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A Probabilistic Inventory Model for Deteriorating Items with Ramp Type Demand Rate under Inflation

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Abstract In this paper we developed a general inventory model for deteriorating items with constant deterioration rate and ramp type demand under stock dependent consumption rate. Shortages are allowed and partially backlogged. The backlogging rate of unsatisfied demand is assumed as a decreasing function of waiting time. The effect of inflation and time value of money is introduced into the model. The deterioration of the product is assumed as probabilistic to make the model more realistic one. The partially backorder rate is assumed as a decline exponential function of waiting time. The purpose of our study is to determine the optimal replenishment policy for maximize the total profit. Numerical examples are also given to demonstrate the developed model.

Keywords Probabilistic deterioration, Partial-backlogging, Shortage, Ramp type demand rate, Inflation and stock dependent consumption rate

1. Introduction

In real life situations there is a realistic phenomenon of the deterioration of goods in a storage period. Deterioration is defined as damage, decay or spoilage of the items that are stored for future use always loose part of their value with passage of time, so deterioration cannot be avoided in any business organization. Covert and Philip [1973] developed an inventory model for exponentially decaying items. Buzacott [1975] developed an economic order quantity model and first time he assumed the effect of inflation in his inventory model. Donaldson [1979] determine an optimal replenishment policy for finding the computational solution of the inventory model of deteriorating items. Deb and Chaudhuri [1987] extended the Donaldson [1979] model by allowing shortages.

In classical inventory models the demand rate is either constant, linearly increasing, decreasing, exponentially increasing, decreasing function of time or stock dependent. Later it has been observed that in the super market the above demand pattern do not precisely depict the demand of certain items such as newly launched products, fashionable garments, hardware devices, cosmetics, electronic items, mobiles etc. increases with time and after some time it becomes constant. In such cases the concept of ramp-type demand is introduced. Ramp-type demand is a demand which increases up to a certain time and after a certain time it becomes constant. Dutta and Pal [1991] discussed the effect of inflation and time value of money in his inventory model by considering a linearly time dependent demand rate. Mandal and Pal [1998] were the first authors who discussed the ramp-type demand rate in his order level inventory model for deteriorating items. Chang and Dye [1999] developed an EOQ model for deteriorating items in which backlogging rate is reciprocal of a linear function of waiting time. Wu, K.S., Ouyang, L.Y. (2000) A replenishment policy for deteriorating items with ramp-type demand rate. Wu [2001] formulated an EOQ model for weibull deteriorating items with ramp-type demand rate and partial backlogging. Teng et al. [2002] determines an optimal replenishment policy for deteriorating items with time varying demand rate by allowing shortage. Manna, S.K. and Choudhuri, K.S. (2003) An EOQ model with ramp-type demand rate, time dependent deterioration rate, unit production cost and shortages. Jaggi et al. [2006] determine an optimal order policy for deteriorating items under inflation and discounted cash flow approach over a finite planning horizon. Dye et al. [2006] considered an inventory model for deteriorating items with time varying demand rate and shortage. Deng et al. (2007) A note on inventory models for deteriorating items with ramp-type demand rate. Panda et al. (2008) Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand. Kun-Shan et al. (2008) developed a retailer's optimal ordering policy for deteriorating items with ramp-type demand under stock-dependent consumption rate. Chung and Wee [2008] formulated a policy pricing integrated production inventory model for deteriorating items by considering imperfect production, inspection planning and stock dependent

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demand rate. Skouri et al. [2009] developed an inventory model for weibull deteriorating items with ramp-type demand rate and shortage. Cardenas Barron [2009] proposed an inventory model with a rework process at a single stage manufacturing system with planned backorders. Jain, S. and Kumar, M. (2010) formulated an EOQ model for three parameter weibull deteriorating items with ramp-type demand and shortages. Sana [2010] developed a multi item EOQ model for both deteriorating and ameliorating items. Sarkar et al. [2010] described a production policy to find out an optimal safety stock, production lot size and reliability parameters. Sana [2010] formulated a lot size inventory model for deteriorating items with time varying deterioration rate and partial backlogging. Chang, C.T. (2011) developed an inventory model for weibull deteriorating items with ramp-type demand rate and partial backlogging. Wee et al. [2011] determines an optimal replenishment cost of life analysis of deteriorating green products. Cardenas- Barron [2011] considered an inventory model with shortage and find out an approximate solution by using basic algebraic procedure. Sarkar and Moon [2011] extended the economic production quantity model in an imperfect production system. Sett et al. [2012] formulated a two warehouse inventory model for time varying deteriorating items and stock dependent demand rate. Ahmad et al. (2013) developed an inventory model with ramp-type demand rate, partial backlogging and general deterioration rate. Cardenas et al. [2013] determines an improved solution procedure of the replenishment policy for the EMQ model with rework and multiple shipments. Sarkar and Majumder [2013] developed an integrated vendor buyer supply chain inventory model with the reduction of vendors set up cost. Cardenas et al. [2013] derived two easy and improved algorithms to determine jointly the replenishment lot size and number of shipments for an EPQ model. Karmakar, B. and Choudhuri, K.D. (2014) developed an inventory model for deteriorating items with ramp-type demand rate, partial backlogging and time varying holding cost. Sarkar et al. [2014] developed an inventory model with trade credit policy and variable deterioration rate for fixed life time products. Sarkar et al. [2014] developed an EMQ model with price and time dependent demand under inflation. Sarkar et al. [2015] derived a continuous review manufacturing inventory model with set up cost reduction, quality improvement and a service level constraint. Kumar et al. (2015) developed a two warehouse partially backlogging inventory model for deteriorating items with ramp-type demand rate.

The tables 1 and 2 show that the variation of total profit when $t_1 \le \mu$ with respect to the change in deterioration parameter θ and inflation parameter ρ, the tables 3 and 4 show that the variation of total profit when $t_1 \ge \mu$ with respect to the change in deterioration parameter θ and inflation parameter ρ .The figures I and II are correspond to the developed model in two cases $t_1 \le \mu$ and $t_1 \ge \mu$. The figures 3 and 4 show that the variation of total profit when $t_1 \le \mu$ with respect to the change in deterioration parameter θ and inflation parameter ρ and the figures 5 and 6 show that the variation of total profit when $t_1 \ge \mu$ with respect to the change in deterioration parameter θ and inflation parameter ρ.

In this paper we developed a partially backlogging inventory model for deteriorating items with probabilistic deterioration rate and ramp type demand under stock dependent consumption rate. Shortages are allowed and completely backlogged, the backlogging rate of unsatisfied demand is assumed as a function of waiting time. The effect of inflation and time value of money is introduced in the model. The ramp type demand is a demand which increases up to a certain time and after that it becomes stable or constant. In the case of real estate, electronics items, fashionable garments, cosmetics, hardware devices, food grains etc. the demand is unknown with certainty while the supply is dependent on consumers need so in this paper we consider an inventory model with deterministic demand.

Although there are so many research papers related to the ramp-type demand rate of newly launched products in the super market. This paper deals with the same type problem and the purpose of our study is to provide an approximate solution procedure for an optimal replenishment policy to maximizing the total profit.

2. Assumptions and Notations

We consider the following assumptions and notations corresponding to the developed model.

1. The ramp type demand rate is $R(t) = ae^{b[t-(t-\mu)H(t-\mu)]}$, where a is the initial demand rate and b a constant governing exponential demand rate and $H(t-\mu)$ is the Heaviside unit step function of time defined by

$$H(t-\mu) = \begin{cases} 1, t \ge \mu \\ 0, t \prec \mu \end{cases}$$

2. The selling rate is influenced by the demand rate and on hand inventory stock by,

$$S(t) = \begin{cases} R(t) + kI(t), & I(t) \succ 0 \\ R(t), & I(t) \le 0 \end{cases}$$

3. Where, k is a positive constant.
4. θ is the probabilistic deterioration rate.
5. δ is the backlogging parameter.
6. O_C is the ordering cost per order.
7. h_C is the holding cost per unit
8. s_C is the shortage cost.
9. p_C is the purchase cost per unit.
10. c_1 is the lost sell cost per unit.
11. c_2 is the selling price per unit.

12. r is the discount rate represent the time value of money.
13. i is the inflation rate per unit time.
14. $\rho = r - i$ is the discount rate minus inflation rate.
15. μ is the parameter of ramp-type demand rate.
16. $I(t)$ is the inventory level at any time in [0,T].
17. t_1 is the time of zero inventory level.
18. T is the length of each ordering cycle.
19. Q is the order quantity per cycle.
20. The replenishment rate is infinite.
21. Shortages are allowed and partially backlogged.
22. The lead time is zero.
23. $TP_1(\mu, t_1, T)$ is the total profit per unit time for model I.
24. $TP_2(\mu, t_1, T)$ is the total profit per unit time for model II.

3. Mathematical Formulation

Suppose an inventory system consists the maximum inventory level at time t=0 in the beginning of each cycle. During the time interval $[0, t_1]$ the inventory level decreases due to both demand and deterioration and becomes zero at $t = t_1$. The shortage starts at $t = T_1$ and shortage interval is the end of current order cycle. During the shortage interval $[t_1, T]$ shortages are allowed and partially backlogged. The unsatisfied demand is backlogged at rate of $\delta(T-t)$, where t is the waiting time and the positive constant δ is the backlogging parameter.

The instantaneous inventory level at any time t in $[0, T]$ is governed by the following differential equations

$$\frac{dI}{dt} = -S(t), \quad 0 \le t \le t_1 \tag{1}$$

with boundary condition

$$I(0) = I_{\max}$$

$$\frac{dI}{dt} = -S(t)\delta(T-t), \quad t_1 \le t \le T \tag{2}$$

with boundary condition

$$I(t_1) = 0$$

The solution of these equations is affected by the selling rate. Now we discuss the following two cases $t_1 \le \mu$ and $t_1 \ge \mu$

Case I When $t_1 \le \mu$ then in the interval $[0, T]$ the selling rate $S(t) = \begin{cases} R(t) + kI(t), & I(t) \succ 0 \\ R(t), & I(t) \le 0 \end{cases}$ is defined as

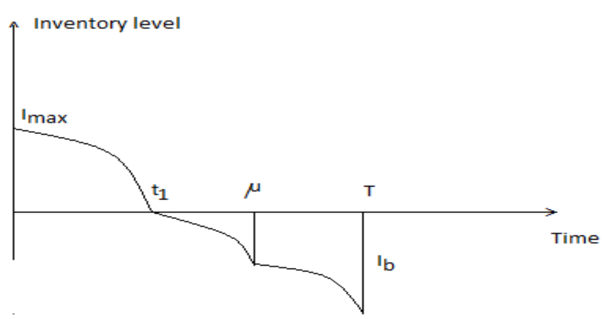

Figure 1. For case I

$$S(t) = \begin{cases} ae^{bt} + kI(t), & 0 \le t \le t_1 \\ ae^{bt}, & t_1 \le t \le \mu \\ ae^{b\mu}, & \mu \le t \le T \end{cases}$$

Using $S(t)$ in equations 1 and 2

$$\frac{dI}{dt} = -[ae^{bt} + kI(t)], \quad 0 \le t \le t_1 \tag{3}$$

with boundary condition

$$I(0) = I_{\max}$$

$$\frac{dI}{dt} = -ae^{bt}\delta(T-t), \quad t_1 \le t \le \mu \tag{4}$$

with boundary condition

$$I(t_1) = 0$$

$$\frac{dI}{dt} = -ae^{bt}\delta(T-t), \quad \mu \le t \le T \tag{5}$$

with boundary condition

$$I(T) = -q$$

The equation (3) can also be written as

$$\frac{dI}{dt} + kI(t) = -ae^{bt}, \quad 0 \le t \le t_1$$

For the solution of equation (3) the integrating factor is

$I.F. = e^{\int kdt}$ and the solution of equation (3) is

$$I*(I.F.) = \int (-ae^{bt})*I.F.dt$$

The solution of equation (3) is

$$I = -a[t + \frac{(b-\theta-k)}{2}t^2 - \frac{(\theta+k)(b+\theta+k)}{2}t^3] + I_{\max}\{1-(\theta+k)t\}, \quad 0 \le t \le t_1 \tag{6}$$

For the solution of equation (4) it can be written as

$$I = -\int_{t_1}^{\mu} ae^{bt}\delta(T-t)dt,$$

The solution of equation (4) is

$$I = a\delta[t^2 - tt_1], \quad t_1 \le t \le \mu \tag{7}$$

For the solution of equation (5) it can be written as

$$I = -\int_{\mu}^{T} ae^{bt}\delta(T-t)dt,$$

The solution of equation (5) is

$$I = [\frac{a\delta(1+b\mu)}{2}\{T^2 - 2Tt + t^2\} - q], \quad \mu \le t \le T \tag{8}$$

Using the condition $I(t_1) = 0$ and from the equation (6) we obtain

$$I_{\max} = a[t_1 + \frac{(b+\theta+k)}{2}t_1^2 - (\theta+k)^2 t_1^3] \ , \tag{9}$$

Using $I_{\max}$ in the equation (6) we obtain the solution of equation (3)

$$I = a[t_1 + \frac{(b+\theta+k)}{2}t_1^2 - (\theta+k)tt_1 - (\theta+k)^2 t_1^3 - \frac{(\theta+k)(b+\theta+k)}{2}t_1^2] - a[t + \frac{(b-\theta-k)}{2}t^2 - \frac{(\theta+k)(b+\theta+k)}{2}t^3], \quad 0 \le t \le t_1 \tag{6A}$$

When $t = \mu$ then from the equations (7) and (8)

$$q = \frac{a\delta}{2}[T^2 - 2\mu T - \mu^2 + \mu t_1 + b\mu^3 + b\mu T^2 - 2bT\mu^2], \tag{10}$$

Using the value of q in equation (8) we obtain the solution of equation (5)

$$I = \frac{a\delta}{2}[t^2 + \mu^2 + 2\mu T - 2\mu t_1 - 2tT - b\mu^3 + 2b\mu^2 T + b\mu t^2 - 2b\mu Tt], \quad \mu \le t \le T \tag{8A}$$

The maximum order quantity Q is

$$Q = I_{\max} - q$$

$$Q = \frac{a}{2}[2t_1 + \delta\mu^2 - \delta T^2 + (b+\theta+k)t_1^2 - 2\delta\mu t_1 + 2\delta\mu t - b\delta\mu^3 - 2(\theta+k)^2 t_1^3 - b\delta\mu T^2 + 2b\delta\mu^2 T] \tag{11}$$

The ordering cost per cycle is

$$O_C = o_C \int_0^T e^{-\rho t} dt$$

$$O_C = o_C[T - \frac{\rho}{2}T^2], \tag{12}$$

The holding cost per cycle is

$$H_C = h_C \int_0^{t_1} I(t) e^{-\rho t} dt$$

$$H_C = \frac{ah_C}{6}[3t_1^2 + (2b+\theta+k-\rho)t_1^3], \tag{13}$$

The shortage cost per cycle is

$$S_C = -s_C[\{\int_{t_1}^{\mu} I(t) + \int_{\mu}^{T} I(t)\} e^{-\rho t} dt]$$

$$S_C = -\frac{a\delta s_C}{6}[t_1^3 - 2\mu^3 + 2T^3 + 6\mu T^2 + 3\mu^2 t_1 - 6\mu t_1 T], \tag{14}$$

The purchase cost per cycle is

$$P_C = p_C \int_0^T Q e^{-\rho t} dt$$

$$P_C = \frac{ap_C}{2}[2Tt_1 + \delta\mu^2 T - \delta T^3 + (b+\theta+k)Tt_1^2 - 2\delta\mu t_1 T + \delta\mu T^2 - \rho T^2 t_1], \tag{15}$$

The lost sales cost per cycle is

$$LS_C = ac_1[\int_{t_1}^{\mu} e^{bt} e^{-\rho t}\{1 - \delta(T-t)\}dt + \int_{\mu}^{T} e^{b\mu} e^{-\rho t}\{1 - \delta(T-t)\}dt]$$

$$LS_C = \frac{ac_1}{6}[6T - 6t_1 - 3(b+\delta-\rho)t_1^2 - 3(\delta+\rho)T^2 + 6b\mu T + 6\delta t_1 T + b(\rho-\delta)\mu^3 + 3b\delta T\mu^2$$
$$+2(b\rho - b\delta + \delta\rho)t_1^3 + 3b(\delta+\rho)T^2\mu + \delta\rho T^3 + 3\delta(b-\rho)Tt_1^2], \tag{16}$$

The sales revenue per cycle is

$$S_R = c_2[\int_0^{t_1} S(t)e^{-\rho t}dt + \int_{t_1}^{\mu} S(t)\delta(T-t)e^{-\rho t}dt + \int_{\mu}^{T} S(t)\delta(T-t)e^{-\rho t}dt]$$

$$S_R = \frac{ac_2}{6}[6t_1 + 3(1+b+\delta-2\rho)t_1^2 + 3T^2 - 3(b+\delta-\rho)Tt_1 + b\delta\mu^3 - 6b\mu^2 T$$
$$+3b\delta\mu T^2 - \delta\rho T^3 + (2b+\theta+k+2\rho-2b\rho+2b\delta-2\delta\rho)t_1^3], \quad (17)$$

The total profit per unit time under the effect of inflation and time value of money is

$$TP_1(\mu,t_1,T) = \frac{1}{T}[S_R - (O_C + H_C + S_C + P_C + LS_C]$$

$$TP_1(\mu,t_1,T) = \frac{1}{T}[a(c_1+c_2)t_1 - (o_C + ac_1)T + \frac{a}{2}\{c_2(1+b+\delta-2\rho) - h_C + c_1(b+\delta-\rho)\}t_1^2$$
$$+\frac{1}{2}\{a\delta(c_1+c_2) + \rho(o_C+ac_1)\}T^2 - \frac{a}{2}\{2p_C + 2\delta c_1 + \delta c_2(b+\delta-\rho)\}Tt_1$$
$$-abc_1\mu T + \frac{a}{6}\{b\delta(c_1+c_2) - 2\delta s_C - b\rho\}\mu^3 + \frac{a}{6}\{c_2(2b+\theta+k+2\rho-2b\rho$$
$$+2b\delta-2\delta\rho) - h_C(2b+\theta+k-\rho) + \delta s_C - 2c_1(b\rho - b\delta + \delta\rho)\}t_1^3 - \frac{a}{2}\{b(2c_2+\delta c_1)$$
$$-\delta p_C\}T\mu^2 - \frac{a\delta}{6}\{\rho(c_1+c_2) + (2s_C + 3p_C)\}T^3 + \frac{a}{2}\{b\delta(c_2-c_1) + \delta(2s_C - p_C)$$
$$-b\rho c_1\}T^2\mu - \frac{ah_C}{2}\mu^2 t_1 + \frac{a\rho p_C}{2}T^2 t_1 + a(h_C + \delta p_C)\mu T t_1 - \frac{a}{2}\{p_C(b+\theta+k)$$
$$+\delta(b-\rho)c_1\}Tt_1^2], \quad (18)$$

Now our objective is to determine the optimal value of t_1 and T for which the total profit $TP_1(\mu,t_1,T)$ is maximum.

The necessary condition for $TP_1(\mu,t_1,T)$ to be maximum is that

$\frac{\partial TP_1(\mu,t_1,T)}{\partial t_1} = 0$ *and* $\frac{\partial TP_1(\mu,t_1,T)}{\partial T} = 0$ and solving these equations we find the optimum values of t_1 *and* T say for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2 TP_1(\mu,t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TP_1(\mu,t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TP_1(\mu,t_1,T)}{\partial t_1 \partial T}\right)^2 \succ 0 \ and \left(\frac{\partial^2 TP_1(\mu,t_1,T)}{\partial t_1^2}\right) \prec 0$$

$$\frac{\partial TP_1(\mu,t_1,T)}{\partial t_1} = \frac{a}{T}[(c_1+c_2) + \{c_2(1+b+\delta-2\rho) - h_C + c_1(b+\delta-\rho)\}t_1 - \frac{1}{2}\{2p_C + 2\delta c_1 + \delta c_2(b+\delta$$
$$-\rho\}T + \frac{1}{2}\{c_2(2b+\theta+k+2\rho-2b\rho+2b\delta-2\delta\rho) - h_C(2b+\theta+k-\rho) + \delta s_C$$
$$-2c_1(b\rho+\delta\rho-b\delta)\}t_1^2 - \frac{h_C}{2}\mu^2 + \frac{\rho p_C}{2}T^2 + a(h_C+\delta p_C)\mu T - \{p_C(b+\theta+k)$$
$$+\delta c_1(b-\rho)\}T], \quad (19)$$

$$\frac{\partial^2 TP_1(\mu,t_1,T)}{\partial t_1^2} = \frac{a}{T}[c_2(1+b+\delta-2\rho) + c_1(b+\delta-\rho) - h_C + \{c_2(2b+\theta+k+2\rho-2b\rho+2b\delta-2\delta\rho)$$
$$-h_C(2b+\theta+k-\rho) + \delta s_C - 2c_1(b\rho+\delta\rho-b\delta)\}t_1], \quad (20)$$

$$\frac{\partial(TP_1(\mu,t_1,T))}{\partial T} = \frac{1}{T}[-(o_C+ac_1) + \{a\delta(c_1+c_2) + \rho(o_C+ac_1)\}T - \frac{a}{2}\{2p_C + 2\delta c_1 + \delta c_2(b+\delta-\rho)\}t_1$$
$$-abc_1\mu - \frac{a}{2}\{b(2c_2+\delta c_1) - \delta p_C\}\mu^2 - \frac{a\delta}{2}\{\rho(c_1+c_2) + (2s_C+3p_C)\}T^2 + a\{b\delta(c_2-c_1)$$

$$+\delta(2s_C-p_C)-b\rho c_1\}T\mu+a\rho p_C Tt_1+a(h_C+\delta p_C)\mu t_1-\frac{a}{2}\{p_C(b+\theta+k)$$

$$+\delta(b-\rho)c_1\}t_1^2]-\frac{1}{T^2}[a(c_1+c_2)t_1-(o_C+ac_1)T+\frac{a}{2}\{c_2(1+b+\delta-2\rho)-h_C$$

$$+c_1(b+\delta-\rho)\}t_1^2+\frac{1}{2}\{a\delta(c_1+c_2)+\rho(o_C+ac_1)\}T^2-\frac{a}{2}\{2p_C+2\delta c_1+\delta c_2(b+\delta-\rho)\}Tt_1$$

$$-abc_1\mu T+\frac{a}{6}\{b\delta(c_1+c_2)-2\delta s_C-b\rho\}\mu^3+\frac{a}{6}\{c_2(2b+\theta+k+2\rho-2b\rho$$

$$+2b\delta-2\delta\rho)-h_C(2b+\theta+k-\rho)+\delta s_C-2c_1(b\rho-b\delta+\delta\rho)\}t_1^3-\frac{a}{2}\{b(2c_2+\delta c_1)$$

$$-\delta p_C\}T\mu^2-\frac{a\delta}{6}\{\rho(c_1+c_2)+(2s_C+3p_C)\}T^3+\frac{a}{2}\{b\delta(c_2-c_1)+\delta(2s_C-p_C)$$

$$-b\rho c_1\}T^2\mu-\frac{ah_C}{2}\mu^2 t_1+\frac{a\rho p_C}{2}T^2 t_1+a(h_C+\delta p_C)\mu Tt_1-\frac{a}{2}\{p_C(b+\theta+k)$$

$$+\delta(b-\rho)c_1\}Tt_1^2] \qquad (21)$$

$$\frac{\partial^2(TP_1(\mu,t_1,T))}{\partial T^2}=\frac{1}{T}[\{a\delta(c_1+c_2)+\rho(o_C+ac_1)\}-\frac{a\delta}{2}\{\rho(c_1+c_2)+(2s_C+3p_C)\}T^2+a\{b\delta(c_2-c_1)$$

$$+\delta(2s_C-p_C)-b\rho c_1\}\mu+a\rho p_C t_1]-\frac{1}{T^2}[-2(o_C+ac_1)+2\{a\delta(c_1+c_2)+\rho(o_C+ac_1)\}T$$

$$-a\{2p_C+2\delta c_1+\delta c_2(b+\delta-\rho)\}t_1-2abc_1\mu-a\{b(2c_2+\delta c_1)-\delta p_C\}\mu^2$$

$$-2a\delta\{\rho(c_1+c_2)+(2s_C+3p_C)\}T^2+2a\{b\delta(c_2-c_1)+\delta(2s_C-p_C)-b\rho c_1\}T\mu$$

$$+3a\rho p_C Tt_1+2a(h_C+\delta p_C)\mu t_1-a\{p_C(b+\theta+k)+\delta(b-\rho)c_1\}t_1^2]$$

$$+\frac{2}{T^3}[a(c_1+c_2)t_1-(o_C+ac_1)T+\frac{a}{2}\{c_2(1+b+\delta-2\rho)-h_C+c_1(b+\delta-\rho)\}t_1^2$$

$$+\frac{1}{2}\{a\delta(c_1+c_2)+\rho(o_C+ac_1)\}T^2-\frac{a}{2}\{2p_C+2\delta c_1+\delta c_2(b+\delta-\rho)\}Tt_1-abc_1\mu T$$

$$+\frac{a}{6}\{b\delta(c_1+c_2)-2\delta s_C-b\rho\}\mu^3+\frac{a}{6}\{c_2(2b+\theta+k+2\rho-2b\rho+2b\delta-2\delta\rho)$$

$$-h_C(2b+\theta+k-\rho)+\delta s_C-2c_1(b\rho-b\delta+\delta\rho)\}t_1^3-\frac{a}{2}\{b(2c_2+\delta c_1)-\delta p_C\}T\mu^2$$

$$-\frac{a\delta}{6}\{\rho(c_1+c_2)+(2s_C+3p_C)\}T^3+\frac{a}{2}\{b\delta(c_2-c_1)+\delta(2s_C-p_C)-b\rho c_1\}T^2\mu$$

$$-\frac{ah_C}{2}\mu^2 t_1+\frac{a\rho p_C}{2}T^2 t_1+a(h_C+\delta p_C)\mu Tt_1-\frac{a}{2}\{p_C(b+\theta+k)+\delta(b-\rho)c_1\}Tt_1^2] \qquad (22)$$

$$\frac{\partial^2 TP_1(\mu,t_1,T)}{\partial T\partial t_1}=\frac{a}{T}[-\frac{1}{2}\{2p_C+2\delta c_1+\delta c_2(b+\delta-\rho)\}+\rho p_C T+a(h_C+\delta p_C)\mu-\{p_C(b+\theta+k)$$

$$+\delta c_1(b-\rho)\}]-\frac{a}{T^2}[(c_1+c_2)+\{c_2(1+b+\delta-2\rho)-h_C+c_1(b+\delta-\rho)\}t_1$$

$$-\frac{1}{2}\{2p_C+2\delta c_1+\delta c_2(b+\delta-\rho)\}T+\frac{1}{2}\{c_2(2b+\theta+k+2\rho-2b\rho+2b\delta-2\delta\rho)$$

$$-h_C(2b+\theta+k-\rho)+\delta s_C-2c_1(b\rho+\delta\rho-b\delta)\}t_1^2-\frac{h_C}{2}\mu^2+\frac{\rho p_C}{2}T^2$$

$$+a(h_C+\delta p_C)\mu T-\{p_C(b+\theta+k)+\delta c_1(b-\rho)\}T] \tag{23}$$

Case II When $t_1 \geq \mu$ then in the interval $[0,T]$ the selling rate $S(t)=\begin{cases} R(t)+kI(t), & I(t)\succ 0 \\ R(t), & I(t)\leq 0 \end{cases}$ is defined as

$$S(t)=\begin{cases} ae^{bt}+kI(t), & 0\leq t\leq \mu \\ ae^{b\mu}+kI(t), & \mu\leq t\leq t_1 \\ ae^{b\mu}, & t_1\leq t\leq T \end{cases}$$

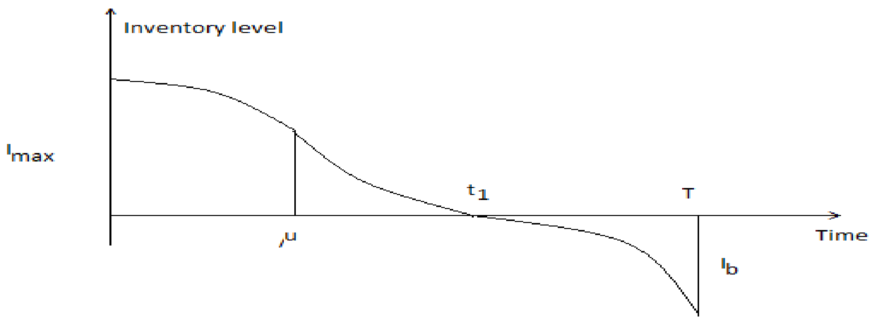

Figure 2. For case II

Then the instantaneous inventory level at any time t in $[0,T]$ are governed by the following differential equations

$$\frac{dI}{dt}+\theta I=-[ae^{bt}+kI(t)], \qquad 0\leq t\leq \mu \tag{24}$$

With boundary condition

$$I(0)=I_{\max}$$

$$\frac{dI}{dt}+\theta I=-[ae^{b\mu}+kI(t)], \qquad \mu\leq t\leq t_1 \tag{25}$$

With boundary condition

$$I(t_1)=0$$

$$\frac{dI}{dt}=-ae^{b\mu}\delta(T-t) \qquad , \qquad t_1\leq t\leq T \tag{26}$$

With boundary condition

$$I(t_1)=0$$

The solution of the equation (21) is

$$I=-a[t+\frac{(b-\theta-k)}{2}t^2-\frac{(\theta+k)(b+\theta+k)}{2}t^3]+I_{\max}\{1-(\theta+k)\}, \qquad 0\leq t\leq \mu \tag{27}$$

The solution of the equation (22) is

$$I=a[(1+b\mu)t_1-\frac{3(\theta+k)(1+b\mu)}{2}t_1^2-(\theta+k)(1+b\mu)tt_1+\frac{3(\theta+k)^2(1+b\mu)}{2}tt_1^2]-a[(1+b\mu)t$$

$$-\frac{(\theta+k)(1+b\mu)}{2}t^2-\frac{(\theta+k)^2(1+b\mu)}{2}t^3], \qquad \mu\leq t\leq t_1 \tag{28}$$

$$I = a\delta[Tt_1 - Tt + \frac{1}{2}t^2 - \frac{1}{2}t_1^2 - \frac{b}{2}\mu t^2 - \frac{b}{2}\mu t_1^2 + b\mu Tt_1 - b\mu Tt], \qquad t_1 \le t \le T \tag{29}$$

From the continuity of $I(t)$, putting $t = \mu$ in the equation (24)

$$I_{\max} = a[\mu + t_1 - t + \frac{(b+\theta+k)}{2}\mu^2 - \frac{3(\theta+k)}{2}t_1^2 + \frac{(\theta+k)}{2}t^2 - (b+\theta+k)\mu t + (1+b)\mu t_1$$
$$-(\theta+k)tt_1 - (\theta+k)^2\mu^3 + \frac{(\theta+k)^2}{2}t^2 - \frac{3(\theta+k)(b+\theta+k)}{2}\mu t_1^2 + \frac{3(\theta+k)^2}{2}tt_1^2$$
$$+\frac{(\theta+k)(1+b)}{2}\mu t^2 + b\mu^2 t_1 - b(\theta+k)\mu^2 t - (\theta+k)(b+\theta+k)\mu tt_1], \tag{30}$$

Using $I_{\max}$ in the equation (24), the solution of the equation (21) is

$$I = a[\mu + t_1 - 2t + \frac{(4\theta+4k-b)}{2}t^2 + \frac{(b+\theta+k)}{2}\mu^2 - \frac{3(\theta+k)}{2}t_1^2 + (1+b)\mu t_1 - 2(\theta+k)tt_1$$
$$-(b+2\theta+2k)\mu t + \frac{(\theta+k)(b+\theta+k)}{2}t^3 - \frac{3(\theta+k)(b+\theta+k)}{2}\mu t_1^2 + 3(\theta+k)^2 tt_1^2$$
$$+(\theta+k)^2 t^2 t_1 + \frac{(3b+2\theta+2k+1)(\theta+k)}{2}\mu t^2 - \frac{(3b+\theta+k)}{2}\mu^2 t + b\mu^2 t_1$$
$$-(\theta+k)(2b+\theta+k)\mu tt_1], \qquad 0 \le t \le \mu \tag{21A}$$

The maximum amount of demand backlogged per cycle is obtained by putting $t = T$ in the equation (26)

$$q = \frac{a\delta}{2}[2Tt_1 - t_1^2 + 2b\mu Tt_1 - b\mu t_1^2 - T^2 - 3b\mu T^2], \tag{31}$$

The maximum order quantity is $Q = I_{\max} - q$

$$Q = a[\mu + t_1 - t + \frac{(b+\theta+k)}{2}\mu^2 - \frac{3(\theta+k)}{2}t_1^2 + \frac{(\theta+k)}{2}t^2 - (b+\theta+k)\mu t + (1+b)\mu t_1$$
$$-(\theta+k)tt_1 - (\theta+k)^2\mu^2 + \frac{(\theta+k)^2}{2}t^3 + \frac{3(\theta+k)^2}{2}tt_1^2 - \frac{3(\theta+k)(b+\theta+k)}{2}\mu t_1^2$$
$$+\frac{(1+b)(\theta+k)}{2}\mu t^2 + b\mu^2 t_1 - b(\theta+k)\mu^2 t - (\theta+k)(b+\theta+k)\mu tt_1] - \frac{a\delta}{2}[2Tt_1 - t_1^2$$
$$+2b\mu Tt_1 - b\mu t_1^2 - T^2 - 3b\mu T^2], \tag{32}$$

The ordering cost per cycle is

$$O_C = o_C\int_0^T e^{-\rho t}dt$$

$$O_C = o_C[T - \frac{\rho}{2}T^2], \tag{33}$$

The holding cost per cycle is

$$H_C = h_C[\int_0^{\mu} I(t)e^{-\rho t}dt + \int_{\mu}^{t_1} I(t)e^{-\rho t}dt]$$

$$H_C = \frac{ah_C}{6}[3t_1^2 + 3\mu^2 + 2b\mu^3 - 11(\theta+k)t_1^3 + 3b\mu t_1^2 - 3(\theta+k-2)\mu^2 t_1 - \frac{\rho}{6}(\mu^3 + t_1^3)], \tag{34}$$

The shortage cost per cycle is

$$S_C = -s_C[\int_{t_1}^{T} I(t)e^{-\rho t}dt]$$

$$S_C = -a\delta s_C[T^2t_1 - Tt_1^2 - \frac{1}{3}T^3 + \frac{1}{3}t_1^3], \tag{35}$$

The purchase cost per cycle is

$$P_C = p_C\int_0^T Qe^{-\rho t}dt$$

$$P_C = \frac{ap_C}{6}[6\mu T + 6Tt_1 - 3t^2 + 3(b+\theta+k)T\mu^2 - 9(\theta+k)Tt_1^2 + (\theta+k)T^3 - 3(b+\theta+k)\mu T^2$$
$$+6(1+b)\mu Tt_1 - 3(\theta+k)T^2t_1 - 3\delta(2T^2t_1 - Tt_1^2 - T^3) - \rho(3\mu T^2 + 3T^2t_1 - 2T^3)], \tag{36}$$

The lost sales cost per cycle is

$$LS_C = ac_1\int_{t_1}^{T} e^{b\mu}e^{-\rho t}\{1-\delta(T-t)\}dt$$

$$LS_C = \frac{ac_1}{6}[6T - 6t_1 - 3(\delta+\rho)T^2 - 3(\delta-\rho)t_1^2 + 6\delta Tt_1 + 6b\mu T - 6b\mu t_1 - 5\delta\rho T^3 + 3\delta\rho Tt_1^2$$
$$+2\delta\rho t_1^3 - 3b(\delta-\rho)\mu T^2 - 3b(\delta-\rho)\mu t_1^2 + 6b\delta\mu Tt_1], \tag{37}$$

The sales revenue per cycle is

$$S_R = c_2[\int_0^{t_1} S(t)e^{-\rho t}dt + \int_{t_1}^{T} S(t)\delta(T-t)e^{-\rho t}dt]$$

$$S_R = ac_2[t_1 + (b+k+k\theta+k^2)\mu t_1 - \frac{b}{2}\mu^2 - \frac{\rho}{2}t_1^2 + \frac{1}{6}(3b+6k+6\theta-8k\theta-5kb+b\rho+\rho k-\rho$$
$$-8k^2)\mu^3 + \frac{1}{6}(7\rho-11k\theta-kb-11k^2)t_1^3 + \frac{1}{2}(2k+kb-2k\theta-2k^2-b-1)\mu t_1^2$$
$$+\frac{(kb-k\theta-2k^2-4\rho k^2-\rho)}{2}\mu^2t_1] + \frac{a\delta c_2}{6}[3T^2 + 3t_1^2 - 6Tt_1 - \rho T^3 - 2\rho t_1^3 + 3b\mu T^2$$
$$+3b\mu t_1^2 + 3\rho Tt_1^2], \tag{38}$$

The total profit per unit time is

$$TP_2(\mu,t_1T) = \frac{1}{T}[a(c_1+c_2)t_1 - (c_0+ac_1)T - \frac{a}{2}\{bc_2 - 7p_C(b+\theta+k) - h_C\}\mu^2 + \frac{a}{2}\{\delta(c_1+c_2)$$
$$-\rho(c_1+c_2) - h_C\}t_1^2 + \frac{1}{2}\{a\delta(c_1+c_2) + a\rho c_1 + ap_C + \rho c_0\}T^2 + a\{c_2(b+k+k\theta$$
$$+k^2) + bc_1\}\mu t_1 - a\{\delta(c_1+c_2) + p_C\}Tt_1 - a\{p_C + bc_1\}\mu T + \{ac_2(\frac{b}{2} + k + \theta - \frac{4k\theta}{3}$$
$$-\frac{5kb}{6} + \frac{b\rho}{6} + \frac{\rho k}{6} - \frac{\rho}{6} - \frac{4k^2}{3}) + \frac{ah_C}{36}(\rho - 12b)\}\mu^3 + a\{c_2(\frac{7\rho - 11k\theta - kb - 11k^2)}{6}$$
$$-\frac{\delta\rho c_2}{3} + \frac{11(\theta+k)h_C}{6} + \frac{\rho h_C}{36} + \frac{\rho s_C}{3} - \frac{\delta\rho c_1}{3}\}t_1^3 + a\{\frac{5\delta\rho c_1}{3} - \frac{5\delta p_C}{6}\}T^3 + \frac{a}{2}\{c_2(bk$$
$$+2k - 2k\theta - 2k^2 - b - 1) + b\delta(c_1+c_2) - b(h_C + \rho c_1)\}\mu t_1^2 + \frac{a}{2}\{c_2(kb - k\theta - 2k^2$$

$$-4\rho k^2-\rho)+h_C(\theta+k-2)\}\mu^2 t_1+\frac{a}{2}\{b\delta(c_1+c_2)+p_C(b+\theta+k+\delta+\rho)$$

$$-b\rho c_1\}\mu T^2+\frac{a}{2}\{\delta\rho(c_2-c_1)+p_C(3\theta+3k-\delta)-2\delta s_C\}Tt_1^2-a\{b(p_C+\delta c_1)$$

$$+p_C\}\mu Tt_1+\frac{a}{2}\{p_C(\theta+k+\rho+3\delta)+2\delta s_C\}T^2t_1], \quad (39)$$

Now our objective is to determine the optimal value of t_1 for which the total profit $TP_2(\mu,t_1,T)$ is maximum.

The necessary condition for $TP_2(\mu,t_1,T)$ to be maximum is that

$\frac{\partial TP_2(\mu,t_1,T)}{\partial t_1}=0$ *and* $\frac{\partial TP_2(\mu,t_1,T)}{\partial T}=0$ and solving these equations we find the optimum values of t_1 *and* T say for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2 TP_2(\mu,t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TP_2(\mu,t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TP_2(\mu,t_1,T)}{\partial t_1\partial T}\right)^2\succ 0 \ and \left(\frac{\partial^2 TP_2(\mu,t_1,T)}{\partial t_1^2}\right)\prec 0$$

$$\frac{\partial TP_2}{\partial t_1}=\frac{a}{T}[(c_1+c_2)+\{\delta(c_1+c_2)-\rho(c_1+c_2)-h_C\}t_1+\{c_2(b+k+k\theta+k^2)+bc_1\}\mu-\{\delta(c_1+c_2)$$

$$+p_C\}T+\frac{1}{2}\{\frac{c_2(7\rho-11k\theta-kb-11k^2)}{2}-\delta\rho c_2+\frac{11h_C(\theta+k)}{2}+\frac{\rho h_C}{12}+\delta s_C-\delta\rho c_1\}t_1^2$$

$$+\{c_2(kb+2k-2k\theta-2k^2-b-1)+b\delta(c_1+c_2)-b(h_C+\rho c_1)\}\mu t_1+\frac{1}{2}\{c_2(kb-k\theta$$

$$-2k^2-4\rho k^2-\rho)+h_C(\theta+k-2)\}\mu^2+\{\delta\rho(c_2-c_1)+p_C(3\theta+3k-\delta)-2\delta s_C\}Tt_1$$

$$-\{b(p_C+\delta c_1)+p_C\}\mu T+\frac{1}{2}\{p_C(\theta+k+3\delta+\rho)+2\delta s_C\}T^2], \quad (40)$$

$$\frac{\partial^2 TP_2}{\partial t_1^2}=\frac{a}{T}[\delta(c_1+c_2)-\rho(c_1+c_2)-h_C+\{c_2(7\rho-11k\theta-kb-11k^2)-2\delta\rho c_2+11(\theta+k)h_C+\frac{\rho h_C}{6}$$

$$+2\delta(s_C-\rho c_1)\}t_1+\{c_2(bk+2k-2k\theta-2k^2-b-1)+b\delta(c_1+c_2)-b(h_C+\rho c_1)\}\mu$$

$$+\{\delta\rho(c_2-c_1)+p_C(3\theta+3k-\delta)-2\delta s_C\}T], \quad (41)$$

$$\frac{\partial TP_2(\mu,t_1T)}{\partial T}=\frac{1}{T}[-(c_0+ac_1)+\{a\delta(c_1+c_2)+a\rho c_1+ap_C+\rho c_0\}T-a\{\delta(c_1+c_2)+p_C\}t_1$$

$$-a\{p_C+bc_1\}\mu+a\{b\delta(c_1+c_2)+p_C(b+\theta+k+\delta+\rho)-b\rho c_1\}\mu T+\frac{a}{2}\{\delta\rho(c_2-c_1)$$

$$+p_C(3\theta+3k-\delta)-2\delta s_C\}t_1^2-a\{b(p_C+\delta c_1)+p_C\}\mu t_1+a\{p_C(\theta+k+\rho+3\delta)$$

$$+2\delta s_C\}Tt_1]-\frac{1}{T^2}[a(c_1+c_2)t_1-(c_0+ac_1)T-\frac{a}{2}\{bc_2-7p_C(b+\theta+k)-h_C\}\mu^2$$

$$+\frac{a}{2}\{\delta(c_1+c_2)-\rho(c_1+c_2)-h_C\}t_1^2+\frac{1}{2}\{a\delta(c_1+c_2)+a\rho c_1+ap_C+\rho c_0\}T^2+a\{c_2(b+k$$

$$+k\theta+k^2)+bc_1\}\mu t_1-a\{\delta(c_1+c_2)+p_C\}Tt_1-a\{p_C+bc_1\}\mu T+\{ac_2(\frac{b}{2}+k+\theta-\frac{4k\theta}{3}$$

$$-\frac{5kb}{6}+\frac{b\rho}{6}+\frac{\rho k}{6}-\frac{\rho}{6}-\frac{4k^2}{3})+\frac{ah_C}{36}(\rho-12b)\}\mu^3+a\{c_2(\frac{7\rho-11k\theta-kb-11k^2)}{6}$$

$$-\frac{\delta\rho c_2}{3}+\frac{11(\theta+k)h_C}{6}+\frac{\rho h_C}{36}+\frac{\rho s_C}{3}-\frac{\delta\rho c_1}{3}\}t_1^3+a\{\frac{5\delta\rho c_1}{3}-\frac{5\delta p_C}{6}\}T^3+\frac{a}{2}\{c_2(bk$$

$$+2k-2k\theta-2k^2-b-1)+b\delta(c_1+c_2)-b(h_C+\rho c_1)\}\mu t_1^2+\frac{a}{2}\{c_2(kb-k\theta-2k^2$$

$$-4\rho k^2-\rho)+h_C(\theta+k-2)\}\mu^2 t_1+\frac{a}{2}\{b\delta(c_1+c_2)+p_C(b+\theta+k+\delta+\rho)$$

$$-b\rho c_1\}\mu T^2+\frac{a}{2}\{\delta\rho(c_2-c_1)+p_C(3\theta+3k-\delta)-2\delta s_C\}Tt_1^2-a\{b(p_C+\delta c_1)$$

$$+p_C\}\mu Tt_1+\frac{a}{2}\{p_C(\theta+k+\rho+3\delta)+2\delta s_C\}T^2t_1] \tag{42}$$

$$\frac{\partial^2 TP_2(\mu,t_1T)}{\partial T^2}=\frac{1}{T}[\{a\delta(c_1+c_2)+a\rho c_1+ap_C+\rho c_0\}-a\{p_C+bc_1\}\mu+a\{b\delta(c_1+c_2)+p_C(b+\theta$$

$$+k+\delta+\rho)-b\rho c_1\}\mu+a\{p_C(\theta+k+\rho+3\delta)+2\delta s_C\}t_1]-\frac{2}{T^2}[-(c_0+ac_1)$$

$$+\{a\delta(c_1+c_2)+a\rho c_1+ap_C+\rho c_0\}T-a\{\delta(c_1+c_2)+p_C\}t_1-a\{p_C+bc_1\}\mu$$

$$+a\{b\delta(c_1+c_2)+p_C(b+\theta+k+\delta+\rho)-b\rho c_1\}\mu T+a\{\delta\rho(c_2-c_1)+p_C(3\theta$$

$$+3k-\delta)-2\delta s_C\}t_1^2-a\{b(p_C+\delta c_1)+p_C\}\mu t_1+a\{p_C(\theta+k+\rho+3\delta)+2\delta s_C\}Tt_1]$$

$$+\frac{2}{T^3}[a(c_1+c_2)t_1-(c_0+ac_1)T-\frac{a}{2}\{bc_2-7p_C(b+\theta+k)-h_C\}\mu^2$$

$$+\frac{a}{2}\{\delta(c_1+c_2)-\rho(c_1+c_2)-h_C\}t_1^2+\frac{1}{2}\{a\delta(c_1+c_2)+a\rho c_1+ap_C+\rho c_0\}T^2+a\{c_2(b+k$$

$$+k\theta+k^2)+bc_1\}\mu t_1-a\{\delta(c_1+c_2)+p_C\}Tt_1-a\{p_C+bc_1\}\mu T+\{ac_2(\frac{b}{2}+k+\theta-\frac{4k\theta}{3}$$

$$-\frac{5kb}{6}+\frac{b\rho}{6}+\frac{\rho k}{6}-\frac{\rho}{6}-\frac{4k^2}{3})+\frac{ah_C}{36}(\rho-12b)\}\mu^3+a\{c_2(\frac{7\rho-11k\theta-kb-11k^2)}{6}$$

$$-\frac{\delta\rho c_2}{3}+\frac{11(\theta+k)h_C}{6}+\frac{\rho h_C}{36}+\frac{\rho s_C}{3}-\frac{\delta\rho c_1}{3}\}t_1^3+a\{\frac{5\delta\rho c_1}{3}-\frac{5\delta p_C}{6}\}T^3+\frac{a}{2}\{c_2(bk$$

$$+2k-2k\theta-2k^2-b-1)+b\delta(c_1+c_2)-b(h_C+\rho c_1)\}\mu t_1^2+\frac{a}{2}\{c_2(kb-k\theta-2k^2$$

$$-4\rho k^2-\rho)+h_C(\theta+k-2)\}\mu^2 t_1+\frac{a}{2}\{b\delta(c_1+c_2)+p_C(b+\theta+k+\delta+\rho)$$

$$-b\rho c_1\}\mu T^2+\frac{a}{2}\{\delta\rho(c_2-c_1)+p_C(3\theta+3k-\delta)-2\delta s_C\}Tt_1^2-a\{b(p_C+\delta c_1)$$

$$+p_C\}\mu Tt_1+\frac{a}{2}\{p_C(\theta+k+\rho+3\delta)+2\delta s_C\}T^2t_1] \tag{43}$$

$$\frac{\partial TP_2}{\partial T\partial t_1}=\frac{a}{T}[-\{\delta(c_1+c_2)+p_C\}+\{\delta\rho(c_2-c_1)+p_C(3\theta+3k-\delta)-2\delta s_C\}t_1-\{b(p_C+\delta c_1)+p_C\}\mu$$

$$+\{p_C(\theta+k+3\delta+\rho)+2\delta s_C\}T]-\frac{a}{T^2}[(c_1+c_2)+\{\delta(c_1+c_2)-\rho(c_1+c_2)-h_C\}t_1$$

$$+\{c_2(b+k+k\theta+k^2)+bc_1\}\mu-\{\delta(c_1+c_2)+p_C\}T+\frac{1}{2}\{\frac{c_2(7\rho-11k\theta-kb-11k^2)}{2}-\delta\rho c_2$$

$$+\frac{11h_C(\theta+k)}{2}+\frac{\rho h_C}{12}+\delta s_C-\delta\rho c_1\}t_1^2+\{c_2(kb+2k-2k\theta-2k^2-b-1)+b\delta(c_1+c_2)$$
$$-b(h_C+\rho c_1)\}\mu t_1+\frac{1}{2}\{c_2(kb-k\theta-2k^2-4\rho k^2-\rho)+h_C(\theta+k-2)\}\mu^2+\{\delta\rho(c_2-c_1)$$
$$+p_C(3\theta+3k-\delta)-2\delta s_C\}Tt_1-\{b(p_C+\delta c_1)+p_C\}\mu T+\frac{1}{2}\{p_C(\theta+k+3\delta+\rho)+2\delta s_C\}T^2] \quad (44)$$

4. Numerical Parameters

Let us consider the following parameters in the appropriate units and two different values of the ramp-type demand parameter μ

$a=20, b=0.5, o_C=\$15/order,$

$h_C=\$5/unit/unit\,time, s_C=\$8/unit,$

$p_C=\$10/unit, c_1=\$4/unit,$

$c_2=\$6/unit, \theta=0.05, \delta=2, k=3, \rho=0.3, \mu=0.4$

Numerical example 1- when $t_1\le\mu$ then solving the equations $\frac{\partial TP_1}{\partial t_1}=0$ and $\frac{\partial TP_1}{\partial T}=0$ we find the optimum value of t_1 and T for different values of θ and ρ.

Table 1. Variation in total profit with respect to the change in deterioration parameter θ

θ	t_1	T	TP
0.05	0.0482171	0.183668	-226.4435077
0.10	0.0479426	0.181986	-226.6504760
0.15	0.0476980	0.180344	-226.8382140
0.20	0.0474420	0.178725	-227.0359760
0.25	0.0471883	0.177135	-227.2350910
0.30	0.0469371	0.175572	-227.4343680

Since the sign of total profit comes out to be negative and the value of $\frac{\partial^2 TP_1}{\partial t_1^2}\succ 0$ so the total profit is minimum. As we increase the deterioration parameter θ then the total profit decreases.

Table 2. Variation in total profit with respect to the change in inflation parameter ρ

ρ	t_1	T	TP
0.3	0.0482171	0.183668	-226.4435077
0.6	0.0517967	0.195779	-221.1549705
0.9	0.0557286	0.209625	-215.6210758
1.2	0.0599322	0.225688	-209.9036964
1.5	0.0641455	0.244776	-204.1475730
1.8	0.0675702	0.268513	-198.7271973

Since the sign of total profit comes out to be negative and the value of $\frac{\partial^2 TP_1}{\partial t_1^2}\succ 0$ so the total profit is minimum. As we increase the inflation parameter ρ then the total profit increases.

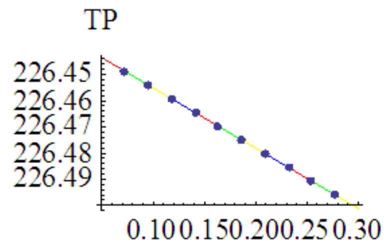

Figure 3. With respect to θ

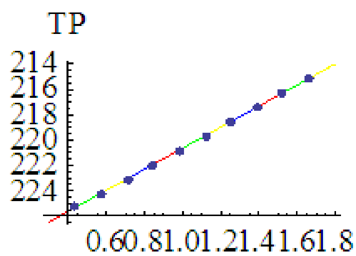

Figure 4. With respect to ρ

Numerical example 2- when $t_1\ge\mu$ then solving the equations $\frac{\partial TP_1}{\partial t_1}=0$ and $\frac{\partial TP_1}{\partial T}=0$ we find the optimum value of t_1 and T for different values of θ and ρ.

Table 3. Variation in total profit with respect to the change in deterioration parameter θ

θ	t_1	T	TP
0.05	0.446380	2.14232	1047.603331
0.10	0.444914	2.15555	1057.758120
0.15	0.443541	2.16850	1068.037214
0.20	0.442254	2.18131	1078.438840
0.25	0.441049	2.19395	1088.963065
0.30	0.439918	2.20642	1099.600578

As we increase the deterioration parameter θ then the value of the total profit increases.

Table 4. Variation in total profit with respect to the change in inflation parameter ρ

ρ	t_1	T	TP
0.3	0.446380	2.14232	1047.603331
0.6	0.418701	2.07680	1391.116096
0.9	0.393017	2.01720	1695.559346
1.2	0.369062	1.96273	1967.665828
1.5	0.346616	1.91276	2212.845355
1.8	0.325498	1.86675	2435.439588

As we increase the inflation parameter ρ then the value of the total profit increases.

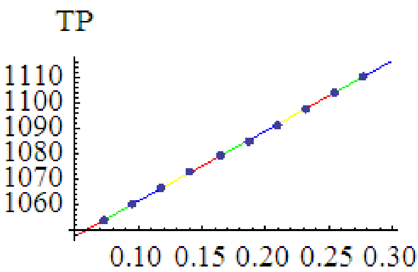

Figure 5. With respect to θ

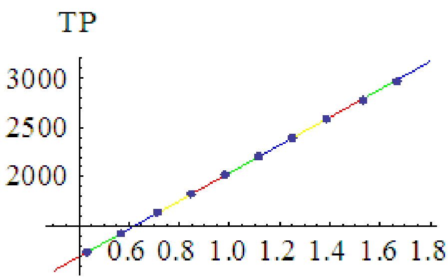

Figure 6. With respect to ρ

5. Sensitivity Analysis

Thus from the tables 1, 2, 3 and 4 we see that the parameters θ and ρ are more sensitive in case of $t_1 \geq \mu$ in comparison of $t_1 \leq \mu$ because in the case I shortage interval is greater than the shortage interval in case II and so the total profit is minimum in case I and the total profit is maximum in case II.

6. Conclusions

In this paper we developed a probabilistic inventory model for deteriorating items with ramp-type demand rate under the effect of inflation. Shortages are allowed and partially backlogged. We studied the above developed model in two cases $t_1 \leq \mu$ and $t_1 \geq \mu$. In case I from the tables 1 and 2 we see that as we increase the parameters θ and ρ then the total profit is minimum. In case II from the tables 3 and 4 we see that as we increase the parameters θ and ρ then the total profit is maximum. Thus when $t_1 \geq \mu$ then the deterioration parameter θ and the inflation parameter ρ become more sensitive in comparison of deterioration parameter θ and inflation parameter ρ in case of $t_1 \leq \mu$ because in the case I the shortage interval is greater than the shortage interval in case II. Further this model can be generalized by considering the fuzzy type demand rate.

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