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Recurrent Type Fuzzy Time Series Forecasting Method Based on Artificial Neural Networks

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Abstract The approaches of fuzzy time series are used commonly for the analysis of real life time series whose observations include uncertainty. Because of the fact that forecasting methods of fuzzy time series do not need many constraints in the approaches of classic time series, the interest towards this method is increasing. The forecasting methods of fuzzy time series in the literature focus on the models connected to the fuzzy autoregressive (AR) variables. In the models in which the methods of classic time series are used, there are not only autoregressive variables of time series but also moving average (MA) variables of time series. However in the forecasting method of fuzzy time series proposed in the literature, there are no using of MA variables except for only two studies. In this study, by defining a new first-order forecasting model of fuzzy time series which include not only fuzzy AR variables but also MA variables, an analysis of algorithm that depends on artificial neural networks is proposed. The new proposed method is applied to Istanbul Stock Exchange (IMKB) national 100 index time series, gold prices of the Central Bank of the Republic of Turkey and two simulated chaotic time series and compared with the other methods in the literature with regard to forecasting performance.

Keywords Fuzzy time series, Forecasting, Fuzzy sets, Feed forward neural Networks, ARMA models

1. Introduction

The methods of fuzzy time series are based on the theory of fuzzy set of Zadeh [28]. In the literature, Song and Chissom [21] defined several forecasting methods of fuzzy time series for the first time and classified the fuzzy time series into two types as time variant and time invariant. Also, Song and Chissom [22] proposed an analysis method based on resultant actions for the time invariant fuzzy time series. In Song and Chissom's study [23], a similar analysis method was proposed for the time variant fuzzy time series. Although time invariant fuzzy time series were proposed for the analysis in most of the studies in the literature, there are also methods offered for the analysis of variant fuzzy time series. These studies were followed by the methods based on the tables of fuzzy group relation developed by Chen [2] which depends on Markov model presented by Sullivan and Woodall [24]. Furthermore, Hwang et al. [15], Huarng and Yu [12], Yu [26] and Yu [27] and Chen and Hwang [4] proposed first order forecasting model of fuzzy time series. In the literature, high order forecasting model of fuzzy time series have also been proposed. The first high order forecasting model of fuzzy time series was introduced by Chen [3]. Some other high order forecasting models of fuzzy time series in the literature can be listed as Aladag et al. [1] and Egrioglu et al. [8].

Huarng and Yu [13] introduced the first order forecasting models of fuzzy time series in the step of determination of fuzzy relation, as well as fuzzy time series of artificial neural networks used for the first time in the step of defuzzification in literature by Song and Chissom [23]. Moreover, Aladag et al. [1] and Egrioglu et al. [7] utilized artificial neural networks in the analysis of high order and multivariate models respectively in the step of determination of fuzzy relation. As the usage of tables of fuzzy groups relation in high order and multivariate models require quite intensive and complex actions, the usage of artificial neural networks in the step of the determination of fuzzy relation removes the complexity especially in the high order and multivariate models.

The most of the studies in the literature are those in which interval length is determined intuitively, diverse interval lengths are tried and then the best forecasts are achieved. Huarng [11] showed in his study that the selection of the interval length used in the division of the universal set into equal intervals is a critical decision and directly affects the forecasting performance, and determined the optimal interval length by recommending two separate approaches based on average and distribution for the selection of the interval length. In this way, the analysis was performed in a single application without trying diverse interval lengths.

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Huarng [11]'s approaches base on average and rate brought systematicity to the selection of the interval length, but could not guarantee the achievement of the best forecasting performance. For this reason, there are many studies [5, 6, 8, 13, 18, 25] for the determination of the optimal interval length in a single application by using statistical theory.

All of the methods of fuzzy time series developed in the literature have used forecasting models depend on AR (Autoregressive) variables except for only two studies [9,16] until year 2013. These primary studies [9, 16] are first order fuzzy ARMA type models that one of them [9] is based on particle swarm optimization and the other [16] is based on fuzzy logic relation tables. Proposed method in this study also is first-order ARMA type model which is based on artificial neural network. The main difference of the proposed method is using artificial neural network for determining fuzzy relations. The proposed method is the first method employee artificial neural network for a fuzzy ARMA type models.

In the modeling of most of the real life time series, MA (Moving Avarage) variables are also required. In this regard, using only AR variables where MA variables are also required for the analysis of fuzzy time series, could remain insufficient for both in the determination of fuzzy relations and forecasting performance. Moreover, the usage of MA variables with AR variables in the analysis of fuzzy time series can increase forecasting performance because of the fact that it could give opportunity to make application with much more knowledge.

In this study, for forecasting time invariant fuzzy time series, a new forecasting method of fuzzy time series based on a model including MA variables is proposed. The advantages of proposed algorithm are stated as;

- The analysis depend on the forecasting model of fuzzy time series is more real method for real life time series, as it also include MA variables,
- The usage of artificial neural networks in the determination of fuzzy relation remove the complexity of fuzzy logic group relation tables,
- In comparison with the other methods developed in the literature, it has a higher forecasting performance.
- The proposed method has better forecasting performance than other Fuzzy ARIMA methods [9, 16].
- Because of using artificial neural network, unknown patterns can be easily forecasted.
- It is difficult that the methods based on fuzzy relation tables are designed for high order models but the proposed method can be easily extended for high order models like Aladag et al. [1].

In section 2, the main definitions related with fuzzy time series and in Section 3, main information about artificial neural networks are presented. In section 4, the forecasting model of fuzzy time series proposed is defined and the analysis of algorithm is stated. In section 5, the results of the application of proposed method with this study and other methods to Istanbul Stock Exchange (IMKB) national 100 index time series and the prices of gold (Turkey daily gold prices series can be obtained from the website of Central Bank of The Turkish Republic) are presented. In section 6, the results are discussed.

2. Definition of Fuzzy Time Series

The definition of fuzzy time series was firstly introduced by Song and Chissom [21, 22]. General definitions of fuzzy time series are given as follows:

Definition 1. Let $Y(t)(t = \ldots,0,1,2,\ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t),\ldots$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2. Fuzzy time series relationships assume that $F(t)$ is caused only by $F(t-1)$, the model can be expressed as:

$$F(t-1) \rightarrow F(t) \tag{1}$$

This is called as a first-order fuzzy time series forecasting model. Then this relation can be expressed as $F(t) = F(t-1) \circ R(t,t-1)$ where $R(t,t-1)$ is the fuzzy relationship between $F(t-1)$ and $F(t)$.

Definition 3. Suppose $R(t,t-1)$ is a first-order model of $F(t)$. If for any *t*, $R(t,t-1)$ is independent of *t*, $R(t,t-1) = R(t-1,t-2)$, then $F(t)$ is called a time-invariant fuzzy time series otherwise it is called a time-variant fuzzy time series.

Definition 4. Fuzzy time series relationships assume that $F(t)$ is caused only by $F(t-1)$, then the model can be expressed as:

$$F(t-1), F(t-2),\ldots, F(t-m) \rightarrow F(t) \tag{2}$$

This is called as a m^{th}-order fuzzy time series forecasting model [3].

3. Feed Forward Artificial Neural Networks

The technology of artificial neural networks is an information processing mechanism which emerges in the simulation of human neuron and nervous system at the computer environment. The most important feature of the artificial neural networks is its ability of learning from the samples. Although artificial neural networks have simpler structure than human nervous system, it is successful in solving lots of problem such as forecasting, pattern recognition and classification [10, 20, 29].

Although there are several types of artificial neural networks in the literature, only feed forward artificial neural network is used for the problem of forecasting fuzzy time series. Feed forward artificial neural network is formed of input layer, hidden layer or layers and output layer. The sample of architecture of feed forward artificial neural network is shown in Figure 1. Each layer is formed of units named as neuron and there is no relationship between the neurons belonging to the same layer. The neurons of different layers are connected to each other by their weights. In Figure 1, each weight is shown by directional arrows. In the feed forward artificial neural network, the connections shown by directional arrows are forward and one-way. In both the hidden layer and output layer of the feed forward artificial neural network, an activation function is used for each neuron. When the outputs of the neurons of the previous layer are multiplied with the related weights and these multiplication results are added to each other, the inputs that come to the neurons of hidden and output layers are formed. The information come to these neurons are passed through an activation function and the neuron output is formed. Activation function provides non-linear match. For this reason non-linear activation functions are used for the units of the hidden layer [10].

Learning in the feed forward artificial neural network is the determination of weights which produce the nearest outputs towards the target values corresponding to the outputs of artificial neural networks. Learning is provided by the optimization of the total error according to weights. There are many training algorithm proposed in the literature for the learning of feed forward artificial neural network. One of these widespread training algorithms that are employed in also this study is Levenberg-Marquardt (LM) algorithm. The detailed information about artificial neural network can be attained from [10, 17].

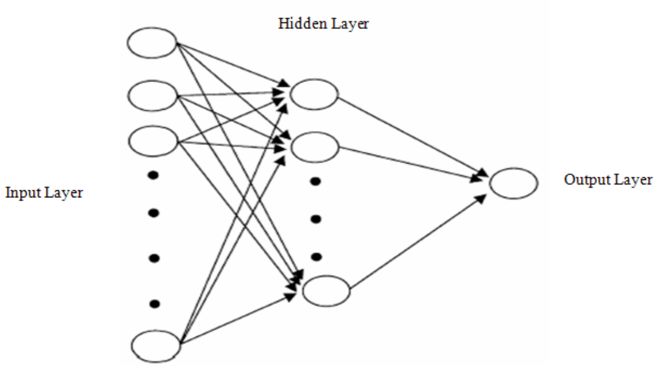

Figure 1. Multilayer feed forward artificial neural network with one output neuron

4. The Proposed Method

The forecasting models of fuzzy time series proposed are the fuzzy AR (Autoregressive) type models except for only two studies [9, 16] in the literature. However, It is necessary that ARMA type models are used to many fuzzy time series in the real life. The definition of first-order ARMA (autoregressive moving average) type fuzzy time series model made by Egrioglu et al. [9] and Kocak [16] is defined below.

Definition 5. Fuzzy time series relationships assume that $F(t)$ is caused by $F(t-1)$ and $\varepsilon(t-1)$, then the relationship can be expressed as:

$$\varepsilon(t-1), F(t-1) \rightarrow F(t) \tag{3}$$

This is called as an first-order fuzzy time ARMA (1, 1) fuzzy time series forecasting model.

In this study, an algorithm is proposed to analyze the ARMA (1, 1) model of fuzzy time series as defined in Eq. (3). In the proposed algorithm, firstly a fuzzy AR(1) defined in Eq. (1) is obtained. Then, the errors are calculated by subtracting the observed values and deffuzzified forecasts with using the results obtained from this fuzzy AR(1) model. The ARMA (1, 1) model given in Eq. (3) is forecasted by using time series and errors. The algorithm of the proposed approach is given below.

The algorithm of the proposed method

Step 1. The universe of discourse (U) and the partitions of $U(u_i, i = 1,2,...,b)$ are defined.

The beginning and the ending points of universe of discourse for time series are determined. These points are selected in the way that will include the possible values of time series. Then, universe of discourse is divided into sub-intervals (partitions) according to proper interval length. In this method, the determination of the interval length depends on the researcher. It should not be forgotten that the interval length determined is effective on the number of fuzzy sets.

Step 2. The fuzzy sets (A_i) are defined in accordance with universe of discourse (U) and the sub-intervals $(u_i, i = 1,2,\dots,b)$.

$$A_i = a_{i1}/u_1 + a_{i2}/u_2 + \cdots + a_{ib}/u_b \quad i = 1,2,\dots,b \tag{4}$$

Where,

$$a_{ik} = \begin{cases} 1 & , k = i \\ 0.5 & , k = i-1, i+1 \\ 0 & , o.w. \end{cases} , \quad i = 1,2,\dots,b \tag{5}$$

Step 3. The observations are made fuzzy.

Sub-interval is determined for each observation. The fuzzy set that has the highest membership value of the determined sub-interval is determined. The fuzzy value of the observation is this determined fuzzy set.

Step 4. Fuzzy relations are determined by the feed forward artificial neural network (FANN).

In the determination of relation by the feed forward artificial neural network, one lagged fuzzy variable $F(t-1)$ is the input of artificial neural network, $F(t)$ is target value and $\hat{F}(t)$ is the output of artificial neural network. The learning samples of artificial neural network consist of the sequence number of fuzzy sets. For instance, the observations of fuzzy time series are respectively stated as A5, A1, A4, A3, A2. In Table 1, the input and the target values of artificial neural network for this example is shown. In the determination of fuzzy relation, the architectural structure of artificial neural network is given in Figure 2.

Table 1. An example for using of FANN for determination of fuzzy relationship

$F(t-1)$	$F(t)$	Input of ANN	Target of ANN
A5	A1	5	1
A1	A4	1	4
A4	A3	4	3
A3	A2	3	2

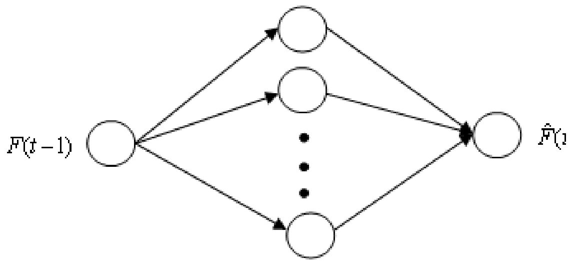

Figure 2. Architecture of ANN for determination of fuzzy relationship

Step 5. Fuzzy forecasts are obtained.

The output of the feed forward artificial neural network that has the architectural structure given in Figure 2 is the sequence numbers of fuzzy sets and the fuzzy sets that have these sequence numbers form the fuzzy forecasts. For instance, when the output of artificial neural network is rounded up as 5, the fuzzy forecast is A_5.

Step 6. Deffuzzified forecasts are obtained for the time series.

In this step centralization method is utilized. When the fuzzy forecast is A_i, deffuzzified forecast is the middle point of u_i interval which has the highest membership value in A_i fuzzy set.

Step 7. Errors are calculated.

When the deffuzzified forecast values $\hat{x}(t)$ obtained in the Step 6 are subtracted from the values observed in the time series $x(t)$, the errors are obtained. The error series $e(t)$ is calculate as,

$$e(t) = x(t) - \hat{x}(t) \quad (6)$$

Step 8. The universe of discourse (V) and its sub-intervals $(v_j, j = 1,2,...,c)$ are defined for the errors.

The universe of discourse for the errors is defined and the universe of discourse is divided in accordance with a determined interval length.

Step 9. For errors, the fuzzy sets (B_j) are defined based on universe of discourse (V) and sub-intervals $(v_j, j = 1,2,\dots,c)$.

$$B_j = b_{j1}/u_1 + b_{j2}/u_2 + \cdots + b_{jc}/u_c \; j = 1,2,\dots,c \quad (7)$$

Where

$$b_{jk} = \begin{cases} 1 & ,k = j \\ 0.5 & ,k = j-1, j+1 \\ 0 & ,o.w. \end{cases} , \quad j = 1,2,\dots,c \quad (8)$$

Step 10. The error series ($e(t)$) is made fuzzy.

Sub-interval is determined for each observation. The fuzzy set that has the highest membership value of the determined sub-interval is determined. The fuzzy value of the observation is this determined fuzzy set.

Step 11. The fuzzy relations are determined by the feed forward artificial neural network (FANN).

In the determination of relation by the feed forward artificial neural network, one lagged fuzzy variable $F(t-1)$ and one lagged fuzzy error series $\varepsilon(t-1)$ are the inputs of artificial neural network, $F(t)$ is target value and $\hat{F}(t)$ is the output of artificial neural network. The learning samples of artificial neural network consist of the sequence number of fuzzy sets. For instance, the observations of fuzzy time series are respectively stated as A5, A1, A4, A3, A2 and the observations of fuzzy error series are respectively stated as B1, B3, B7, B1, B4. In the table below, the input and the target values of artificial neural network for this example is shown.

Table 2. An example for using of FANN for determination of fuzzy relationship

$\varepsilon(t-1)$	$F(t-1)$	$F(t)$	Input 1 of ANN	Input 1 of ANN	Target of ANN
B1	A5	A1	5	1	1
B3	A1	A4	1	3	4
B7	A4	A3	4	7	3
B1	A3	A2	3	1	2

In the determination of fuzzy relation the architectural structure of artificial neural network is given in Figure 3.

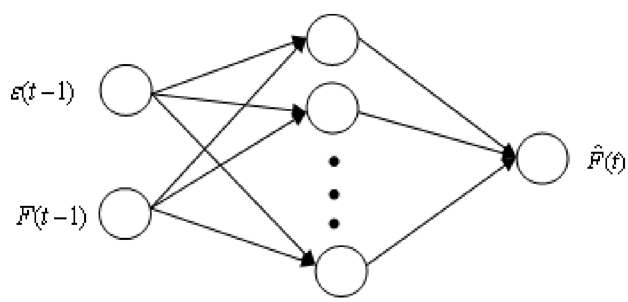

Figure 3. Architecture of FANN for determination of fuzzy relationship

Step 12. Fuzzy forecasts are obtained.

The output of the feed forward artificial neural network that has the architectural structure given in Figure 3 is the sequence number of fuzzy sets and the fuzzy sets that have these sequence numbers form the forecasts. For instance, when the output of artificial neural network is rounded up as 5, the fuzzy forecast is A_5.

Step 13. Defuzzified forecasts are obtained.

In this step centralization method is used. When the fuzzy forecast is A_i, defuzzified forecast is the middle point of u_i interval which has the highest membership value in A_i fuzzy set.

5. Application

In the application, two different real world data sets and two simulated chaotic data sets were used. The real world time series are data of Istanbul Stock Exchange and the prices of Gold. In the application of the proposed method and the methods in the literature, the fuzzy set number of the time series was increased by 1 unit between 5 and 35 and 31 different fuzzy set numbers were tried. If the method requires calculating according to interval length, the interval lengths are determined by using the formula given in the way that the fuzzy set numbers are between 5 and 35.

$$Interval\ Length = \frac{\max(Data) - \min(Data)}{Fuzzy\ Set\ Number} \tag{9}$$

At step 8 of the algorithm, 31 different fuzzy set numbers were tried also by increasing the fuzzy set number of the error series by 1 unit between 5 and 35. The hidden layer unit number in the artificial neural network used at step 4 and step 11 of the algorithm of the proposed method was modified between 1 and 10. In training the artificial neural network, the Levenberg-Marquardt method was used. In the hidden layer units and output layer, logistic activation function given in Eq. (10) was used.

$$f(x) = \frac{1}{1+\exp(-x)} \tag{10}$$

The forecasts that minimize the Root of Mean Square Error (RMSE) given in the Eq. (11) were considered the best result of the method.

$$RMSE = \left(\frac{\sum_{i=1}^{T} (y_i - \hat{y}_i)^2}{T} \right)^{1/2} \tag{11}$$

For the best results of all methods as per RMSE, MAPE (mean absolute percentage error) and DA (direction accuracy) values are calculated as in Eqs. (12) and (13) as well.

$$MAPE = \frac{1}{T} \sum_{i=1}^{T} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \tag{12}$$

$$DA = \frac{1}{T} \sum_{i=1}^{T} a_i, a_i = \begin{cases} 1 & if \quad (y_{i+1} - y_i)(\hat{y}_{i+1} - y_i) > 0 \\ 0 & otherwise. \end{cases} \tag{13}$$

5.1. Analysis of IMKB Data in the Period of 01.10.2010 - 23.12.2010

The first data used for comparing the future forecasting performances of the methods is IMKB close prices time series with 53 observations between the dates of 01.10.2010 - 23.12.2010 as seen in Table 3 and Figure 4.

Table 3. IMKB time series in the period of 01.10.2010 - 29.12.2010

Date	IMKB	Date	IMKB	Date	IMKB	Date	IMKB
01.10.2010	64883	21.10.2010	70450	11.11.2010	70085	09.12.2010	65914
04.10.2010	65415	22.10.2010	71007	12.11.2010	69553	10.12.2010	64759
05.10.2010	66283	25.10.2010	70924	15.11.2010	69998	13.12.2010	66380
06.10.2010	66880	26.10.2010	70747	22.11.2010	67926	14.12.2010	66510
07.10.2010	66759	27.10.2010	68589	23.11.2010	66335	15.12.2010	65499
08.10.2010	67217	28.10.2010	60404	24.11.2010	67231	16.12.2010	64429
11.10.2010	69000	01.11.2010	69074	25.11.2010	67149	17.12.2010	63524
12.10.2010	69675	02.11.2010	68522	26.11.2010	66148	20.12.2010	63502
13.10.2010	70167	03.11.2010	68605	29.11.2010	64072	21.12.2010	64820
14.10.2010	69226	04.11.2010	70905	30.11.2010	65351	22.12.2010	65440
15.10.2010	70101	05.11.2010	70779	01.12.2010	66156	23.12.2010	66219
18.10.2010	70458	08.11.2010	70941	02.12.2010	66939		
19.10.2010	69839	09.11.2010	71543	03.12.2010	66860		
20.10.2010	69365	10.11.2010	70561	08.12.2010	67705		

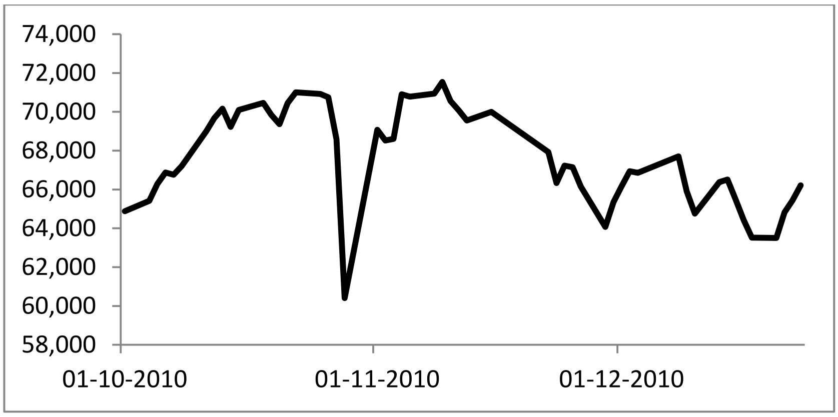

Figure 4. IMKB time series graph in the period of 01.10.2010 - 23.12.2010

Two different applications were performed by taking the last 7 observations as the test set in the first analysis of IMKB data and the last 15 observations as the test set in the second analysis as seen in Table 3. For both applications performed,

- In Song and Chissom [21] and Egrioglu et al. [9]'s method, the test set forecast having the minimum RMSE value from 31 different cases obtained by increasing the fuzzy set number by 1 unit between 5 and 35 was determined as the best result of the method.
- For each of Chen [2] ,Chen [3], Aladag et al. [1] and Kocak [16] methods, RMSE values of the different cases, which were obtained by increasing the interval length by 100 units between 300 and 2300 (in the way that the fuzzy set number is between 5 and 35 according to Eq. (9)), was found. From these forecasts, the test set forecasts having the minimum RMSE value were determined as the best results of the methods.
- By using the optimal interval lengths calculated by the distribution-based approach and average-based approach of Huarng [11], the analysis was performed with the first order fuzzy time series method of Chen [2]. In this way, the best results of the distribution-based approach and average-based approach for the test set were obtained in a single application.
- In the application of rate-based approach of Huarng and Yu [14], the best result of the method for the test set was obtained in a single application by taking the alpha parameter as 0.50.

For two different applications performed by taking the test set number as 7 and 15, in the analysis of IMKB data in the period of 01.10.2010 - 23.12.2010 by using the fuzzy ARMA(1,1) first order forecasting method which proposed;

- In the fuzzification step, 21 different numbers were tried by increasing the interval length by 100 units between 300 and 2300 (so that the fuzzy set number is between 5 and 35 according to Eq. (9)).
- Different forecasts were obtained by increasing the hidden layer unit number in the artificial neural network by 1 unit between 1 and 10 (Step 4) for each of 21 different interval lengths used.
- The error value of the first observation of data has been assumed as 0 and the error values of other observations have been calculated with the Eq. (6) for 21 different forecasts obtained.
- In the fuzzification step of the errors, 31 different numbers were tried by increasing the fuzzy set number of e(t) error series by 1 unit between 5 and 35.
- When determining the fuzzy relations in step 11 for the second time, feed-forward artificial neural network with 2 inputs and 1 output as seen in Figure 3 was used.
- When determining the fuzzy relations for the second time, forecasts having different performances were obtained by increasing the interval length of the time series by 100 between 300 and 2300 and the hidden layer unit number in the artificial neural network by 1 between 1 and 10 for each of the fuzzy set numbers (between 5 and 35) of the error series.
- From these forecasts, the forecast of the test set having the minimum RMSE value was determined as the best result of the proposed method.

In the applications of the methods by taking the last 7 observations of IMKB time series as the test set, the fuzzy set numbers that give the best results were determined as 9 in Song and Chissom method [21], 37 (the interval length is 300) in Chen 's method [2], 11 (the interval length is 1000) in the distribution-based approach and 55 (the interval length is 200) in the average-based approach of Huarng [11], which are the first order methods. In the analyses performed by using the high order models, the fuzzy set numbers that give the best forecasts were found as 5 (the interval length is 2200) in the 3rd order model of Chen [3] 18 (the interval length is 600) in the 2nd order model of Aladag et al. [1], 7 fuzzy sets were used in Egrioglu et al. [9] and 2200 length of interval was use in Kocak [16]. In the application of the proposed method. The case in which the best results were obtained, When the interval length of time series is 900, the fuzzy set number of errors is 33 and the hidden layer unit number of the artificial neural network is 10. The architecture achieved for the value 33 of the fuzzy set number of error series that gives the best result of the proposed method is submitted in Table 4. As a result of the analysis of IMKB data in the period of 01.10.2010 - 23.12.2010, the best forecasts and forecasting performances of all methods for the test set with

7 observations are summarized in Table 5.

When Table 5 is examined, it is seen that the proposed method has the best forecasting performance with minimum RMSE value of 706.84, minimum MAPE value of 0.769% and maximum direction accuracy value of 66.67% as compared with the other methods. That even RMSE value of 928.70, which was calculated by Chen method [3] having the best performance following the proposed method, is a value that is considerably higher than RMSE value of 706.84 of the proposed method indicates that the proposed method has a significantly higher performance than the other methods in the literature except Kocak [16]. The graphs of the test set forecasts obtained by using the last 7 observations of IMKB data in the period of 01.10.2010-23.12.2010 and proposed method are given together in Figure 5.

In the applications of methods by taking the last 15 observations of IMKB time series as the test set, the fuzzy set numbers that give the best results were determined as 20 by using Song and Chissom [21] method, 12 (the interval length is 900) by using Chen [2] method, 11 (the interval length is 1000) with the distribution-based approach and 55 (the interval length is 200) with the average-based approach of Huarng [11], which are the first order methods. In the analysis performed by using the high order models, the fuzzy set numbers that give the best forecasts were found as 8 (the interval length is 1400) in the 2nd order model of Chen [3], 7 (the interval length is 1500) in the 2nd order model of Aladag et al. [1], 7 fuzzy sets were used in Egrioglu et al. [9] and 1400 length of interval was used in Kocak [16]. In the application of the proposed method, the case in which the best results were obtained, when the interval length of time series is 2100, the fuzzy set number of errors is 28 and the hidden layer unit number of the artificial neural network is 8. As a result of the analysis of IMKB data in the period of 01.10.2010 - 23.12.2010, the best forecasts and forecasting performances of all methods for the test set with 15 observations are summarized in Table 6.

When Table 6 is examined, it is seen that the proposed method has the best forecasting performance with minimum RMSE value of 761.91, minimum MAPE value of 0.894% and maximum direction accuracy value of 85.714% as compared with the other methods. That even RMSE value of 896.96, which was calculated by the rate-based approach of Huarng and Yu [14] method having the best performance following the proposed method, is a value that is considerably higher than RMSE value of 761.91 of the proposed method indicates that the proposed method has a significantly higher performance than the other methods in the literature. The graphs of the test set forecasts obtained by using the last 15 observations of IMKB data in the period of 01.10.2010-23.12.2010 and the first order time series model of the proposed method are given together in Fig 6.

Table 5. The forecasts and performances of 7-observation test set for IMKB data

Date	Test Set	Song and Chissom [21]	Chen [2]	Chen [3]	Huarng [11]'s Distribution Approach.	Huarng [11]'s Avarage Approach.	Huarng and Yu [14]	Aladag et al. [1]	Egrioglu et al. [9]	Kocak [16]	Proposed Method*
15.12.2010	65499	65356	66550	65900.0	67400	66500	66784.7	67300	66177.1	66266.7	65650
16.12.2010	64429	65356	66250	65900.0	65400	66300	66178.0	64900	65332.1	63700	65650
17.12.2010	63524	65975	64450	64800.0	65900	64500	65878.7	64900	64841.9	63700	63550
20.12.2010	63502	64737	63550	64066.7	64900	63500	63514.3	64300	64841.9	63700	63550
21.12.2010	64820	64737	63550	63700.0	64900	63500	63514.3	63700	64841.9	65900	63550
22.12.2010	65440	65975	65800	64800.0	65900	65500	65878.7	65500	64841.9	65900	65650
23.12.2010	66219	65356	66250	65900.0	65400	66300	66178.0	66700	66412.8	66266.7	65650
	RMSE	1161.91	1001.70	928.70	1365.14	1014.73	1317.77	1034.06	862.21	606.07	706.84
	MAPE	0.01387	0.01217	0.01283	0.01773	0.01175	0.01593	0.01350	0.0112	0.00762	0.00769
	DA	0.50000	0.50000	0.33333	0.50000	0.50000	0.66667	0.66667	0.6667	0.83333	0.66667

*The Best Forecasts

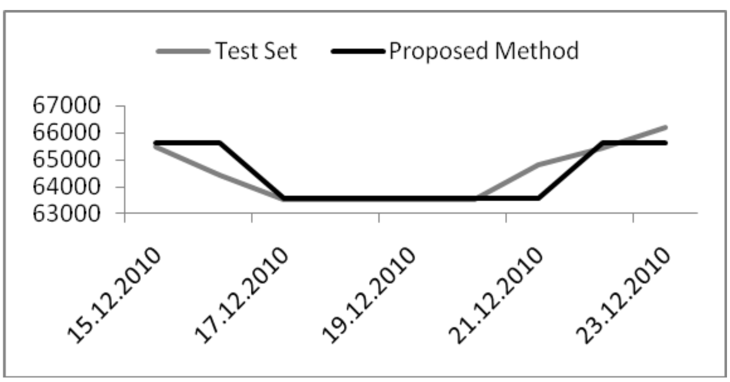

Figure 5. The graph of 7-observation test set and the forecasts obtained by using proposed method for IMKB data

Table 6. The forecasts and performances of 15-observation test set for IMKB data

Date	Test Set	Song and Chissom [21]	Chen [2]	Chen [3]	Huarng [11]'s Distribution Approach.	Huarng [11]'s Avarage Approach.	Huarng and Yu [14]	Aladag et al. [1]	Egrioglu et al. [9]	Kocak [16]	Proposed Method*
01.12.2010	66156	65695.5	66250	64833.3	65900.0	65300	66328.8	65650	66177	66000	66250
02.12.2010	66939	66438.2	65800	66700.0	65566.7	64100	66164.2	67150	66177	67633	67150
03.12.2010	66860	67088.0	67450	66700.0	67233.3	66700	67143.8	67150	66177	66700	67150
08.12.2010	67705	67088.0	67450	66700.0	67233.3	66700	67143.8	67150	66177	66700	67150
09.12.2010	65914	66252.5	66250	67633.3	65900.0	67700	66328.8	67150	66177	68100	67150
10.12.2010	64759	65695.5	65800	66233.3	65566.7	65900	65791.4	64150	66177	65300	65350
13.12.2010	66380	65695.5	65350	66700.0	65900.0	64700	65258.3	65650	66177	65300	65350
14.12.2010	66510	66438.2	65800	66700.0	65566.7	67100	66164.2	65650	66177	66700	67150
15.12.2010	65499	66438.2	65800	66700.0	67233.3	66500	66164.2	67150	66177	66700	66250
16.12.2010	64429	65695.5	66250	65766.7	65566.7	66300	66328.8	64150	66026	65300	64450
17.12.2010	63524	65695.5	65350	64366.7	65900.0	64500	65258.3	65650	66177	63900	65350
20.12.2010	63502	65138.5	63550	63900.0	64900.0	63500	63684.8	65650	64841	63900	62650
21.12.2010	64820	65138.5	63550	63900.0	64900.0	63500	63684.8	65650	66177	63900	65350
22.12.2010	65440	65695.5	65350	64833.3	65900.0	65500	65258.3	65650	66177	65300	65350
23.12.2010	66219	65695.5	66250	66700.0	65566.7	66300	66328.8	65650	65509	66000	66250
	RMSE	919.47	925.42	954.17	1052.90	1283.18	896.96	1060.12	1164.8	865.28	761.91
	MAPE	0.01124	0.01081	0.01245	0.01285	0.01558	0.01085	0.01312	0.0146	0.0102	0.0089
	DA	0.71429	0.57143	0.64286	0.50000	0.57143	0.64286	0.64286	0.5714	0.7142	0.8571

*The Best Forecasts

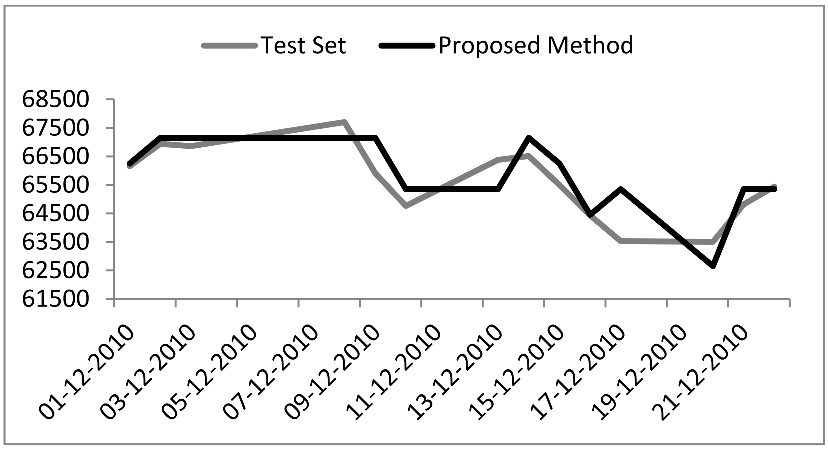

Figure 6. The graph of 15-observation test set and the forecasts obtained by using proposed method for IMKB data

5.2. Analysis of Gold Prices

The second data used for comparing the future forecasting performances of the methods is the time series of the gold prices with 248 observations between the dates of 02.01.2009 - 31.12.2009, which were taken from the website of the Central Bank of the Republic of Turkey as seen in Table 7 and Figure 7.

Two different applications were performed by taking the last 15 observations as the test set in the first analysis and the last 30 observations as the test set in the second analysis of the gold prices data of 2009 as seen in Table 8. For both applications performed, 31 different situations were tried by increasing the fuzzy set number of the time series and the fuzzy set number of the error series by 1 unit between 5 and 35. If the method is analyzed into the interval lengths, the interval lengths were determined in a way to being in the range of 5-35 according to Eq. (9).

Table 7. The gold prices of 2009

Tarih	TL/KGG	Tarih	TL/KGG	Tarih	TL/KGG	Tarih	TL/KGG	Tarih	TL/KGG
02.01.2009	42455	13.03.2009	50510	28.05.2009	47840	07.08.2009	45915	20.10.2009	49810
05.01.2009	42030	16.03.2009	50100	29.05.2009	48295	10.08.2009	45050	21.10.2009	49570
06.01.2009	41075	17.03.2009	50385	01.06.2009	48045	11.08.2009	45600	22.10.2009	49970
07.01.2009	42060	18.03.2009	49985	02.06.2009	48080	12.08.2009	45700	23.10.2009	50162
08.01.2009	42020	19.03.2009	51250	03.06.2009	48200	13.08.2009	45570	26.10.2009	50355
09.01.2009	42310	20.03.2009	51430	04.06.2009	47930	14.08.2009	45710	27.10.2009	50020
12.01.2009	42460	23.03.2009	50820	08.06.2009	47675	17.08.2009	45290	30.10.2009	50030
13.01.2009	42000	24.03.2009	49070	09.06.2009	47620	18.08.2009	45200	02.11.2009	50700
14.01.2009	42010	25.03.2009	49470	10.06.2009	47750	19.08.2009	45200	03.11.2009	51230
15.01.2009	42320	26.03.2009	49900	11.06.2009	47450	20.08.2009	45365	04.11.2009	52490
16.01.2009	42675	27.03.2009	49775	12.06.2009	46585	21.08.2009	45255	05.11.2009	52150
19.01.2009	43840	30.03.2009	49665	15.06.2009	46550	24.08.2009	45550	06.11.2009	52125
20.01.2009	44830	31.03.2009	49310	16.06.2009	46485	25.08.2009	45580	09.11.2009	52500
21.01.2009	45600	01.04.2009	49115	17.06.2009	46730	26.08.2009	45570	10.11.2009	52165
22.01.2009	44770	02.04.2009	47100	18.06.2009	47225	27.08.2009	45750	11.11.2009	52560
23.01.2009	46630	03.04.2009	46285	19.06.2009	46900	28.08.2009	45940	12.11.2009	52972
26.01.2009	47620	06.04.2009	44365	22.06.2009	46750	31.08.2009	45865	13.11.2009	52800
27.01.2009	46580	07.04.2009	45350	23.06.2009	46770	01.09.2009	46150	16.11.2009	53500
28.01.2009	45930	08.04.2009	45435	24.06.2009	46875	02.09.2009	46900	17.11.2009	53935
29.01.2009	45570	09.04.2009	44345	25.06.2009	47065	03.09.2009	47600	18.11.2009	54550
30.01.2009	48400	10.04.2009	44600	26.06.2009	46955	04.09.2009	47870	19.11.2009	54495
02.02.2009	48215	13.04.2009	45300	29.06.2009	46570	07.09.2009	47435	20.11.2009	54830
03.02.2009	47780	14.04.2009	45000	30.06.2009	46290	08.09.2009	48025	23.11.2009	55950
04.02.2009	47275	15.04.2009	45985	01.07.2009	46030	09.09.2009	47825	24.11.2009	56285
05.02.2009	48277	16.04.2009	45800	02.07.2009	45870	10.09.2009	47820	25.11.2009	56430
06.02.2009	48030	17.04.2009	45155	03.07.2009	46170	11.09.2009	48385	01.12.2009	57635
09.02.2009	46534	20.04.2009	46193	06.07.2009	46080	14.09.2009	48300	02.12.2009	58330
10.02.2009	46980	21.04.2009	47300	07.07.2009	46015	15.09.2009	47795	03.12.2009	58150
11.02.2009	48440	24.04.2009	47115	08.07.2009	46095	16.09.2009	48300	04.12.2009	56630
12.02.2009	49609	27.04.2009	47445	09.07.2009	45620	17.09.2009	48320	07.12.2009	54820
13.02.2009	49175	28.04.2009	46620	10.07.2009	45660	18.09.2009	48255	08.12.2009	55660
16.02.2009	49940	29.04.2009	46180	13.07.2009	45700	23.09.2009	48435	09.12.2009	55110
17.02.2009	52200	30.04.2009	45580	14.07.2009	45985	24.09.2009	48350	10.12.2009	54180
18.02.2009	52320	04.05.2009	45650	15.07.2009	45925	25.09.2009	47697	11.12.2009	54580
19.02.2009	52450	05.05.2009	45236	16.07.2009	46155	28.09.2009	47760	14.12.2009	54190
20.02.2009	54350	06.05.2009	45850	17.07.2009	45920	29.09.2009	47570	15.12.2009	54120
23.02.2009	53267	07.05.2009	45830	20.07.2009	46350	30.09.2009	47740	16.12.2009	54855
24.02.2009	53560	08.05.2009	45700	21.07.2009	45950	01.10.2009	48483	17.12.2009	54430
25.02.2009	51950	11.05.2009	45600	22.07.2009	45810	02.10.2009	48350	18.12.2009	53750
26.02.2009	50985	12.05.2009	45950	23.07.2009	45790	05.10.2009	48151	21.12.2009	54570
27.02.2009	52210	13.05.2009	46240	24.07.2009	45520	06.10.2009	48635	22.12.2009	53400
02.03.2009	52330	14.05.2009	46885	27.07.2009	45600	07.10.2009	49330	23.12.2009	52990
03.03.2009	51030	15.05.2009	46550	28.07.2009	45225	08.10.2009	49550	24.12.2009	53575
04.03.2009	50700	18.05.2009	46470	29.07.2009	44950	09.10.2009	49430	25.12.2009	53450
05.03.2009	51485	20.05.2009	45980	30.07.2009	44640	12.10.2009	49500	28.12.2009	53795
06.03.2009	53510	21.05.2009	46405	31.07.2009	44650	13.10.2009	49980	29.12.2009	53515
09.03.2009	54530	22.05.2009	47200	03.08.2009	45315	14.10.2009	49398	30.12.2009	53095
10.03.2009	51590	25.05.2009	47300	04.08.2009	45220	15.10.2009	48990	31.12.2009	52920
11.03.2009	50170	26.05.2009	47710	05.08.2009	45535				
12.03.2009	51955	27.05.2009	47450	06.08.2009	46080				

*TL: Turkish Lira, KG: Kilogram

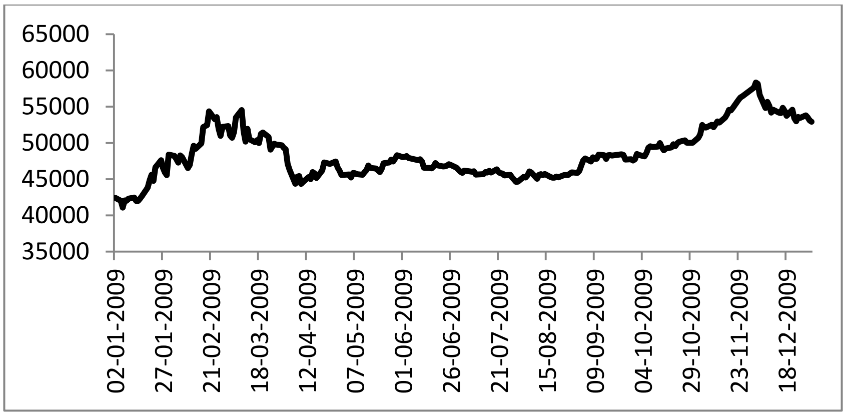

Figure 7. The gold prices graph of 2009

In the analysis of the methods by taking the last 15 observations of the gold prices of 2009 as the test set, the fuzzy set numbers that give the best results were determined as 33 by using Song and Chissom [21] method, 29 (the interval length is 600) by using Chen [2] method, 43 (the interval length is 400) with the distribution-based approach and 86 (the interval length is 200) with the average-based approach of Huarng [11] which are the first order methods. In the analyses performed by using the high order models, the fuzzy set numbers that give the best forecasts were found as 22 (the interval length is 800) in the 5th order model of Chen [3] and 19 (the interval length is 900) in the 2nd order model of Aladag [1], 7 fuzzy sets were used in Egrioglu et al [9] and 300 length of interval was used in Kocak [16]. In the application of the proposed method, The case in which the best result were obtained when the interval length of time series is 1000, the fuzzy set number of errors is 28 and the hidden layer unit number of the artificial neural network is 6. As a result of the analysis of the time series of the gold prices of 2009, the best forecasts and forecasting performances of all methods for the test set with 15 observations are summarized in Table 8.

When Table 8 is examined, it is seen that the proposed method has the best forecasting performance with minimum rmse value of 404.24, minimum mape value of 0.563% and maximum direction accuracy (da) value of 71.43% as compared with the other methods. The graphs of the test set and forecasts the proposed method are given together in Figure 8.

In the analysis of the methods by taking the last 30 observations of the gold prices of 2009 as the test set, the fuzzy set numbers that give the best results were determined as 10 by using Song and Chissom [21] method, 29 (the interval length is 600) by using Chen [2] method, 43 (the interval length is 400) with the distribution-based approach and 86 (the interval length is 200) with the average-based approach of Huarng [11], which are the first order methods. In the analyses performed by using the high order models, the fuzzy set numbers that give the best forecasts were found as 22 (the gap length is 800) in the 3td order model of Chen [3], 29 (the interval length is 600) in the 5th order model of Aladag [1], 7 fuzzy sets were used in Egrioglu et al. [9] and 800 length of interval was used in Kocak [16]. In the application of the proposed method, The case in which the best result were obtained when the interval length of time series is 1700, the fuzzy set number of errors is 16 and the hidden layer unit number of the artificial neural network is 9. As a result of the analysis of the time series for the gold prices of 2009, the best forecasts and forecasting performances of all methods for the test set with 30 observations are summarized in Table 9.

When Table 9 is examined, it is seen that the proposed method has the best forecasting performance with minimum RMSE value of 714.61, minimum MAPE value of 1.132% and maximum direction accuracy value of 51.72% as compared with the other methods. That even RMSE value of 857.34, which was calculated by using Chen (2002)' method having the best performance following the proposed method, is a value that is considerably higher than RMSE value of 761.91 of the proposed method indicates that the proposed method has a significantly higher performance than the other methods in the literature. The graphs of the test set and forecasts of proposed method are given together in Figure 9.

Moreover, two simulated time series were used to compare proposed methods with other alternatives. Simulated time series are chaotic series and they generated by following formula [19]:

$$I_t = \sin(12t) + \varepsilon_t \tag{14}$$

Number of Observations of these series are 250. The last 45 data was taken as test set, other observations were used as training data. Similar applications were made for both of simulated series and the obtained results of applications were given Table 10 and 11. When Table 10 and 11 are examined, it is clearly seen that the proposed method produces the best result for both simulated series.

Table 8. The forecasts and performances of 15-observation test set for Gold Prices

Date	Test Set	Song and Chissom [21]	Chen [2]	Chen [3]	Huarng [11]'s Distribution Approach.	Huarng [11]'s Avarage Approach.	Huarng and Yu [14]	Aladag et al. [1]	Egrioglu et al. [9]	Kocak [16]	Proposed Method*
11.12.2009	54580	53818.8	54500	55000.0	54400.0	54100.0	53941.0	54050	54192	54300	54500
14.12.2009	54190	53818.8	53600	54626.7	53500.0	53633.3	54343.5	54050	54192	54300	54500
15.12.2009	54120	53818.8	54500	54413.3	54400.0	54100.0	53941.0	54050	54192	54300	54500
16.12.2009	54855	54411.5	54500	54253.3	54400.0	54100.0	54704.2	54050	54192	54300	54500
17.12.2009	54430	54376.6	54800	54466.7	55800.0	55800.0	54343.5	54050	54192	55000	54500
18.12.2009	53750	53818.8	53600	54413.3	53500.0	53633.3	53941.0	54050	54192	54300	53500
21.12.2009	54570	54411.5	54500	54093.3	53466.7	53700.0	54704.2	54050	54192	54300	53500
22.12.2009	53400	53818.8	53600	54093.3	53500.0	53633.3	54343.5	54050	54192	54300	53500
23.12.2009	52990	53178.7	53450	53826.7	53466.7	53433.3	53566.9	53150	53205	52900	53500
24.12.2009	53575	53104.0	53150	53346.7	53200.0	53200.0	53555.3	53150	52218	52900	53500
25.12.2009	53450	53178.7	53450	53293.3	53466.7	53433.3	53566.9	53150	53205	52340	53500
28.12.2009	53795	53178.7	53450	53293.3	53466.7	53433.3	53566.9	53150	53205	54300	53500
29.12.2009	53515	54411.5	54500	53293.3	53466.7	53700.0	54704.2	54050	54192	54300	53500
30.12.2009	53095	53178.7	53450	53346.7	53466.7	53433.3	53566.9	53150	53205	52900	53500
31.12.2009	52920	53104.0	53450	53400.0	53600.0	53100.0	53555.3	53150	52218	52900	53500
	RMSE	429.59	425.65	471.57	579.62	545.40	508.47	442.35	569.98	550.55	404.24
	MAPE	0.00654	0.00657	0.00781	0.00830	0.00777	0.00711	0.00710	0.0085	0.00841	0.00563
	DA	0.78571	0.64286	0.64286	0.64286	0.57143	0.71429	0.71429	0.7143	0.71429	0.71429

*The Best Forecasts

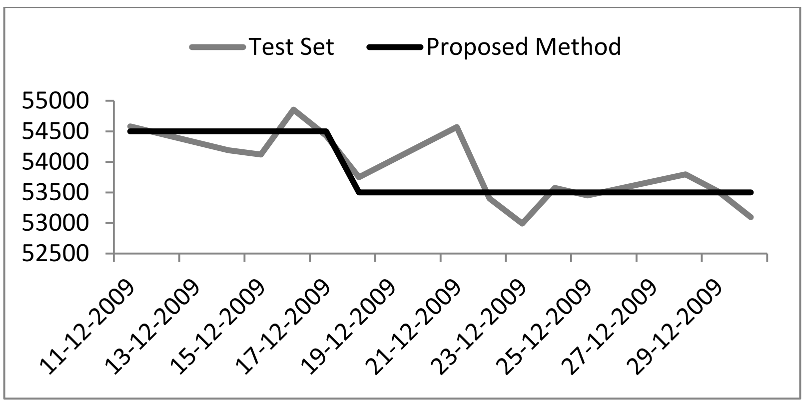

Figure 8. The graph of 15-observation test set and the forecasts obtained by using proposed method for gold prices

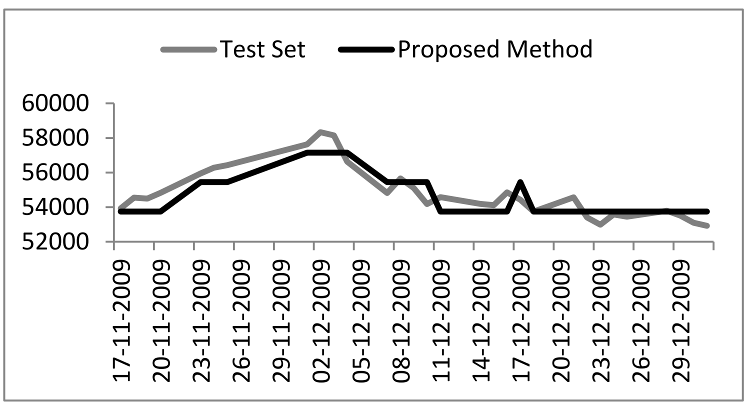

Figure 9. The graph of 30-observation test set and the forecasts obtained by using proposed method for gold prices

Table 9. The forecasts and performances of 30-observation test set for gold prices

Date	Test Set	Song and Chissom [21]	Chen [2]	Chen [3]	Huarng [11]'s Distribution Approach.	Huarng [11]'s Avarage Approach.	Huarng and Yu [14]	Aladag et al. [1]	Egrioglu et al. [9]	Kocak [16]	Proposed Method*
17.11.2009	53935	53157	53300	53000.0	53200	53200	53356.4	53300	52987	53400	53750
18.11.2009	54550	53157	53900	53666.7	54000	53900	53894.0	53900	52987	54200	53750
19.11.2009	54495	53157	52400	54066.7	52400	51500	51715.7	54500	52987	54200	53750
20.11.2009	54830	53157	52400	54200.0	52400	51500	51715.7	53900	52987	54200	53750
23.11.2009	55950	53157	55100	54600.0	54800	54900	55017.3	54500	52987	55000	55450
24.11.2009	56285	52294	55700	55266.7	56000	55900	55779.2	55100	52987	55800	55450
25.11.2009	56430	52294	56300	56066.7	56400	56300	56164.1	55700	52987	56600	55450
01.12.2009	57635	52294	56300	56466.7	56400	56500	56551.6	55100	52987	56600	57150
02.12.2009	58330	53157	57500	57000.0	57600	57700	57730.3	55700	52987	57400	57150
03.12.2009	58150	53157	58100	57666.7	58400	58300	58529.8	56300	52987	58200	57150
04.12.2009	56630	53157	58100	58066.7	58000	58100	58128.7	56300	52987	58200	57150
07.12.2009	54820	53157	56900	57400.0	56800	56700	56551.6	55100	52987	56600	55450
08.12.2009	55660	53157	55100	56066.7	54800	54900	51715.7	55100	52987	55000	55450
09.12.2009	55110	52294	55700	55666.7	55600	55700	55779.2	55100	52987	55800	55450
10.12.2009	54180	52294	55100	55266.7	55200	55100	55017.3	54500	52987	55000	55450
11.12.2009	54580	53157	53900	54733.3	54000	54100	53157.9	53900	48982	54200	53750
14.12.2009	54190	53157	52400	54333.3	52400	51500	51715.7	54500	52987	54200	53750
15.12.2009	54120	53157	53900	54200.0	54000	54100	53157.9	53300	48974	54200	53750
16.12.2009	54855	53157	53900	54200.0	54000	54100	53157.9	53900	52987	54200	53750
17.12.2009	54430	53157	55100	54600.0	54800	54900	55017.3	54500	52987	55000	55450
18.12.2009	53750	53157	52400	54466.7	52400	51500	53157.9	54500	52987	54200	53750
21.12.2009	54570	53157	53900	53933.3	53200	53700	53894.0	53300	52987	53400	53750
22.12.2009	53400	53157	52400	53933.3	52400	51500	51715.7	54500	52987	54200	53750
23.12.2009	52990	53157	53300	53666.7	53200	53200	53356.4	53300	52987	53400	53750
24.12.2009	53575	52294	53150	53133.3	53200	53200	53524.7	52700	52987	53400	53750
25.12.2009	53450	53157	53300	53133.3	53200	53200	53356.4	53300	52987	53400	53750
28.12.2009	53795	53157	53300	53266.7	53200	53200	53356.4	53300	52987	53400	53750
29.12.2009	53515	53157	53900	53400.0	53200	53700	53894.0	53900	52987	53400	53750
30.12.2009	53095	53157	53300	53400.0	53200	53200	53356.4	52700	52987	53400	53750
31.12.2009	52920	52294	53300	53400.0	53600	53100	53524.7	53300	52987	53400	53750
	RMSE	2410.96	1031.12	857.34	1045.23	1288.80	1412.66	1003.50	2687	707.71	714.61
	MAPE	0.03339	0.01512	0.01245	0.01530	0.01713	0.01935	0.01382	0.0375	0.01028	0.01132
	DA	0.55172	0.55172	0.51724	0.55172	0.62069	0.55172	0.48276	0.5517	0.62069	0.51724

*The Best Forecasts

Table 10. Application results for first simulated time series

Song and Chissom [21]	Chen [2]	Chen [3]	Huarng [11]'s Distribution Approach.	Huarng [11]'s Avarage Approach.	Huarng and Yu [14]	Aladag et al. [1]	Egrioglu et al. [9]	Kocak [16]	Proposed Method*
1.13633	1.12305	1.30221	1.41151	1.67611	1.16162	1.03824	1.2384	1.0926	1.03261
0.04166	0.04131	0.04977	0.06070	0.05163	0.08947	0.03951	1.2713	1.4807	0.03909
0.75000	0.79545	0.70455	0.63636	0.63636	0.50000	0.79545	0.6591	0.7500	0.81818

Table 11. Application results for second simulated time series

Song and Chissom [21]	Chen [2]	Chen [3]	Huarng [11]'s Distribution Approach.	Huarng [11]'s Avarage Approach.	Huarng and Yu [14]	Aladag et al. [1]	Egrioglu et al. [9]	Kocak [16]	Proposed Method*
0.95767	0.94760	1.02005	0.96481	1.18411	0.97050	1.00771	1.0026	0.9201	0.87876
0.00968	0.00928	0.02063	0.00930	0.01807	0.07810	2.38074	1.1373	1.0695	0.00835
0.77273	0.68182	0.61364	0.65909	0.61364	0.27273	0.54545	0.6136	0.7045	0.77273

6. Discussion and Conclusions

The models used in the classical time series methods include not only Autoregressive (AR) variables, but also Moving Average (MA) variables. However, the forecasting models of the fuzzy time series proposed in the literature do not included MA variables except for two studies. In this study, an analysis algorithm based on the artificial neural networks was proposed by identifying a new fuzzy time series forecasting model of ARMA(1,1) type that includes the fuzzy MA variables along with the fuzzy AR variables. In the proposed method, finding MA variable reveals a more realistic approach since the majority of the real life time series should be analyzed by using the models including MA variables.

Furthermore, it was seen as a result of the application that the inclusion of MA variables into the model increases the forecasting performance. Moreover, it is remarkable that the proposed method has considerably high direction accuracy (DA) as well as it provides lower RMSE and MAPE values than the other methods in the literature. In the proposed method, the use of feed forward artificial neural networks at the step of determining the fuzzy relation make easer the process of determining the fuzzy relation. In the future studies, it can be ensured that the proposed method has more systematical and higher forecasting performance through the improvements to be made at various steps.

REFERENCES

[1] Aladag, C.H., Basaran, M.A., E. Egrioglu, E., Yolcu, U., Uslu, V.R. (2009) Forecasting in high order fuzzy time series by using neural networks to define fuzzy relations, Expert Systems with Applications, 36, 3, 4228-4231.

[2] Chen, S.M. (1996) Forecasting enrollments based on fuzzy time-series, Fuzzy Sets and Systems, 81, 3, 311-319.

[3] Chen, S.M. (2002) Forecasting enrollments based on high order fuzzy time series, Cybernetics and Systems, 33, 1, 1-16.

[4] Chen, S.M., Hwang, J.R. (2000) Temperature prediction using fuzzy time series, IEEE Transaction on Systems Man and Cybernetics, Part B, 30, 2, 263-275.

[5] Cheng, C., Cheng, G., Wang, J. (2008) Multi-attribute fuzzy time series method based on fuzzy clustering, Expert Systems with Applications, 34, 2, 1235-1242.

[6] Egrioglu, E., Aladag, C.H., Basaran, M.A., Uslu, V.R., Yolcu, U. (2011) A New approach based on the optimization of the length of intervals in fuzzy time series, Journal of Intelligent and Fuzzy Systems, 22, 1, 15-19.

[7] Egrioglu, E., Aladag, C.H., Yolcu, U., Uslu, V.R., Basaran, M.A. (2009) A new approach based on artificial neural networks for high order multivariate fuzzy time series, Expert Systems with Applications, 36, 7, 10589-10594

[8] Egrioglu, E., Aladag, C.H., Yolcu, U., Uslu, V.R., Basaran, M.A. (2010) Finding an optimal interval length in high order fuzzy time series, Expert Systems with Applications, 37, 7, 5052-5055.

[9] Egrioglu, E., Yolcu, U., Aladag, C.H., Kocak, C. (2013) An ARMA type fuzzy time series forecasting method based on particle swarm optimization, Mathematical Problems in Engineering 2013 (Article ID 935815), 12 pages.

[10] Gunay, S., Egrioglu, E., Aladag, C.H. (2007) An introduction to single variable time series analysis, Hacettepe University Press, Ankara, Turkey.

[11] Huarng K. (2001) Effective length of intervals to improve forecasting in fuzzy time series, Fuzzy Sets and Systems, 123, 3, 387-394.

[12] Huarng, K., Yu, H. (2005) A type 2 fuzzy time series model for stock index forecasting, Physica A, 353, 445-462.

[13] Huarng, K., Yu, H. (2006) The application of neural networks to forecast fuzzy time series, Physica A, 363, 2, 481-491.

[14] Huarng, K., Yu, T.H. (2006) Ratio-based lengths of intervals to improve fuzzy time series forecasting. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 36, 2, 328-340.

[15] Hwang, J.R., Chen, S.M., Lee, C.H. (1998) Handling forecasting problems using fuzzy time series, Fuzzy Sets and Systems, 100, 1-3, 217-228.

[16] Kocak, C. (2013) First-Order ARMA type fuzzy time series method based on logic relation tables, Mathematical Problems in Engineering 2013 (Article ID 769125), 12 pages.

[17] Kısın, O. (2004) Multi-layer perceptrons with Levenberg-Marquardt training algorithm for suspended sediment concentration prediction and estimation, Hydrolojical Sciences–Journal – des Sciences Hydrologiques, 49, 6, 1025-1040.

[18] Li, S., Cheng, Y., Lin, S. (2008) A FCM-based deterministic forecasting model for fuzzy time series, Computers and Mathematics with Applications, 56, 12, 3052-3063.

[19] Maia, A.L.S., De Carvalho, F.A.T. (2011) Holt's exponential smoothing and neural network models for forecasting

internal-valued time series, İnternational Journal of Forecasting, 27, 3, 740-759.

[20] Oztemel, E. (2006) Artificial neural networks Papatya Publications, İstanbul, Turkey.

[21] Song, Q., Chissom, B.S. (1993) Fuzzy time series and its models, Fuzzy Sets and Systems, 54, 3, 269-277.

[22] Song, Q., Chissom, B.S. (1993) Forecasting enrollments with fuzzy time series- Part I, Fuzzy Sets and Systems, 54, 1, 1-9.

[23] Song, Q., Chissom, B.S. (1994) Forecasting enrollments with fuzzy time series-Part II, Fuzzy Sets and Systems, 62, 1, 1-8.

[24] Sullivan, J., Woodall, W.H. (1994) A comparison of fuzzy forecasting and markov modeling, Fuzzy Sets and Systems, 64,3, 279 - 293.

[25] Yolcu, U., Egrioglu, E., Uslu, V.R., Basaran, M.A., Aladag, C.H. (2009) A New Approach for determining the length of intervals for fuzzy time series, Applied Soft Computing, 9, 2, 647-651.

[26] Yu, H. (2005) A refined fuzzy time series model for forecasting, Physica A, 346, 3-4, 657-681.

[27] Yu, H. (2005) Weighted fuzzy time series models for TAIEX forecasting, Physica A, 349, 3-4, 609-624.

[28] Zadeh, L.A. (1965) Fuzzy Sets, Inform and Control, 8, 3, 338-353.

[29] Zurada, J.M. (1992) Introduction of artificial neural systems, West Publishing, N.Y.