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Optimization of Fuzzified Economic Order Quantity Model Allowing Shortage and Deterioration with Full Backlogging

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Abstract In this paper, an attempt has been made to optimize fuzzified economic order quantity model allowing shortage and deterioration with full backlogging. Triangular membership function using Zadeh's extension principle and method of centroid for defuzzification has been used to develop a system of non-linear equations which is further solved by using fast converging Newton-Raphson (N-R) method. The optimized and fuzzified total cost of economic order quantity model as an important performance measure has been computed and sensitivity analysis has been also presented to demonstrate the use of the model under consideration.

Keywords Optimization of Fuzzified EOQ Control System, Fuzzification and Defuzzification, System of Nonlinear Equations, Computational Approach

1. Introduction

Inventory items are elaborated by man, material, machine, money and method to optimize the cost of inventory control system, (see Ackoff and Sasieni (1993)). The first inventory model of inventory control system was developed by Harris (1915). Later, significant amount of research has been conducted in the area of inventory control system including fuzzified inventory control system. Attention of several authors has been drawn to this area of research, most notably that of and ultimately culminated into notable reporting of results of Zadeh (1965, 1965a) who introduced the concept of fuzzy set as an extension of classical set theory. In fuzzy set theory elements of the set have degrees of membership. Moreover, the fuzzy set theory was extended to various systems. Zadeh (1978) propounded the theory of possibility on the basis of fuzzy set as a revolutionary area of knowledge in scientific applications.

A mathematical model on decision making in a fuzzy environment was developed by Zadeh and Bellman (1970). Jain (1976) developed a fuzzy decision model using fuzzy variables and some important operations were discussed by Dubois and Prade (1978). An application of fuzzy sets in the area of operations research was suggested by Zimmerman (1983). It gradually proliferated and embraced the inventory management problems for its wider spectrum of accessibility. A fresh attempt was made by Kacpryzk and Staniewski (1982) who dwelt upon a model on long-term inventory policy-making through fuzzy-decision making models.

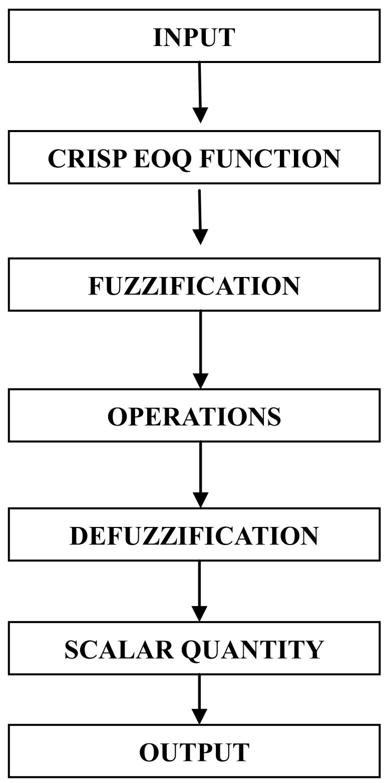

Figure 1.1. Fuzzified EOQ Control System

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A fuzzy set theoretical interpretation of economic order quantity inventory problem was presented by Park (1987). Considering inventory cost as a fuzzy number, an economic order quantity was discussed by Vujosevic et al. (1996). A fuzzified inventory control system is given in figure 1.1 as under.

The above figure in the context of present problem consists of several processes. Crisp EOQ function is input; it is fuzzified by membership function and its relevant extension principle given by Zadeh. Further, operations on fuzzy system are introduced and later it is defuzzified by some suitable method of defuzzification which ultimately provides single scalar quantity as output. This output may be one of performance measures of the fuzzified EOQ control system under consideration, as per Van and Kerre (1999).

A fuzzy inventory model with or without backorder for fuzzy order quantity with trapezoidal fuzzy number was investigated by Yao and Lee (1999). Yao and Lee (1999a) also proposed and discussed an economic order quantity model in fuzzy sense for inventory without backorder model.

Zimmermann (2001) conceptualized the fuzzy set and developed its applications. Kao and Hsu (2002) made an attempt to discuss an inventory model with fuzzy demand under single period and Hsieh (2002) analyzed optimization of fuzzified production for an inventory model. An inventory without back order with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance was discussed by Yao and Chiang (2003). They compared the optimal results obtained by both the defuzzification methods. Deterioration of an item in the inventory system is one of integral characteristics which needs to be explored at deeper level. If these characterisrtics are fuzzified then its applications are more realistic and meaningful. Inventory model involving fuzzy demand rate and fuzzy deterioration rate was attempted by Kumar et al. (2003). A fuzzified deteriorating inventory model with breakdown of machine and shortage cost was investigated by Mishra et al. (2004).

Lengari (2005) has attempted to deal with fuzzy logic intelligence control system of information. Syed and Aziz (2007) formulated an inventory model without shortage and fuzzified it by applying signed distance method. Mahata and Goswami (2007) investigated an EOQ model for deteriorating items over trade credit financing in the fuzzy sense. A production inventory model with remanufacturing for defective and usable items in fuzzy environment was reported by Roy et al. (2008). Roy and Maity (2009) proposed an inventory model with remanufacturing. Mishra and Mishra (2011) investigated an inventory model for deterioration item under cobweb phenomenon and permissible delay in payment for fuzzy environment. A fuzzy inventory model without shortages using triangular fuzzy number was formulated and analyzed by De and Rawat (2011). A technique of possibility and necessity approach was developed by Pathak and Sarkar (2012) for fuzzy production model of deteriorating inventory allowing shortage subject to time dependent learning and forgetting. Phase wise supply chain model of EOQ with normal life time for queued customers using a computational approach was analyzed at length by Mishra and Mishra (2012).

A fuzzy inventory model for deteriorating items with time-varying demand and shortages was discussed by Jaggi et al. (2012). Similarly, a fuzzy EOQ model for time dependent deteriorating items and time dependent demand with shortages was investigated by Saha and Chakrabarti (2012). A fuzzy inventory model without shortage was discussed by Dutta and Pawan (2012) by using trapezoidal fuzzy number and further by Dutta and Pawan (2013) who considered fuzziness in demand, holding cost and ordering cost for an inventory model without shortages.

Pathak and Mondal (2013) considered an EOQ model for random Weibull deterioration with Ramp-type demand, partial backlogging and inflation under trade credit financing in the fuzzy sense. Prasath and Seshaiah (2013) discussed an inventory model allowing shortage for fuzzy production distribution. Mishra (2014) developed intelligent index for fuzzified inventory of supply chain by using neural computing. Further, Mishra et al. (2015) analyzed an inventory flow in supply chain with deteriorating items for customers in queue for computation of profit optimization in fuzzy environment. Recently, Dutta and Pawan (2013) attempted Fuzzy inventory model for deteriorating items with shortage under the fully backlogged condition. Kumar and Rajput (2015) attempted to describe fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. Sen and Malakar (2015) discussed a fuzzy inventory model with shortages using different fuzzy numbers. Dinagar and Kannan (2015) dealt with fuzzy inventory model with allowable shortage.

In the present paper, we attempt optimization of fuzzified economic order quantity model allowing shortage and deterioration with full backlogging using different methods of fuzzification and defuzzification to compute fuzzified total optimal cost of the system as to explore more efficient results. The present method is sought to employ the Zadeh's extension principle using triangular fuzzy membership and defuzzified by centroid method so as to develop a system of non-linear equations and to solve it by using fast converging Newton-Raphson (N-R) method by using C++ language, as in Jeffery (2004). Proposed work intends to leverage the edge of consistency of centroid method of defuzzification and fast convergence of N-R method over previous works in order to yield more efficient and stable results, as in Saneifard and Saneifard (2011), Abbasbandy and Ghanbari (2013). A sensitivity analysis has been also presented to demonstrate the use of the model under consideration.

2. Notations and Assumptions

The proposed inventory model uses the following notations and assumptions:

2.1. Notations

The following notations are used in the present paper:

i. c_0 : Ordering cost per order.
ii. h : Holding cost per unit per unit time.
iii. S : Shortage cost per unit time.
iv. p_c : Purchasing cost per unit per unit time.
v. r : Demand rate at any time t per unit time.
vi. α : Deterioration rate function, where $0<\alpha<1$.
vii. T : Length of each ordering cycle.
viii. q : Order quantity per unit.
ix. C_S : Total shortage cost per unit time.
x. C : Total cost per unit time.
xi. $\tilde{C}$: Total cost per unit time in fuzzy environment.
xii. $I(t)$: The inventory level at any instant of time t, $0\leq t \leq T$

2.2. Assumptions

The following assumptions are made in the paper:

i. The inventory system involves only one items.
ii. Demand rate r is constant.
iii. Lead time is zero.
iv. The replenishment rate is infinite.
v. Shortages are allowed and fully backlogged. Therefore, the lost sale cost is zero.
vi. There is no replacement of the deteriorated items during the production cycle.

3. Mathematical Formulation

The status of inventory model is shown in figure 3.1 as given below:

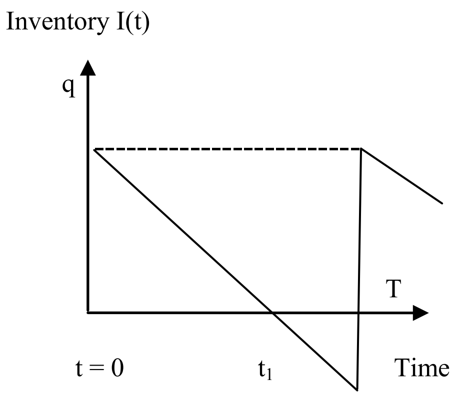

Figure 3.1. Graphical Representation of EOQ model

Crisp model given by Dutta and Pawan (2013) uses the inventory level $I(t)$ at time t, which is the rate of change of inventory and is governed by the following differential equation:

$$\frac{d\,I(t)}{dt} = \alpha\, I(t) = -\,r, \text{ for } 0 \leq t \leq t_1 \quad (3.1)$$

$$\frac{d\,I(t)}{dt} = -\,r, \quad \text{for } t_1 \leq t \leq T \quad (3.2)$$

with the initial condition $I(0)$ = q, and the boundary condition

$$I(t_1) = 0 \quad (3.3)$$

Solving equation (3.1) and (3.2) using the boundary condition in (3.3), the final solution is given by

$$I(t) = \frac{r}{\alpha}\left[e^{(t_1-t)\alpha} - 1\right],\ for\ 0 \leq t \leq t_1 \quad (3.4)$$

and

$$I(t) = r[t_1 - t], \quad for\ t_1 \leq t \leq T \quad (3.5)$$

using exponential series expansion in (3.4). Since $0<\alpha<1$, we ignore higher power of term of α, and get

$$I(t) = \frac{r}{\alpha}[1 + (t_1-t)\alpha + (t_1-t)^2\,\alpha^2 - 1],\ for\ 0 \leq t \leq t_1$$

$$= r[t_1 - t + \alpha t_1^2 + \alpha t^2 - 2\alpha t_1 t],\ for\ 0 \leq t \leq t_1 \quad (3.6)$$

Using the initial condition $I(0)$ = q in (3.6), we get

$$q = r[t_1 + \alpha t_1^2] \quad (3.7)$$

Total average number of holding units due to the period [0,T] is given by

$$T_h = \int_0^{t_1} I(t)dt = r\left[\frac{1}{2}\,t_1^2 + \frac{1}{3}\,\alpha t_1^3\right] \quad (3.8)$$

Total number of deteriorated units due to the period [0, T] is given by

$$T_d = q - \text{total demand}$$

$$= q - \int_0^{t_1} r\,dt = r\alpha t_1^2 \quad (3.9)$$

Total average number of shortage units due to the period [0, T] is given by

$$I_s = -\int_{t_1}^{T} I(t)dt$$

$$= \frac{-r}{2}[2Tt_1 - T^2 - t_1^2] = \frac{r}{2}(T-t_1)^2 \quad (3.10)$$

Total shortage cost per unit time

$$C_S = \frac{1}{T}[S\,I_s] = \frac{(T-t_1)^2}{2T}\,r\,S \quad (3.11)$$

Therefore, the total cost of the system per unit time is given by

$$C = \frac{1}{T}[c_0 + h\,T_h + p_c\,T_d + S\,I_s]$$

$$= \frac{1}{T}\left[c_0 + h\,r\left(\frac{1}{2}\,t_1^2 + \frac{1}{3}\,\alpha t_1^3\right) + p_c\,r\alpha t_1^2 + \frac{r}{2}\,S(T-t_1)^2\right]$$

$$C = \frac{1}{T}\left[c_0 + \frac{1}{2}h\,rt_1^2 + \frac{r}{2}S\,(T-t_1)^2\right] + \frac{\alpha}{T}\left[\frac{1}{3}\,h\,rt_1^3 + p_c\,rt_1^2\right] \quad (3.12)$$

This can be also written as $f_1 + \alpha\, f_2 = C$, where (3.13)

$f_1 = \frac{1}{T}\left[c_0 + \frac{1}{2}h\,rt_1^2 + \frac{r}{2}S\,(T-t_1)^2\right]$ and

$f_2 = \frac{1}{T}\left[\frac{1}{3}\,h\,rt_1^3 + p_c\,rt_1^2\right]$

$$C = f_1 + \alpha\, f_2 \quad (3.14)$$

where, α is rate of deteriorating during production

4. Formulation in Fuzzy Environment

In case of fuzzification, deterioration rate α changes into fuzzified deterioration rate $\emptyset$, and equation (3.14) becomes

$$\tilde{C} = f_1 + \emptyset\, f_2 \quad (4.1)$$

Let $\xi_t(\emptyset) = f_1 + \emptyset\, f_2 = \tilde{C}$ and let traingular membership function of fuzzified dereriotion rate $\emptyset$ be given as

$$G_{\emptyset}(\alpha) = \begin{cases} \dfrac{\alpha - \alpha_1}{\alpha_0 - \alpha_1} \text{ if } \alpha_1 \le \alpha \le \alpha_0 \\ \dfrac{\alpha_2 - \alpha}{\alpha_2 - \alpha_1} \text{ if } \alpha_1 \le \alpha \le \alpha_2 \\ 0 \text{ elsewhere} \end{cases}$$

where $\alpha_1, \alpha_0, \& \ \alpha_2$ are the positive variables and $0 \le \alpha_1 \le \alpha_0 \le \alpha_2$ and the centroid of $G_{\emptyset}(\alpha)$ is given as

$$M_d(\alpha_1, \alpha_2, \alpha_3) = \frac{\alpha_1 + \alpha_2 + \alpha_3}{3}$$

We further observe that

$$\emptyset = \frac{f_1 - f_2}{f_2} \ge 0; \text{ for } f_2 \ne 0, C \ge f_1$$

This shows that $f_1 \le C_1 \le C_0 \le C_2$, where C_1, C_0, C_2 are lower, middle and upper cost at time t from above equation and we can express membership function for cost at time t

$$G_{\xi_t(\emptyset)}(C) = \begin{cases} \dfrac{C - f_1 - \alpha_1 f_2}{(\alpha_0 - \alpha_1) f_2} \ \ if\ C_1 \le C \le C_0 \\ \dfrac{f_1 + \alpha_2 f_2 - C}{\alpha_2 - \alpha_0) f_2} \ \ if\ C_1 \le C \le C_2 \\ 0 \quad elsewhere \end{cases}$$

As we know the extension principle of Zadeh is a very important tool in the fuzzy set theory for providing procedure to fuzzify a crisp function which is given below. Let f: M → N be a crisp function and F(m) (respectively F(n)) be the set of all fuzzy sets (called fuzzy power set) of M (respectively n). The function f: M → N induces two functions f: F (M) → F (N) and f^1: F (N) → F (M) and the extension principal of Zadeh gives formulas to compute the membership function of fuzzy sets f (A) in N (respectively f^1 (B) in M) in terms of membership function of fuzzy set A in M (respectively B in N).

Definition 4.1 (Zadeh's Extension Principle): In terms of the notations introduced above, extension principle of Zadeh states that

(i) $\mu_{f(A)}(n) = \underset{m \in M, f(m) = n}{SUP}(\mu A(m))$ for all A ∈ F (M), and

(ii) $\mu_{f^{-1}(B)}(m) = \mu_B(f(m))$, for all B ∈ F (N).

Hence, from the above theorem, we get the following equation as

$$G_{\xi_t(\emptyset)}(C) = \underset{\emptyset \in \xi_t^{-1}(C)}{SUP} G_{\emptyset}(C)$$

Now, we find the quantities π and π_0 so that we can find centroid for defuzzification as

$$\pi = \int_{-\infty}^{\infty} T\,\mu(\emptyset)\,(C)\,dC$$

$$\pi_0 = \int_{-\infty}^{\infty} C\,T\,\mu(\emptyset)\,(C)\,dC$$

The centroid to $T_{\mu(\phi)}(C)$ is given by $\frac{\pi_0}{\pi}$ which gives the total cost in fuzzified environment.

We have π and π_0 as

$$\pi_0 = \frac{1}{(\alpha_0 - \alpha_1) f_2} \int_{C_1}^{C_0} C\{C - f_1 - \alpha_1 f_2) dC + \frac{1}{(\alpha_2 - \alpha_0) f_2} \int_{C_0}^{C_2} C\{f_1 + \alpha_2 f_2 - C)\, dC$$

$$\pi = \frac{1}{(\alpha_0 - \alpha_1) f_2} \int_{C_1}^{C_0} \{C - f_1 - \alpha_1 f_2) dC + \frac{1}{(\alpha_2 - \alpha_0) f_2} \int_{C_0}^{C_2} \{f_1 + \alpha_2 f_2 - C)\, dC$$

After evaluating integrals of π_0 and π from the above equations, we get the centroid of $T_{\xi_t(\emptyset)}(C)$ as $\tilde{C} = \frac{\pi_0}{\pi}$, which finally turns out to be

$$\tilde{C} = \frac{\frac{1}{(\alpha_0 - \alpha_1) f_2} \int_{C_1}^{C_0} C\{C - f_1 - \alpha_1 f_2) dC + \frac{1}{(\alpha_2 - \alpha_0) f_2} \int_{C_0}^{C_2} C\{f_1 + \alpha_2 f_2 - C)\, dC}{\frac{1}{(\alpha_0 - \alpha_1) f_2} \int_{C_1}^{C_0} \{C - f_1 - \alpha_1 f_2) dC + \frac{1}{(\alpha_2 - \alpha_0) f_2} \int_{C_0}^{C_2} \{f_1 + \alpha_2 f_2 - C)\, dC}$$

For obtaining optimal solution of $\tilde{C}$, i,e for total cost of the model, we take first order partial derivatives w. r. t. α_1 and α_2 and equating them to zero with the condition of $\frac{\partial \tilde{c}}{\partial \alpha_1} = 0$. Note that $\frac{\partial \tilde{c}}{\partial \alpha_1}$ is given by as

$$\frac{\left[(\alpha_2 - \alpha_0)\left(\frac{C_0^2}{2} - C_0 f_1 - \alpha_1 f_2 C_0 - \frac{C_1^2}{2} + C_1 f_1 + \alpha_1 f_2 C_1\right) + (\alpha_0 - \alpha_1)\left(C_2 f_1 + \alpha_2 f_2 C_2 - \frac{C_2^2}{2} - C_0 f_1 - \alpha_2 f_2 C_0 + \frac{C_0^2}{2}\right)\right] \frac{\partial}{\partial \alpha_1}\left[(\alpha_2 - \alpha_0)\left(\frac{C_0^3}{3} - \frac{C_0^2}{2} f_1 - \alpha_1 f_1 \frac{C_0^2}{2} - \frac{C_1^3}{3} + \frac{C_1^2}{2} f_1 + \alpha_1 f_2 \frac{C_1^2}{2}\right) + (\alpha_0 - \alpha_1)\left(\frac{C_0^2}{2} f_1 + \alpha_2 f_2 \frac{C_0^2}{2} - \frac{C_0^3}{3} - \frac{C_1^2}{2} f_1 - \alpha_2 f_2 \frac{C_1^2}{2} + \frac{C_1^3}{3}\right)\right] \frac{\partial}{\partial \alpha_1}\left[(\alpha_2 - \alpha_0)\left(\frac{C_0^2}{2} - C_0 f_1 - \alpha_1 f_2 C_0 - \frac{C_1^2}{2} + C_1 f_1 + \alpha_1 f_2 C_1\right) + (\alpha_0 - \alpha_1)\left(C_2 f_1 + \alpha_2 f_2 C_2 - \frac{C_2^2}{2} - C_0 f_1 - \alpha_2 f_2 C_0 + \frac{C_0^2}{2}\right)\right]}{\left[(\alpha_2 - \alpha_0)\left(\frac{c_0^2}{2} - C_0 f_1 - \alpha_1 f_2 C_0 - \frac{c_1^2}{2} + C_1 f_1 + \alpha_1 f_2 C_1\right) + (\alpha_0 - \alpha_1)\left(C_2 f_1 + \alpha_2 f_2 C_2 - \frac{C_2^2}{2} - C_0 f_1 - \alpha_2 f_2 c_0 + \frac{C_0^2}{2}\right)\right]^2}$$

$$\frac{\begin{array}{c}\left[(\alpha_2-\alpha_0)\left(\frac{C_0^2}{2}-C_0 f_1-\alpha_1 f_2 C_0-\frac{C_1^2}{2}+C_1 f_1+\alpha_1 f_2 C_1\right)+(\alpha_0-\alpha_1)\left(C_2 f_1+\alpha_2 f_2 C_2-\frac{C_2^2}{2}-C_0 f_1-\alpha_2 f_2 C_0+\frac{C_0^2}{2}\right)\right]\\ \left[(\alpha_2-\alpha_0)\left(f_2\frac{C_1^2}{2}-\alpha_1 f_1\frac{C_0^2}{2}\right)-\left(\frac{C_0^2}{2}f_1+\alpha_2 f_2\frac{C_0^2}{2}-\frac{C_0^3}{3}-\frac{C_1^2}{2}f_1-\alpha_2 f_2\frac{C_1^2}{2}+\frac{C_1^3}{3}\right)\right]\\ -\left[(\alpha_2-\alpha_0)\left(\frac{C_0^3}{3}-\frac{C_0^2}{2}f_1-\alpha_1 f_1\frac{C_0^2}{2}-\frac{C_1^3}{3}+\frac{C_1^2}{2}f_1+\alpha_1 f_2\frac{C_1^2}{2}\right)+(\alpha_0-\alpha_1)\left(\frac{C_0^2}{2}f_1+\alpha_2 f_2\frac{C_0^2}{2}-\frac{C_0^3}{3}-\frac{C_1^2}{2}f_1-\alpha_2 f_2\frac{C_1^2}{2}+\frac{C_1^3}{3}\right)\right]\\ \left[(\alpha_2-\alpha_0)(f_2 C_1-f_2 C_0)-\left(C_2 f_1+\alpha_2 f_2 C_2-\frac{C_2^2}{2}-C_0 f_1-\alpha_2 f_2 C_0+\frac{C_0^2}{2}\right)\right]\end{array}}{\left[(\alpha_2-\alpha_0)\left(\frac{c_0^2}{2}-c_0 f_1-\alpha_1 f_2 c_0-\frac{c_1^2}{2}+c_1 f_1+\alpha_1 f_2 c_1\right)+(\alpha_0-\alpha_1)\left(c_2 f_1+\alpha_2 f_2 c_2-\frac{c_2^2}{2}-c_0 f_1-\alpha_2 f_2 c_0+\frac{c_0^2}{2}\right)\right]^2}$$

Similarly, we can differentiate w. r. t. α_2, and get

$$\frac{\begin{array}{c}\left[(\alpha_2-\alpha_0)\left(\frac{C_0^2}{2}-C_0 f_1-\alpha_1 f_2 C_0-\frac{C_1^2}{2}+C_1 f_1+\alpha_1 f_2 C_1\right)+(\alpha_0-\alpha_1)\left(C_2 f_1+\alpha_2 f_2 C_2-\frac{C_2^2}{2}-C_0 f_1-\alpha_2 f_2 C_0+\frac{C_0^2}{2}\right)\right]\\ \frac{\partial}{\partial\alpha_2}\left[(\alpha_2-\alpha_0)\left(\frac{C_0^3}{3}-\frac{C_0^2}{2}f_1-\alpha_1 f_1\frac{C_0^2}{2}-\frac{C_1^3}{3}+\frac{C_1^2}{2}f_1+\alpha_1 f_2\frac{C_1^2}{2}\right)+(\alpha_0-\alpha_1)\left(\frac{C_0^2}{2}f_1+\alpha_2 f_2\frac{C_0^2}{2}-\frac{C_0^3}{3}-\frac{C_1^2}{2}f_1-\alpha_2 f_2\frac{C_1^2}{2}+\frac{C_1^3}{3}\right)\right]\\ -\left[(\alpha_2-\alpha_0)\left(\frac{C_0^3}{3}-\frac{C_0^2}{2}f_1-\alpha_1 f_1\frac{C_0^2}{2}-\frac{C_1^3}{3}+\frac{C_1^2}{2}f_1+\alpha_1 f_2\frac{C_1^2}{2}\right)+(\alpha_0-\alpha_1)\left(\frac{C_0^2}{2}f_1+\alpha_2 f_2\frac{C_0^2}{2}-\frac{C_0^3}{3}-\frac{C_1^2}{2}f_1-\alpha_2 f_2\frac{C_1^2}{2}+\frac{C_1^3}{3}\right)\right]\\ \frac{\partial}{\partial\alpha_2}\left[(\alpha_2-\alpha_0)\left(\frac{C_0^2}{2}-C_0 f_1-\alpha_1 f_2 C_0-\frac{C_1^2}{2}+C_1 f_1+\alpha_1 f_2 C_1\right)+(\alpha_0-\alpha_1)\left(C_2 f_1+\alpha_2 f_2 C_2-\frac{C_2^2}{2}-C_0 f_1-\alpha_2 f_2 C_0+\frac{C_0^2}{2}\right)\right]\end{array}}{\left[(\alpha_2-\alpha_0)\left(\frac{c_0^2}{2}-c_0 f_1-\alpha_1 f_2 c_0-\frac{c_1^2}{2}+c_1 f_1+\alpha_1 f_2 c_1\right)+(\alpha_0-\alpha_1)\left(c_2 f_1+\alpha_2 f_2 c_2-\frac{c_2^2}{2}-c_0 f_1-\alpha_2 f_2 c_0+\frac{c_0^2}{2}\right)\right]^2}$$

$$\frac{\begin{array}{c}\left[(\alpha_2-\alpha_0)\left(\frac{C_0^2}{2}-C_0 f_1-\alpha_1 f_2 C_0-\frac{C_1^2}{2}+C_1 f_1+\alpha_1 f_2 C_1\right)+(\alpha_0-\alpha_1)\left(C_2 f_1+\alpha_2 f_2 C_2-\frac{C_2^2}{2}-C_0 f_1-\alpha_2 f_2 C_0+\frac{C_0^2}{2}\right)\right]\\ \left[\left(\frac{C_0^3}{3}-\frac{C_0^2}{2}f_1-\alpha_1 f_1\frac{C_0^2}{2}-\frac{C_1^3}{3}+\frac{C_1^2}{2}f_1+\alpha_1 f_2\frac{C_1^2}{2}\right)+(\alpha_0-\alpha_1)\left(f_2\frac{C_0^2}{2}-f_2\frac{C_1^2}{2}\right)\right]\\ -\left[(\alpha_2-\alpha_0)\left(\frac{C_0^3}{3}-\frac{C_0^2}{2}f_1-\alpha_1 f_1\frac{C_0^2}{2}-\frac{C_1^3}{3}+\frac{C_1^2}{2}f_1+\alpha_1 f_2\frac{C_1^2}{2}\right)+(\alpha_0-\alpha_1)\left(\frac{C_0^2}{2}f_1+\alpha_2 f_2\frac{C_0^2}{2}-\frac{C_0^3}{3}-\frac{C_1^2}{2}f_1-\alpha_2 f_2\frac{C_1^2}{2}+\frac{C_1^3}{3}\right)\right]\\ \left[\left(\frac{C_0^2}{2}-C_0 f_1-\alpha_1 f_2 C_0-\frac{C_1^2}{2}+C_1 f_1+\alpha_1 f_2 C_1\right)+(\alpha_0-\alpha_1)(f_2 C_2-f_2 C_0)\right]\end{array}}{\left[(\alpha_2-\alpha_0)\left(\frac{c_0^2}{2}-c_0 f_1-\alpha_1 f_2 c_0-\frac{c_1^2}{2}+c_1 f_1+\alpha_1 f_2 c_1\right)+(\alpha_0-\alpha_1)\left(c_2 f_1+\alpha_2 f_2 c_2-\frac{c_2^2}{2}-c_0 f_1-\alpha_2 f_2 c_0+\frac{c_0^2}{2}\right)\right]^2}$$

After solving both partial derivatives with respect to α_1 and α_2 for fixed values of α_0 and t, we get a system of non-linear equations for α_1 and α_2 respectively as follows:

$$F(\alpha_1, \alpha_2) = (X\alpha_1 + Y)\alpha_2^2 + Z\alpha_2 + \psi\alpha_1 + \eta\alpha_1\alpha_2 + \mu = 0 \tag{4.2}$$

$$G(\alpha_1, \alpha_2) = (a\alpha_2 - b)\alpha_1^2 + \Upsilon\alpha_1 + \delta\alpha_2 + \lambda\alpha_1\alpha_2 + \omega = 0 \tag{4.3}$$

where, $X = -4f_2^2\frac{C_0^3}{2}$, $Y = \frac{f_1 f_2 C_0^3}{3} - \frac{C_0^3}{2}f_2$, $Z = \frac{\alpha_0 f_1 f_2 C_0^3}{2} - \frac{\alpha_0 f_2 C_0^3}{3} + \frac{f_2 C_0^3}{3} + \frac{f_1 C_0^3}{2} - \frac{C_0^5}{6}$

$$\Psi = \frac{C_0^5}{6} + \frac{C_0^4}{3} - \frac{f_1 C_0^3}{3} - \frac{2\alpha_0 f_2 C_0^4}{3},\ \eta = \frac{2\alpha_0 f_2^2 C_0^3}{2} + f_2\left(\frac{C_0^3}{2} + \frac{C_0^4}{3}\right) - \frac{f_1 f_2 C_0^3}{2} - \frac{C_0^4}{4}$$

$$\mu = -\frac{\alpha_0 C_0^5}{2} - \frac{\alpha_0 f_1^2 C_0^3}{2},\ a = \frac{f_2}{16},\quad b = \frac{-3f_1 f_2}{16} + \frac{37 f_2}{192} - \alpha_0 f_2^2,\ \gamma = \frac{95\alpha_0 f_2}{96} + \frac{\alpha_0 f_1 f_2}{4} + \frac{\alpha_0^2 f_2^2}{8}$$

$$\lambda = \frac{-23 f_2}{24} - \frac{\alpha_0 f_2}{16} + \frac{\alpha_0 f_2^2}{4},\ \omega = \frac{\alpha_0^2 f_2}{4} + \frac{\alpha_0^2}{96}$$

This system of non-linear equations in α_1 and α_2 given by (4.2) and (4.3) is solved by using fast converging N-R method which is programmed in C++.

5. Computing Algorithm

We use the following algorithm with C++ language to compute the optimal results.

Step i: Begin.

Step ii: Input data.

Step iii: Compute the value of F, G, X, Y, Z, ψ, η, μ, a, b, ϒ, δ, λ, ω.

Step iv: Compute the lower cost.

Step v: Compute the middle cost.

Step vi: Compute the upper cost.

Step vii: Compute the coefficient of first non-linear equation by Jacobi method.

Step viii: Compute the coefficient of second non-linear equation by Jacobi method.

Step ix: Compute optimal value of alpha one.

Step x: Compute optimal value of alpha two.

Step xi: Compute optimal value of cost in fuzzy case.

Step xii: End.

6. Sensitivity Analysis of the Model

A sensitivity analysis of the inventory model seeks to study the model impact due to variation of various parameters involved therein. Optimal performance measure of the model under consideration is jointly affected by these parameters and thus inputting these data can fix their variational range for obtaining optimal solution in the form of performance measure as total optimal cost of the fuzzified model in reference. We generate the data for input by using the simulation technique based on Gaussian distribution using R-software. Setting this kaleidoscopic pattern of data is intended to draw future line of action of attempting the findings of the model under consideration. Values of various parameters are computed in the following tables.

It is evident from table (6.1) that the purchasing cost, shortage cost and ordering costs increase when level of inventory keeps increasing indicating a positive correlation between them in the fuzzy environment. From table (6.2) we can draw the conclusion that as time for transaction cycle diminishes pressure for demand increases and it amounts to greater holding cost leading to negative correlation between them when environment is fuzzy. The values of all constants involved in the model are computed in order to produce the optimal values of deterioration of items and their corresponding total cost and are given in tables (6.3), (6.4), (6.5) and (6.6) respectively. The optimal values of deteriorations and total cost in fuzzy environment are given in table (6.7).

Table 6.1.

p_c	c_0	q	S
20	200	100	15
21	205	105	16
22	210	110	17
23	215	115	18
24	220	120	19
25	225	125	20

Table 6.2.

h	r	t_1	T
5	110	0.72	1.005
6	115	0.68	0.958
7	120	0.66	0.917
8	125	0.63	0.880
9	130	0.61	0.848
10	135	0.59	0.868

Table 6.3.

α_0	F	G	X
0.004	0.0004	0.0002	7.5827
0.005	0.0013	0.0004	7.2697
0.006	1.3402	0.1498	7.7096
0.007	1.8450	1.6337	7.5908
0.008	4.9311	2.8799	7.8574
0.009	5.3402	8.0838	8.0464

Table 6.4.

Y	Z	ψ	η
9.5452	-1.1560	-5.3973	1.5165
9.5740	-1.4520	-5.5123	1.8174
1.0437	-1.9031	-5.8175	2.3128
1.0626	-2.2641	-5.9523	2.6567
1.1268	-2.7483	-6.1860	3.1429
1.1805	-3.2446	-6.3872	3.6209

Table 6.5.

μ	a	b	γ
3.0776	5.4424	-1.9053	5.1363
4.8211	5.3289	-1.0005	6.4508
7.5777	5.4877	-1.0987	8.4543
1.0511	5.4455	-1.1257	1.0057
1.4573	5.5401	-1.2014	1.2207
1.9345	5.6063	-1.2665	1.4411

Table 6.6.

δ	λ	ω	f_1
-7.5827	7.5827.	3.4831	5.8810
-9.0872	9.0872	5.3289	6.0314
-1.1564	1.1564	7.9024	6.3927
-1.3283	1.3283	1.0672	6.5605
-1.5714	1.5714	1.4182	6.8505
-1.8104	1.8104	1.8164	7.1005

Table 6.7.

f_2	α_1^*	α_2^*	$\tilde{C}$
8.7078	0.7519	2.0164	25.7352
8.5262	0.4935	0.8960	49.4390
8.7804	0.3489	0.9682	9.3348
8.7125	0.0274	1.0571	9.8392
8.8641	0.0846	1.1977	10.3699
8.9702	0.8864	1.2168	10.9432

7. Conclusions

Fuzzified EOQ control system under consideration allows us to nicely discuss the fuzzified total optimum cost of the system with deteriorating items with shortage under full backlogging. The relevant result is computed and displayed in table (6.7). This result can be used as future guideline for an enterprising decision system for a better supply network of inventory items. Zadeh's extension principle and membership function in together have become conscious approach of fuzzification to the model; a popular and versatile centroid method of defuzzification provides the computational insight to attain its optimum solution for various parameters involved therein, as seen in tables (6.1-6.7).

The present research presents the state-of-art application of nuanced technique of fuzzified EOQ control system. A fast converging numerical computational technique has been sought to interface with the fuzzified system to yield the stable results of the model unfolding the point of realization towards seeking solution techniques in the area. The present problem may lead to future research for multi-ware house for classified customers queued for EOQ items.

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