7^2	1829.6315	132.3799	5.5038	355.3634	667623576	35794395	3098.4067	1419946
\hat{S}_8^2	2125.7591	134.1848	7.8275	359.8641	668911625	35795495	3427.1850	1429077
\hat{S}_9^2	1963.6570	131.6458	5.7705	354.8875	668182833	35793951	3133.3256	1418424
\hat{S}_{10}^2	1663.3086	127.6040	3.6276	348.1975	666910707	35791562	2878.5603	1398150
\hat{S}_{YM}^2	-475.706	-196.716	-6.168	-193.893	403977243	16417438	2078.2435	714592

Table 3. Percentage Relative Efficiency (PRE) of different estimators with respect to $t_0 = s_y^2$

Estimator	PRE			
	Pop-I	Pop-II	Pop-III	Pop-IV
$t_0 = s_y^2$	100.00	100.00	100.00	100.00
\hat{S}_1^2	573.93	107.48	140.03	92.76
\hat{S}_2^2	574.12	107.48	147.37	92.82
\hat{S}_3^2	573.92	107.48	138.30	92.76
\hat{S}_4^2	574.05	107.48	150.57	92.84
\hat{S}_5^2	574.63	107.49	154.91	91.98
\hat{S}_6^2	576.84	107.49	185.36	93.23
\hat{S}_7^2	576.30	107.49	174.01	92.50
\hat{S}_8^2	575.19	107.49	157.32	91.91
\hat{S}_9^2	575.82	107.49	172.07	92.60
\hat{S}_{10}^2	576.91	107.50	187.30	93.94
\hat{S}_{YM}^2	952.41	234.35	259.43	183.80

5. Observation and Conclusions

Table-1 shows the Bias, MSE and Constants for previously existing estimators given by different researchers. Proposed estimator which has been subjected to comparisons with previous estimators nothing but is a ratio estimator. In this paper, we have been able to develop more efficient estimator whose bias is significantly **far much less** as compared to previous estimators as evidently presented in table-2. Similarly, MSE is also **far much less** as compared to previous estimators given in the table-2. Moreover, table-3 shows that relative efficiency is comparatively much higher. Thus, we can finally conclude with passing remarks that from theoretical discussions in section-3 and the results in table-2 and 3, we infer that the proposed estimator is much better than the previously existing estimators of population mean in simple random sampling scheme, therefore proposed estimator should be preferred for the estimation of population variance.

ACKNOWLEDGMENTS

The authors are very much thankful to the editor of American Journal of Operational Research and the anonymous referees for critically examining the manuscript and giving the valuable suggestions to improve the manuscript in present form.

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